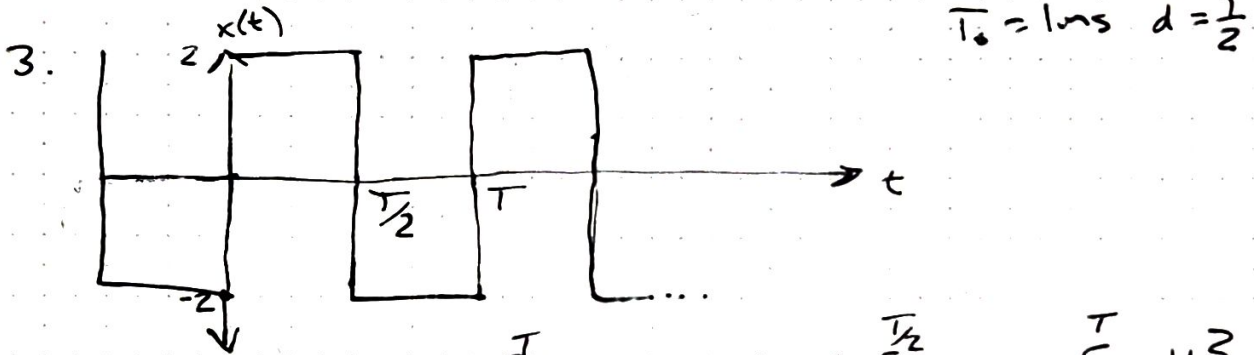1. $x(t) = 2 \cos(2\pi (1000t))$

$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T V_m^2 \cos^2(\omega t) \, dt} = V_{PK} \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \boxed{1.414 \, V}$$

$$dBV = 20 \log_{10}(V_{RMS}) = \boxed{3.010 \, dBV}$$

2. $x(t) = 1 \cos(2\pi \, 3000t)$

$$V_{RMS} = \frac{1}{\sqrt{2}} = 0.707 \qquad dBV = 20\log_{10}(0.707) = \boxed{-3.010 \, dBV}$$

3.

$T_0 = 1 ms \quad d = \frac{1}{2}$

$$X_0 = C_0 = a_0 = \frac{1}{T} \int_0^T x(t) \, dt = \frac{1}{T} \left\{ \int_0^{T/2} 2 \, dt + \int_{T/2}^T -2 \, dt \right\}$$

$$= \frac{1}{T} \left\{ 2t \Big|_0^{T/2} - 2t \Big|_{T/2}^T \right\}$$

$$= \frac{1}{T} \left\{ \frac{2T}{2} - 0 - 2T + \frac{2T}{2} \right\}$$

$$= \boxed{0}$$

$$X_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} \, dt = \frac{1}{T} \left\{ \int_0^{T/2} 2 e^{-jn\omega_0 t} \, dt - \int_{T/2}^T 2 e^{-jn\omega_0 t} \, dt \right\}$$

$$= \frac{1}{T} \left\{ \frac{2}{jn\omega_0} \left(-e^{-jn\omega_0 t} \Big|_0^{T/2} + e^{-jn\omega_0 t} \Big|_{T/2}^T \right) \right\}$$

$$= \frac{1}{T} \frac{2}{jn\frac{2\pi}{T}} \left(-e^{-jn\frac{2\pi}{T}\frac{T}{2}} + e^{-j0} + e^{-jn\frac{2\pi}{T}T} - e^{-jn\frac{2\pi}{T}\frac{T}{2}} \right)$$

$$= \frac{1}{jn\pi} \left(-e^{-jn\pi} + 1 + 1 - e^{-jn\pi} \right)$$

$$= \frac{1}{jn\pi}\left(2-2e^{-jn\pi}\right) = \frac{2}{jn\pi}\left(1-e^{-jn\pi}\right)$$

$$= \frac{4}{jn2\pi}\left(e^{jn\pi/2}-e^{-jn\pi/2}\right)e^{-jn\pi/2}$$

$$= \frac{4}{n\pi}\sin\left(n\frac{\pi}{2}\right)e^{-jn\pi/2}$$

$$= 2\;\frac{\sin\left(n\frac{\pi}{2}\right)}{n\frac{\pi}{2}}e^{-jn\pi/2}$$

$$= 2\,\text{sinc}\left(\frac{n}{2}\right)e^{-jn\pi/2}$$

n	0	1	2	3	4	5	6	7	8	9	10
C_n	0	2.	1.27	0	0.4244	0	0.2506	0	0.1817	0	0.1415
θ_n	0	0	-90	-180	-90	-180	-90	-180	-90	-180	-90
dBV	$-\infty$	3.0+3	-0.1121	-325.173	-10.45	-325.173	-14.79	-325.173	-17.74	-35.17	-19.9769

