Lab03

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Question 1

(a)

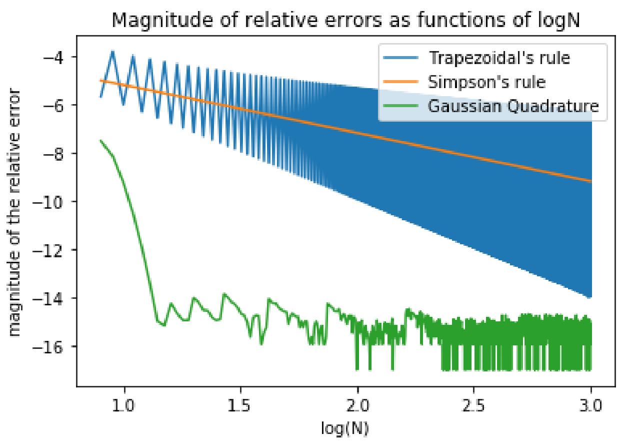
The results of the three methods for a range of slices is shown in the following table.

Number of slices	Trapezoidal's rule	Simpson's rule	Gaussian Quadrature
8	0.9999687596667604	0.9999759576770337	0.9999779385594257
108	0.9999778557995493	0.9999779094202548	0.9999779095030012
208	0.9999778950204392	0.9999779094969785	0.9999779095029999
308	0.999977902897661	0.9999779095017486	0.9999779095030021
408	0.9999779057386927	0.9999779095025946	0.9999779095030022
508	0.9999779070748153	0.9999779095028325	0.9999779095030022
608	0.9999779078078638	0.9999779095029181	0.9999779095030008
708	0.9999779082528947	0.999977909502957	0.9999779095030015
808	0.9999779085431753	0.9999779095029744	0.9999779095030025
908	0.9999779087429504	0.9999779095029845	0.9999779095030015

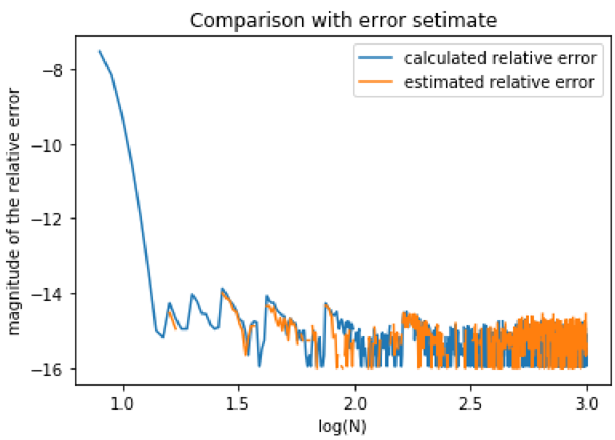
From this table, we could see that the Gaussian Quadrature gives the best performance since the first 13 digits of its results became stable after just 108 slices.

(b)

From the log-log graph on the right, we could verify the statement we made in (a), Gaussian Quadrature truly have much better performance than Trapezoidal's rule and Simpson's rule. Once the number of slices exceed 10^1.5, the magnitude of relative errors would keep smaller than 10^-13.



And, compared with the error estimate, we could see that the estimation is not that accurate since the relative errors sometimes are very close to 10^-17 which is recognized as 0 in Python.

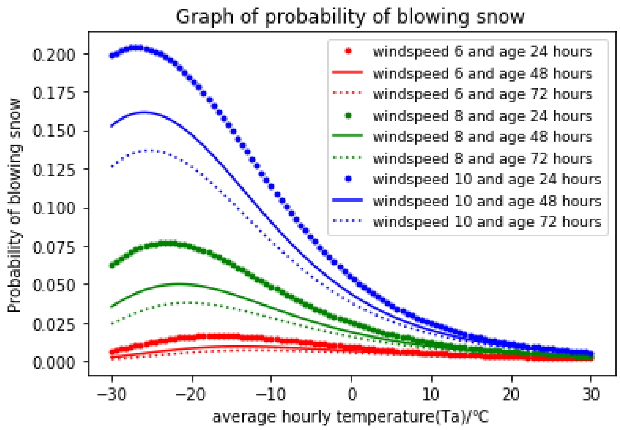


(c) First, lets re-cast equation(3) in terms of the error function:

$$\mathrm{P}(u_{10}, T_a, t_h) = \frac{1}{\sqrt{2\pi}\delta}\int_0^{u_{10}} e^{[-\frac{(\bar{u}-u)^2}{2\delta^2}]} du$$

