EFA refrences :

https://towardsdatascience.com/a-gentle-introduction-to-dimensionality-reduction-21b3fa63f1ca (https://towardsdatascience.com/a-gentle-introduction-to-dimensionality-reduction-21b3fa63f1ca)

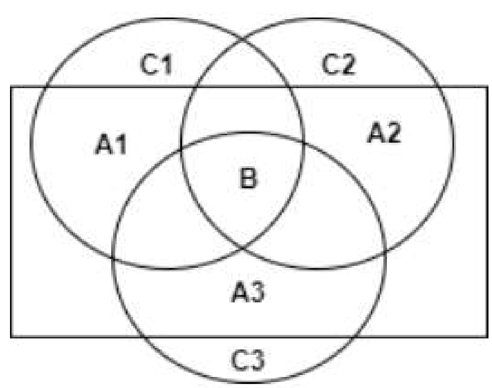
https://towardsdatascience.com/exploratory-factor-analysis-vs-principal-components-from-concept-to-application-b67bbbdb82c4 (https://towardsdatascience.com/exploratory-factor-analysis-vs-principal-components-from-concept-to-application-b67bbbdb82c4)

Import the necessary libaries

In [246]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import seaborn as sns
```

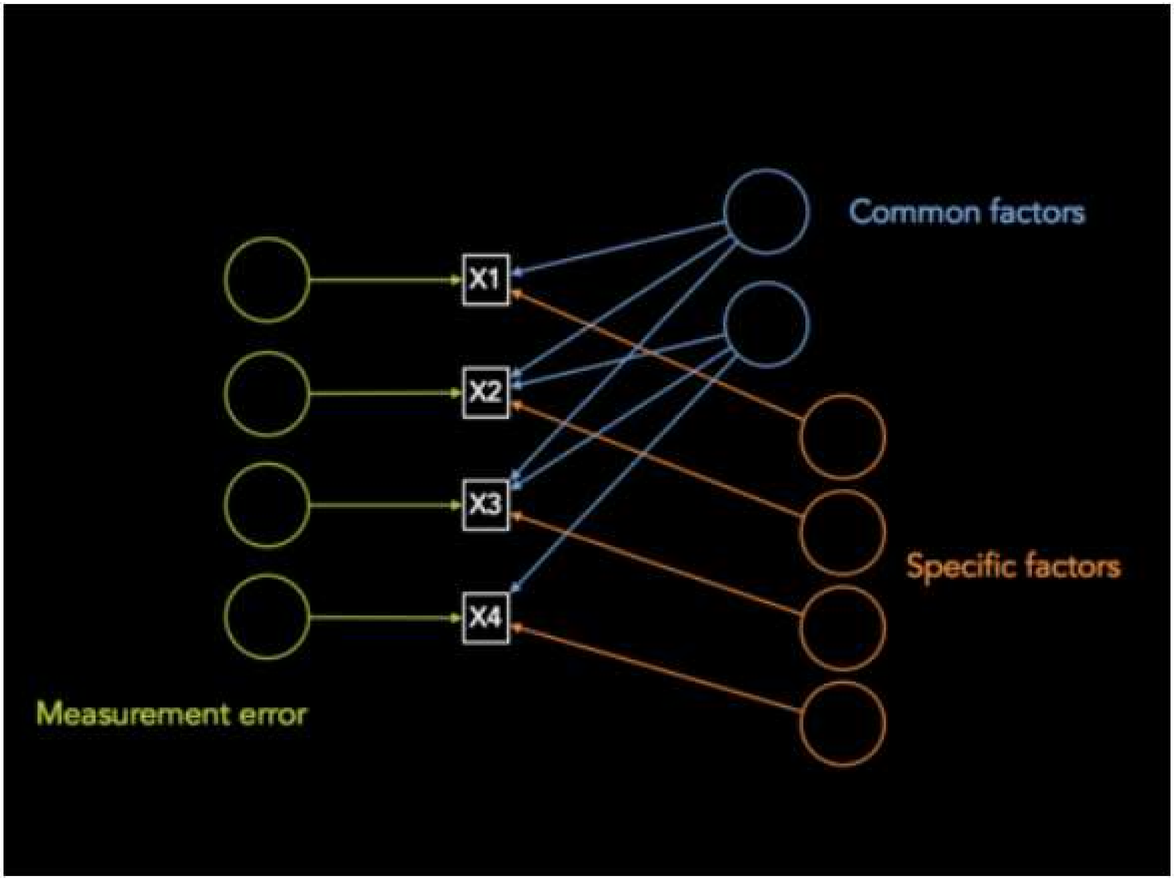
Factor analysis is a way to take a large data set and shrinking it to a smaller data set that is more manageable and more understandable. It’s a way to find hidden patterns, show how those patterns overlap and show what characteristics are seen in multiple patterns.

In PCA, when we retain a component, we take into account both specific variance and common variance. While in EFA we only take into account common variance. Seeing the next figure, we can think that A’s are specific variances, B is the common variance, and C’s are error variances. In PCA we use A’s + B while in EFA we only use B



