

Berechnen Sie die Steigung für die nachfolgenden Funktionen an den Stellen $x_0 = -2$, $x_0 = 0$ und $x_0 = -4$

(a) $f(x) = -2x^2$

$x_0 = -2$	$x_0 = 0$	$x_0 = -4$
$ \begin{aligned} m_t &= \lim_{h \rightarrow 0} \frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(-2+h)-f(-2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(-2)*(-2+h)^2 - (-2)*(-2)^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{(-2)*(4-4h+h^2) - (-2)*4}{h} \\ &= \lim_{h \rightarrow 0} \frac{(-8)+8h-2h^2+8}{h} \\ &= \lim_{h \rightarrow 0} \frac{8h-2h^2}{h} \\ &= \lim_{h \rightarrow 0} 8 - 2h \\ &= 8 \end{aligned} $	$ \begin{aligned} m_t &= \lim_{h \rightarrow 0} \frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(0+h)-f(0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(-2)*h^2 - (-2)*0^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2h^2-0}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2h^2}{h} \\ &= \lim_{h \rightarrow 0} -2h \\ &= 0 \end{aligned} $	$ \begin{aligned} m_t &= \lim_{h \rightarrow 0} \frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(-4+h)-f(-4)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(-2)*(-4+h)^2 - (-2)*(-4)^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{(-2)*(16-8h+h^2) - (-2)*16}{h} \\ &= \lim_{h \rightarrow 0} \frac{(-32)+16h-2h^2+32}{h} \\ &= \lim_{h \rightarrow 0} \frac{16h-2h^2}{h} \\ &= \lim_{h \rightarrow 0} 16 - 2h \\ &= 16 \end{aligned} $

(b) $f(x) = 3x^2 + 4$

$x_0 = -2$	$x_0 = 0$	$x_0 = -4$
$ \begin{aligned} m_t &= \lim_{h \rightarrow 0} \frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(-2+h)-f(-2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[3*(-2+h)^2+4] - [3*(-2)^2+4]}{h} \\ &= \lim_{h \rightarrow 0} \frac{[3*(4-4h+h^2)+4] - [3*4+4]}{h} \\ &= \lim_{h \rightarrow 0} \frac{[12-12h+3h^2+4] - [12+4]}{h} \\ &= \lim_{h \rightarrow 0} \frac{-12h+3h^2}{h} \\ &= \lim_{h \rightarrow 0} -12 + 3h \\ &= -12 \end{aligned} $	$ \begin{aligned} m_t &= \lim_{h \rightarrow 0} \frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(0+h)-f(0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[3*h^2+4] - [3*0^2+4]}{h} \\ &= \lim_{h \rightarrow 0} \frac{[3*h^2+4] - 4}{h} \\ &= \lim_{h \rightarrow 0} \frac{3h^2}{h} \\ &= \lim_{h \rightarrow 0} 3h \\ &= 0 \end{aligned} $	$ \begin{aligned} m_t &= \lim_{h \rightarrow 0} \frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(-4+h)-f(-4)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[3*(-4+h)^2+4] - [3*(-4)^2+4]}{h} \\ &= \lim_{h \rightarrow 0} \frac{[3*(16-8h+h^2)+4] - [3*16+4]}{h} \\ &= \lim_{h \rightarrow 0} \frac{[48-24h+3h^2+4] - [48+4]}{h} \\ &= \lim_{h \rightarrow 0} \frac{-24h+3h^2}{h} \\ &= \lim_{h \rightarrow 0} -24 + 3h \\ &= -24 \end{aligned} $

(c) $f(x) = \frac{x^2}{3}$

$x_0 = -2$	$x_0 = 0$	$x_0 = -4$
$ \begin{aligned} m_t &= \lim_{h \rightarrow 0} \frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(-2+h)-f(-2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(-2+h)^2}{3} - \frac{(-2)^2}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{4-4h+h^2}{3} - \frac{4}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{4-4h+h^2-4}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{-4h+h^2}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{4h+h^2}{3h} \\ &= \lim_{h \rightarrow 0} \frac{4+h}{3} \\ &= \frac{4}{3} \end{aligned} $	$ \begin{aligned} m_t &= \lim_{h \rightarrow 0} \frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(0+h)-f(0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(0+h)^2}{3} - \frac{0^2}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{h^2}{3} - \frac{0}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{h^2-0}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{h^2}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{h^2}{3h} \\ &= \lim_{h \rightarrow 0} \frac{h}{3} \\ &= 0 \end{aligned} $	$ \begin{aligned} m_t &= \lim_{h \rightarrow 0} \frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(-4+h)-f(-4)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(-4+h)^2}{3} - \frac{(-4)^2}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{16-8h+h^2}{3} - \frac{16}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{16-8h+h^2-16}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{-8h+h^2}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{-8h+h^2}{3h} \\ &= \lim_{h \rightarrow 0} \frac{-8+h}{3} \\ &= \frac{-8}{3} \end{aligned} $