ECE5470-11

ECE 5740 Digital Image Processing

Morphological Image Processing II

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Outline

Some Basic Morphological Algorithms

- Boundary Extraction
- Region Filling
- Extraction of connected components
- Convex hull
- Thinning
- Thickening
- Extension to Gray-Scale images
  - Dilation & Erosion
  - Opening and closing
  - Morphological smoothing
  - Textural segmentation

Boundary Extraction

- The boundary of a set A can be obtained by first eroding A by B and then performing the set difference between A and its erosion

$$\beta(A) = A - (A \ominus B)$$

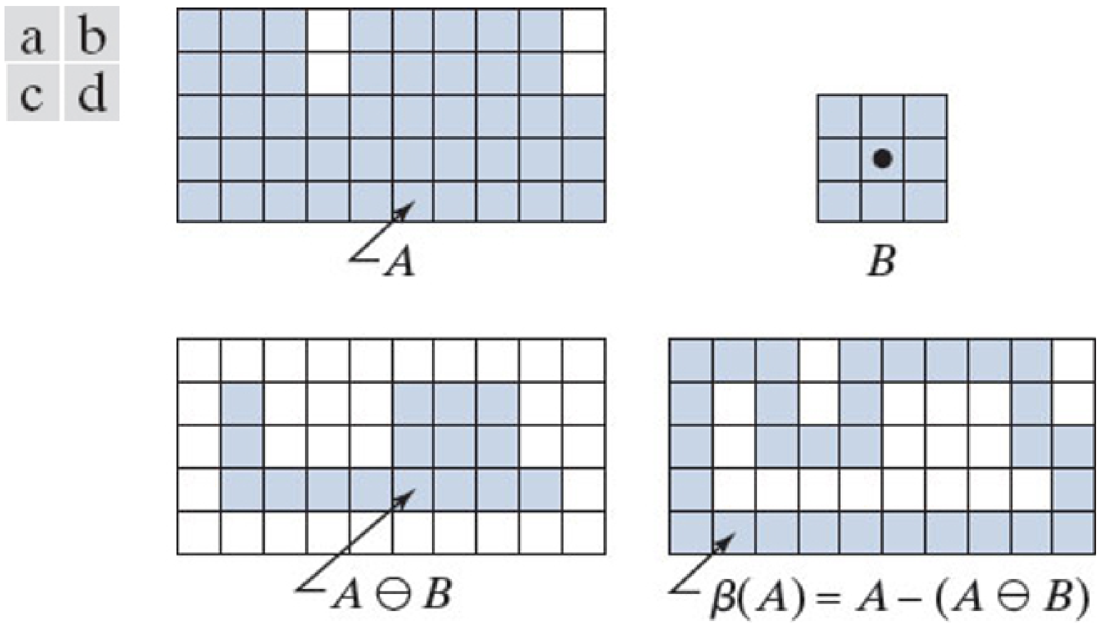
- Where B is a suitable structure element

a) Set A, of foreground pixels

b) Structuring element B

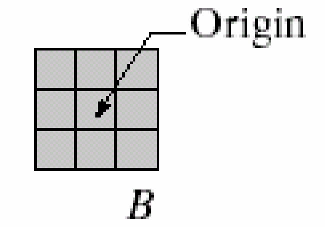
c) A eroded by B

d) Boundary, given by the set difference between a and its erosion



Example:

- A simple binary image, with 1's represented in white
- Result of using Boundary extraction with the structure element B



Region Filling

Based on set dilations, complementation and intersection

Hole filling

a) Set A, (shown shaded) contained in image I

b) Complement of image I

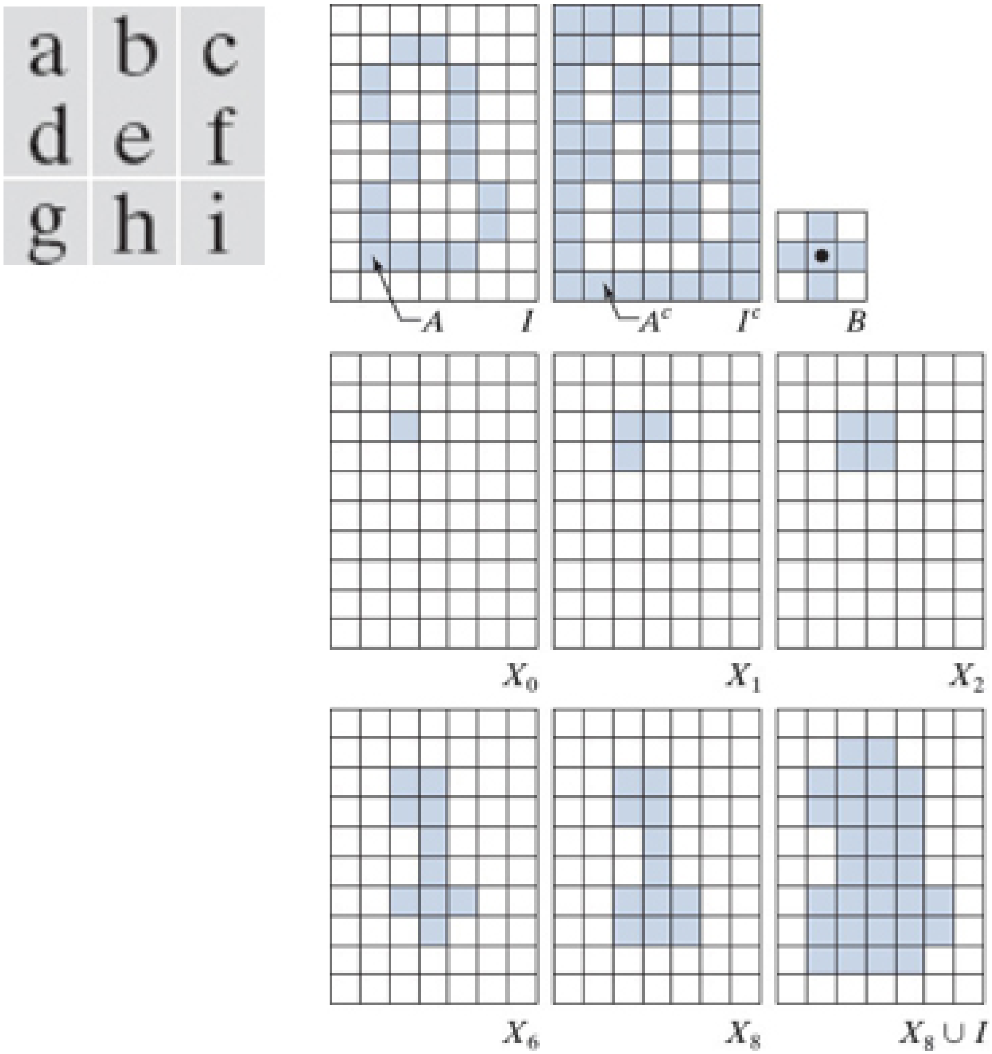
c) Structuring element B. **Only** the foreground elements are used in computations

d) Initial point inside the boundary X_0

e) Various steps of

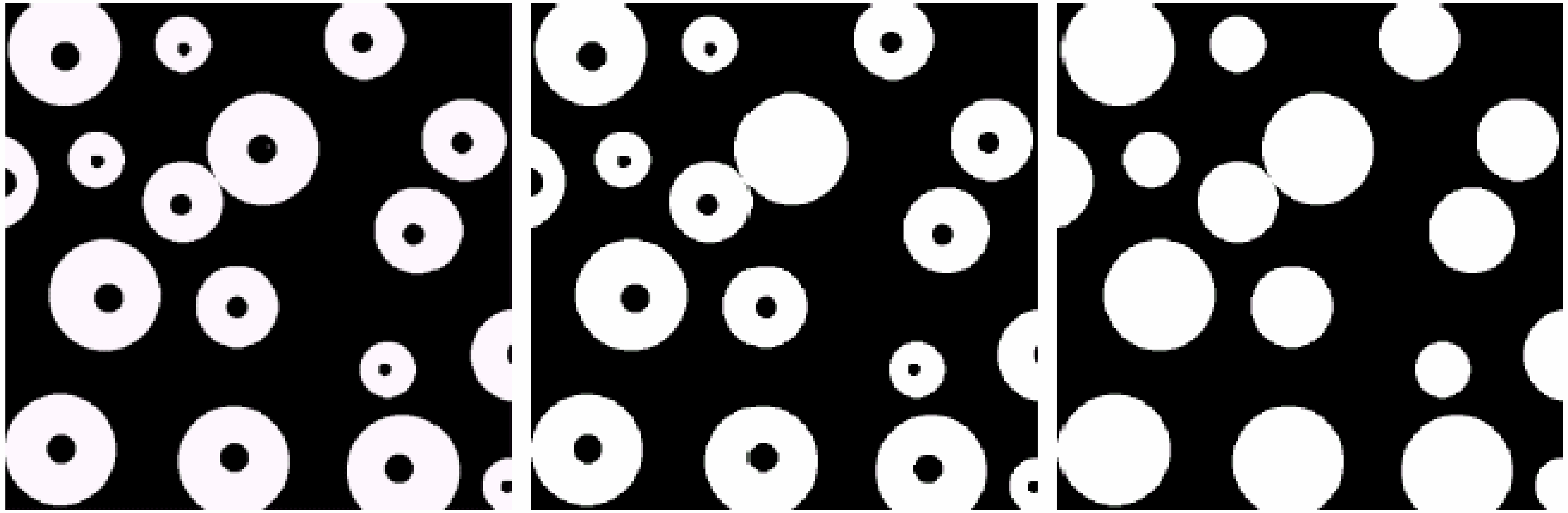
$$X_k = (X_{k-1} \oplus B) \cap I^c \quad k = 1,2,3,\ldots$$

(i) Final result [union of (a) and (h)].



