ECE 5740 Digital Image Processing

Morphological Image Processing

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Outline

- Mathematic Morphology
- Basic Set Theory
- Logical Operations
- Dilation
- Erosion
- Opening
- Closing
- Duality
- Hit-or-Miss Transformation

Mathematic Morphology

- It is used to extract image components that are useful in the representation and description of region shape, such as
  - boundaries extraction
  - skeletons
  - convex hull
  - morphological filtering
  - thinning
  - pruning

Z^2 and Z^3

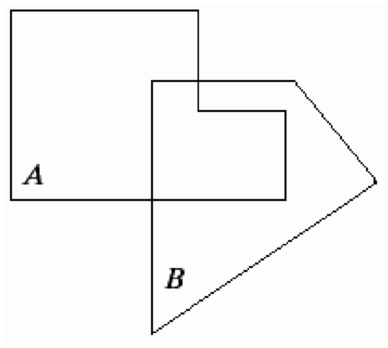
- The language of mathematical morphology is a set theory
- Sets in mathematic morphology represent objects in an image
  - binary image : the element of the set is the coordinates (x,y) of pixel belong to the object ⇨ Z^2
  - gray-scaled image : the element of the set is the coordinates (x,y) of pixel belong to the object and the gray levels ⇨ Z^3

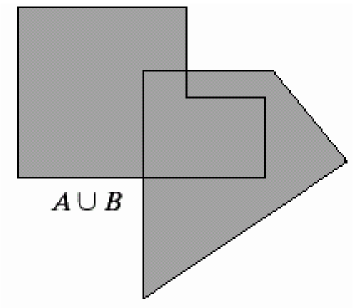
Basic Set Theory

- Let A and B be a set of in Z^2.
- The union of two sets A and B, denoted by

$$C = A \bigcup B$$

  - C is the set of all elements belonging to either A, B or either



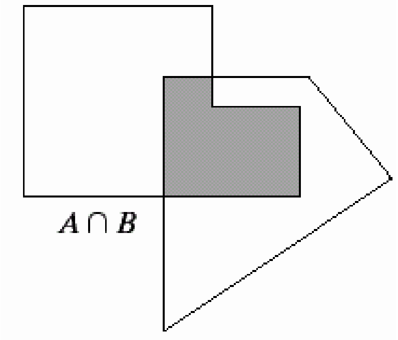


- The intersection of two sets A and B, denoted by

$$C = A \bigcap B$$

  - C is the set of all elements belonging to both A and B
  - Two set A and B are disjoint or mutually exclusive if they have no common element

$$A \bigcap B = \varnothing$$



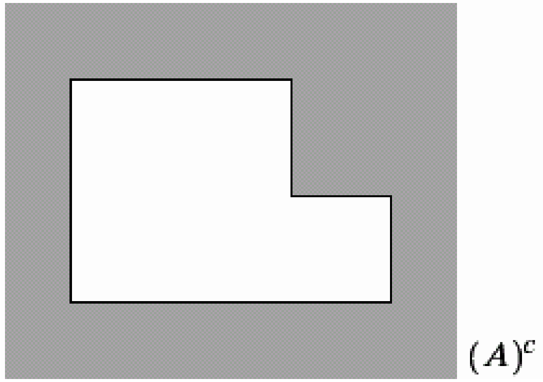
Basic Set Theory

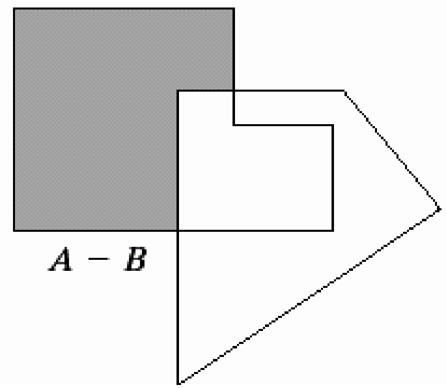
- The complement of a set A is the set of element not contained in A:

$$A^c = \{w \mid w \notin A\}$$

- The difference of two set A and B, denoted A-B, is define as

$$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$$

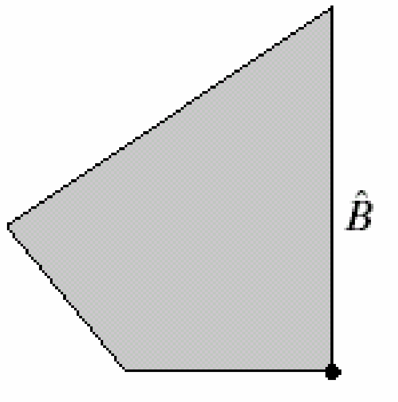




Basic Set Theory

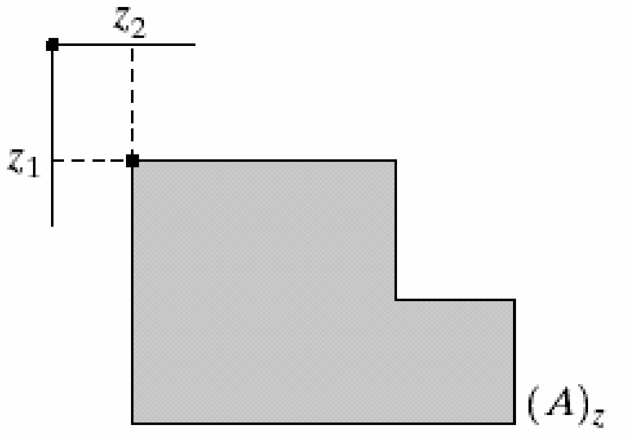
- The reflection of set B, denoted by $\hat{B}$, is defined as

$$\hat{B} = \{w \mid w = -b, \ \text{for} \ \ b \in B\}$$



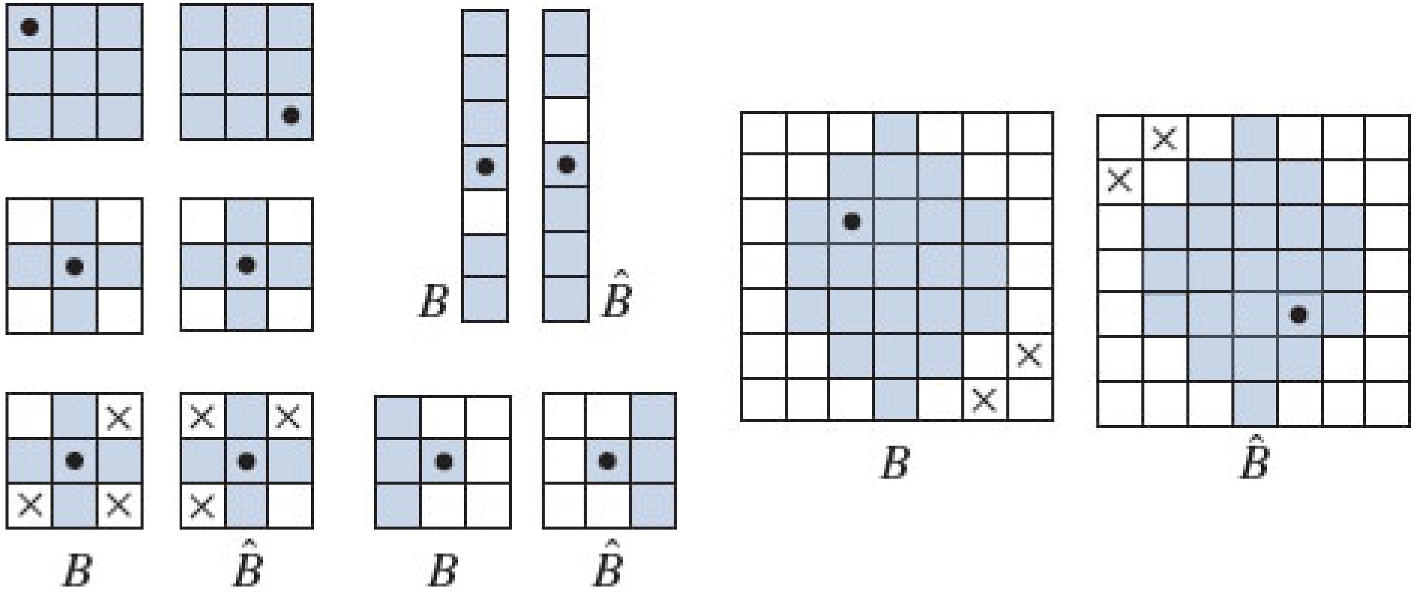
- The translation of set A by point $z=(z_1, z_2)$, denoted by $(A)_z$, is defined as

$$(A)_z = \{c \mid c = a + b, \ \text{for} \ \ a \in A\}$$



Basic Set Theory

Structuring elements and their reflections about the origin (the X's are don't care elements, and the dots denote the origin). Reflection is rotation by 180 degree of an SE about its origin



