Simple Doppler Estimation for MIMO Radar with Co-Located Antennas

I. Problem Formulation

Consider a MIMO radar system in which there are N_T transmit antennas and N_R receive antennas, with interelement spacing d and transmission wavelength λ. The received signal at antenna p and time index n due to a moving target located at angle θ with complex reflection coefficient $\beta(\theta)$ can be written as

$$y_p(n) = \beta(\theta) e^{j\frac{2(p-1)\pi d}{\lambda}\sin(\theta)} e^{j2\pi f_D n} \sum_{q=1}^{N_t} x_q(n) e^{j\frac{2(q-1)\pi d}{\lambda}\sin(\theta)} + w_p(n), \tag{1}$$

where f_D represents the resulting doppler shift caused by the moving target, $x_q(n)$ represents the baseband signal transmitted from antenna q, and $w_p(n)$ is circularly symmetric white Gaussian noise. Assuming the transmitted probing signals are narrowband and that the propagation is nondispersive, the received baseband data vector due to a target at location θ can be described by

$$\mathbf{y}(n) = \beta(\theta)\mathbf{a}_R(\theta)\mathbf{a}_T^T(\theta)\mathbf{x}(n)e^{j2\pi f_D n} + \mathbf{w}(n), \tag{2}$$

where

$$\mathbf{y}(n) = \begin{bmatrix} y_1(n) & y_2(n) & \ldots & y_{N_R}(n) \end{bmatrix}^T, \tag{3}$$

$$\mathbf{x}(n) = \begin{bmatrix} x_1(n) & x_2(n) & \ldots & x_{N_T}(n) \end{bmatrix}^T, \tag{4}$$

and

$$\mathbf{w}(n) = \begin{bmatrix} w_1(n) & w_2(n) & \ldots & w_{N_R}(n) \end{bmatrix}^T, \tag{5}$$

while

$$\mathbf{a}_T(\theta) = \begin{bmatrix} 1 & e^{j\frac{2\pi d}{\lambda}\sin(\theta)} & \ldots & e^{j\frac{2(N_T-1)\pi d}{\lambda}\sin(\theta)} \end{bmatrix}^T \tag{6}$$

and

$$\mathbf{a}_R(\theta) = \begin{bmatrix} 1 & e^{j\frac{2\pi d}{\lambda}\sin(\theta)} & \ldots & e^{j\frac{2(N_R-1)\pi d}{\lambda}\sin(\theta)} \end{bmatrix}^T \tag{7}$$

represent the transmit and receive steering vectors, and $(\cdot)^T$ denotes the transpose. The total N samples can be collected into a single matrix

$$\mathbf{Y} = \beta(\theta)\mathbf{A}(\theta)\mathbf{X}\mathbf{D}(f_D) + \mathbf{W}, \tag{8}$$

where

$$\mathbf{A}(\theta) = \mathbf{a}_R(\theta)\mathbf{a}_T^T(\theta), \tag{9}$$

$$\mathbf{X} = \left[\mathbf{x}(0) \quad \mathbf{x}(1) \quad \ldots \quad \mathbf{x}(N-1) \right], \tag{10}$$

$\mathbf{D}(f_D)$ is the $N \times N$ diagonal matrix whose main diagonal elements consist of the doppler shifts $e^{j2\pi f_D(0:N-1)}$, and $\mathbf{W}$ is the $N_R \times N$ noise matrix. Let the vector of unknown variables be $\psi = [\theta \ f_D]^T$. We vectorize $\mathbf{Y}$ by stacking its columns on top of one another in order to create the received data vector $\mathbf{y}$. Assuming $\mathbf{W}$ has covariance matrix $\mathbf{C_W} = \sigma_w^2 \mathbf{I}$, where $\mathbf{I}$ denotes the identity matrix of corresponding dimension, the received signal follows a Gaussian probability density function (pdf) with mean $\mu(\psi) = \mathrm{vec}(\beta(\theta)\mathbf{A}(\theta)\mathbf{X}\mathbf{D}(f_D))$ and covariance matrix $\mathbf{C_y} = \sigma_w^2 \mathbf{I}$. Let

$$\mathbf{v}(\psi) = \mathrm{vec}(\beta(\theta)\mathbf{A}(\theta)\mathbf{X}\mathbf{D}(f_D)). \tag{11}$$