Example

a) Binary Image (the white dot inside one of the regions is starting point for the region-filling algorithm)
b) Result of filling that region
c) Result of filling all region



a b c

Extraction of Connected Components

Extraction of connected components in a binary image.

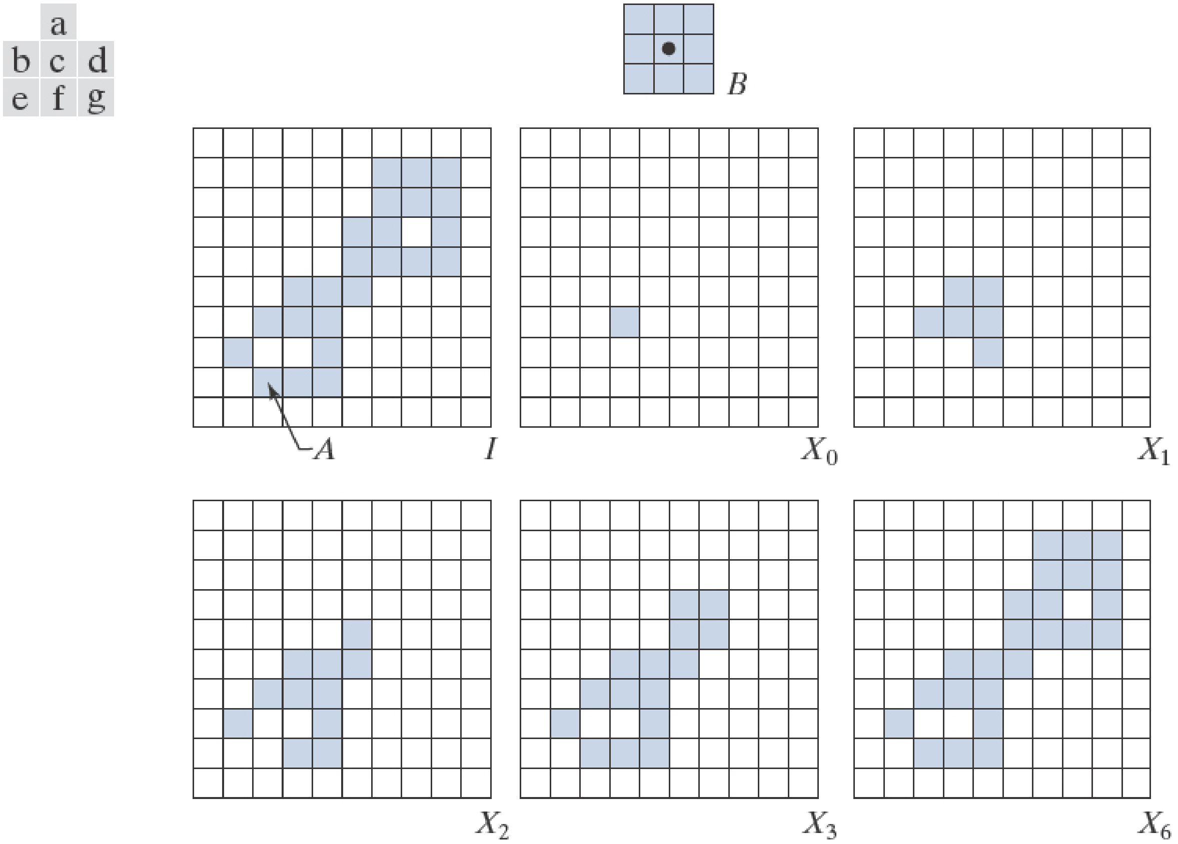
a. Structuring element

b. Image containing a set with one connected component.

c. Initial array containing a 1 in the region of the connected component.

d-g Various steps in the iteration of Eq

$$X_k = (X_{k-1} \oplus B) \cap I \quad k = 1,2,3,\ldots$$



Example

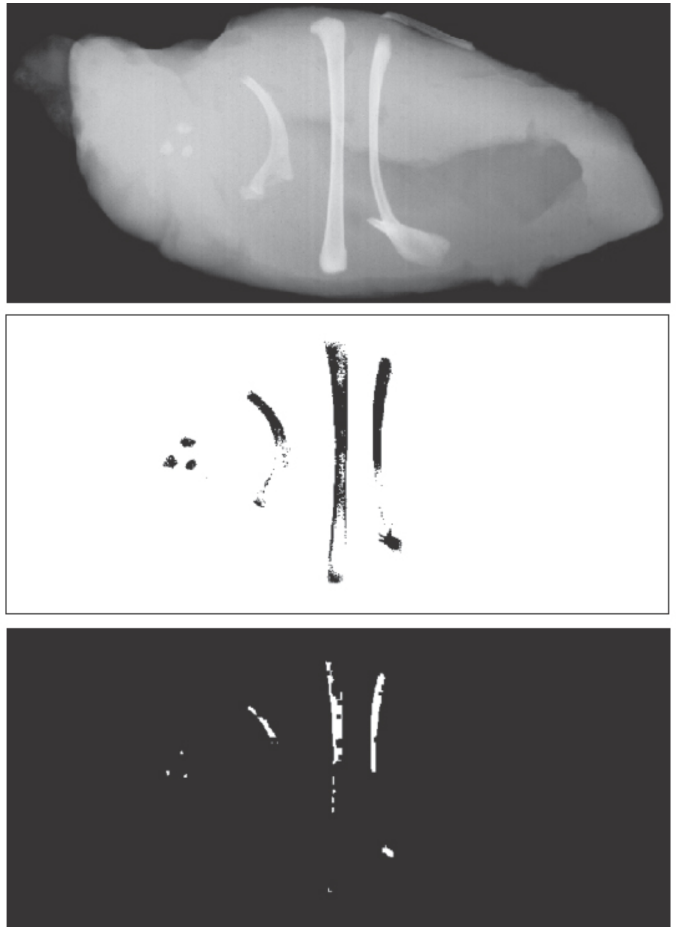
a) X-ray image of a chicken filet with bone fragments

b) Threshold image (shown as the negative for clarity).

c) Image eroded with a 5x5 structuring elements of 1's

d) Number of pixel in the connected components of c

a
b
c d



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex Hull

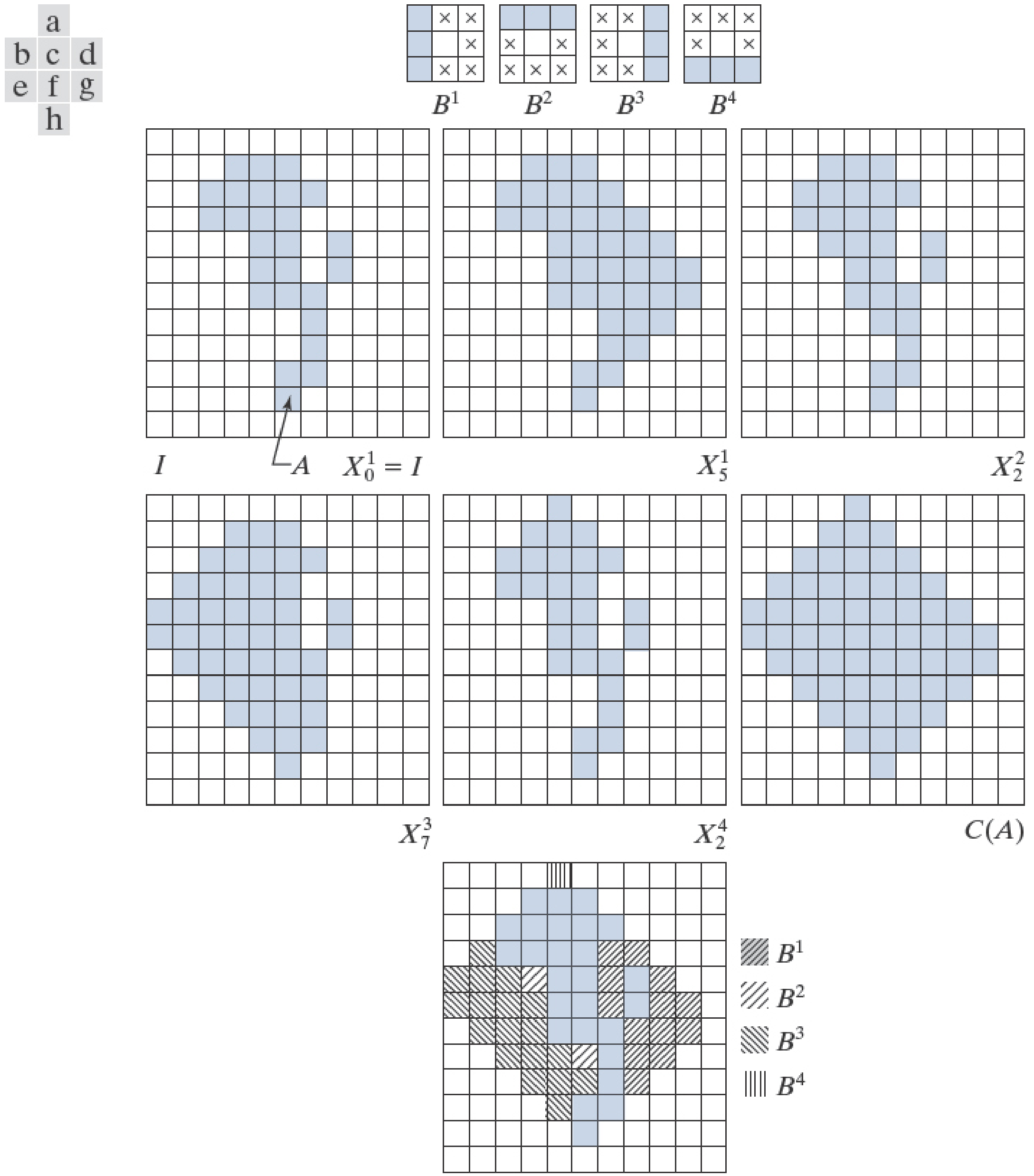
- A set *A* is said to be convex if the straight line segment joining any two points in *A* lies entirely within *A*.
- The convex hull *H* of an arbitrary set *S* is the smallest convex set containing *S*.
- The set difference *H-S* is called the convex deficiency of *S*
- If B_i, i=1,2,3,4 represent the four structuring elements
- The representation is

$$X_k^i = (X_{k-1}^i \circledast B^i) \cup X_{k-1}^i \quad i = 1,2,3,4 \;\text{ and } k = 1,2,3,\ldots$$