Example of the variance of three items and its relations with a hypothetical factor.

$$Total\ Variance = Common\ Variance + Unique\ Variance\ + error$$



EFA is based on the common factor model. In this model, manifest variables are expressed as a function of common factors, unique factors, and errors of measurement. Each unique factor influences only one manifest variable, and does not explain correlations between manifest variables. Common factors influence more than one manifest variable and "factor loadings" are measures of the influence of a common factor on a manifest variable.For the EFA procedure, we are more interested in identifying the common factors and the related manifest variables.

Estimating Latent/Hidden Factors From Data

$$\boldsymbol{x} = \begin{bmatrix} English\ Score\ (ES) \\ Maths\ Score\ (MS) \\ Physics\ Score\ (PS) \\ Chemistry\ Score\ (CS) \\ Humanities\ Score\ (HS) \\ Lab\ Exam\ Score\ (LS) \\ Sports\ Score\ (SS) \\ Movies\ Watched\ (MW) \\ Parents\ Separated\ (PP) \end{bmatrix}$$

$$\boldsymbol{z} = \begin{bmatrix} Studious\ (st) \\ Intelligent\ (ig) \\ Mental\ Stability\ (ms) \\ Physical\ Fitness\ (pf) \\ Disciplined\ (ds) \end{bmatrix}$$

Latent Factors

Observed Data $\boldsymbol{x} = V\boldsymbol{z} + \boldsymbol{\eta}$ Additional Source

