Lab04

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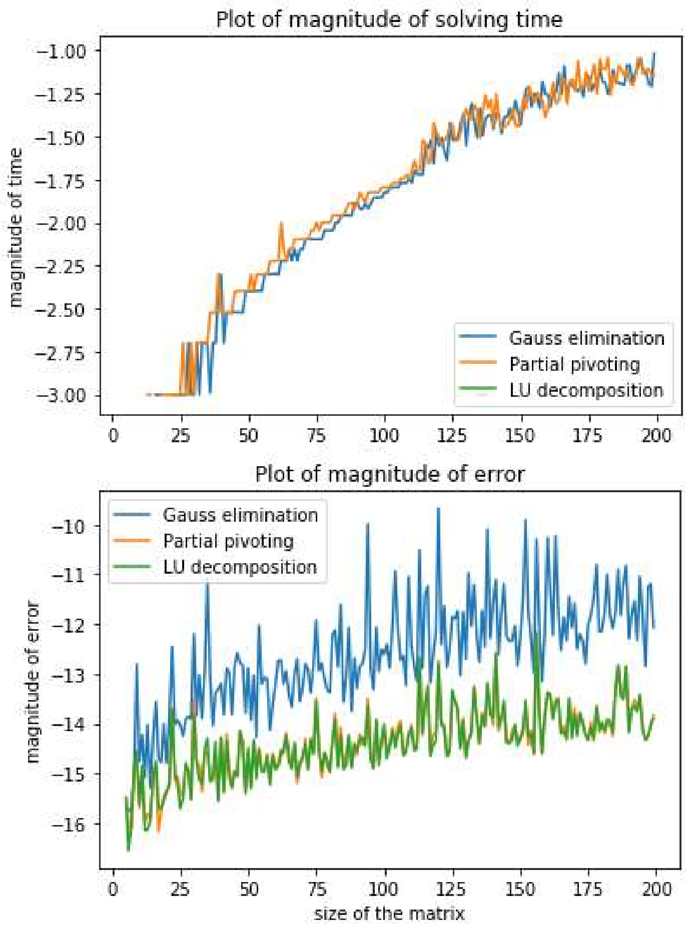
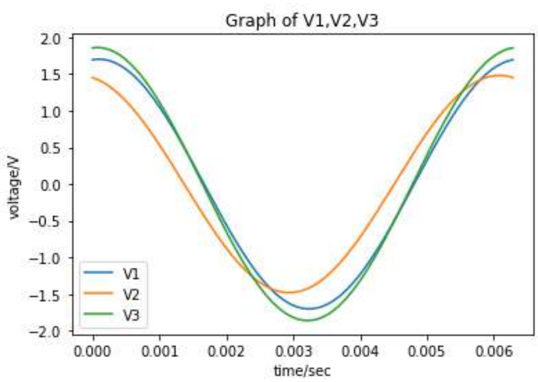
Question 1



(b) To compare the three different approaches , we calculated the time and accuracy of solving random linear systems with a range of sizes for the three methods. And we created two diagrams: one for magnitudes of time vs size of the random matrices, one for magnitudes of error vs size of the random matrices, which are both shown on the right. Then it is easy to see from the diagram, that Gaussian elimination takes the roughly same time usage as partial pivoting whereas LU decomposition is way faster than the other two, where we use the numpy.linalg.solve function which is more efficient. On the other hand, on the stage of error, LU decomposition and partial pivoting are just on the same level where Gaussian elimination isn't doing as good as the other two which is slightly higher than them.



(c) Firstly, using the partial pivoting package we wrote in Q1(a), we got the answer of V1,V2,V3 for the electric circuit under the initial conditions. The printed output of the amplitudes and phases of the three voltages is as follows:

The amplitudes of V1,V2,V3 are [1.70143907 1.48060535 1.86076932]

The phases of V1,V2,V3 are [-0.09545371 0.20216879 -0.07268725]

Then, we could make V1,V2,V3 into functions of time by just multiplying $e^{i\omega t}$ and taking the real parts of them. The graph of these three functions is shown on the right.

Finally, we replaced the R6 with an inductor L as required and repeated all the steps.

