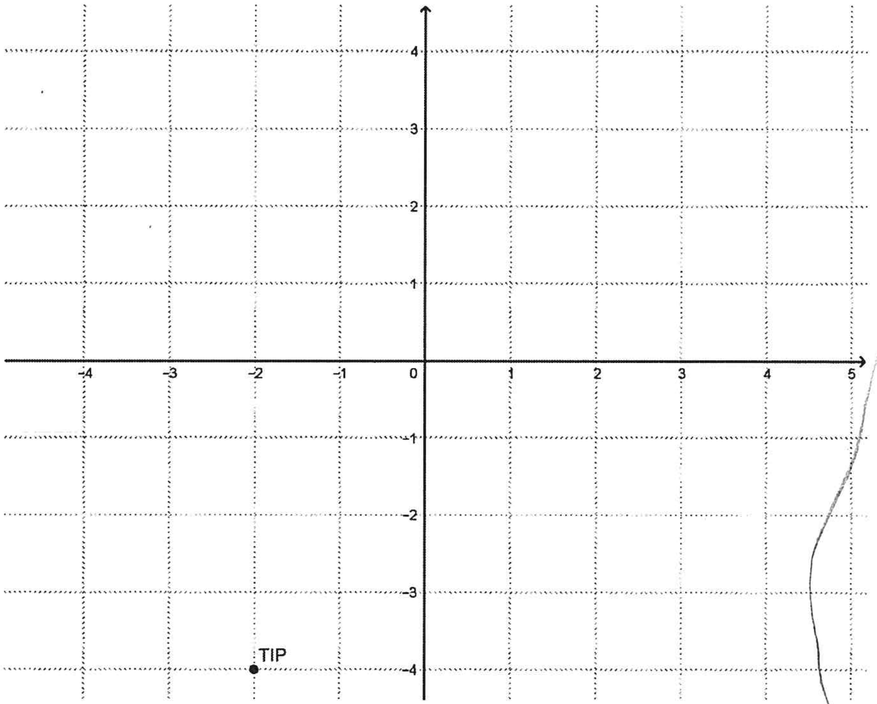
- Funktion dritten Grades
- Punktsymmetrisch
- $TIP(-2|-4)$



- Funktion dritten Grades $\rightarrow f(x) = ax^3 + bx^2 + cx + d$
- im Ursprung die Steigung 9 $\rightarrow f'(0) = 9 \qquad f(0) = 0$
- Wendepunkt bei W(4|4) $\rightarrow f''(4) = 0 \qquad f(4) = 4$

$f(x) = ax^3 + bx^2 + cx + d$
$f'(x) = 3ax^2 + 2bx + c$
$f''(x) = 6ax + 2b$

$f'(0) = 9 \qquad f(0) = 0$

$\times \; 0 = a \cdot 0^3 + b \cdot 0^2 + c \cdot 0 + d$

$\Rightarrow d = 0$

$\times \; 9 = 3a \cdot 0^2 + 2b \cdot 0 + c$

$\Rightarrow c = 9$

$f''(4) = 0 \qquad f(4) = 4$

$\times \; 4 = a \cdot 4^3 + b \cdot 4^2 + 9 \cdot 4$

$4 = 64a + 16b + 36 \quad | -4$

$0 = 64a + 16b + 32$

$\times \; 0 = 6a \cdot 4 + 2b$

$0 = 24a + 2b \quad | -24a$

$-24a = 2b \quad | : 2$

$b = -12a$

einsetzen

einsetzen

$0 = 64a + 16 \cdot (-12a) + 32$

$0 = 64a - 192a + 32 \quad | -32$

$-32 = -128a \quad | : (-128)$

$a = \frac{1}{4}$

$b = -12 \cdot \frac{1}{4} = -3$

$\Rightarrow f(x) = \frac{1}{4}x^3 - 3x^2 + 9x$

• Funktion vierten Grades $\rightarrow$ $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

• $S(0|-2{,}75)$ Sattelpunkt $\rightarrow$ $f''(0)=0$ $f'(0)=0$ $f(0)=-2{,}75$

• $H(-3|4)$ Hochpunkt $\rightarrow$ $f'(-3)=0$ $f(-3)=4$

$f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$f''(x) = 12ax^2 + 6bx + 2c$