Factor Loading Matrix

EFA process

Exploratory Factor Analysis

Input

$$x \sim P_X(x) \; ; \; x \in \mathbb{R}^D$$

$$E(x) = \mu \quad , \; cov(x) = C_{(D \times D)}$$

$$\tilde{x} = x - \mu \; , \; E(\tilde{x}) = 0 \; , \; cov(\tilde{x}) = C$$

Latent Factor

$$Z \sim P_Z(Z) \; ; \; Z \in \mathbb{R}^M$$

$$E(Z) = 0 \quad , \; Cov(Z) = I(M \times M)$$

$$Var(Z[m]) = 1 \; ; \; m = 1 \ldots M$$

Additional Source

$$\eta \sim P_H(\eta) \; ; \; \eta \in \mathbb{R}^D$$

$$E(\eta) = 0 \quad , \; cov(\eta) = S_{(D \times D)} \; , \; Var(\eta[i]) = \sigma_i^2$$

$$i = 1, \ldots, D.$$

$$Cov(\eta[i], Z(m)) = 0 \; ; \; \forall \; i, m$$

$$Cov(\eta[i], \eta[j]) = 0 \; ; \; i \neq j.$$

Means a diagonal Matrix

$$\tilde{x} = V Z + \eta$$

η ← Source vector

Z ← Latent Factor

V ← Factor Loading Matrix

Derivations

$$Cov(\tilde{x}, Z) = V$$

$$V = E_1 \sqrt{\lambda_1}$$ → taking larger part of eigen values.

$$Z_n = W \tilde{x}_n$$ → Dimensionality Transformation

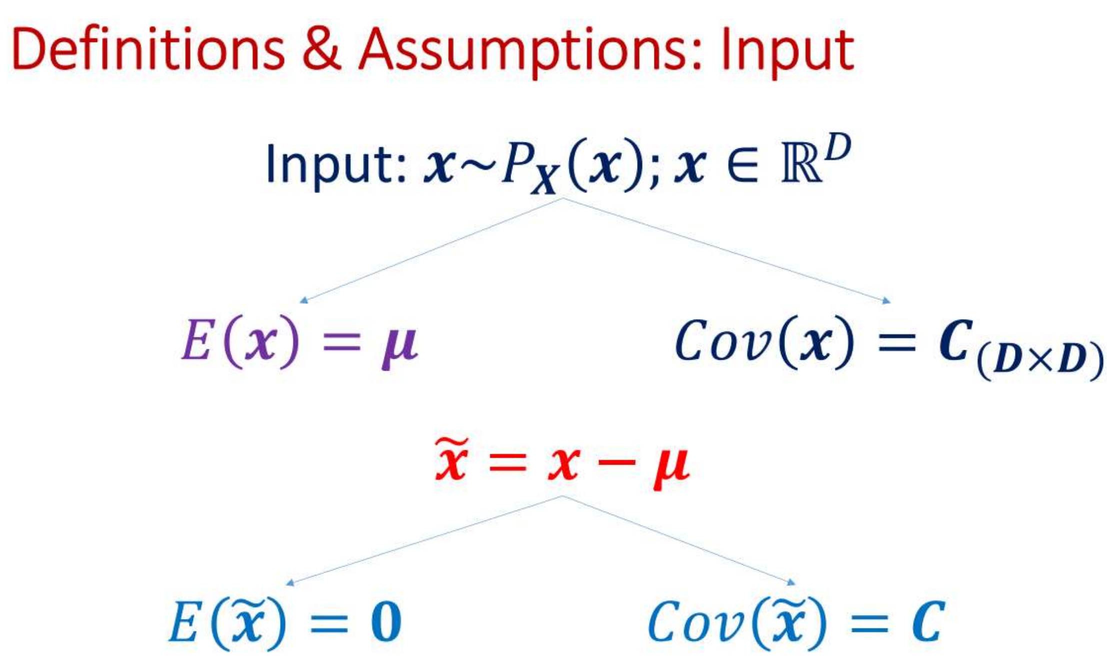
$$W = \hat{V}^T C^{-1}$$

P(X), P(Z), P(η) ==> are probability distributions for X, Z and η respectively

Generate the dataframe from the excel file

In [247]:
```
df = pd.read_excel("beer_rnd_reduced.xlsx")
print(df.shape)
print(df)
```

