Berechnen Sie die Steigung für die nachfolgenden Funktionen an den Stellen $x_0 = -2$ $x_0 = 0$ und $x_0 = -4$

(a) $f(x) = -2x^2$

$x_0 = 2$

$$\begin{aligned} m_t &= \lim_{h\to 0} \tfrac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0} \tfrac{f(2+h)-f(2)}{h} \\ &= \lim_{h\to 0} \tfrac{(-2)*(2+h)^2-(-2)*(2)^2}{h} \\ &= \lim_{h\to 0} \tfrac{(-2)*(4+4h+h^2)-(-2)*4}{h} \\ &= \lim_{h\to 0} \tfrac{(-8)-8h-2h^2+8}{h} \\ &= \lim_{h\to 0} \tfrac{-8h-2h^2}{h} \\ &= \lim_{h\to 0} -8-2h \end{aligned}$$

= -8

$x_0 = 0$

$$\begin{aligned} m_t &= \lim_{h\to 0} \tfrac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0} \tfrac{f(0+h)-f(0)}{h} \\ &= \lim_{h\to 0} \tfrac{(-2)*h^2-(-2)*0^2}{h} \\ &= \lim_{h\to 0} \tfrac{-2h^2-0}{h} \\ &= \lim_{h\to 0} \tfrac{-2h^2}{h} \\ &= \lim_{h\to 0} -2h \\ &= 0 \end{aligned}$$

$x_0 = -4$

$$\begin{aligned} m_t &= \lim_{h\to 0} \tfrac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0} \tfrac{f(-4+h)-f(-4)}{h} \\ &= \lim_{h\to 0} \tfrac{(-2)*(-4+h)^2-(-2)*(-4)^2}{h} \\ &= \lim_{h\to 0} \tfrac{(-2)*(16-8h+h^2)-(-2)*16}{h} \\ &= \lim_{h\to 0} \tfrac{(-32)+16h-2h^2+32}{h} \\ &= \lim_{h\to 0} \tfrac{16h-2h^2}{h} \\ &= \lim_{h\to 0} 16-2h \\ &= 16 \end{aligned}$$

(b) $f(x) = 3x^2 + 4$

$x_0 = 2$

$$\begin{aligned} m_t &= \lim_{h\to 0} \tfrac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0} \tfrac{f(2+h)-f(2)}{h} \\ &= \lim_{h\to 0} \tfrac{[3*(2+h)^2+4]-[3*2^2+4]}{h} \\ &= \lim_{h\to 0} \tfrac{[3*(4+4h+h^2)+4]-[3*4+4]}{h} \\ &= \lim_{h\to 0} \tfrac{[12+12h+3h^2+4]-[12+4]}{h} \\ &= \lim_{h\to 0} \tfrac{12h+3h^2}{h} \\ &= \lim_{h\to 0} 12+3h \\ &= 12 \end{aligned}$$

$x_0 = 0$

$$\begin{aligned} m_t &= \lim_{h\to 0} \tfrac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0} \tfrac{f(0+h)-f(0)}{h} \\ &= \lim_{h\to 0} \tfrac{[3*h^2+4]-[3*0^2+4]}{h} \\ &= \lim_{h\to 0} \tfrac{[3*h^2+4]-4}{h} \\ &= \lim_{h\to 0} \tfrac{3h^2}{h} \\ &= \lim_{h\to 0} 3h \\ &= 0 \end{aligned}$$

$x_0 = -4$

$$\begin{aligned} m_t &= \lim_{h\to 0} \tfrac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0} \tfrac{f(-4+h)-f(-4)}{h} \\ &= \lim_{h\to 0} \tfrac{[3*(-4+h)^2+4]-[3*(-4)^2+4]}{h} \\ &= \lim_{h\to 0} \tfrac{[3*(16-8h+h^2)+4]-[3*16+4]}{h} \\ &= \lim_{h\to 0} \tfrac{[48-24h+3h^2+4]-[48+4]}{h} \\ &= \lim_{h\to 0} \tfrac{-24h+3h^2}{h} \\ &= \lim_{h\to 0} -24+3h \end{aligned}$$

= -24

(c) $f(x) = \frac{x^2}{3}$

$x_0 = 2$

$$\begin{aligned} m_t &= \lim_{h\to 0} \tfrac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0} \tfrac{f(2+h)-f(2)}{h} \\ &= \lim_{h\to 0} \frac{\frac{(2+h)^2}{3}-\frac{(2)^2}{3}}{h} \\ &= \lim_{h\to 0} \frac{\frac{4+4h+h^2}{3}-\frac{4}{3}}{h} \\ &= \lim_{h\to 0} \frac{\frac{4+4h+h^2-4}{3}}{h} \\ &= \lim_{h\to 0} \frac{\frac{4h+h^2}{3}}{h} \\ &= \lim_{h\to 0} \tfrac{4h+h^2}{3h} \\ &= \lim_{h\to 0} \tfrac{4+h}{3} \\ &= \tfrac{4}{3} \end{aligned}$$

$x_0 = 0$

$$\begin{aligned} m_t &= \lim_{h\to 0} \tfrac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0} \tfrac{f(0+h)-f(0)}{h} \\ &= \lim_{h\to 0} \frac{\frac{(0+h)^2}{3}-\frac{0^2}{3}}{h} \\ &= \lim_{h\to 0} \frac{\frac{h^2}{3}-\frac{0}{3}}{h} \\ &= \lim_{h\to 0} \frac{\frac{h^2-0}{3}}{h} \\ &= \lim_{h\to 0} \frac{\frac{h^2}{3}}{h} \\ &= \lim_{h\to 0} \tfrac{h^2}{3h} \\ &= \lim_{h\to 0} \tfrac{h}{3} \\ &= 0 \end{aligned}$$

$x_0 = -4$

$$\begin{aligned} m_t &= \lim_{h\to 0} \tfrac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0} \tfrac{f(-4+h)-f(-4)}{h} \\ &= \lim_{h\to 0} \frac{\frac{(-4+h)^2}{3}-\frac{(-4)^2}{3}}{h} \\ &= \lim_{h\to 0} \frac{\frac{16-8h+h^2}{3}-\frac{16}{3}}{h} \\ &= \lim_{h\to 0} \frac{\frac{16-8h+h^2-16}{3}}{h} \\ &= \lim_{h\to 0} \frac{\frac{-8h+h^2}{3}}{h} \\ &= \lim_{h\to 0} \tfrac{-8h+h^2}{3h} \\ &= \lim_{h\to 0} \tfrac{-8+h}{3} \\ &= \tfrac{-8}{3} \end{aligned}$$