Logical Operations

- The primary logic operations used in image processing are AND, OR, and NOT.

p	q	p AND q
0	0	0
0	1	0
1	0	0
1	1	1

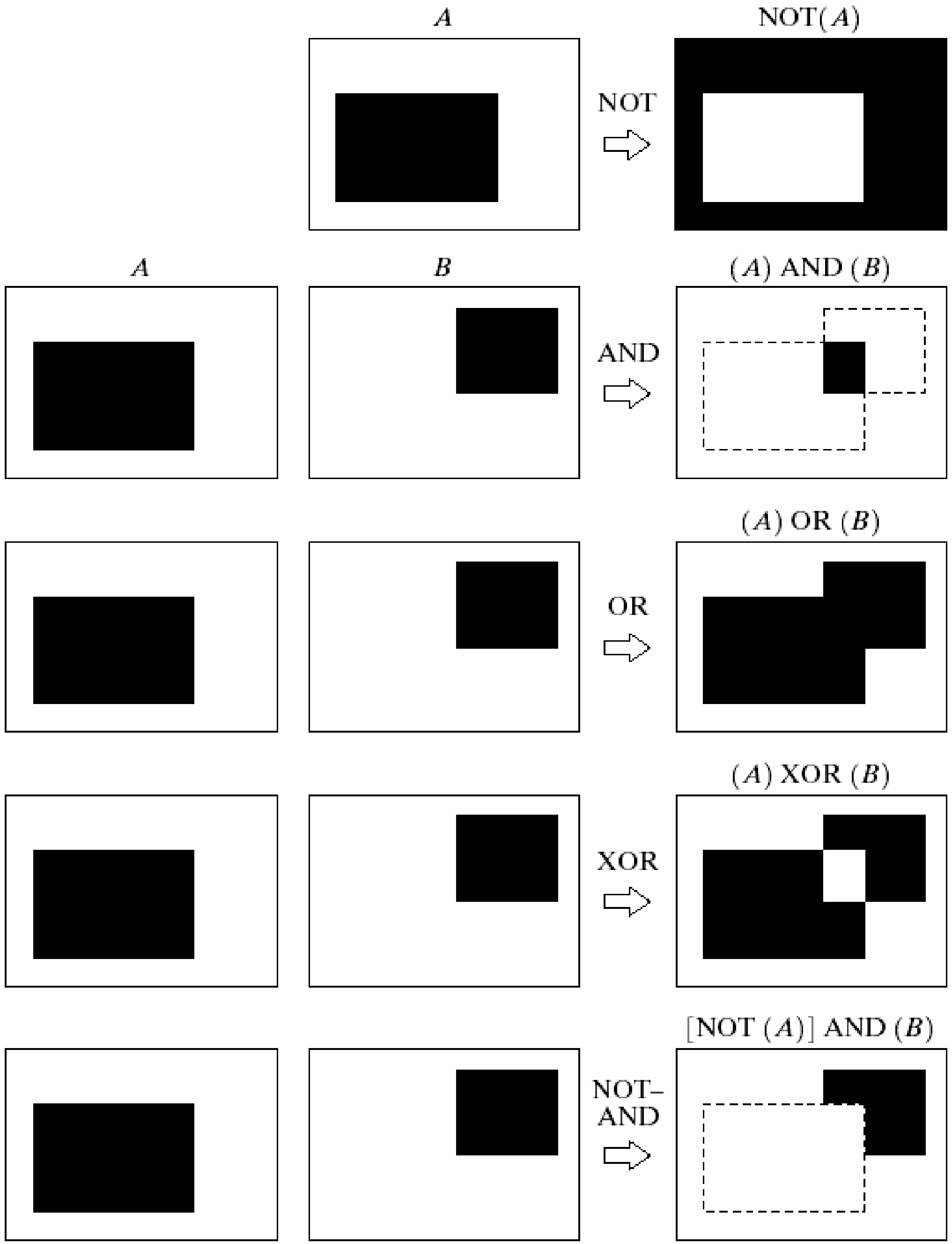
p	q	p OR q
0	0	0
0	1	1
1	0	1
1	1	1

p	NOT q
0	1
1	0

Set Operation	in MATLAB	
$A \cap B$	A & B	% A AND B
$A \cup B$	A | B	% A OR B
B^C	~B	% NOT B
A-B	A & ~B	% Difference

Example:

- Some logic operations between binary images
- Black represents binary 1 and white represents binary 0.

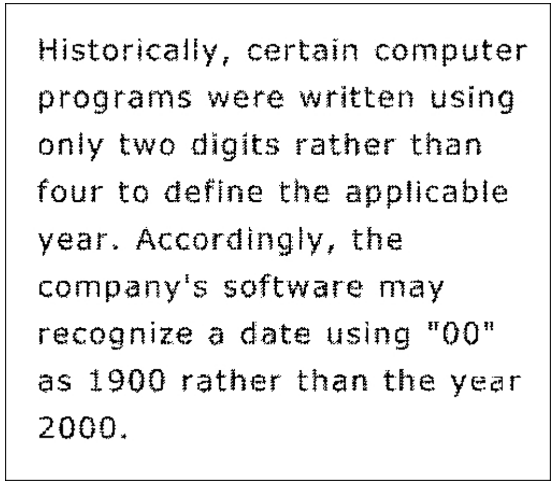


MATLAB

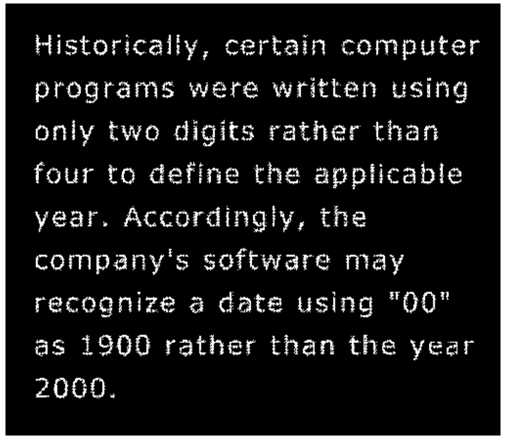
```
f=imread('Fig9.05.jpg');
>> imshow(f)

>> f2=~f;
>> imshow(f2)

imshow(a)
BW = im2bw(a,level)
```



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Dilation

- With A and B as sets in Z^2, the dilation of A and B is given

$$A \oplus B = \left\{ z \mid (\hat{B})_z \cap A \neq \varnothing \right\}$$

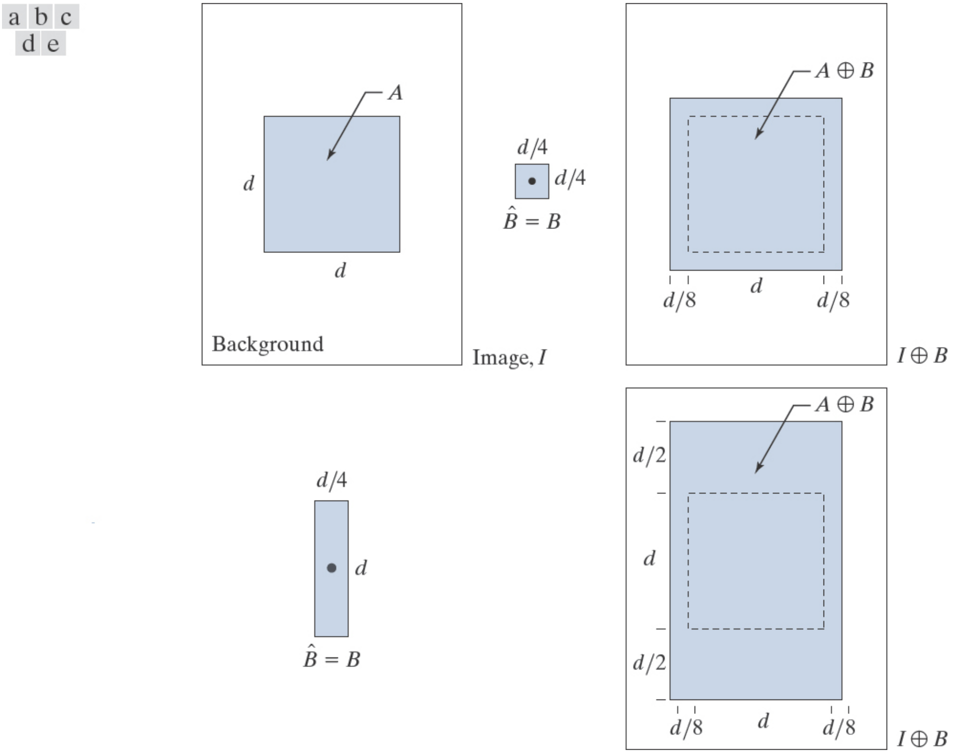
- Set B is commonly referred to as the structuring element.
- The process is convolution based on set operation; "flipping" B about its origin and then successively displacing it so that it slides over set (image) A