- Where $X_i^0 = I$. Now $\quad D_i = X_k^i(\mathit{converges})$
- The convex hull of A is (When $X_k^i = X_{k-1}^i$) $\quad C(A) = \bigcup_{i=1}^{4} D^i$

Example

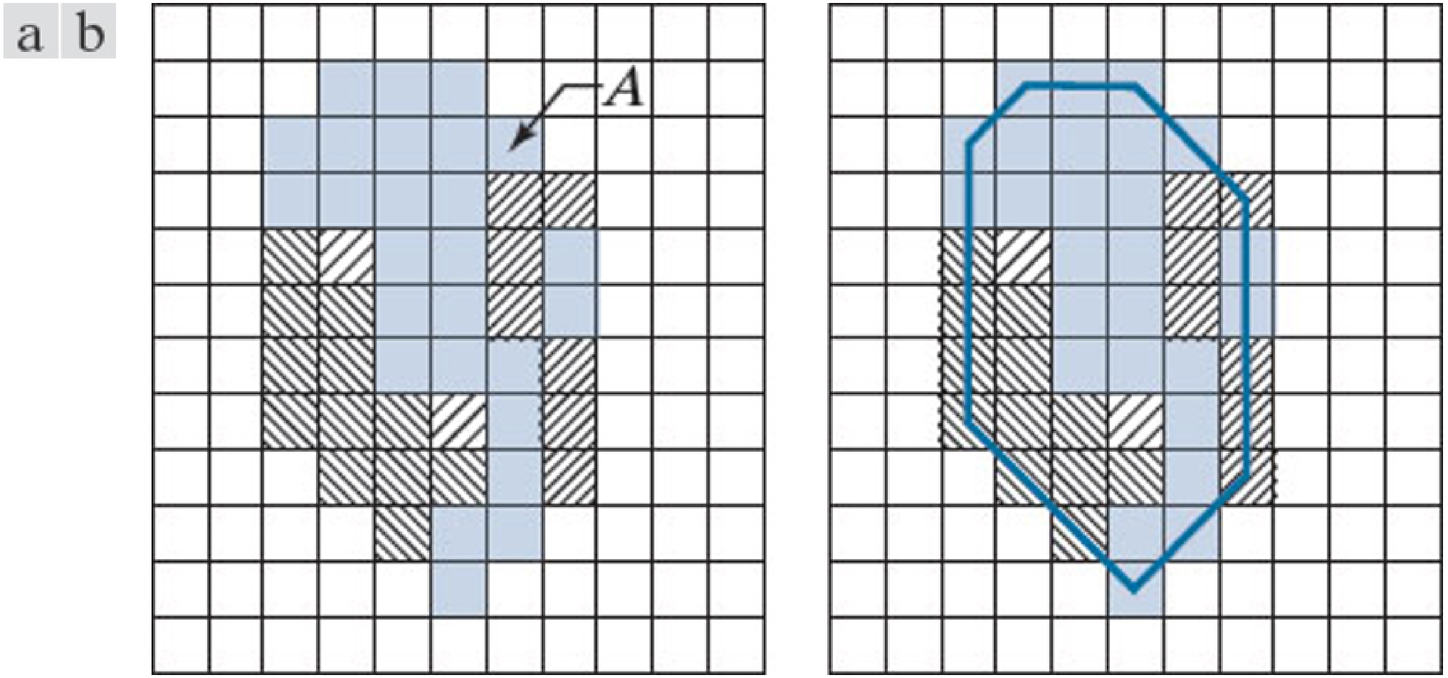
a) Structuring elements
b) Set A
c) Result of convergence with the structuring elements
d) .
e) .
f) .
g) Convex hull
h) Convex hull showing the contributing of each structuring element



Example

a) Result of limiting growth of the convex hull algorithm.

b) Straight lines connecting the boundary points show that the new set is convex also.



Thinning

- The thinning of A by a structuring element B

Hit-or-miss Transformation

$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^C$$

- A sequence of structuring elements

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

- Where B^i is a rotated version of B^{i-1}
- Defining thinning by a sequence of structure elements as

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

- The process is to thin A by one pass with B^1, then thin the result with one pass of B^2, and so on, until B^n.
- Entire process is repeated until no further changes occur

Example

a. Sequence of rotated structuring elements

b. Set A

c. Result of thinning **A** with B^1 (shaded).

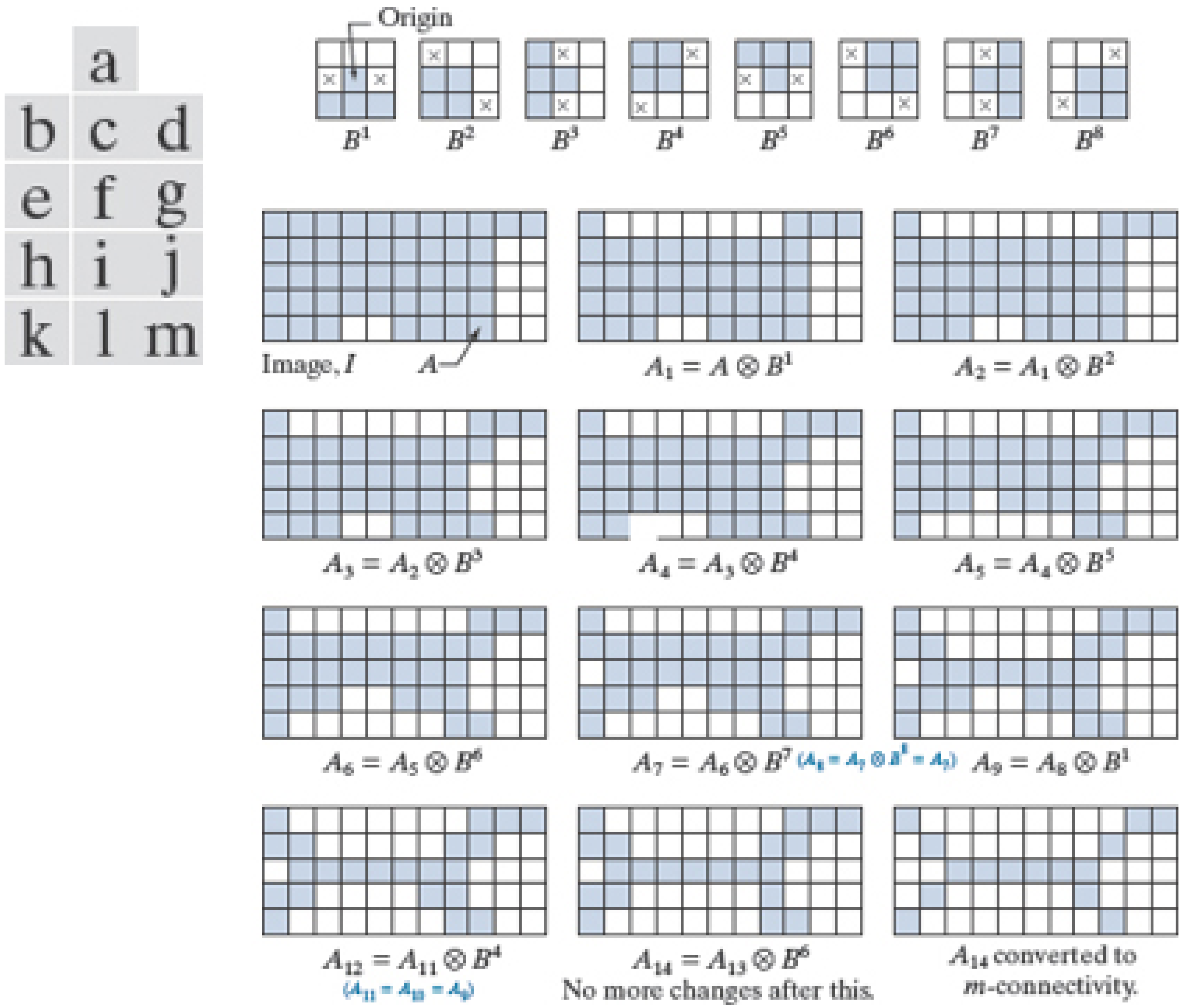
d. Result of thinning A_1 with B^2

e-l Results of thinning with the next six SEs. (There was no change between A_7 with A_7

(j)–(k) Result of using the first four elements again

(l) Result after convergence.

(m) Result converted to **m**-connectivity.



