Hausaufgabe

Untersuchen Sie folgende Funktionen auf Extrema.
Geben Sie die Art der Extrema an (HOP/TIP).

a) $f(x) = x^2 - 3x - 4$

b) $f(x) = -\frac{3}{2}x^2 + x$

c) $f(x) = 8x^3 - 3x^2$

d) $f(x) = -x^3 + 12x$

e) $f(x) = x^3 + 3x^2 - 9x$

f) $f(x) = \frac{1}{8}x^3 + x$

g) $f(x) = -x^3 + 4,5x^2 - 6x + 2$

h) $f(x) = x^4 - 6x^2 + 4$

a) $f(x) = x^2 - 3x - 4$

$f'(x) = 2x - 3$

$f''(x) = 2$

Extremstelle bestimmen:

$f'(x) = 0$

$0 = 2x - 3 \quad | +3$

$3 = 2x \quad | :2$

$x = 1{,}5$

$f''(1{,}5) = 2 > 0 \Rightarrow$ TIP

b) $f(x) = -\frac{3}{2}x^2 + x$

$f'(x) = -3x + 1$

$f''(x) = -3$

$f'(x) = 0$

$0 = -3x + 1 \quad | +3x$

$3x = 1 \quad | :3$

$x = \frac{1}{3}$

$f''(x) = -3 < 0 \Rightarrow$ HOP

c) $f(x) = 8x^3 - 3x^2$

$f'(x) = 24x^2 - 6x = x(24x - 6)$

$f''(x) = 48x - 6$

$f'(x) = 0$

$0 = x(24x - 6)$

$x_1 = 0$

$0 = 24x - 6 \quad | +6$

$6 = 24x \quad | :24$

$x_2 = \frac{1}{4}$

$f''(0) = 48 \cdot 0 - 6 = -6 < 0$ HOP

$f''\left(\frac{1}{4}\right) = 48 \cdot \frac{1}{4} - 6 = 6 > 0$ TIP

d) $f(x) = -x^3 + 12x$

$f'(x) = -3x^2 + 12$

$f''(x) = -6x$

$f'(x) = 0$

$0 = -3x^2 + 12 \quad | + 3x^2$

$3x^2 = 12 \quad | : 3$

$x^2 = 4 \quad | \sqrt{}$

$x_1 = 2 \quad x_2 = -2$

$f''(2) = -6 \cdot 2 = -12 < 0$ HOP

$f''(-2) = -6 \cdot (-2) = 12 > 0$ TIP

e) $f(x) = x^3 + 3x^2 - 9x$

$f'(x) = 3x^2 + 6x - 9$

$f''(x) = 6x + 6$

$f'(x) = 0$

$0 = 3x^2 + 6x - 9 \quad | : 3$

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$x_1 = -3 \quad x_2 = 1$

$f''(-3) = 6 \cdot (-3) + 6 = -12 < 0$ HOP

$f''(1) = 6 \cdot 1 + 6 = 12 > 0$ TIP

f) $f(x) = \frac{1}{8}x^3 + x$

$f'(x) = \frac{3}{8}x^2 + 1$

$f''(x) = \frac{3}{4}x$

$f'(x) = 0$

$0 = \frac{3}{8}x^2 + 1 \quad |-1$

$-1 = \frac{3}{8}x^2 \quad |:\frac{3}{8}$

$-\frac{8}{3} = x^2 \quad |\sqrt{\ }$ ↯

Die Funktion hat keine Extremstellen.

g) $f(x) = -x^3 + 4{,}5x^2 - 6x + 2$

$f'(x) = -3x^2 + 9x - 6$

$f''(x) = -6x + 9$

$f'(x) = 0$

$0 = -3x^2 + 9x - 6 \quad |:(-3)$

$0 = x^2 - 3x + 2$

$0 = (x-1)(x-2)$

$x_1 = 1 \qquad x_2 = 2$

$f''(1) = -6 \cdot 1 + 9 = 3 > 0$ TIP

$f''(2) = -6 \cdot 2 + 9 = -3 < 0$ HOP

h) $f(x) = x^4 - 6x^2 + 4$

$f'(x) = 4x^3 - 12x = 4x(x^2 - 3)$

$f''(x) = 12x^2 - 12$

$f'(x) = 0$

$0 = 4x(x^2 - 3)$ $\quad 0 = x^2 - 3 \quad | + 3$

$x_1 = 0$ $\quad 3 = x^2 \quad | \sqrt{\ }$

$x_2 = \sqrt{3} \quad x_3 = -\sqrt{3}$

$f''(0) = 12 \cdot 0 - 12 = -12 < 0$ HOP

$f''(\sqrt{3}) = 12(\sqrt{3})^2 - 12 = 24 > 0$ TIP

$f''(-\sqrt{3}) = 12 \cdot (-\sqrt{3})^2 - 12 = 24 > 0$ HOP