a) Image **I**, composed of set (object) **A** and background

b) Square SE (the dot is the origin).

c) Dilation of **A** by **B** (shown shaded).

d) Elongated SE. (e) Dilation of **A** by this element. The dotted line in (c) and (e) is the boundary of **A**, shown for reference.

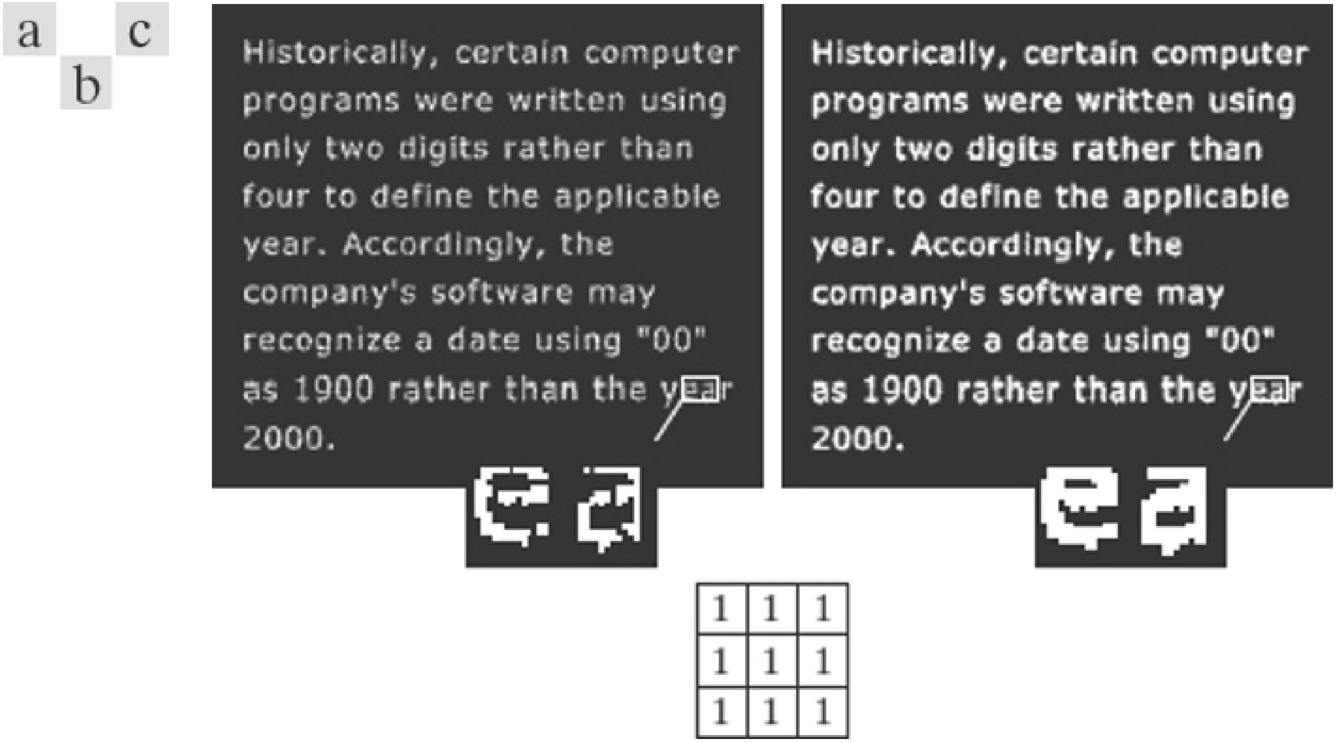


Example:

(a) Sample text of poor resolution with broken characters.

(b) Structuring element

(c) Dilation of (a) by (b), broken segments were joined

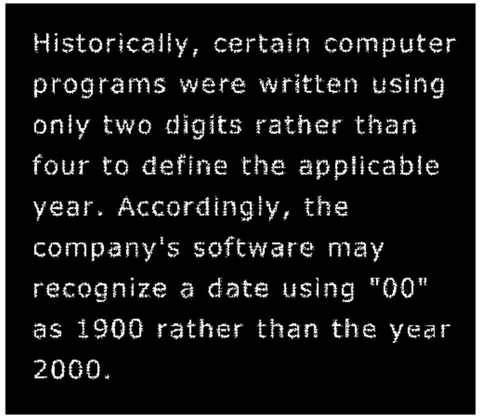


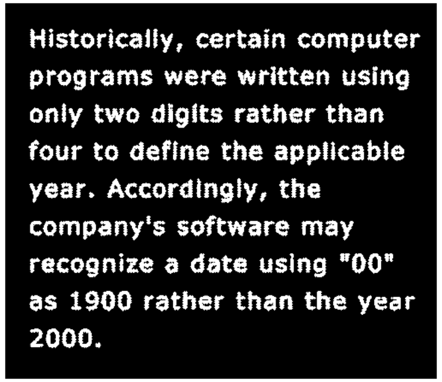
MATLAB

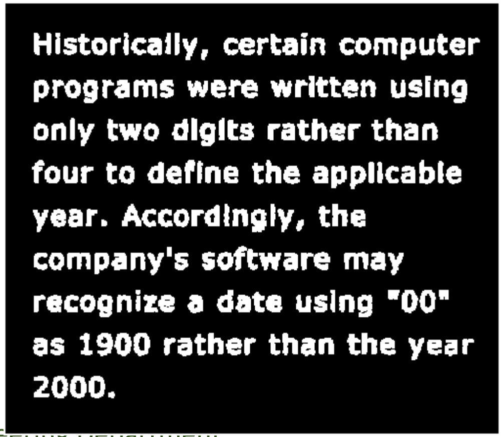
```
A=imread('Fig9.05.jpg');
A2=~A;
imshow(A2)
B=[0 1 0;1 1 1 ;0 1 0];
C=imdilate(A2,B);
figure, imshow(C)
```