MATLAB

- f=imread('Fig9.11.jpg');
- f=logical(f);
- se=strel('square',3);
- f0=imopen(f,se);
- imshow(f0)
- g1=bwmorph(f0,'thin',1);
- figure, imshow(g1)
- g2=bwmorph(f0,'thin',5);
- figure, imshow(g2)

MATLAB

```
f=imread('Fig9.11.jpg');
f=logical(f);
se=strel('square',3);
f0=imopen(f,se);
imshow(f0)

g=bwmorph(f0,'thin',5);
figure, imshow(g)
y=bwmorph(g,'thicken',1);
figure, imshow(y)
```

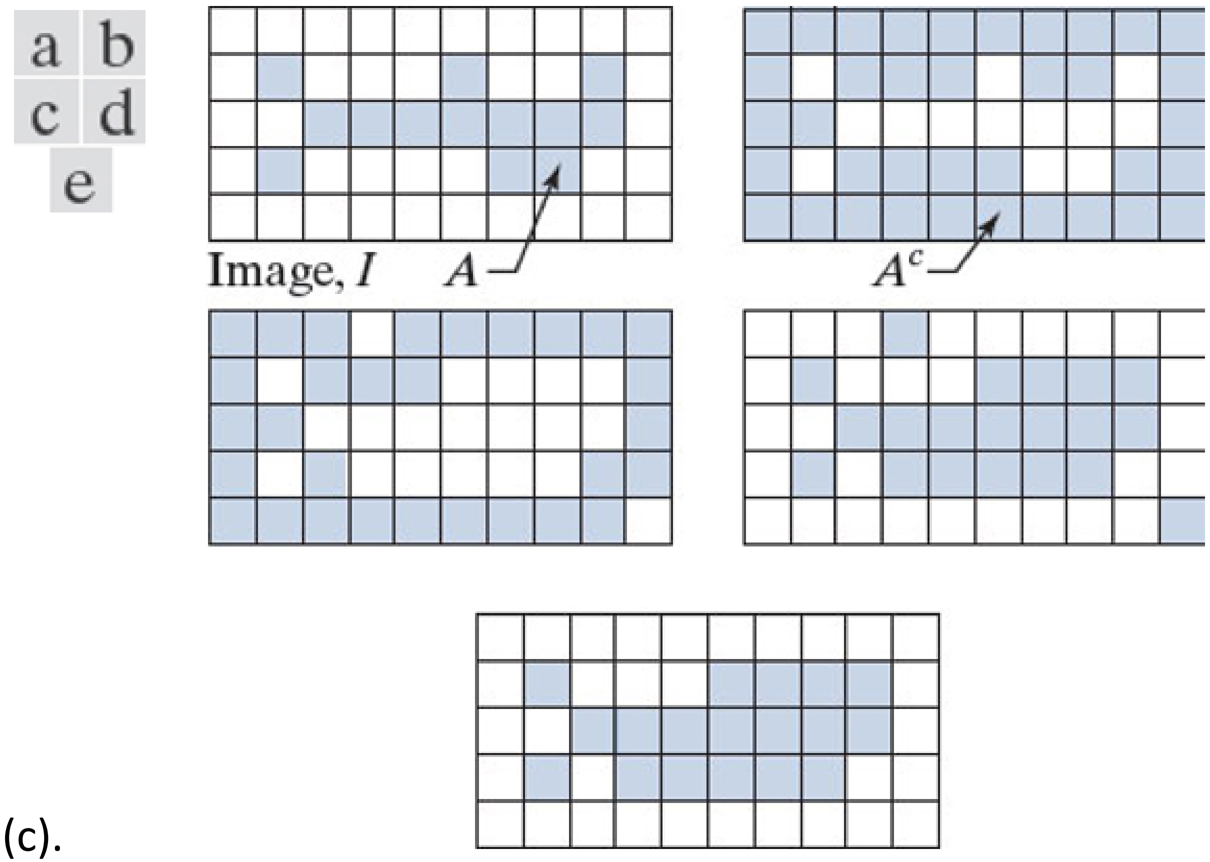
Thickening

- It is dual of thinning

$$A \odot B = A \cup (A \circledast B)$$

- Can be defined as a sequence operation:

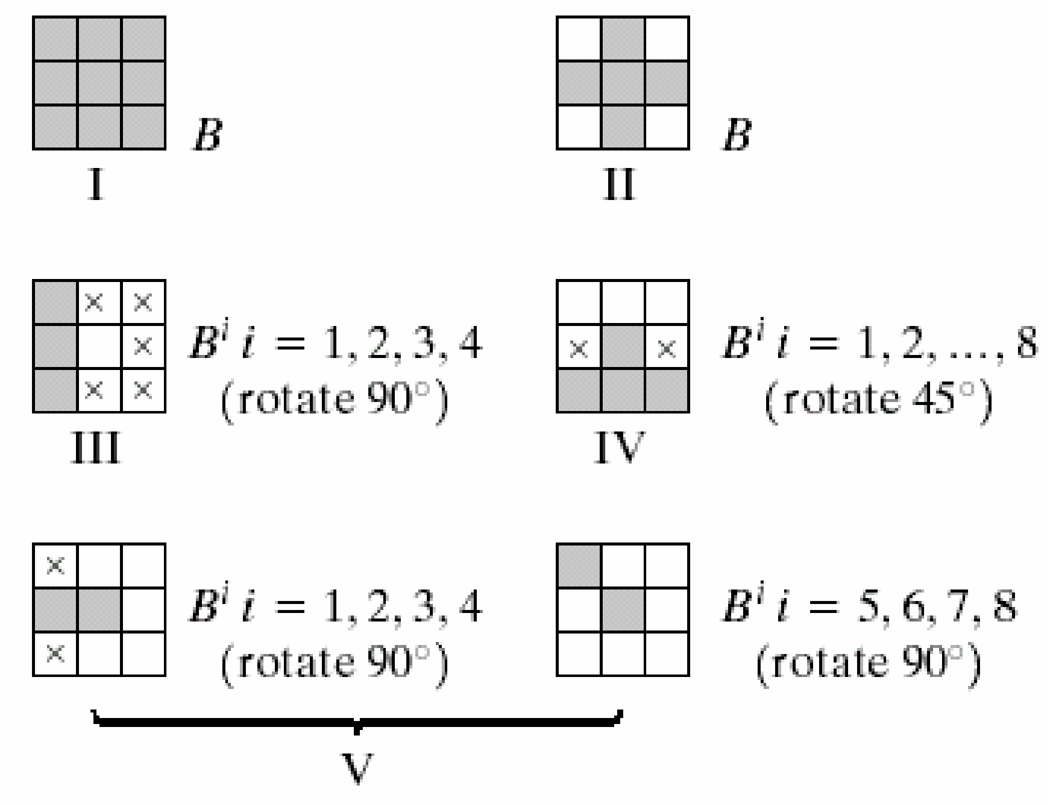
$$\boxed{A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)}$$



(a) Set **A**.
(b) Complement of **A**.
(c) Result of thinning the complement.
(d) Thickened set obtained by complementing (c).
(e) Final result, with no disconnected points

5 Basic Structuring Elements

- Five basic types of structuring elements used for binary morphology.
- The origin of each element is at its center, the x’s don’t care



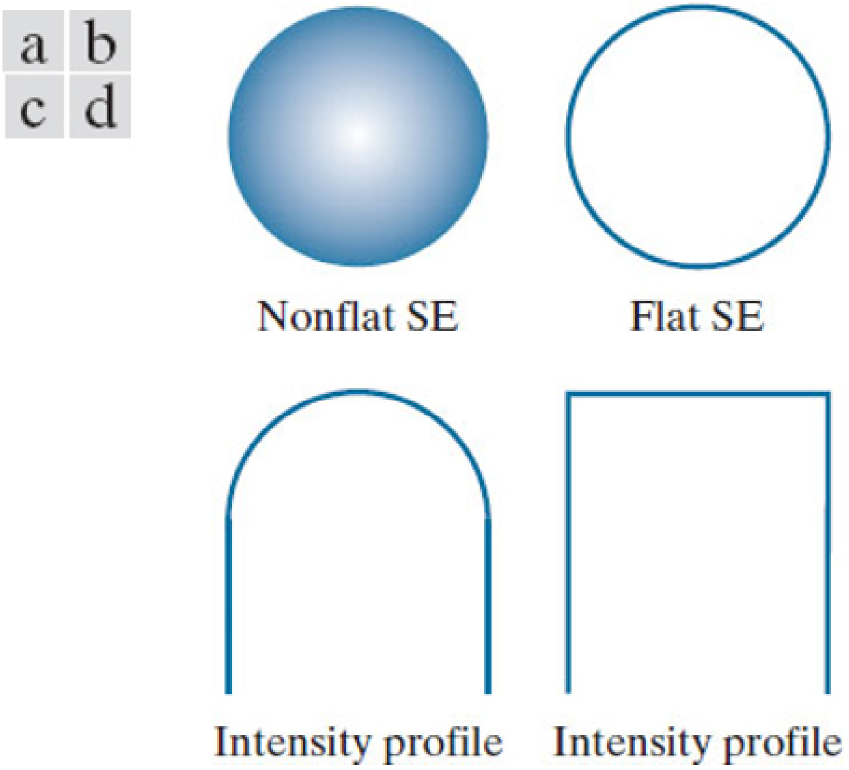
Extension to Gray-Scale images

- deal with digital image function
  - f(x,y) : the input image
  - b(x,y) : a structuring element (a subimage function)
- assumption : these functions are discrete
  - (x,y) are integers
  - f and b are functions that assign a gray-level value (real number or real integer) to each distinct pair of coordinate (x,y)

Dilation

Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their centers.

All examples in this section are based on flat SEs.