4. #3 but duty cycle $\frac{1}{3}$

$$X_0 = C_0 = A_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{T}\left\{\int_0^{T/3}2\,dt - \int_{T/3}^T 2\,dt\right\}$$

$$= \frac{1}{T}\left\{2t\Big|_0^{T/3} - 2t\Big|_{T/3}^T\right\}$$

$$= \frac{1}{T}\left\{2\frac{T}{3} - 0 - 2T + \frac{2T}{3}\right\}$$

$$= \frac{1}{T}\left\{\frac{4T}{3} - \frac{6T}{3}\right\} = \frac{1}{T}\left(-\frac{2T}{3}\right)$$

$$= \boxed{-\frac{2}{3}\text{ V}}$$

$$X_n = \frac{1}{T}\left\{\int_0^{T/3}2e^{-jn\omega_0 t}\,dt - \int_{T/3}^T 2e^{-jn\omega_0 t}\,dt\right\}$$

$$= \frac{1}{T}\left(\frac{2}{jn\frac{2\pi}{T}}\right)\left\{-e^{-jn\omega_0 t}\Big|_0^{T/3} + e^{-jn\omega_0 t}\Big|_{T/3}^T\right\}$$

$$= \frac{2}{jn2\pi}\left(-e^{-jn\frac{2\pi}{T}\frac{T}{3}}+1+e^{-jn\frac{2\pi}{T}T}-e^{-jn\frac{2\pi}{T}\frac{T}{3}}\right)$$

$$= \frac{2}{jn2\pi}\left(2-2e^{-jn\frac{2\pi}{3}}\right) = \frac{4}{jn2\pi}\left(1-e^{-jn\frac{2\pi}{3}}\right)$$

$$= \frac{4}{jn2\pi} \left(e^{jn\frac{\pi}{6}} - e^{-jn\frac{2\pi}{6}} \right) e^{-jn\frac{2\pi}{6}}$$

$$= \frac{4}{n\pi} \sin\left(n\frac{2\pi}{6}\right) e^{-jn\frac{2\pi}{6}}$$

$$= \frac{4}{3} \frac{\sin\left(n \cdot \frac{2\pi}{6}\right)}{n\frac{2\pi}{6}} e^{-jn\frac{2\pi}{6}}$$

$$= \frac{4}{3} \operatorname{sinc}\left(\frac{n}{3}\right) e^{-jn\frac{\pi}{3}}$$

Table and plots on attached page

5. #3 but with duty cycle of $\frac{1}{4}$

$$X_0 = C_0 = A_0 = \frac{1}{T}\int_0^T x(t)\, dt = \frac{1}{T}\left\{ \int_0^{T/4} 2\, dt - \int_{T/4}^T 2\, dt \right\}$$

$$= \frac{1}{T}\left\{ 2t\Big|_0^{T/4} - 2t\Big|_{T/4}^T \right\}$$

$$= \frac{1}{T}\left\{ \frac{2T}{4} - 0 - 2T + \frac{2T}{4} \right\}$$

$$= \frac{1}{T}\left\{ \frac{4T}{4} - 2T \right\} = \frac{1}{T}(-T)$$

$$= \boxed{-1\ V}$$

$$X_n = \frac{1}{T}\left\{ \int_0^{T/4} 2e^{-jn\omega_0 t}\, dt - \int_{T/4}^T 2e^{-jn\omega_0 t}\, dt \right\}$$

$$= \frac{1}{T}\cdot\frac{2}{jn\frac{2\pi}{T}}\left\{ -e^{-jn\omega_0 t}\Big|_0^{T/4} + e^{-jn\omega_0 t}\Big|_{T/4}^T \right\}$$

$$= \frac{2}{jn2\pi}\left\{ -e^{-jn\frac{2\pi}{T}\frac{T}{4}} + 1 + e^{-jn\frac{2\pi}{T}T} - e^{-jn\frac{2\pi}{T}\frac{T}{4}} \right\}$$

$$= \frac{2}{jn2\pi}\left\{ 2 - 2e^{-jn\frac{\pi}{2}} \right\} = \frac{4}{jn2\pi}\left\{ 1 - e^{-jn\frac{\pi}{2}} \right\}$$

$$= \frac{4}{jn2\pi}\left(e^{jn\frac{\pi}{4}} - e^{-jn\frac{\pi}{4}} \right) e^{-jn\frac{\pi}{4}}$$

$$= \frac{4}{n\pi} \sin\left(n\frac{\pi}{4}\right) e^{-jn\frac{\pi}{4}} = 1 \frac{\sin\left(\frac{n\pi}{4}\right)}{\frac{n\pi}{4}} e^{-jn\frac{\pi}{4}}$$

$$= \operatorname{sinc}\left(\frac{n}{4}\right) e^{-jn\frac{\pi}{4}}$$

6. Same as #3 but duty cycle of $\frac{1}{5}$

$$X_0 = C_0 = A_0 = \frac{1}{T} \left\{ \int_0^{T/5} 2 \, dt - \int_{T/5}^{T} 2 \, dt \right\}$$

$$= \frac{1}{T} \left\{ \frac{2T}{5} - 0 - 2T + \frac{2T}{5} \right\}$$

$$= \frac{1}{T} \left\{ \frac{4T}{5} - \frac{10T}{5} \right\} = \boxed{-\frac{6}{5} \, V}$$

$$X_n = V_{pp} \cdot duty \cdot sinc(n \cdot duty) \cdot e^{-jn\pi \, duty}$$

$$= 4 \cdot \frac{1}{5} \cdot sinc\left(\frac{n}{5}\right) \cdot e^{-jn\frac{\pi}{5}}$$

$$= \frac{4}{5} sinc\left(\frac{n}{5}\right) e^{-jn\frac{\pi}{5}}$$

7. $X_n = sinc^2\left(\frac{n}{2}\right)$ $n = 1, 2, 3, \ldots$ $X_0 = 0$

graph and values on attached
page