With

```
B=[1 1 1;1 1 1 ;1 1 1];
```







Erosion

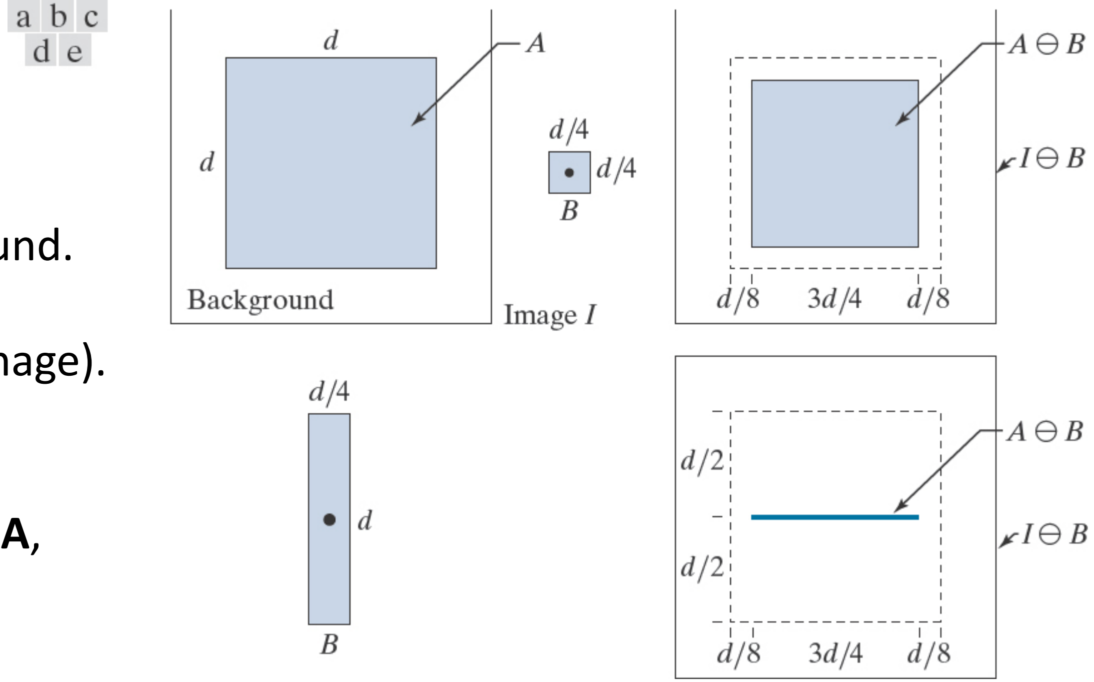
- For sets A and B in Z^2, the erosion of A and B is given Spectrum of

$$A \ominus B = \{ z \mid (B)_z \subseteq A \}$$

- Duality

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

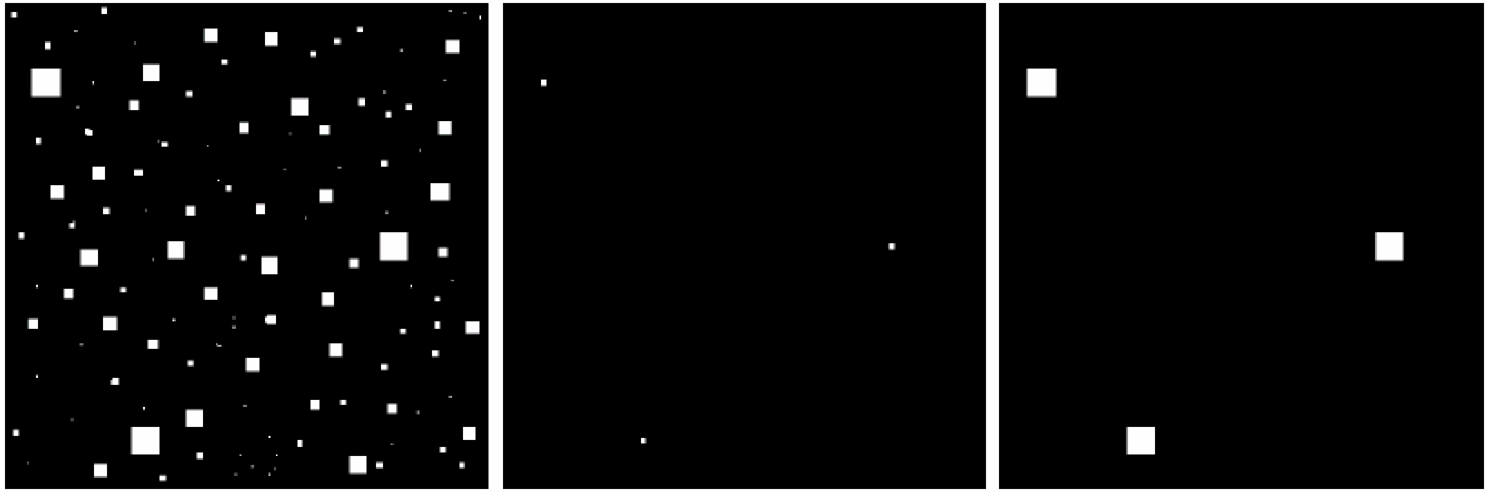
Dilation and erosion are duals of each other



a) Image **I**, consisting of a set (object) **A**, and background.
b) Square SE, **B** (the dot is the origin).
c) Erosion of **A** by **B** (shown shaded in the resulting image).
d) Elongated SE.
e) Erosion of **A** by **B**. (The erosion is a line.)
The dotted border in (c) and (e) is the boundary of **A**, shown for reference.

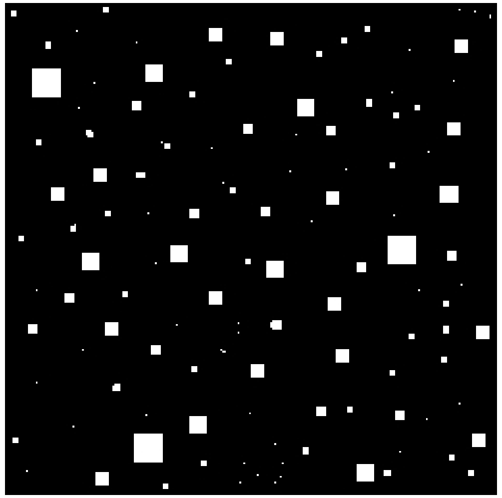
Example:

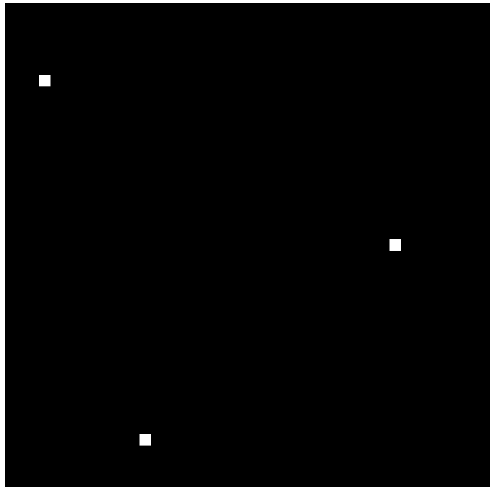
- Erosion : eliminating irrelevant detail
- (a) Image of squares of size 1,3, 5, 7, 9, and 15 pixel on the side.
- (b) Erosion of (a) with a square structure element of 1's 13x13 pixels.
- (c) Dilation of (b) with the same structuring element

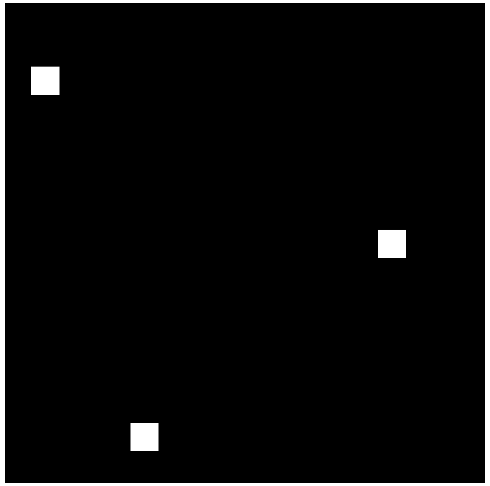


MATLAB

```
A=imread('Fig9.07.jpg');
A=logical(A);
imshow(A)
B=strel('square',10);
C=imerode(A,B);
figure, imshow(C)
C2=imdilate(C, B);
figure, imshow(C2)
```





