PHY254 Problem Set 2, 2017

Due Wed, October 11, 2017 by 5:00 p.m to: Deepak Box 1, Roya Box 2, Matt Box 4 . DropBoxes are located at the base of the tower building. Two problems will be graded.

1. **A damped oscillation: Taylor 5.25**
   Consider a damped oscillator with $\beta < \omega_o$. The motion described by:

   $$x(t) = Ae^{-\beta t} cos(\omega_1 t - \delta) \tag{1}$$

   is not periodic. "Period" τ_1 is approximated as the time between succesive maxima of $x(t)$.
   a) Show that $\tau_1 = \frac{2\pi}{\omega_1}$.
   b) Show that an equivalent definition is that τ_1 is twice the time between succesive zeros of $x(t)$.
   c) If $\beta = \frac{\omega_o}{2}$, by what factor does the amplitude shrink in one period?

2. **Tension in a pendulum string**
   Consider a simple pendulum, undergoing small oscillations.
   Is the time average of the tension in the string of the pendulum larger or smaller than mg? By how much?

3. **Effective spring constant**
   a) Two springs with spring constants k_1 and k_2 are connected in parallel. What is the effective spring constant k_{eff}? k_{eff} is the spring constant of one spring that does the same job as the two springs connecetd in parralel.
   b) The same springs are connected now in series. What is the effective spring constant?

4. **A projectile attached to a spring:** A projectile of mass m is fired from the origin at speed v_0 and angle θ. It is attached to the origin by a spring with spring constant k and relaxed length zero.

   a) Find $x(t)$ and $y(t)$.

   b) Show that for small $\omega = \sqrt{k/m}$, the trajectory reduces to normal projectile motion. Setup a condition for ω that accounts for the statement "small ω".

   Show that for large ω, the trajectory reduces to simple harmonic motion along a line (meaning $y/x \approx$ constant). Setup a condition that accounts for the statement "large ω".

   c) What value should ω take so that the projectile *hits* the ground when it moves straight downward?

5. **A disappearing act**
   The springs from Figure 1 are at equilibrium length. The mass oscillates along the line of the springs with amplitude d. At a moment in time (let this be $t = 0$), when the mass is at position $x = \frac{d}{2}$ and is moving to the right, the right spring is removed.
   a) What is the resulting $x(t)$?
   b) What is the amplitude of the new oscillation?

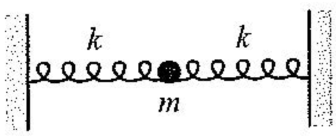

Figure 1: One mass, two springs