

$$1. a. 5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3 = 10n^3 \quad (\text{crossed out})$$

$$5n^3 + 2n^2 + 3n = O(n^3) \quad (n_0 = 1, C = 10)$$

$$b. \sqrt{7n^2 + 2n - 8} < \sqrt{7n^2 + 2n} \leq \sqrt{7n^2 + 2n^2} = \sqrt{9n^2} = 3n$$

$$\sqrt{7n^2 + 2n - 8} \geq \sqrt{7n^2} = \sqrt{7}n \quad (n \geq 4)$$

$$\sqrt{7n^2 + 2n - 8} = \Theta(n) \quad (n_0 = 4, C_1 = \sqrt{7}, C_2 = 3)$$

$$c. d(n) \leq C_1 \cdot f(n) \text{ for } n \geq n_1$$

$$e(n) \leq C_2 \cdot g(n) \text{ for } n \geq n_2$$

assume that $n_1 \geq n_2$

then,

$$d(n)e(n) \leq C_1 C_2 f(n)g(n) \text{ for } n \geq n_1$$

$$d(n)e(n) = O(f(n)g(n)), \text{ where } n_0 = n_1, C = C_1 C_2$$

$$2. (1) \quad \text{crossed out} \quad \Theta(n^2)$$

$$(2) \quad \Theta(n)$$

$$(3) \quad \Theta(n^2)$$

$$(4) \quad \Theta(n \log_2 n)$$