```
(19, 7)
    cost  size  alcohol  reputat  color  aroma  taste
0     10    15       20       85     40     30     50
1    100    70       50       30     75     60     80
2     65    30       35       80     80     60     90
3      0     0       20       30     80     90    100
4     10    25       10      100     50     40     60
5     25    35       30       40     45     30     65
6      5    10       15       65     50     65     85
7     20     5       10       40     60     50     95
8     15    10       25       30     95     80    100
9     10    15       20       85     40     30     50
10   100    70       50       30     75     60     80
11    65    30       35       80     80     60     90
12     0     0       20       30     80     90    100
13    10    25       10      100     50     40     60
14    25    35       30       40     45     30     65
15     5    10       15       65     50     65     85
16    20     5       10       40     60     50     95
17    15    10       25       30     95     80    100
18    10    15       20       85     40     30     50
```



Find $\tilde{x}_n$ **from** x_n where $\tilde{x}$ is the mean centered data set

In [248]:
```
x = df.values
x_mean = np.mean(x,axis=0)  # # axis = 0 takes column means
x_n = x - x_mean # mean centering

std = np.std(x_n)

## x_n = x_n/std  # we are not doing as of now can be done as well

x_n = x_n.T ## Converts row vectors to column vectors
print(x_n.shape)
```

```
(7, 19)
```

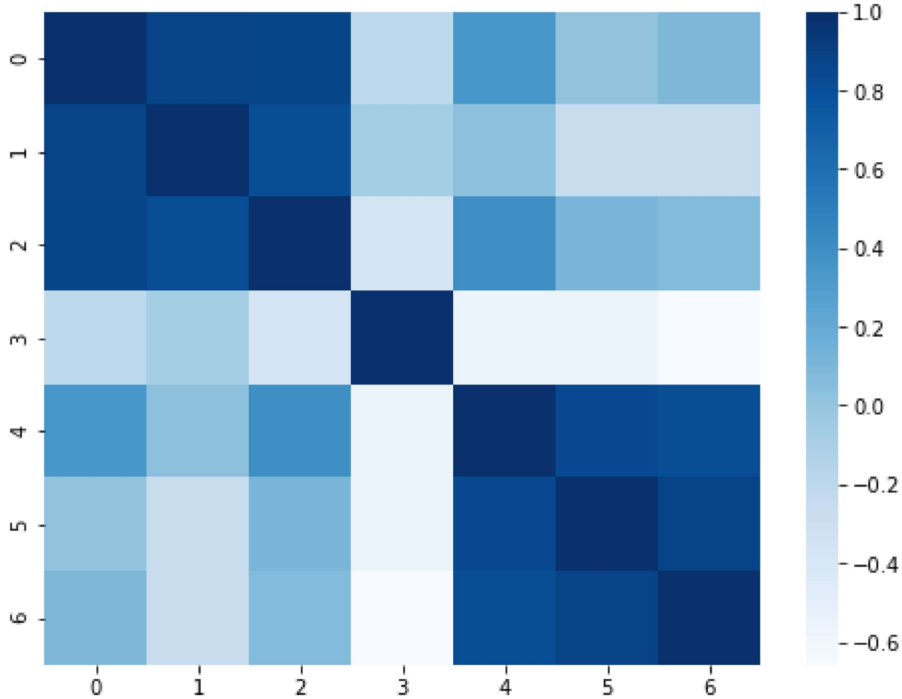
Test whether to perform EFA or not:

Test whether the correlation matrix is an identity matrix, which would indicate that the factor model is inappropriate.

In [249]:
```
C2 = np.corrcoef(x_n)    ## Corr(x,y) = Cov(x,y)/sqrt(Var(x)*Var(y))
print(C2)
```

```
[[ 1.          0.87867423  0.87675773 -0.20052763  0.34212021  0.0115177
   0.09843894]
 [ 0.87867423  1.          0.82468892 -0.07981517  0.03752096 -0.24907249
  -0.2530088 ]
 [ 0.87675773  0.82468892  1.         -0.36629982  0.40073751  0.11443376
   0.07904386]
 [-0.20052763 -0.07981517 -0.36629982  1.         -0.55847769 -0.55644779
  -0.65691684]
 [ 0.34212021  0.03752096  0.40073751 -0.55847769  1.          0.83817565
   0.82281394]
 [ 0.0115177  -0.24907249  0.11443376 -0.55644779  0.83817565  1.
   0.87717777]
 [ 0.09843894 -0.2530088   0.07904386 -0.65691684  0.82281394  0.87717777
   1.        ]]
```

In [250]:
```
plt.figure(figsize=(8,6))
ax = sns.heatmap(C2,cmap='Blues')
```



Now we compute covariance matrix of mean centered data set x_n and find eigen values and eigen vectors

In [251]:
```
C1 = np.cov(x_n)

eig_val,eig_vec = np.linalg.eig(C1)

print("eigen values : ", eig_val)

print("--------------------------------------------------------------")
print("eigen vectors : ", eig_vec)
```