The log-likelihood function can then be written as

$$\Lambda(\psi) = C - \frac{\sigma_w^2}{2} \left(\mathbf{y}^H \mathbf{y} - \mathbf{y}^H \mathbf{v}(\theta, f_D) - \mathbf{v}(\theta, f_D)^H \mathbf{y} + \mathbf{v}(\theta, f_D)^H \mathbf{v}(\theta, f_D) \right), \tag{12}$$

where C is some constant that does not depend on ψ. To find the maximum likelihood (ML) estimate, the log-likelihood function must be maximized with respect to ψ.

$$\hat{\psi} = \arg\max_{\psi} \Lambda(\psi) = \arg\min_{\psi} \mathbf{y}^H \mathbf{y} - \mathbf{y}^H \mathbf{v}(\theta, f_D) - \mathbf{v}(\theta, f_D)^H \mathbf{y} + \mathbf{v}(\theta, f_D)^H \mathbf{v}(\theta, f_D). \tag{13}$$

The first term in the equation does not depend on ψ. The final term is constant with respect to f_D and θ (see Appendix 1 for proof). Thus the estimation simplifies to

$$\hat{\psi} = \arg\max_{\psi} \mathbf{y}^H \mathbf{v}(\theta, f_D) + \mathbf{v}(\theta, f_D)^H \mathbf{y}. \tag{14}$$

The above estimator requires a two-dimensional search over θ and f_D.

II. Simulation Results

Simulation results are shown for a uniform linear array with $N_T = N_R = 10$ antennas and half-wavelenth interelement spacing at both the transmit and receive side. The angular search is performed over a linear meshgrid of 181 points and the frequency search is performed over a linear meshgrid of 101 points. The frequency is normalized to be between -0.5 and 0.5. Two targets with reflection coefficients equal to unity are placed at $30°$ and $-55°$, with $f_D = -0.1$ and $f_D = 0.3$, respectively. Zero-mean circularly symmetric white Gaussian noise with variance $\sigma_w^2 = 0.1$ is added to the received signal. Fig. 1 shows a mesh plot of the magnitude of the resulting location and frequency estimates with data markers placed at the local maxima, where X corresponds to frequency and Y corresponds to location angle. In both cases, the estimator performs perfectly.

III. Appendix 1

Proof of independence of f_D, θ for $\mathbf{v}(\theta, f_D)^H \mathbf{v}(\theta, f_D)$.