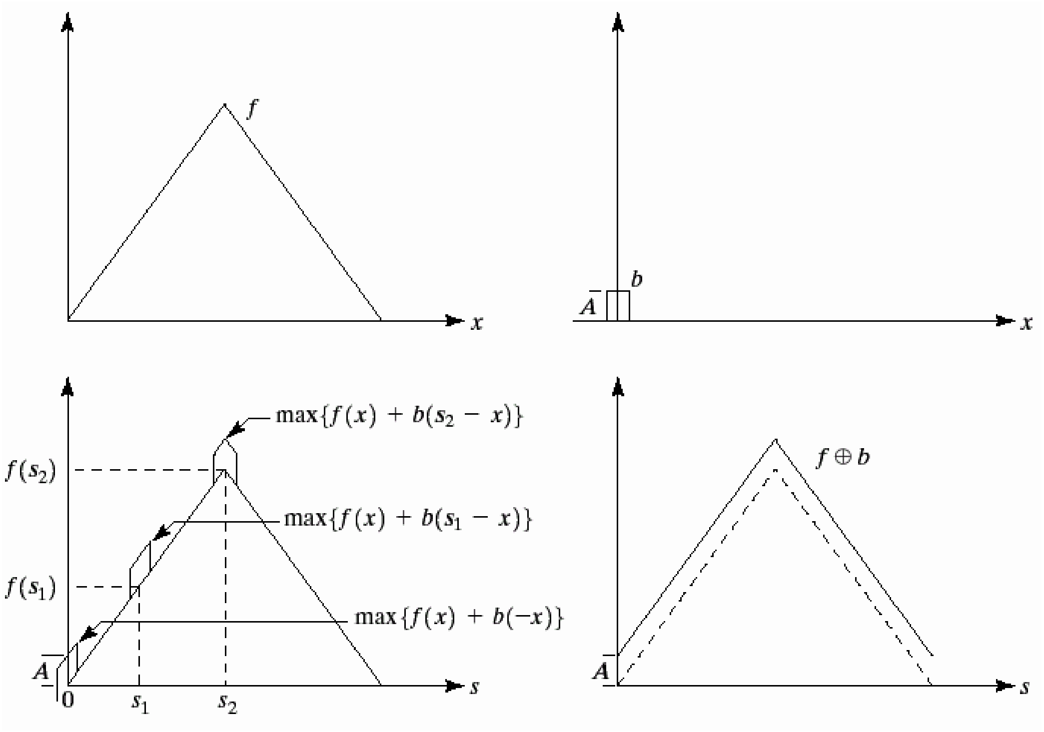


Dilation

- Gray-scale dilation of f by b

$$(f \oplus b)(s,t) = \max\{f(s-x, t-y) + b(x,y) \mid (s-x),(t-y) \in D_f; (x,y) \in D_b\}$$

- D_f and D_b are the domains of *f* and *b*, respectively condition (s - x) and (t - y) have to be in the domain of *f* and (x, y) have to be in the domain of *b* is similar to the condition in binary morphological dilation where the two sets have to overlap by at least one element



Dilation

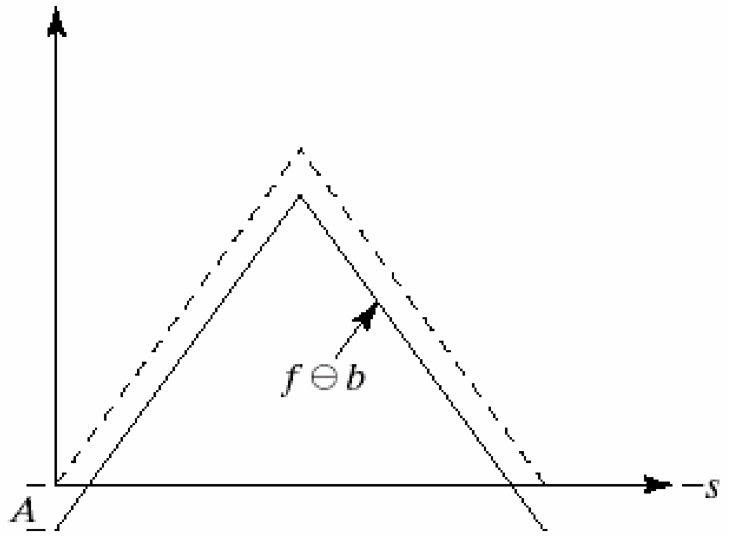
- Similar to 2D convolution
  - f(s-x) : f(-x) is simply f(x) mirrored with respect to the original of the x axis. the function f(s-x) moves to the right for positive s, and to the left for negative s.
  - max operation replaces the sums of convolution
  - addition operation replaces with the products of convolution
- General effect
  - if all the values of the structuring element are positive, the output image tends to be brighter than the input
  - dark details either are reduced or eliminated, depending on how their values and shapes relate to the structuring element used for dilation

Erosion

- Gray-scale erosion is defined as

$$(f \ominus b)(s,t) = \min\{f(s+x, y+t) - b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}$$

- condition (s+x) and (t+y) have to be in the domain of f and (x,y) have to be in the domain of *b* is similar to the condition in binary morphological erosion where the structuring element has to be completely contained by the set being eroded



Erosion

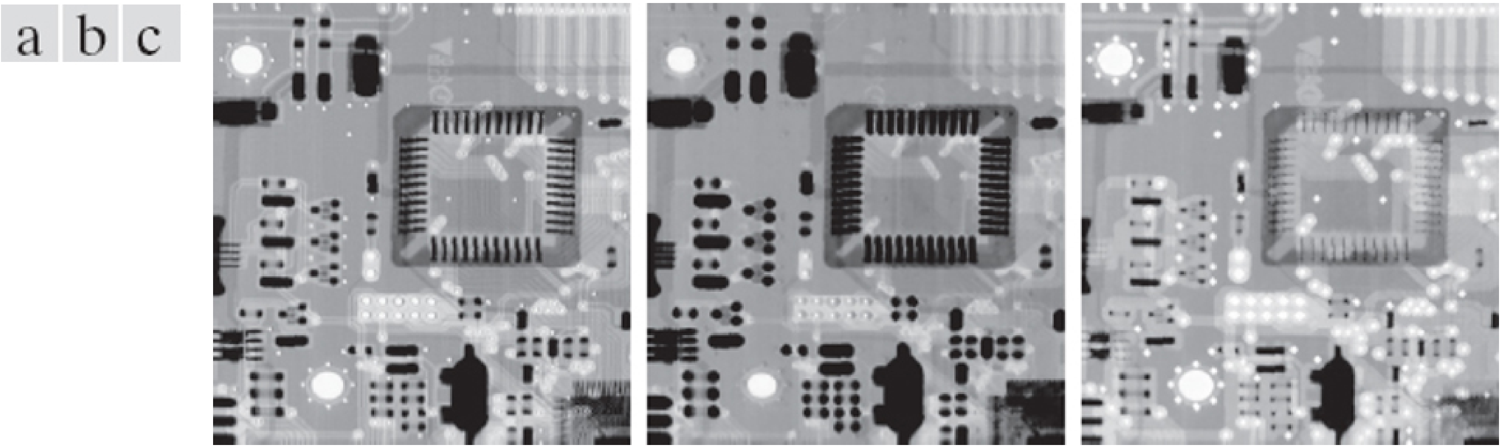
- Similar to 2D correlation
  - $f(s+x)$ moves to the left for positive s and to the right for negative s.
- General effect
  - if all the elements of the structuring element are positive, the output image tends to be darker than the input
  - the effect of bright details in the input image that are smaller in area than the structuring element is reduced, with the degree of reduction being determined by the gray-level values surrounding the bright detail and by the shape and amplitude values of the structuring element itself

Example

a. Gray-scale X-ray image of size 448x425

b. Erosion using a flat disk SE with a radius of 2 pixels.

c. Dilation using the same SE. (Original image courtesy of Lixi, Inc.)



MATLAB

- f=imread(‘Fig0923.tif');
- imshow(f)

- se=strel('square',3);
- gd=imdilate(f,se);
- figure, imshow(gd)

- ge=imerode(f,se);
- figure, imshow(ge)
- y=imsubtract(gd, ge);
- figure, imshow(y)

Dual property

- gray-scale dilation and erosion are duals with respect to function complementation and reflection.

$$(f \ominus b)^c(s,t) = (f^c \oplus \hat{b})(s,t)$$

where

$$f^c = -f(x,y) \text{ and } \hat{b} = b(-x,-y)$$

Example

a) 512x512 original image

b) result of dilation with a flat-top structuring element in the shape of parallelepiped of unit height and size 5x5 pixels

   Note: brighter image and small, dark details are reduced

c) result of erosion

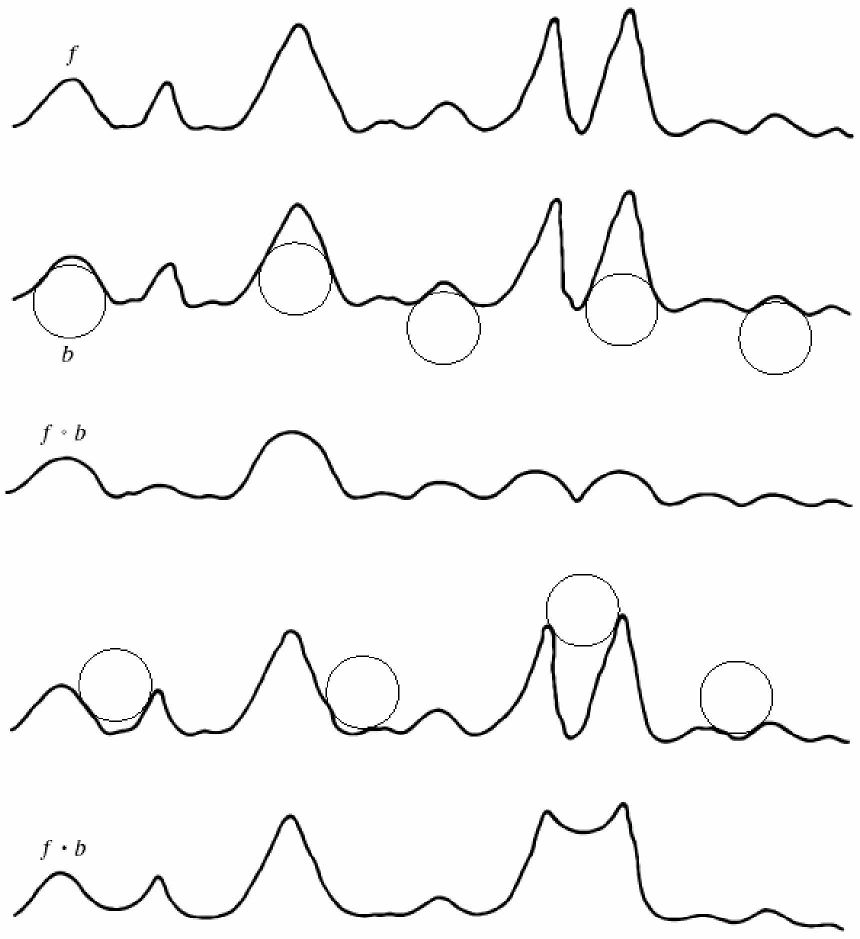
   note: darker image and small, dark details are reduced

Opening and Closing

view an image function f(x,y) in 3D perspective, with the x- and y-axes and the gray-level value axis

a) a gray-scale scan line
b) positions of rolling ball for opening
c) result of opening
d) positions of rolling ball for closing
e) result of closing

$$f \circ b = (f \ominus b) \oplus b$$
$$f \bullet b = (f \oplus b) \ominus b$$



Example

Grayscale opening and closing in one dimension.

a. Original 1-D signal.

b. Flat structuring element pushed up underneath the signal.