The printed output of the amplitudes and phases of the three voltages is as follows:

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After replace R6 with an inductor
The amplitudes of V1,V2,V3 are    [1.56211819 1.49942868 2.81127639]
The phases of V1,V2,V3 are    [-0.07026536    0.37767673    0.25049802]
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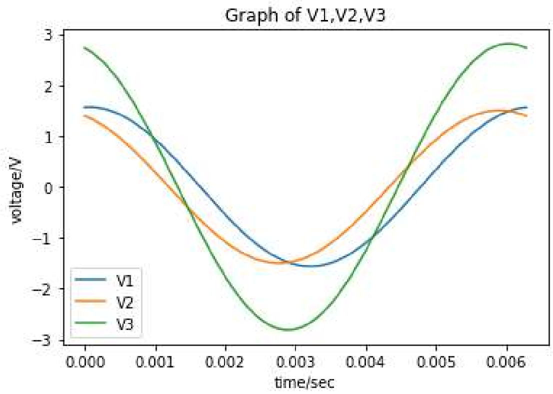
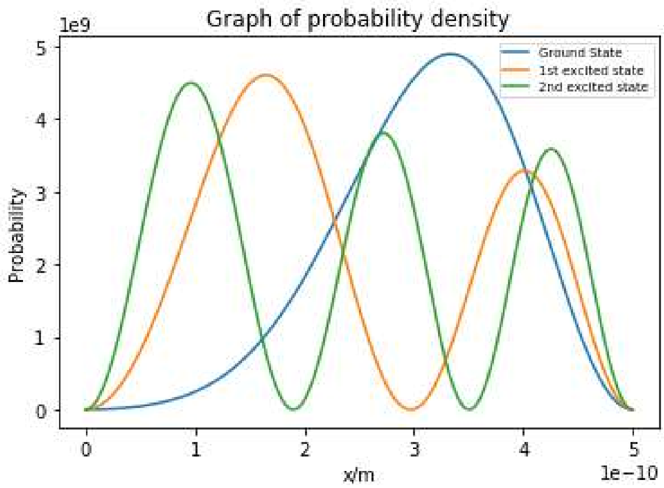
And the graph of functions of V1,V2,V3 vs time is shown on the right.



After replacing R6 with the inductor(which is just replace R6 with iR6 in the linear system), we find that there is no significant change on V1 and V2. But, V3 get a larger amplitude because changing R6 into R6 makes amplitude of the overall impedance around the spot 3 smaller.

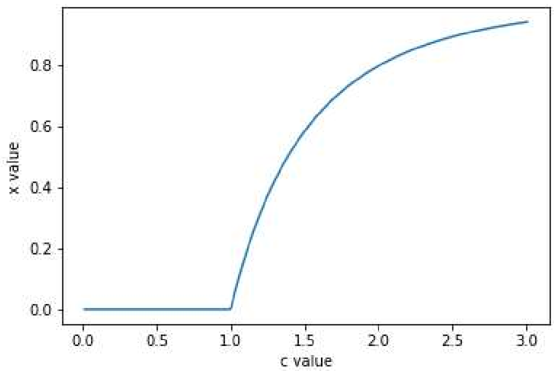
Question 2

As what has been asked, using the method of eigenvalue and eigenvectors to solve the Schrodinger's equation, as the H is not definite size , the first ten results are treated as the target result. The Ground state is supposed to be 5.84eV, and the simulation provides a result with 5.83634271eV which is certainly close enough and acceptable for that. As we expand the eigenvalue to a size of 100 entry, theoretically it should give a more accurate rate of estimation to the energy, however due to the output, not a dramatic difference, what we think as a reason is that the 10x10 array's result is accurate and close to the proper key enough, that expand the size of array didn't give the change that big to be visualized but changes are believed took place. After that we modify to plot ground state and the first two excited state, by normalizing the probability density function of the wave function to fulfill the sum actually equal to one. And by the plot, it matches with the knowledge of the probability density function which proofs it is reasonable and correct.



Q3

A)



The plot to the right is the plot by the method of relaxation of X vs. c as the function of $x = 1 - e^{-cx}$

B)

6.11C) By changing from relaxation method to overrelaxation method, the iteration time dropped from 22 towards 11.

6.11D)

C)

6.13B) The following table is the list of the estimation of three methods and their iteration times. By the table, it is easy to tell the right answer is converge to 5 with Relaxation and Binary search gives the roughly same accuracy while relaxation iterate for fewer times. However, Newton's method is extremely efficient here which not only gives more accuracy but also only iterate for once.

	# of iteration	Output of estimation of X
Relaxation method	10	4.965114223358215
Newton's method	1	4.986259120214346
Binary search	22	4.965114116668701

6.13C) By the estimation of x and all constants can be computed into the equations, the output of the estimation of the temperature of the sun surface is 5772.456138819498 K, which compare to result of 5778K, is quite acceptable.