Aufgabe 1 Geben Sie $u(x)$ und $v(x)$ (Zuordnung: $f(x) = u(v(x))$) an.
Leiten Sie ab und vereinfachen Sie das Ergebnis.

(a) $f(x) = (\frac{1}{3}x + 2)^2$

$u(x) = x^2, v(x) = \frac{1}{3}x + 2 \qquad \Rightarrow f'(x) = \underbrace{2 \cdot (\frac{1}{3}x + 2)}_{u'(v(x))} \cdot \underbrace{(\frac{1}{3})}_{v'(x)} = \frac{2}{3} \cdot (\frac{1}{3}x + 2)$

(b) $f(x) = \frac{1}{18}(3x + 2)^6$

$u(x) = \frac{1}{18}x^6, v(x) = 3x + 2 \qquad \Rightarrow f'(x) = \underbrace{\frac{\cancel{6}^{\,1}}{\cancel{18}_{\,3}} \cdot (3x + 2)^5}_{u'(v(x))} \cdot \underbrace{\cancel{3}}_{v'(x)} = (3x + 2)^5$

(c) $f(x) = \frac{1}{8}(\frac{1}{2} - x^2)^7$

$u(x) = \frac{1}{8}x^7, v(x) = \frac{1}{2} - x^2 \qquad \Rightarrow f'(x) = \underbrace{\frac{7}{\cancel{8}_{\,4}} \cdot (\frac{1}{2} - x^2)^6}_{u'(v(x))} \cdot \underbrace{(-\cancel{2}x)}_{v'(x)} = -\frac{7}{4} \cdot x \cdot (\frac{1}{2} - x^2)^6$

(d) $f(x) = (3 - x)^2$

$u(x) = x^2, v(x) = 3 - x \qquad \Rightarrow f'(x) = \underbrace{2(3 - x)}_{u'(v(x))} \cdot \underbrace{(-1)}_{v'(x)} = -2(3 - x)$

(e) $f(x) = (x + x^2)^3$

$u(x) = x^3, v(x) = x + x^2 \qquad \Rightarrow f'(x) = \underbrace{3(x + x^2)^2}_{u'(v(x))} \cdot \underbrace{(1 + 2x)}_{v'(x)} = 3 \cdot (x + x^2)^2(1 + 2x)$

(f) $f(x) = (2 - 3x + x^2)^3$

$u(x) = x^3, v(x) = 2 - 3x + x^2 \qquad \Rightarrow f'(x) = \underbrace{3 \cdot (2 - 3x + x^2)^2}_{u'(v(x))} \cdot \underbrace{(-3 + 2x)}_{v'(x)} = 3(2 - 3x + x^2)^2(2x - 3)$

(g) $f(x) = (1 - x + x^3)^2$

$u(x) = x^2, v(x) = 1 - x + x^3 \qquad f'(x) = \underbrace{2(1 - x + x^3)}_{u'(v(x))} \cdot \underbrace{(-1 + 3x^2)}_{v'(x)} = 2(1 - x + x^3)(3x^2 - 1)$

(h) $f(x) = (x\sqrt{2} - x^2)^2$

$u(x) = x^2, v(x) = x\sqrt{2} - x^2 \qquad f'(x) = \underbrace{2(x\sqrt{2} - x^2)}_{u'(v(x))} \cdot \underbrace{(\sqrt{2} - 2x)}_{v'(x)} = 2(x\sqrt{2} - x^2)(\sqrt{2} - 2x)$

Aufgabe 2 Vervollständige Sie die Tabelle!

	$f(x) = u(v(x))$	$v(x)$	$u(x)$	$v'(x)$	$u'(x)$	$u'(v(x))$	$f'(x)$
	$(5x-1)^3$	$5x-1$	x^3	5	$3x^2$	$3(5x-1)^2$	$15(5x-1)^2$
(a)	$\mathbf{(2x+3)^2}$	$2x+3$	x^2	$\mathbf{2}$	$\mathbf{2x}$	$\mathbf{2(2x+3)}$	$\mathbf{4(2x+3)}$
(b)	$\frac{2}{(2x+1)^2}$	$\mathbf{2x+1}$	$2x^{-2}$	$\mathbf{2}$	$\mathbf{-4x^{-3}}$	$\mathbf{-4(2x+1)^{-3}}$	$\mathbf{-8(2x+1)^{-3}}$
(c)	$\sqrt{5-x^2}$	$\mathbf{5-x^2}$	$\mathbf{\sqrt{x}}$	$\mathbf{-2x}$	$\mathbf{\frac{1}{2}\cdot x^{-\frac{1}{2}}}$	$\mathbf{\frac{1}{2}(5-x^2)^{-\frac{1}{2}}}$	$\mathbf{-x(5-x^2)^{-\frac{1}{2}}}$

Aufgabe 3 Leiten Sie zweimal ab.