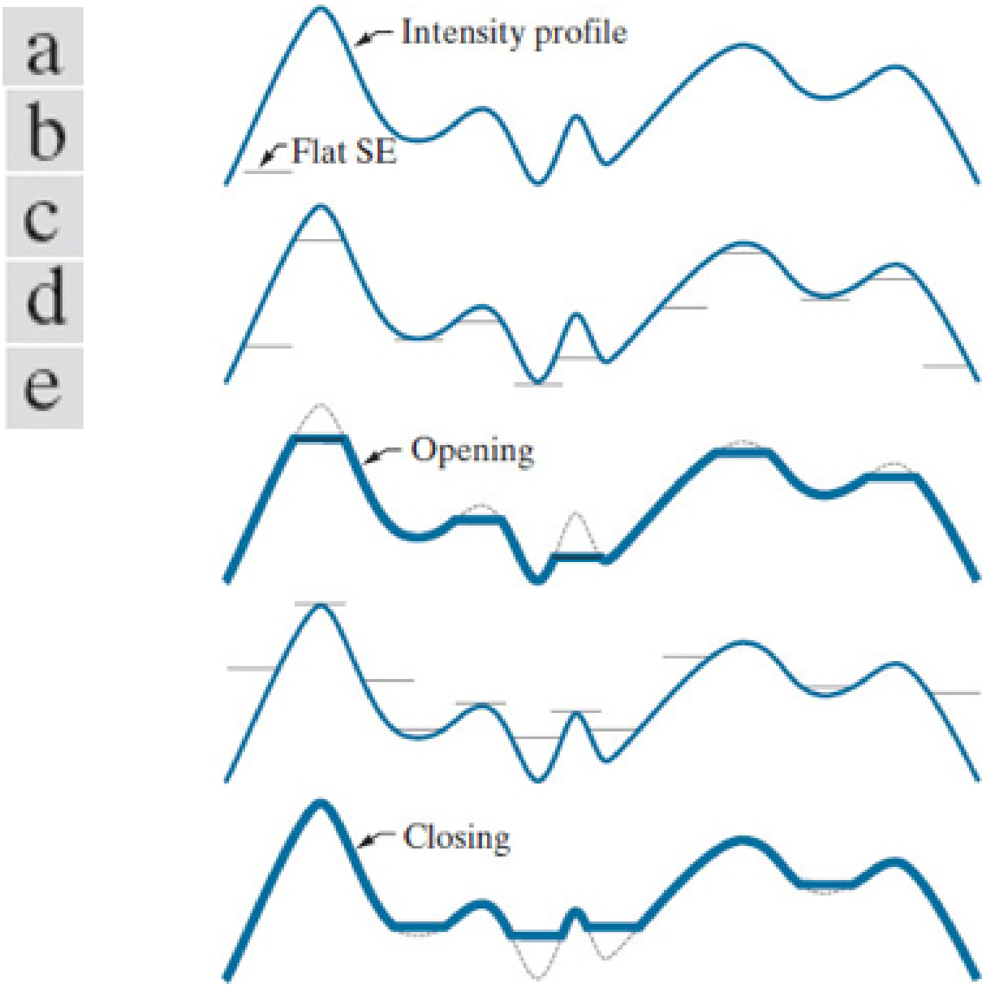
c. Opening.

d. Flat structuring element pushed down along the top of the signal.

e. Closing.

$$f \circ b = (f \ominus b) \oplus b$$

$$f \bullet b = (f \oplus b) \ominus b$$



Opening and Closing Properties

- dual property $(f \bullet b)^c = f^c \circ \hat{b}$
- opening operation satisfies

$$(i) \quad (f \circ b) \dashv f$$
$$(ii) \quad \text{if } f_1 \dashv f_2, \text{ then } (f_1 \circ b) \dashv (f_2 \circ b)$$
$$(iii) \quad (f \circ b) \circ b = (f \circ b)$$

- closing operation satisfies

$$(i) \quad f \dashv (f \bullet b)$$
$$(ii) \quad \text{if } f_1 \dashv f_2, \text{ then } (f_1 \bullet b) \dashv (f_2 \bullet b)$$
$$(iii) \quad (f \bullet b) \bullet b = (f \bullet b)$$

note:
$e \dashv r$ indicates that the domain of *e* is a subset of the domain of *r*, and also that *e* (x,y) ≤ *r* (x,y) for any (x,y) in the domain of *e*

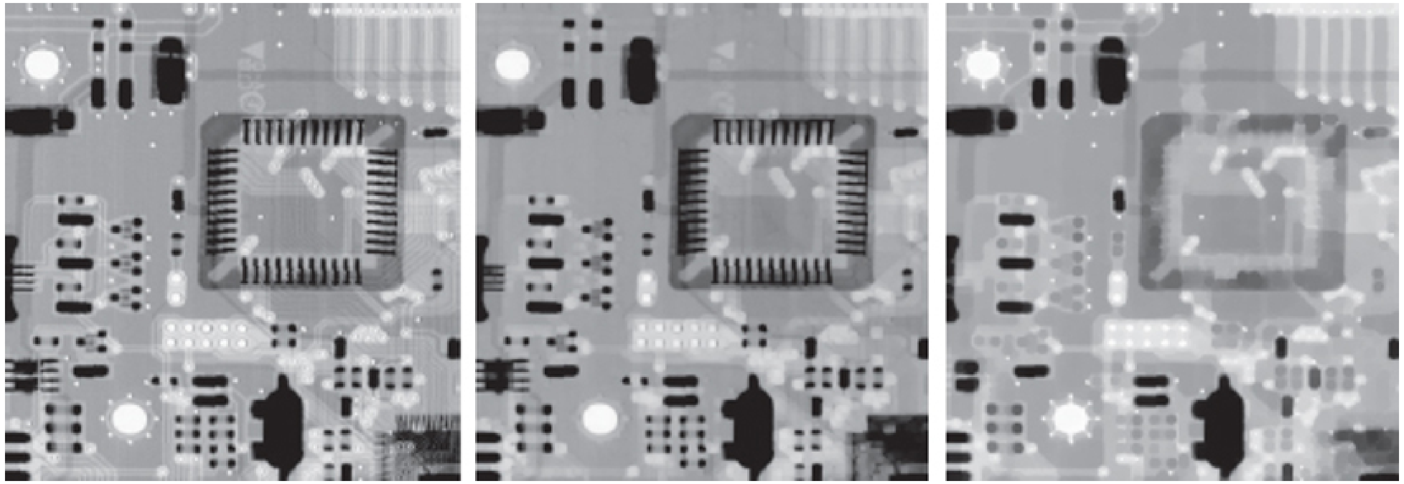
Example

(a) A grayscale X-ray image of size 448x425 pixels.

(b) Opening using a disk SE with a radius of 3 pixels.

(c) Closing using an SE of radius 5.

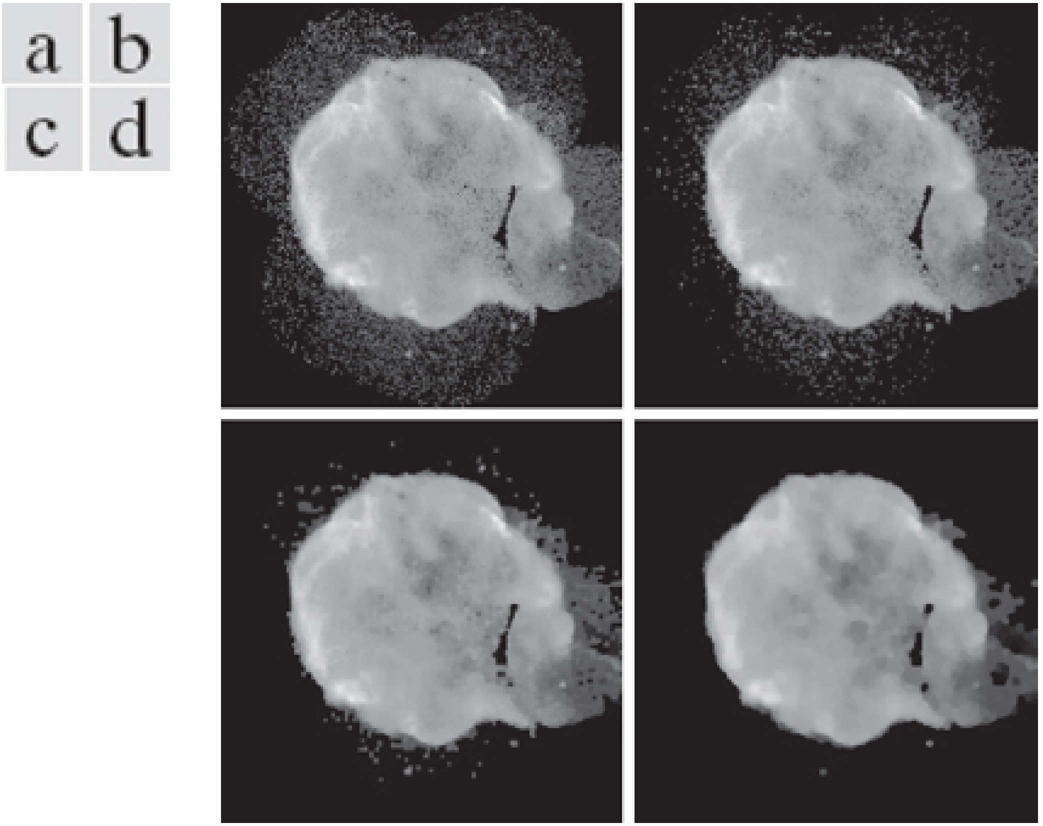
a b c



Example

a. 566x566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope.