```
eigen values :  [1642.95125211 1272.5513268   323.95400255   72.12232663    7.0
9847427
   21.56983247   43.0861185 ]
--------------------------------------------------------------
eigen vectors :  [[ 0.63494469  0.48780829  0.1878345   0.38814994 -0.35060552
-0.21834779
   0.04843719]
 [ 0.3063371   0.42094545 -0.12401137 -0.40484347  0.26997387  0.63272262
   0.276517  ]
 [ 0.24934947  0.14226984 -0.04181088 -0.34519873  0.61010296 -0.62620803
  -0.17975454]
 [-0.43721717  0.38740146  0.79322164 -0.01092102  0.16212564  0.00901466
   0.0554566 ]
 [ 0.34058457 -0.2679333   0.37936514 -0.21859882 -0.08098015  0.29892959
  -0.72428852]
 [ 0.25595332 -0.43791885  0.36352401 -0.446191   -0.25334561 -0.18346995
   0.56003004]
 [ 0.26107252 -0.38854374  0.20572567  0.56505584  0.57873806  0.19169883
   0.21812281]]
```

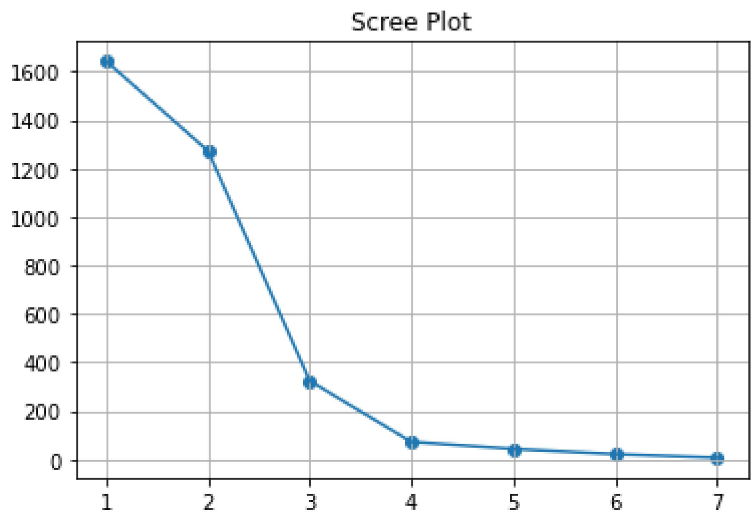
use eigenvalues and scree plot to find number of factors

In [252]:
```
eig_sorted = np.sort(eig_val)[::-1]  # contains sorted eigen values
arg_sort = np.argsort(eig_val)[::-1] # contains the indexes of sorted eigen value

print("sorted eigen values : ", eig_sorted)