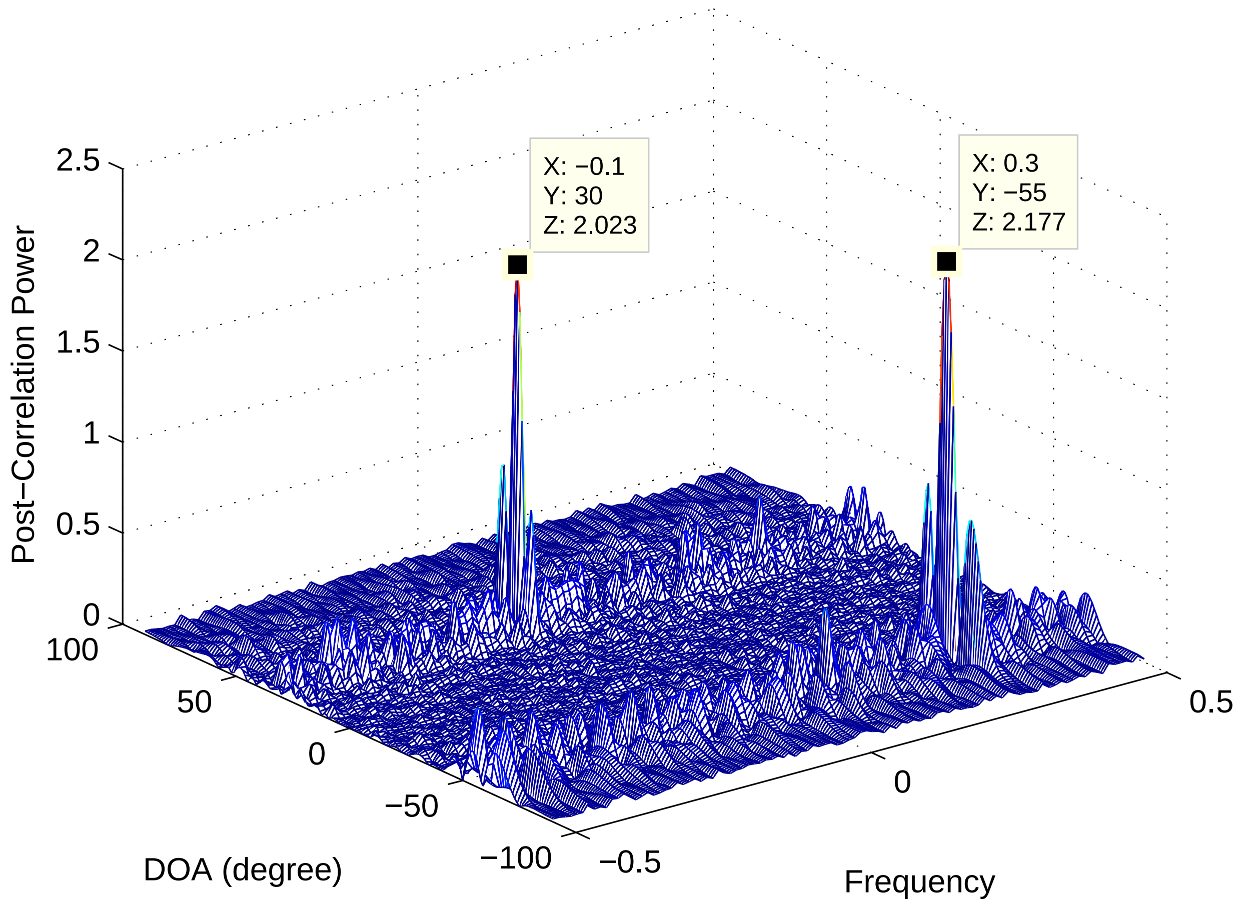

Fig. 1. Location and frequency estimates for two targets where X corresponds to the doppler frequency and Y corresponds to the location angle.

Without any loss of generality, $\beta(\theta)$ is assumed to be unity for compactness. We begin by using the property $\mathrm{vec}(\mathbf{AXB}) = (\mathbf{B}^T \otimes \mathbf{A})\mathrm{vec}(\mathbf{X})$ to rewrite the term as

$$\begin{aligned}
\mathbf{v}(\theta, f_D)^H \mathbf{v}(\theta, f_D) &= \mathrm{vec}(\beta(\theta)\mathbf{A}(\theta)\mathbf{X}\mathbf{D}(f_D))^H \mathrm{vec}(\beta(\theta)\mathbf{A}(\theta)\mathbf{X}\mathbf{D}(f_D)) && (15)\\
&= [(\mathbf{D}(f_D) \otimes \mathbf{A}(\theta))\mathrm{vec}(\mathbf{X})]^H \, [(\mathbf{D}(f_D) \otimes \mathbf{A}(\theta))\mathrm{vec}(\mathbf{X})] && (16)\\
&= \mathrm{vec}(\mathbf{X})^H (\mathbf{D}^*(f_D) \otimes \mathbf{A}^H(\theta))(\mathbf{D}^T(f_D) \otimes \mathbf{A}(\theta))\mathrm{vec}(\mathbf{X}) && (17)\\
&= \mathrm{vec}(\mathbf{X})^H \left[(\mathbf{D}^*(f_D)\mathbf{D}^T(f_D)) \otimes (\mathbf{A}^H(\theta)\mathbf{A}(\theta))\right] \mathrm{vec}(\mathbf{X}) && (18)\\
&= \mathrm{vec}(\mathbf{X})^H \left[I_{N \times N} \otimes (\mathbf{A}^H(\theta)\mathbf{A}(\theta))\right] \mathrm{vec}(\mathbf{X}). && (19)
\end{aligned}$$

As seen above, the frequency term cancels otu completely. Let $\tilde{\mathbf{A}}(\theta) = \mathbf{A}^H(\theta)\mathbf{A}(\theta)$. It can be easily shown that the diagonal elements of $\tilde{\mathbf{A}}(\theta)$ are constants equal to N_T. Note that due to the orthogonal signaling, we have

$\mathbf{E}\left\{x_{ij}^{*}x_{kl}\right\} = 1$ for $i = k$ and $j = l$, and zero otherwise. Expansion of the above product under expectation yields zeros everywhere except at the diagonal elements of $\tilde{\mathbf{A}}(\theta)$. The resulting product is therefore a constant equal to $N_T^2 \times N$.