(b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively.
(Original image courtesy of NASA.)



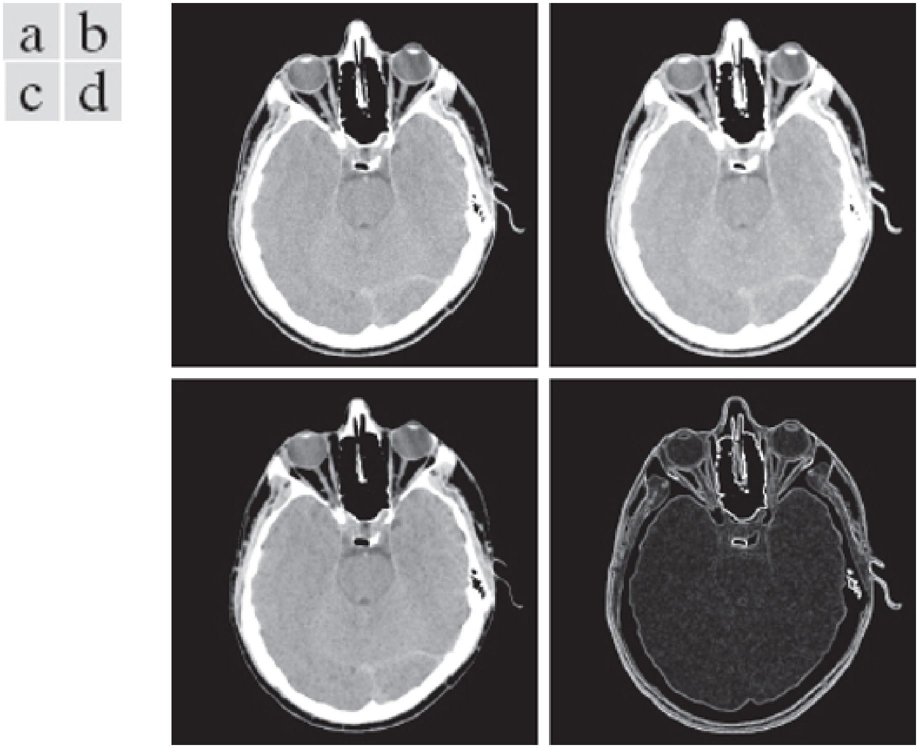
Example

a. 512x512 image of a head CT scan.

b. Dilation.

c. Erosion.

d. Morphological gradient, computed as the difference between (b) and (c).



Example

Using the top-hat transformation for **shading correction**.

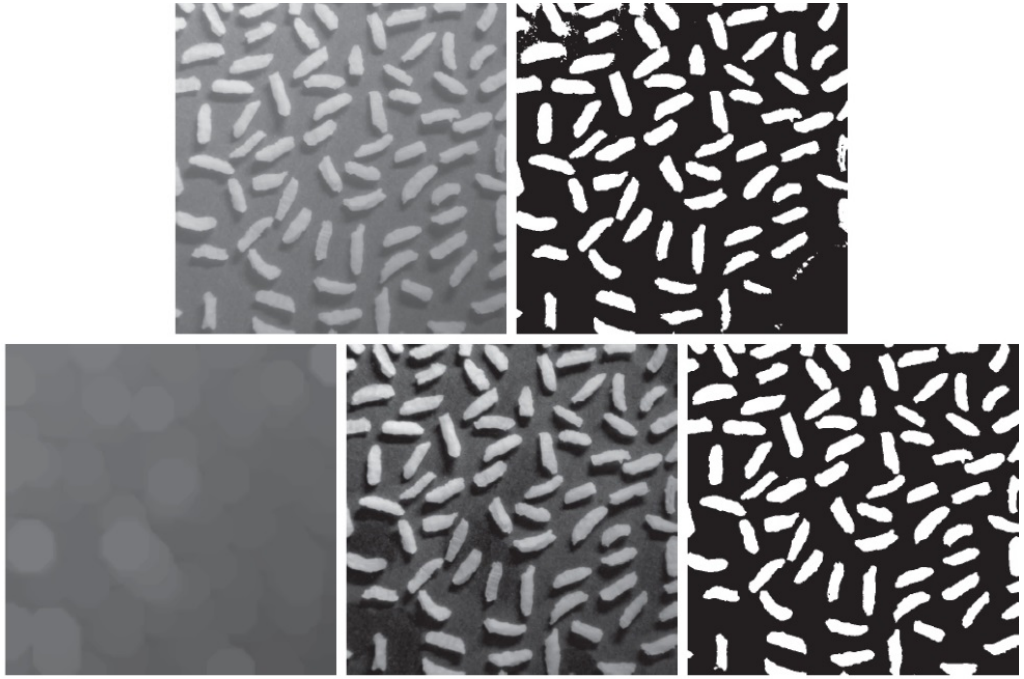
(a) Original image of size 600x600 pixels

(b) Thresholded image.

(c) Image opened using a disk SE of radius 40.

(d) Top-hat transformation (the image minus its opening).

(e) Thresholded top-hat image.



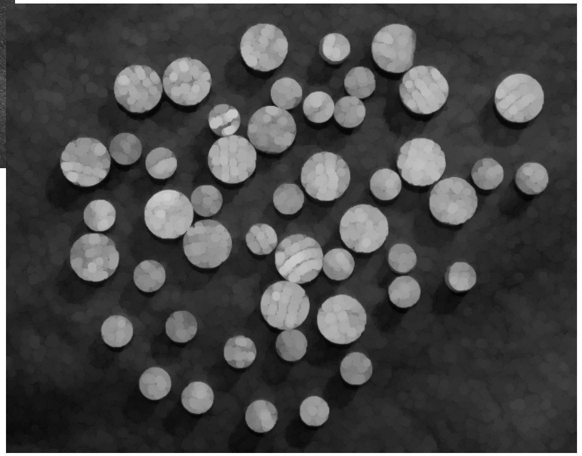
a b
c d e

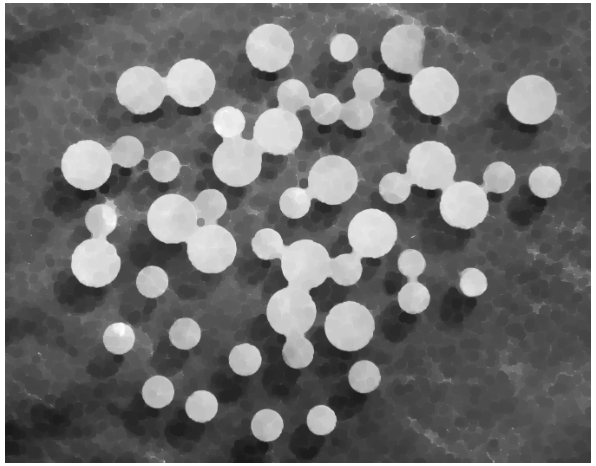
MATLAB

- f=imread('Fig0925.tif');
- imshow(f)

- se=strel('disk',5);
- fo=imopen(f,se);
- figure, imshow(fo)

- gc=imclose(f,se);
- figure, imshow(gc)





Effect of Opening

Opening

- the structuring element is rolled underside the surface of f
- all the peaks that are narrow with respect to the diameter of the structuring element will be reduced in amplitude and sharpness
- so, opening is used to remove small light details, while leaving the overall gray levels and larger bright features relatively undisturbed.
- the initial erosion removes the details, but it also darkens the image.
- the subsequent dilation again increases the overall intensity of the image without reintroducing the details totally removed by erosion

Effect of Closing

Closing

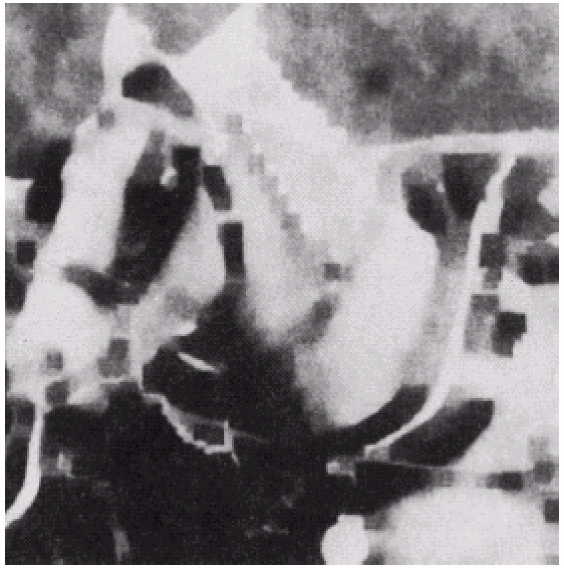
- the structuring element is <u>rolled on</u> <u>top of the surface</u> of f
- peaks essentially are left in their original form (assume that their separation at the narrowest points exceeds the diameter of the structuring element)
- so, closing is used to <u>remove small dark details</u>, while <u>leaving bright features relatively undisturbed</u>.
- the initial <u>dilation removes the dark details and brightens the image</u>
- the subsequent erosion darkens the image without reintroducing the details totally removed by dilation

Example:

- Original image

Opening image

Closing image



Some Applications of Gray-scale Morphology

- Morphological smoothing
- Morphological gradient
- Top-hat transformation
- Textural segmentation
- Granulometry