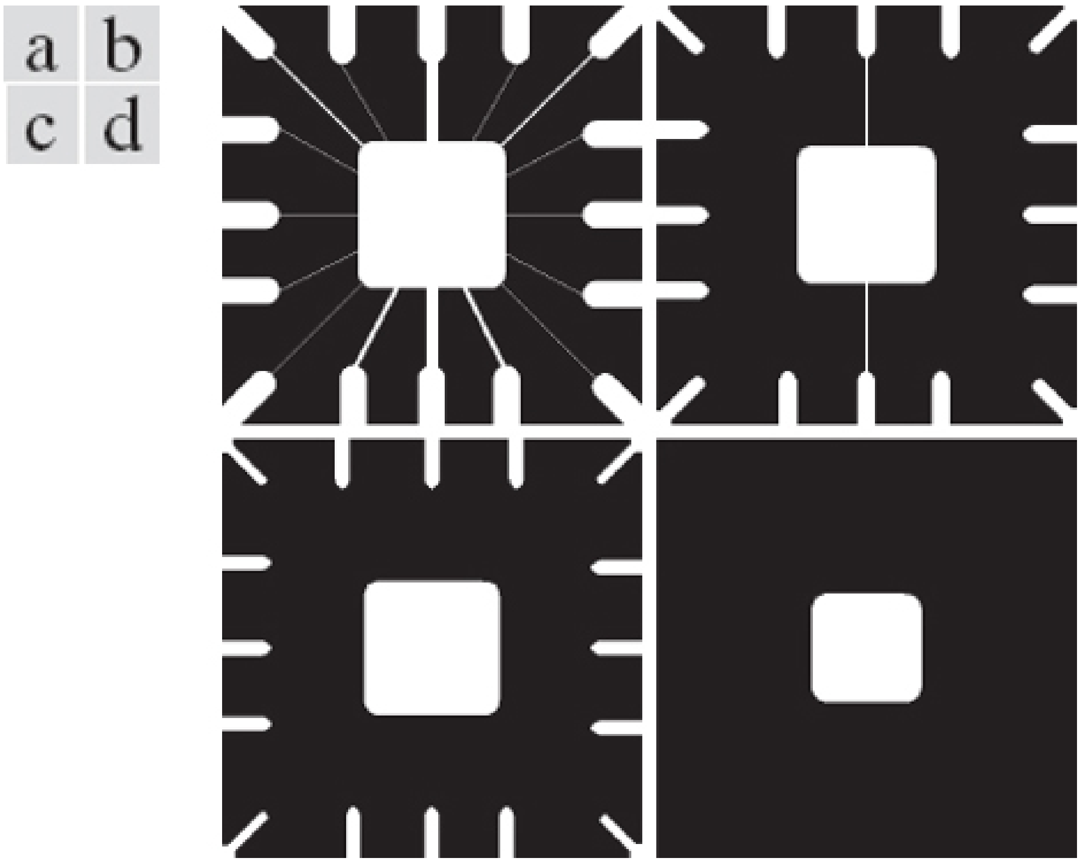


MATLAB

Using erosion to remove image components

a. A 486x486 binary image of a wire-bond mask in which foreground pixels are shown in white.

(b)–(d) Image eroded using square structuring elements of sizes 11x11, 15x15, and 45x45 elements, respectively, all valued 1.



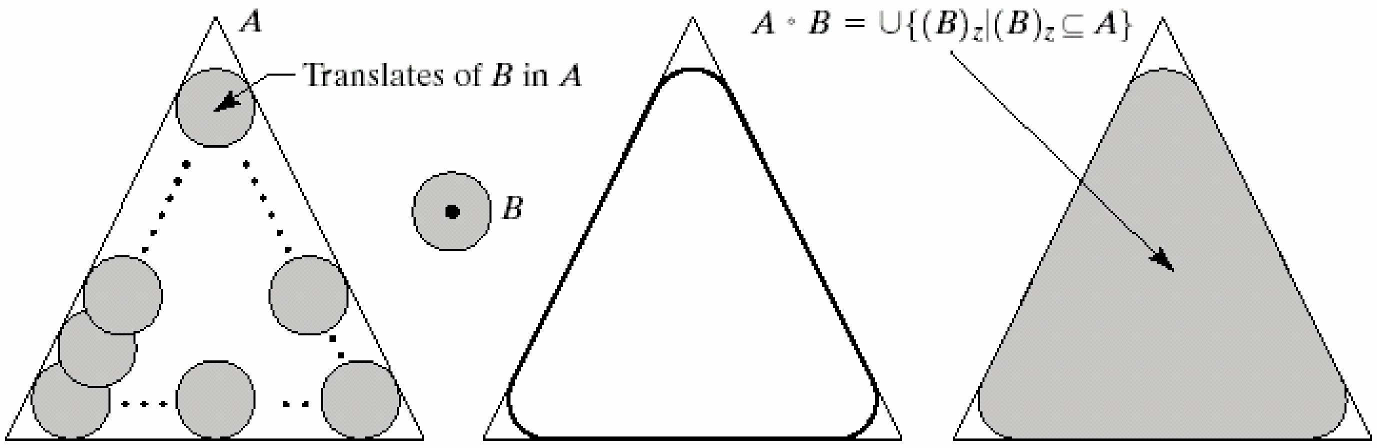
Opening

- Smoothes the contour of the object, breaks narrow isthmuses, and eliminates thin protrusions.
- The opening of set A by structure element B is define as

$$A \circ B = (A \ominus B) \oplus B$$

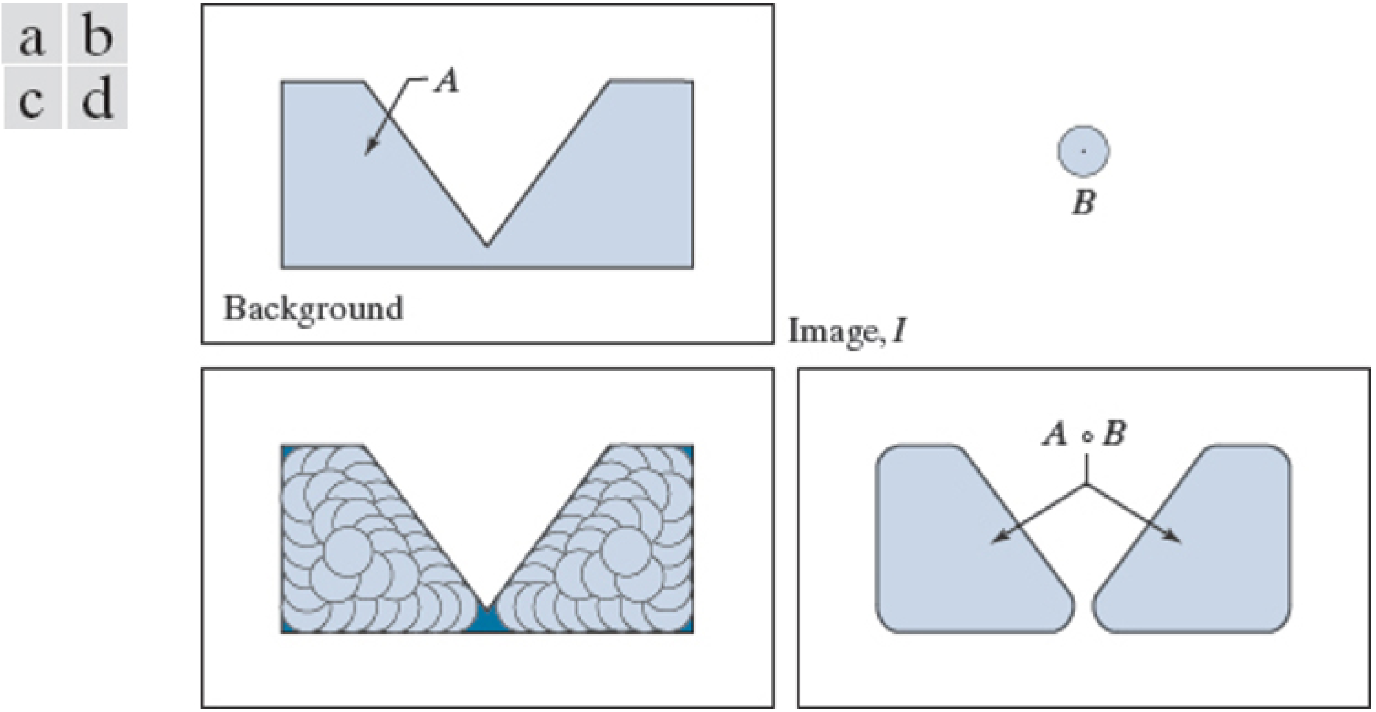
- Obtained by taking the union of all translates of B that fit into A

$$A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$



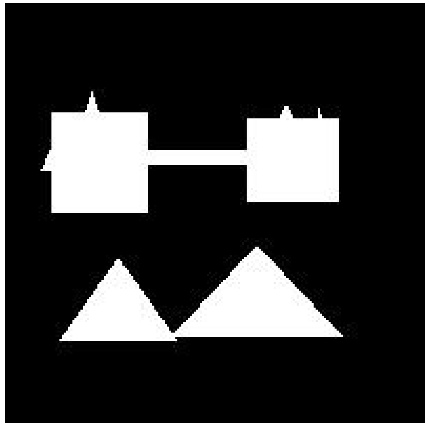
Example

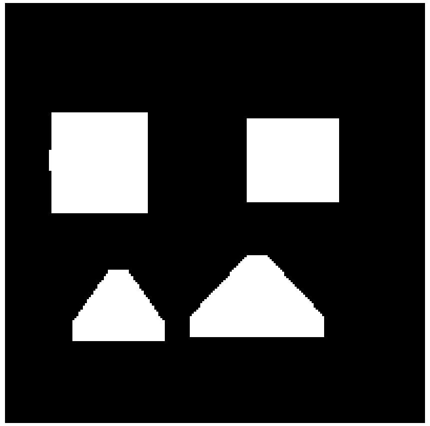
a) Image **I**, composed of set (object) **A** and background.
b) Structuring element, **B**.
c) Translations of **B** while being contained in **A**. (**A** is shown dark for clarity.)
d) Opening of **A** by **B**.