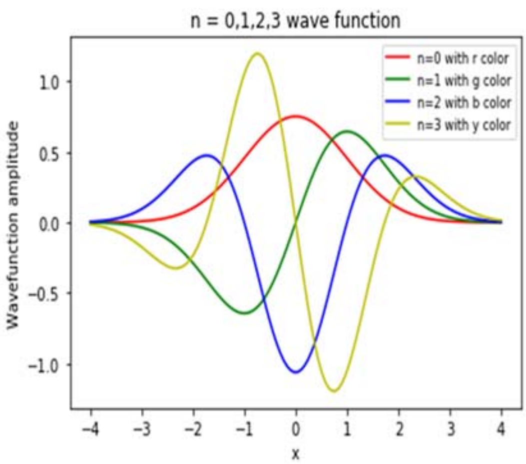
$$= \frac{1}{\sqrt{\pi}}\int_{\frac{-\bar{u}}{\sqrt{2}\delta}}^{\frac{u_{10}-\bar{u}}{\sqrt{2}\delta}} e^{-u'^2} du' = \frac{1}{\sqrt{\pi}} \times (\mathrm{erf}\left(\frac{u_{10}-\bar{u}^2}{\sqrt{2}\delta}\right) + \mathrm{erf}(\frac{\bar{u}}{\sqrt{2}\delta}))$$

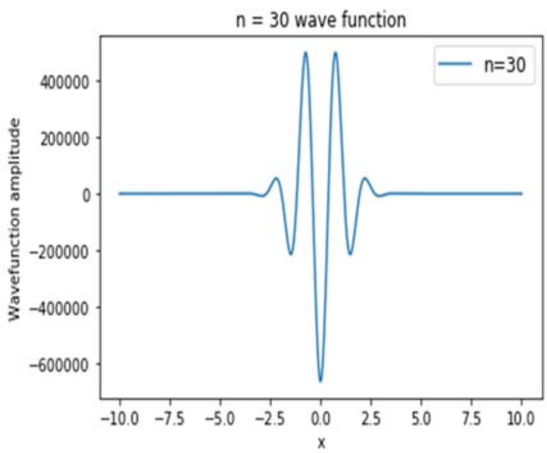
From the plot, we could find that the probability of blowing snow will become larger with faster windspeed u_{10} and shorter snow surface age t_h, which is same as the common sense. And, the highest probability happens when windspeed is 10 and snow surface age is 24 hours with temperature at around -28℃.



Question 2

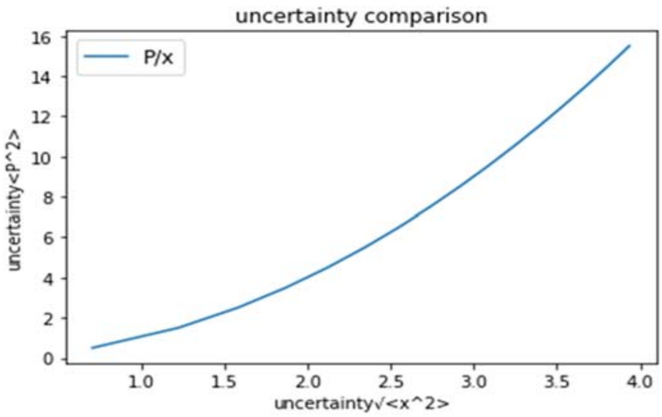
a)& b) We use the equation of ψ_n to compute the estimate of the actual wavefunction, and plot two diagrams. As the first diagram indicates the first four states of the oscillator, it clearly fits the knowledge of harmonic oscillator. And within the same principle, it is easy to find out how the amplitude expands

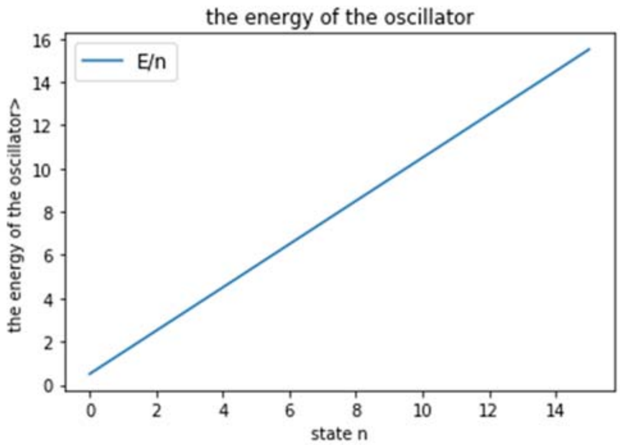




dramatically when state n=30.

c)From the diagram drawn as uncertainty comparison, it's clearly indicated that there is a smooth non-linear relationship between the uncertainty in position and the uncertainty in momentum. As the diagram we provide the analysis within the Energy vs. state, it shows that the energy increases when the state n increases as a perfect linear relationship.