(a) $f(x) = (4x-7)^3$

$$f'(x) = \underbrace{3(4x-7)^2}_{u'(v(x))} \cdot \underbrace{4}_{v'(x)} = 12(4x-7)^2$$

$$f''(x) = \underbrace{2\cdot 12(4x-7)}_{u'(v(x))} \cdot \underbrace{4}_{v'(x)} = 96(4x-7)$$

(b) $f(x) = (7x^3+1)^2$

$$f'(x) = \underbrace{2(7x^3+1)}_{u'(v(x))} \cdot \underbrace{21x^2}_{v'(x)} = \underbrace{42x^2}_{g(x)}\underbrace{(7x^3+1)}_{h(x)}$$

$$f''(x) = \underbrace{84x}_{g'(x)}\underbrace{(7x^3+1)}_{h(x)} + \underbrace{42x^2}_{g(x)}\underbrace{(21x^2)}_{h'(x)}$$

(c) $f(x) = (x-5)^{-3}$

$$f'(x) = \underbrace{-3(x-5)^{-4}}_{u'(v(x))} \cdot \underbrace{1}_{v'(x)} = \underbrace{-3(x-5)^{-4}}_{g(h(x))}$$

$$f''(x) = \underbrace{12(x-5)^{-5}}_{g'(h(x))} \cdot \underbrace{1}_{h'(x)} = 12(x-5)^{-5}$$

Aufgabe 4 Geben Sie $u(x)$ und $v(x)$ (Vorschrift: $f(x) = u(x) \cdot v(x)$) an.
Leiten Sie ab und vereinfachen Sie das Ergebnis.

(a) $f(x) = x^3 \cdot \sqrt{x}$

$u(x) = x^3, v(x) = \sqrt{x}$ $\quad \Rightarrow f'(x) = \underbrace{3x^2}_{u(x)} \cdot \underbrace{\sqrt{x}}_{v(x)} + \underbrace{x^3}_{u(x)} \cdot \underbrace{\frac{1}{2}x^{-\frac{1}{2}}} = \boxed{3x^2 \cdot \sqrt{x} + \frac{x^3}{2\sqrt{x}}}$

(b) $f(x) = x \cdot (x^3 + 1)^3$

$u(x) = x, v(x) = \underbrace{(\overbrace{x^3+1}^{g(x)})^3}_{h(x)}$ $\quad \Rightarrow \quad f'(x) \quad = \quad \underbrace{1}_{u'(x)} \underbrace{(x^3+1)^3}_{v(x)} + \underbrace{x}_{u(x)} \cdot \underbrace{\underbrace{3(x^3+1)^2}_{h'(g(x))} \cdot \underbrace{(3x^2)}_{g'(x)}}_{v'(x)} \quad =$

$\boxed{(x^3+1)^3 + 9x^3(x^3+1)^2}$

(c) $f(x) = (2x^2 - x) \cdot \sqrt{x}$

$u(x) = 2x^2 - x, v(x) = \sqrt{x}$ $\quad \Rightarrow \quad f'(x) \quad = \quad \underbrace{(4x-1)}_{u'(x)} \cdot \sqrt{x} \quad + \quad \underbrace{(2x^2 - x)}_{u(x)} \cdot \underbrace{\frac{1}{2}x^{-\frac{1}{2}}} \quad =$

$\boxed{(4x-1) \cdot \sqrt{x} + \frac{2x^2-x}{2\sqrt{x}}}$

(d) $f(x) = (1-2x) \cdot (3x+1)$

$u(x) = 1 - 2x, v(x) = 3x + 1$ $\quad \Rightarrow f'(x) = \underbrace{-2}_{u'(x)} \cdot \underbrace{(3x+1)}_{v(x)} + \underbrace{(1-2x)}_{u(x)} \cdot \underbrace{3}_{v'(x)} = -2(3x+1) +$

$3(1-2x) = \boxed{-12x + 1}$

(e) $f(x) = (\frac{1}{3}x^3 + x^2) \cdot (-x)$

$u(x) = \frac{1}{3}x^3 + x^2, v(x) = -x$ $\quad \Rightarrow f'(x) = \underbrace{(x^2+2x)}_{u'(x)} \cdot \underbrace{(-x)}_{v(x)} + \underbrace{(\frac{1}{3}x^3 + x^2)}_{u(x)} \cdot \underbrace{(-1)}_{v'(x)} = (-x^3 -$

$2x^2) - (\frac{1}{3}x^3 + x^2) = \boxed{-\frac{4}{3}x^3 - 3x^2}$

(f) $f(x) = x^2 \cdot (2x+1)$

$u(x) = x^2, v(x) = 2x + 1$ $\quad \Rightarrow f'(x) = \underbrace{2x}_{u'(x)} \cdot \underbrace{(2x+1)}_{v(x)} + \underbrace{x^2}_{u(x)} \cdot \underbrace{2}_{v'(x)} = 2x(2x+1) + 2x^2 =$

$\boxed{6x^2 + 2x}$

(g) $f(x) = \frac{(3x^2+4)^4}{(3x^2+4)^3}$ $\quad \Rightarrow f(x) = 3x^2 + 4$

$f'(x) = 6x$

(h) $f(x) = (3x^2 + x - 5)^2 \cdot x^3$

$$u(x) = \underbrace{(\overbrace{3x^2 + x - 5}^{h(x)})^2}_{g(h(x))}, v(x) = x^3 \quad \Rightarrow f'(x) = \underbrace{\underbrace{2(3x^2 + x - 5)}_{g'(h(x))} \cdot \underbrace{(6x+1)}_{h'(x)}}_{u'(x)} \cdot \underbrace{x^3}_{v(x)}$$

$$+ \underbrace{(3x^2 + x - 5)^2}_{u(x)} \cdot \underbrace{3x^2}_{v'(x)} =$$

$$2x^3(3x^2 + x - 5)(6x + 1) + 3x^2(3x^2 + x - 5)^2$$