MATLAB

```
A=imread('image-1.jpg');
A=im2bw(A);
% A=logical(A);
B=strel('square',10);
C=imopen(A,B);
imshow(C)
```





Closing

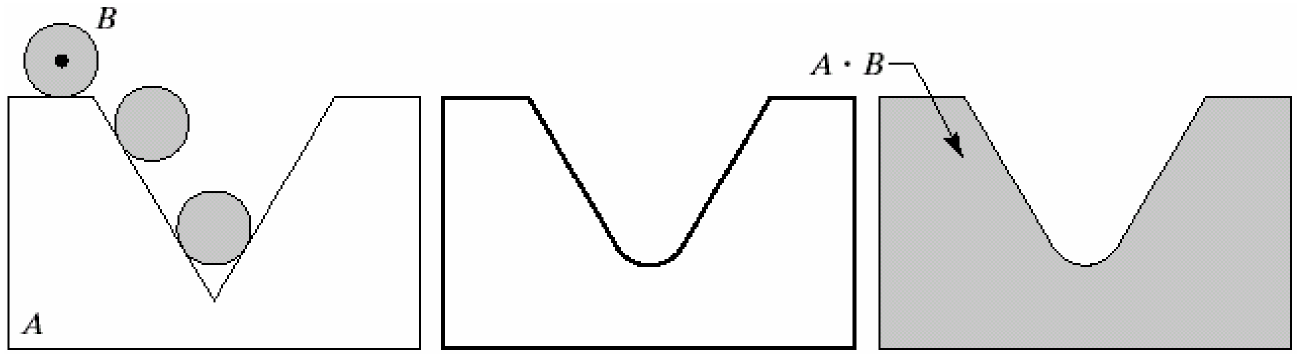
- Smoothes the section of contours but, as opposed to opening, it generally fuses narrow breaks and long thin gulf
- Eliminates small holes, and fills gaps in the contour.

$$A \bullet B = (A \oplus B) \ominus B$$

a) Structuring element B “rolling” on the outer boundary of set A.

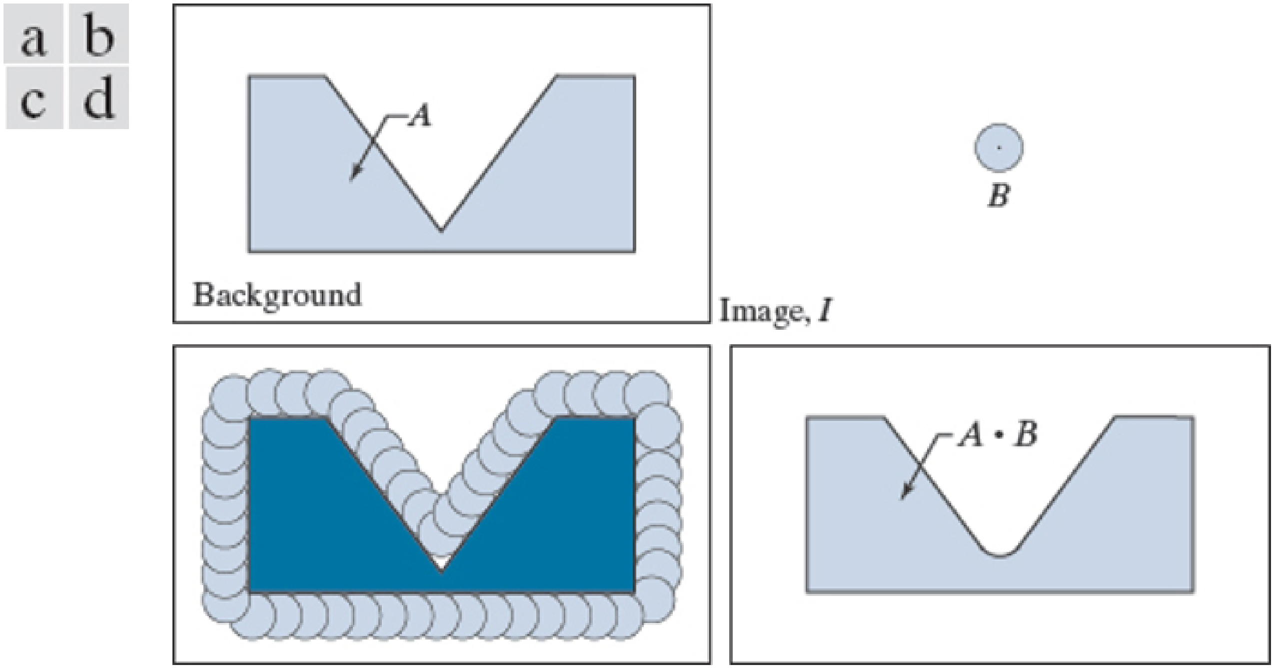
b) Heavy line in the outer boundary of the closing

c) Complete closing (shaded)



Example

a) Image I, composed of set (object) A, and background.
b) Structuring element B.
c) Translations of B such that B does not overlap any part of A. (A is shown dark for clarity.)
d) Closing of A by B.



MATLAB

```
>> A=imread('Fig9.11.jpg');
>> imshow(A)
>> B=strel('square',3);
>> C=imopen(A,B);
>> figure, imshow(C)

>>C2=imclose(C,B);
>> figure, imshow(C2)
```

Duality

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

Properties

Opening

- $A \circ B$ is a subset (subimage) of A
- If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$
- $(A \circ B) \circ B = A \circ B$

Closing

- A is a subset (subimage) of $A \bullet B$
- If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$
- $(A \bullet B) \bullet B = A \bullet B$

Example:

a) Image **I**, composed of a set (object) **A** and background, a solid, circular structuring element is shown also (The dot is the origin.)

b) (b) Structuring element in various positions.

