Calculus Quiz 1

1. (30 points) The disk $(x-3)^2+y^2 \leq 1$ is revolved about the y-axis to generate a solid shaped like a doughnut and called a *torus*. Find its volume by

   (a) (15 points) the washer method.

   (b) (15 points) the shell method.

   Hint: Coming across the integral $\int_{-a}^{a} \sqrt{a^2 - y^2}dy$, use the change of variable $y = a \sin u$.

   Proof. (a) Refer to Figure 1, and then the volume of the torus is

   $$\int_{-1}^{1} \pi \left[\left(3 + \sqrt{1 - y^2}\right)^2 - \left(3 - \sqrt{1 - y^2}\right)^2 \right] dy = \pi \int_{-1}^{1} 6 \cdot 2\sqrt{1 - y^2}dy$$

   $$= 12\pi \int_{-1}^{1} \sqrt{1 - y^2}dy$$

   $$= 12\pi \cdot \frac{\pi \cdot 1^2}{2}$$

   $$= 6\pi^2,$$

   since $\int_{-1}^{1} \sqrt{1 - y^2}dy$ is half the area of the unit circle.

   $\square$

2. (15 points) Find the length of the curve:

   $$x = \int_{0}^{y} \sqrt{\sec^4 t - 1}dt, \quad \frac{-\pi}{4} \leq y \leq \frac{\pi}{4}$$

3. (15 points) Find the area of the surface generated by revolving about the y-axis the curve:

   $$y = \sqrt{2x - x^2}, \quad \frac{1}{2} \leq y \leq \frac{3}{2}$$

4. (15 points) State and prove the fundamental theorem of Calculus part one.

5. (15 points) State and prove the fundamental theorem of Calculus part two.

6. (15 points) State and prove the Mean Value Theorem for derivatives.