$f''(0) = 0$

$0 = 12a \cdot 0^2 + 6b \cdot 0 + 2c$

$\Rightarrow c = 0$

$f'(0) = 0$

$0 = 4a \cdot 0^3 + 3b \cdot 0^2 + d$

$\Rightarrow d = 0$

$f(0) = -2{,}75$

$-2{,}75 = a \cdot 0^4 + b \cdot 0^3 + e$

$\Rightarrow e = -2{,}75$

$f'(-3) = 0$

$0 = 4a \cdot (-3)^3 + 3b \cdot (-3)^2$

$0 = -108a + 27b$ $\quad | + 108a$

$108a = 27b$ $\quad | : 27$

$b = 4a$

einsetzen $\rightarrow$

$f(-3) = 4$

$4 = a \cdot (-3)^4 + 4a \cdot (-3)^3 - 2{,}75$

$4 = 81a - 108a - 2{,}75$

$4 = -27a - 2{,}75$ $\quad | + 2{,}75$

$6{,}75 = -27a$ $\quad | : (-27)$

$a = -\frac{1}{4}$

$b = 4\left(-\frac{1}{4}\right) = -1$ $\leftarrow$ einsetzen

$\Rightarrow f(x) = -\frac{1}{4}x^4 - x^3 - 2{,}75$

- Funktion dritten Grades $\to f(x) = ax^3 + bx^2 + cx + d$
- Sattelpunkt auf y-Achse $\to f'(0) = 0 \quad f''(0) = 0$
- Schneidet x-Achse bei $x = 2$ $\to f(2) = 0$
- geht durch $P(-1|3)$ $\to f(-1) = 3$

$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$f'(0) = 0$

$0 = 3a \cdot 0^2 + 2b \cdot 0 + c$

$\Rightarrow c = 0$

$f''(0) = 0$

$0 = 6a \cdot 0 + 2b$

$\Rightarrow b = 0$

$f(2) = 0$

$0 = a \cdot 2^3 + d$

$0 = 8a + d \quad | -8a$

$d = -8a$

einsetzen

$f(-1) = 3$

$3 = a \cdot (-1)^3 - 8a$

$3 = -a - 8a$

$3 = -9a \quad | : (-9)$

$a = -\frac{1}{3}$

$d = -8 \cdot \left(-\frac{1}{3}\right) = \frac{8}{3}$

$\Rightarrow f(x) = -\frac{1}{3}x^3 + \frac{8}{3}$

- Funktion dritten Grades $\rightarrow f(x) = ax^3 + bx^2 + cx + d$
- geht durch den Ursprung $\rightarrow f(0) = 0$
- in W(-2|2) Wendetangente mit Steigung -3

$\Rightarrow f(-2) = 2 \qquad f'(-2) = -3 \qquad f''(-2) = 0$

$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$f(0) = 0$

$* \; 0 = a \cdot 0^3 + b \cdot 0^2 + c \cdot 0 + d$

$\Rightarrow d = 0$

$f(-2) = 2$

$* \; 2 = a \cdot (-2)^3 + b \cdot (-2)^2 + c \cdot (-2)$

$2 = -8a + 4b - 2c$

$f'(-2) = -3$

$* \; -3 = 3a(-2)^2 + 2b \cdot (-2) + c$

$-3 = 12a - 4b + c$

einsetzen

$f''(-2) = 0$

$* \; 0 = 6a \cdot (-2) + 2b$

$0 = -12a + 2b \qquad | +12a$

$12a = 2b \qquad | :2$

$b = 6a$

einsetzen

$* \; 2 = -8a + 4 \cdot 6a - 2c$

$2 = -8a + 24a - 2c$

$2 = 16a - 2c \qquad | -16a; :(-2)$

$8a - 1 = c$

$* \; -3 = 12a - 4 \cdot 6a + (8a - 1)$

$-3 = 12a - 24a + 8a - 1$

$-3 = -4a - 1 \qquad | +1$

$-2 = -4a \qquad |:(-4)$

$a = \frac{1}{2}$

einsetzen in b, c

$b = 6 \cdot \frac{1}{2} = 3$

$c = 8 \cdot \frac{1}{2} - 1 = 3$

$\Rightarrow f(x) = \frac{1}{2}x^3 + 3x^2 + 3x$