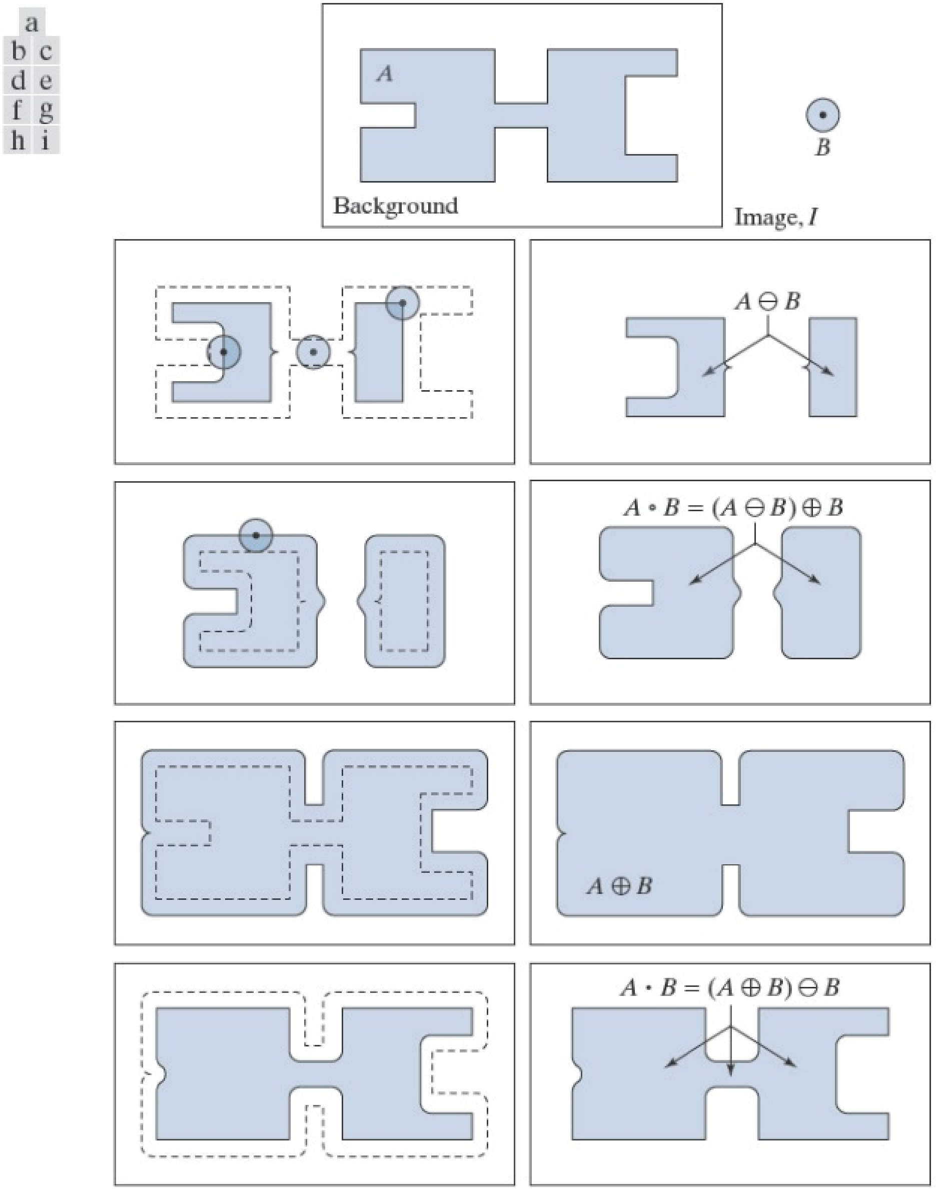
c) Elimination of the bridge between the two main section

d) Shows the process of opening the eroded set

e) show the final result

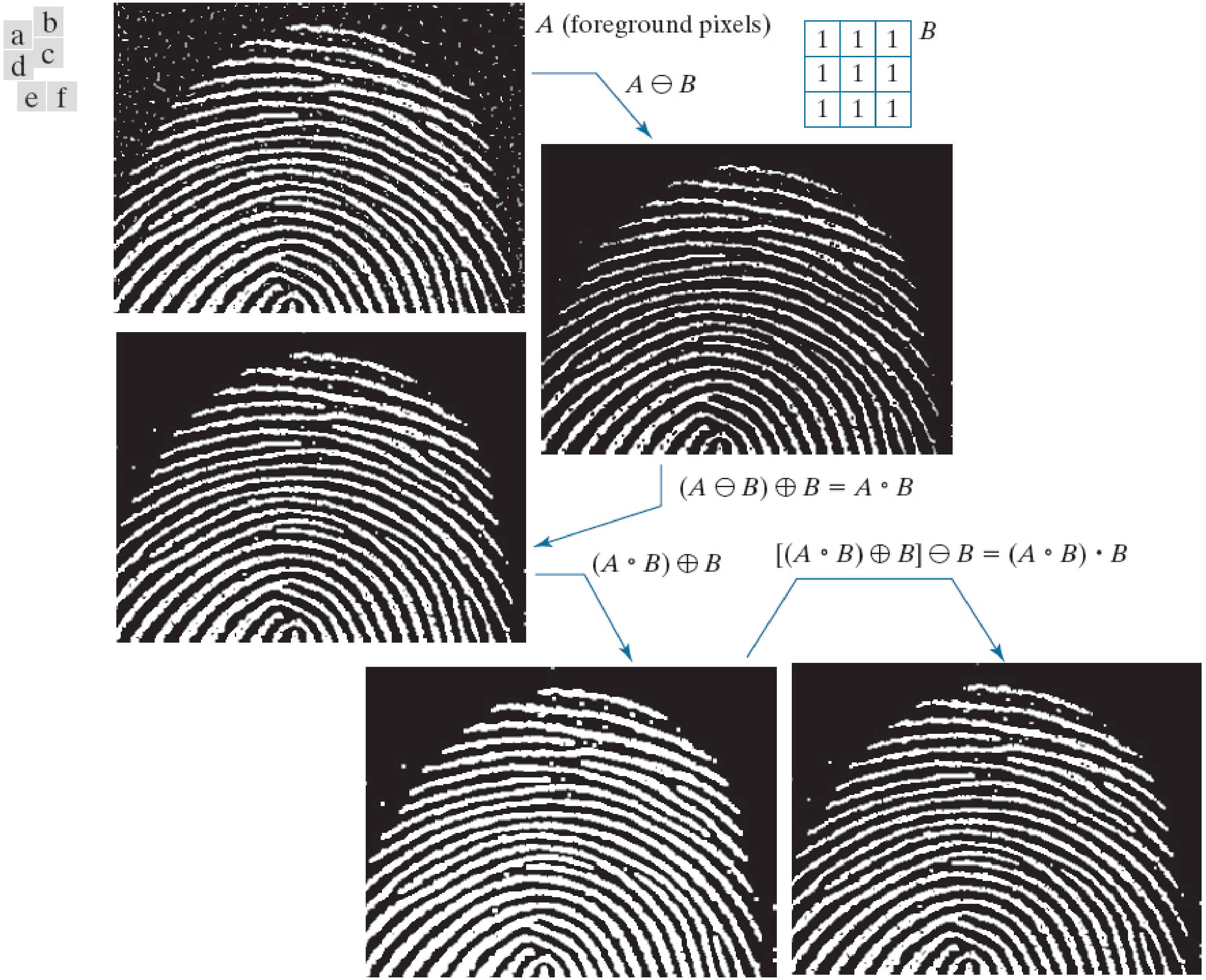
f) Process of dilation

g) Shows the process of closing



Example

a) Noise image
b) Structure B
c) Eroded image
d) Opening of eroded image
e) Dilation of the opening
f) Closing of the opening



Hit-or-Miss Transformation

- The morphological hit-or-mis transform(HTM) is a basic tool for shape detection.
- Let I be a binary Image composed of fore ground (A) and background pixels respectively.
- HMT utilizes two structure elements B1, for detecting shapes in the background, and B2 , for detecting shape in the background.
- The HMT of image I is defined as

$$A \circledast B_{1,2} = \left\{ z \mid (B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^C \right\}$$

$$A \circledast B_{1,2} = \left(A \ominus B_1 \right) \cap \left(A^C \ominus B_2 \right)$$

Where second line follows from the definition of erosion

Hit-or-Miss Transformation

Basic tool for shape detection

a) Image consisting of a foreground (1's) equal to the union, **A**, of set of objects, and a background of 0's.