Printed output:

uncertainty of position is 0.7071067811865476,the uncertainty in momentum is 0.49999999999999994
uncertainty of position is 1.2247448713915987,the uncertainty in momentum is 1.5000000000000242
uncertainty of position is 1.5811388300841383,the uncertainty in momentum is 2.5000000000005147
uncertainty of position is 1.8708286933859621,the uncertainty in momentum is 3.4999999999940195
uncertainty of position is 2.1213203435201113,the uncertainty in momentum is 4.499999999852738
uncertainty of position is 2.3452078797796547,the uncertainty in momentum is 5.500000000009436
uncertainty of position is 2.5495097589880396,the uncertainty in momentum is 6.500000012239947
uncertainty of position is 2.7386128037232598,the uncertainty in momentum is 7.500000040797082
uncertainty of position is 2.915475920103315,the uncertainty in momentum is 8.499999586748158
uncertainty of position is 3.0822064158787934,the uncertainty in momentum is 9.499997628593514
uncertainty of position is 3.240369354450081,the uncertainty in momentum is 10.50000571462408
uncertainty of position is 3.391174088910635,the uncertainty in momentum is 11.500060512951286
uncertainty of position is 3.5355709479251196,the uncertainty in momentum is 12.500002686361363
uncertainty of position is 3.674187669673495,the uncertainty in momentum is 13.499145820917535
uncertainty of position is 3.8073746632833485,the uncertainty in momentum is 14.498840404581093
uncertainty of position is 3.9365739857061115,the uncertainty in momentum is 15.507169992809626
The energy is 0.5 for n =   0
The energy is 1.500000000000024 for n =   1
The energy is 2.500000000000563 for n =   2
The energy is 3.499999999995123 for n =   3

The energy is 4.499999999842511 for n = 4

The energy is 5.49999999969501 for n = 5

The energy is 6.500000011707599 for n = 6

The energy is 7.500000064757028 for n = 7

The energy is 8.499999713725215 for n = 8

The energy is 9.499997009338955 for n = 9

The energy is 10.499999633941659 for n = 10

The energy is 11.500061107125081 for n = 11

The energy is 12.500132307086746 for n = 12

The energy is 13.499400426449142 for n = 13

The energy is 14.497471115596543 for n = 14

The energy is 15.501892368873863 for n = 15

Question 3

(a) The pseudo-code is as following:

#1. open the data file

#2 set the arrays

#3 write the data into the data array by loops, and check the bad points

#4 calculate the partial derivative along y axis

#use forward and backward for borders and central differences for other points

#5 calculate the partial derivative along x axis

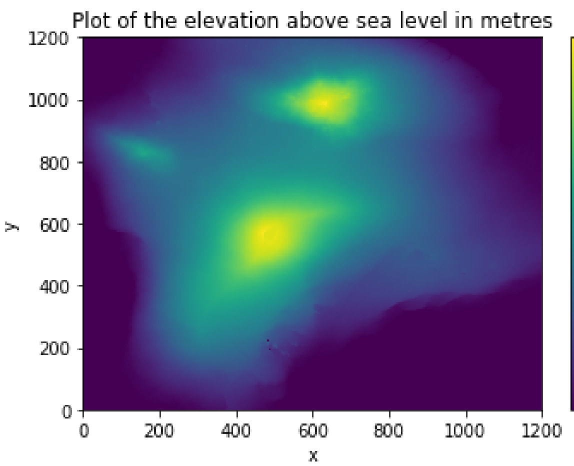
#use forward and backward for borders and central differences for other points

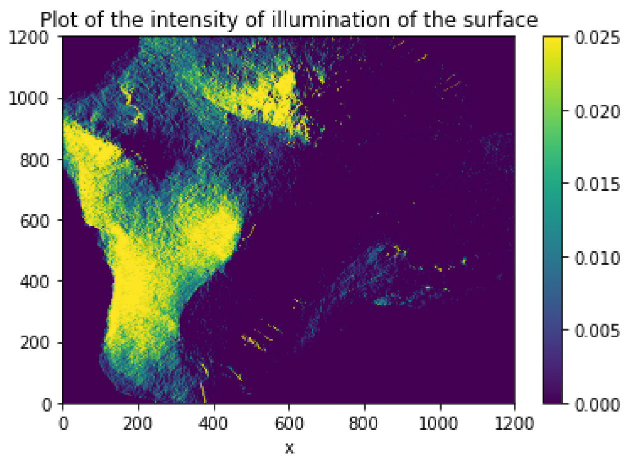
#6 calculate I for every point

#7 plot W and I by pcolormesh

(b)

From the plots of W and I, it is obvious that only the westward surface have positive intensity of illumination, which is same as the common sense, since we are actually doing simulation of the sunset at Hawaii Island.





Changing the φ to 0, we could easily get the plot of intensity of illumination at the sunrise, which is shown on the right.