- Note: the examples shown in this topic are of size 512x512 and processed by using the structuring element in the shape of parallelepiped of unit height and size 5x5 pixels

Morphological Smoothing

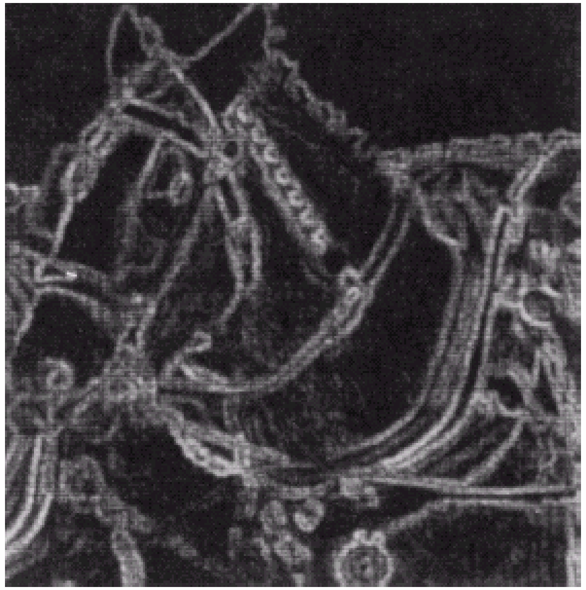
- perform an opening following by a closing
- effect: remove or attenuate both bright and dark artifacts or noise

Morphological smoothing

Morphological gradient

$$g = \underbrace{(f \oplus b)}_{\text{dilation}} - \underbrace{(f \ominus b)}_{\text{erosion}}$$

effect: gradient highlight sharp gray-level transitions in the input image

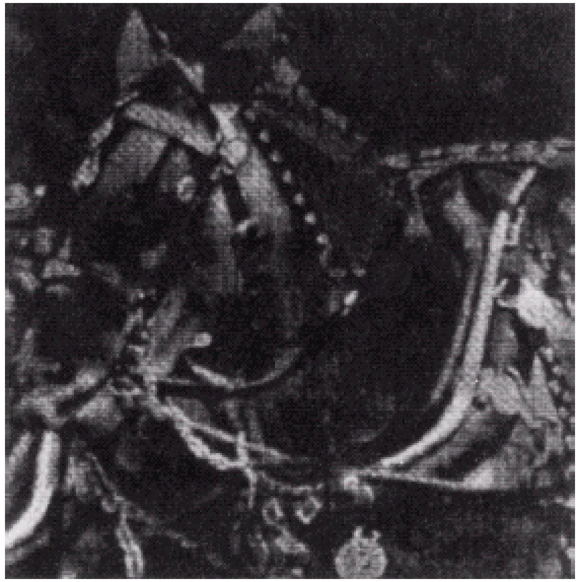


Top-Hat Transformation

opening

$$\boxed{h = f - (f \circ b)}$$

- effect: enhancing detail in the presence of shading
- note: the enhancement of detail in the background region below the lower part of the horse's head.



Textural segmentation

Textural segmentation.

(a) 600x600 image consisting oftwo types of blobs.

(b) Image with small blobs removed by closing (a).

(c) Image with light patches between large blobs removed by opening (b).

(d) Original image with boundary between the two regions in (c) superimposed.
The boundary was obtained using a morphological gradient.

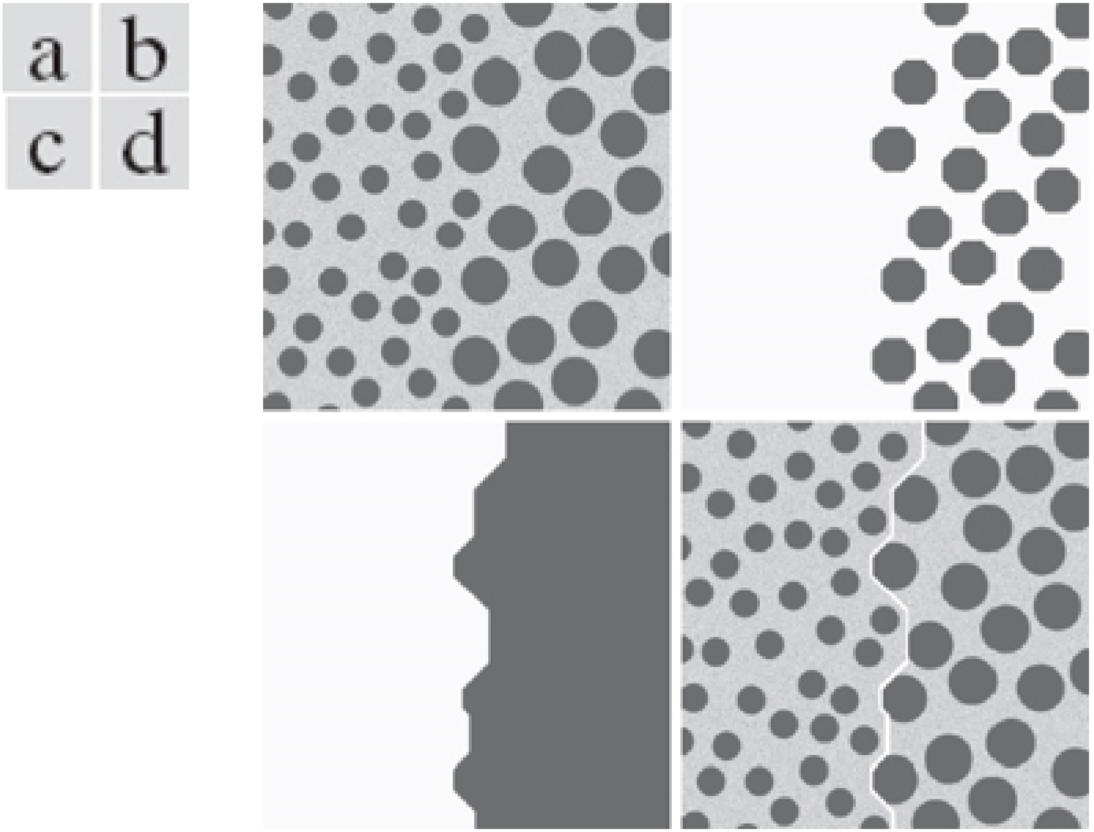


Image showing boundary between regions of different texture

Textural segmentation

Perform

- closing the image by using successively larger structuring elements than small blobs
  - as closing tends to remove dark details from an image, thus the small blobs are removed from the image, leaving only a light background on the left and larger blobs on the right
- opening with a structuring element that is large in relation to the separation between the large blobs
  - opening removes the light patches between the blobs, leaving dark region on the right consisting of the large dark blobs and now equally dark patches between these blobs.
- by now, we have a light region on the left and a dark region on the right, so we can use a simple threshold to yield the boundary between the two textural regions.

Example

(a) Original image of size 1134x1360 pixels

(b) Opening by reconstruction of (a), using a structuring element consisting of a horizontal line 71 pixels long in the erosion.

(c) Opening of (a) using the same SE.

(d) Top-hat by reconstruction.

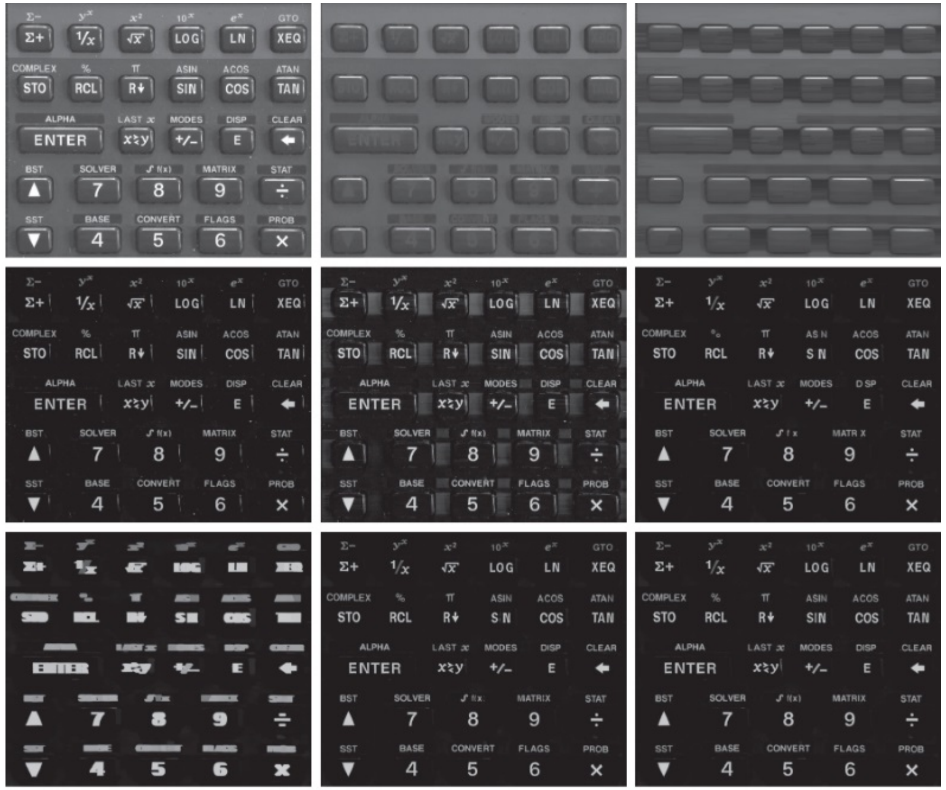
(e) Result of applying just a top-hat transformation.

(f) Opening by reconstruction of (d), using a horizontal line 11 pixels long.

(g) Dilation of (f) using a horizontal line 21 pixels long.

(h) (h) Minimum of (d) and (g).

(i) Final reconstruction result. (Images courtesy of Dr. Steve Eddins, MathWorks, Inc.)



a b c
d e f
g h i