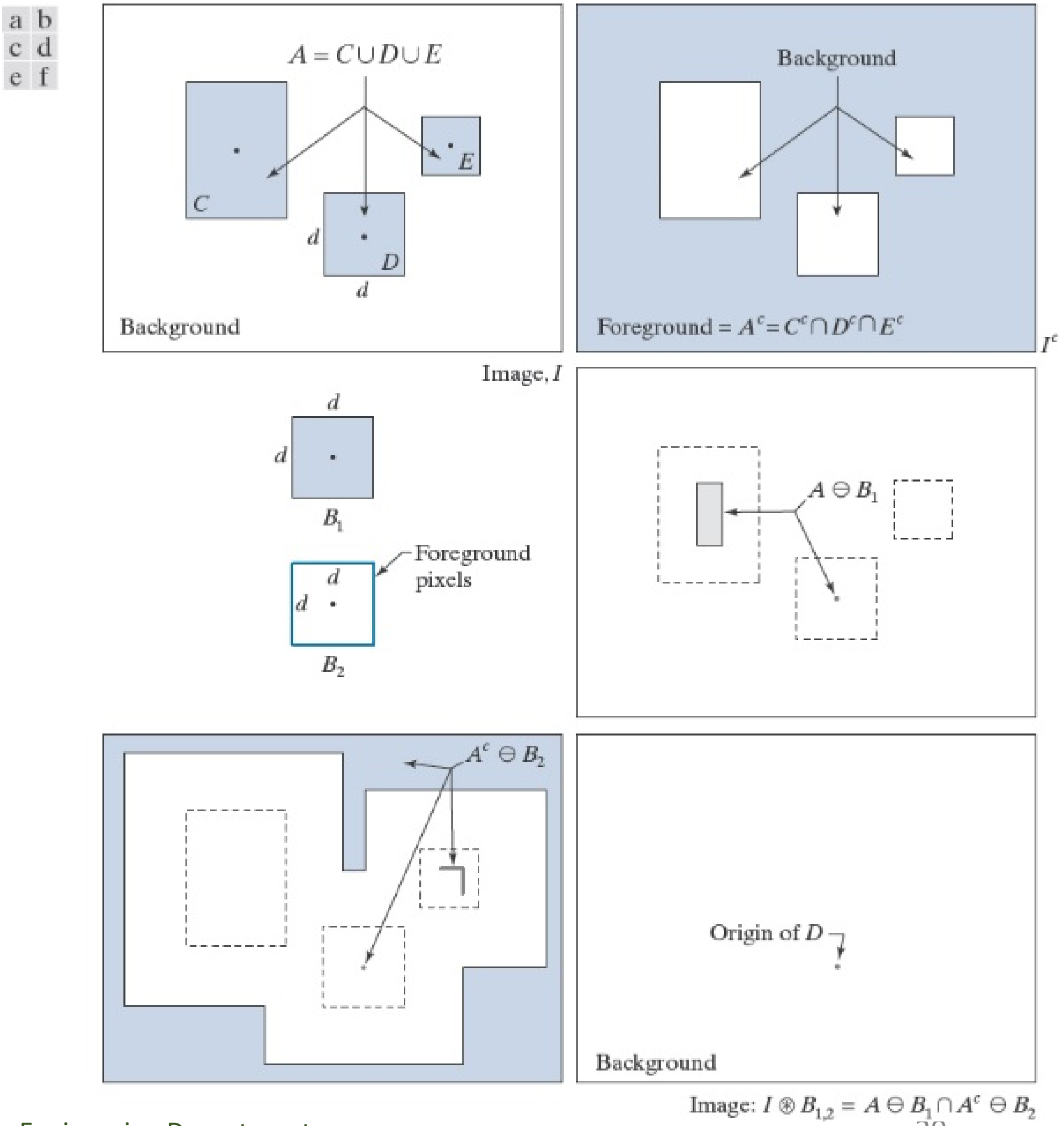
b) Image with its foreground defined as A^C

c) Structuring elements designed to detect object **D**

d) Erosion of A by B_1

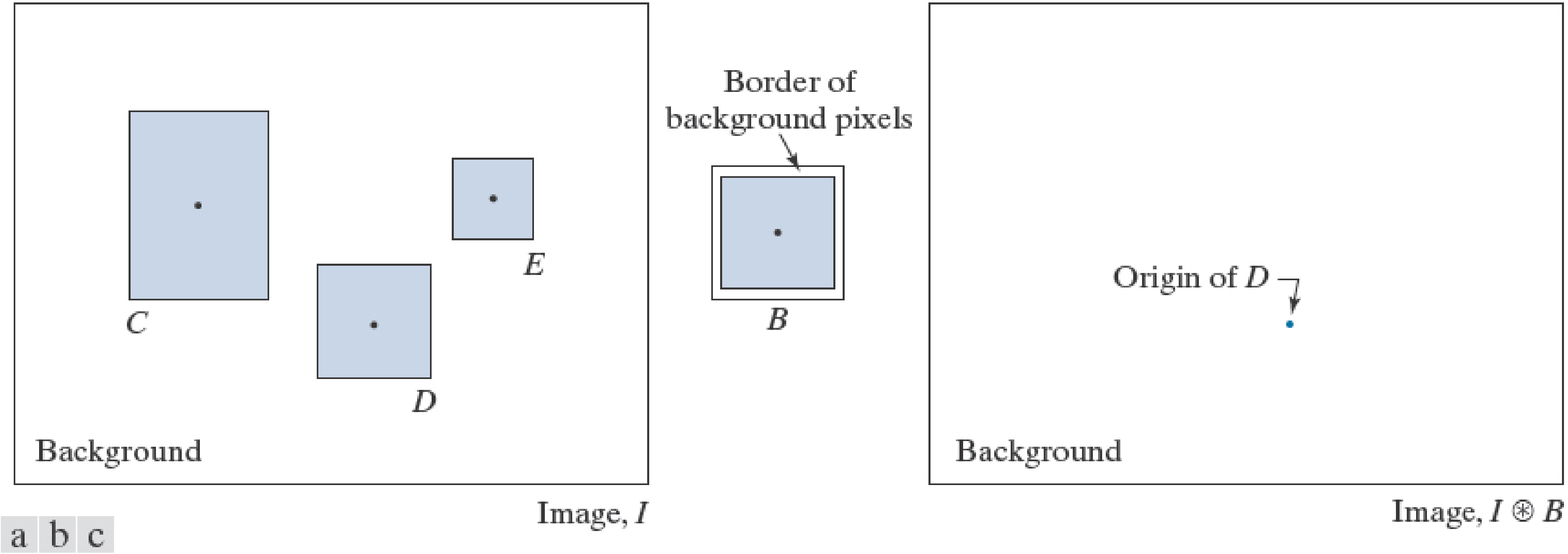
e) Erosion of A^c by B_2.

f) Intersection of (d) and (e) showing the location of the origin of **D**, as desired. The dots indicate the origin of their respective components. Each dot is a single pixel



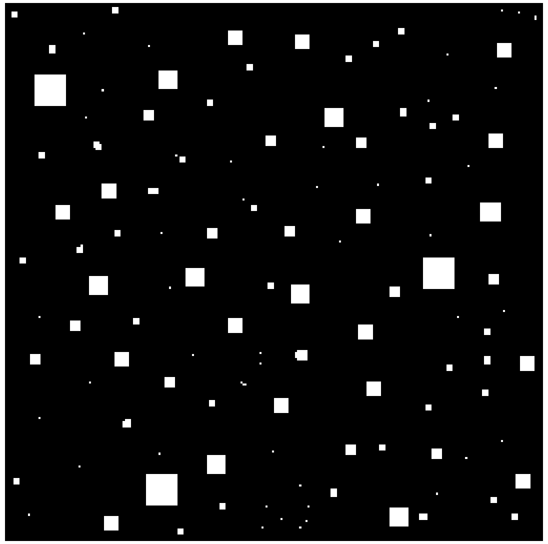
Hit-or-Miss Transformation

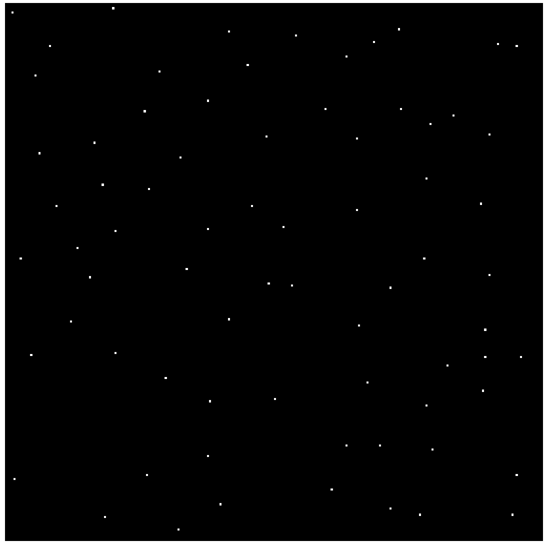
Same solution as in Fig. 9.12, but using Eq. (9-17) with a single structuring element.



MATLAB

```
A=imread('Fig9.07.jpg');
A=logical(A);
imshow(A)
B1=strel([0 0 0 ;0 1 1 ;0 1 0]);
B2=strel([1 1 1 ;1 0 0 ;1 0 0]);
g=bwhitmiss(A, B1, B2);
figure, imshow (g)
```





B1

0	0	0
0	1	1
0	1	0

B2

1	1	1
1	0	0
1	0	0