print("sorted eigen indexes : ", arg_sort)
```

```
sorted eigen values :  [1642.95125211 1272.5513268   323.95400255   72.12232663
43.0861185
   21.56983247    7.09847427]
sorted eigen indexes :  [0 1 2 3 6 5 4]
```

Scree plot to find number of compnents

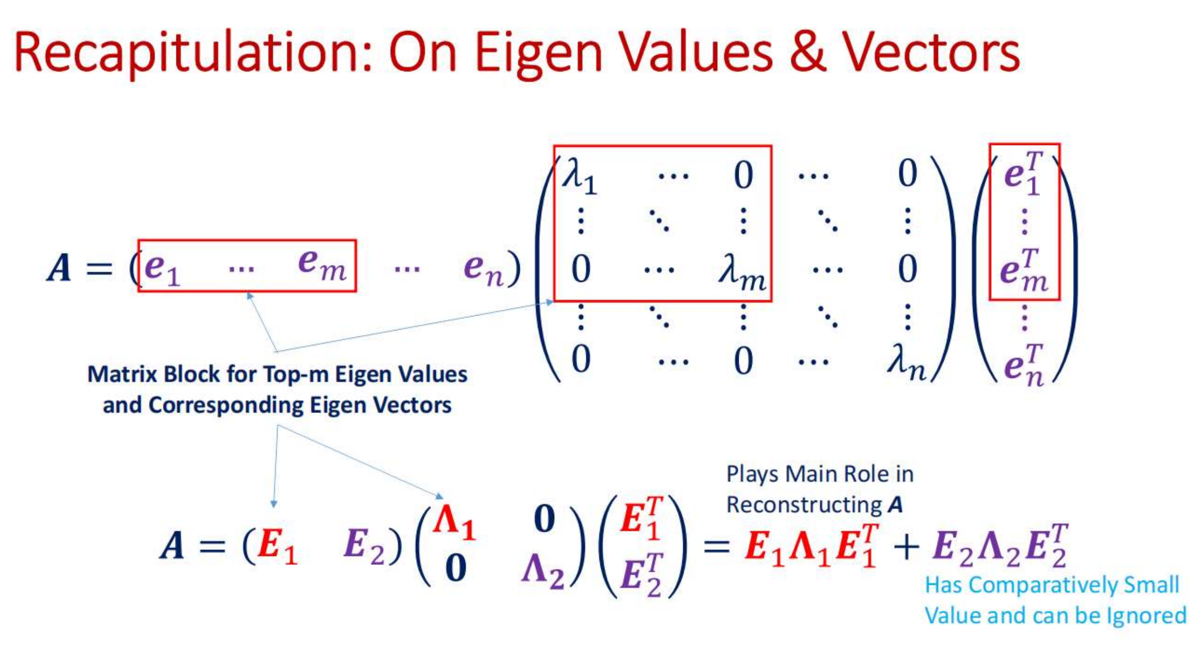
In [253]:
```
fig, ax = plt.subplots()
plt.scatter(range(1,x_n.shape[0]+1),eig_sorted)
ax.plot(range(1,x_n.shape[0] + 1), eig_sorted)
plt.grid()
ax.set_title("Scree Plot")

plt.show()
```



In [254]:
```
######## Sample code to show how much variance is captured with two compenents, t
a = eig_sorted[0] + eig_sorted[1]
tot = sum(eig_sorted)

ratio = a/sum(eig_sorted)
print("ratio :",ratio)
```

```
ratio : 0.8617249001718179
```

Extract the eigen vectors and eigen values based on number of factors choosen



In [255]:

```python
eig_vec_ls = []
eig_val_ls = []
imp_vec = arg_sort[:2]  # we choose two factors based on scree plots, so take fir

for i in imp_vec:
    e_1 = eig_vec[:,i] # get the eigen vector columns based on index
    lambda_1 = eig_val[i] # get the eigen values based on index

    eig_vec_ls.append(e_1) # add the eigen vector to the array
    eig_val_ls.append(lambda_1) # add the eigen value to the array

print("Eigen vectors : ", eig_vec_ls)
print("Eigen values", eig_val_ls)
```

```
Eigen vectors :  [array([ 0.63494469,  0.3063371 ,  0.24934947, -0.43721717,
0.34058457,
        0.25595332,  0.26107252]), array([ 0.48780829,  0.42094545,  0.1422698
4,  0.38740146, -0.2679333 ,
       -0.43791885, -0.38854374])]
Eigen values [1642.951252113008, 1272.5513268016452]
```

Estimating Factor Loading Matrix

Decomposition for Lower Eigen Values

$$Cov(\tilde{x}) = C = EQE^T = E_1\Lambda_1E_1^T + E_2\Lambda_2E_2^T$$

Decomposition for Top-M Eigen Values

$$VV^T \approx E_1\Lambda_1E_1^T = E_1\sqrt{\Lambda_1}\sqrt{\Lambda_1}E_1^T = \{E_1\sqrt{\Lambda_1}\}\{E_1\sqrt{\Lambda_1}\}^T$$

$$V = E_1\sqrt{\Lambda_1}$$

Estimate V the factor loading matrix

In [256]:

```
print("eig_val_arr : ",eig_val_arr)

lambda_1 = np.diag(eig_val_arr) # with diagonalization calculating SQRT is easy

print("lambda 1 : ",lambda_1 )

eig_vec_mat = np.matrix(eig_vec_ls).T # convert to matrix

V = eig_vec_mat@np.sqrt(lambda_1) # E1 dot lambda1, we can use np.dot as well

print("============================  V   ===============================")
print(V)

## formatting as data frame to see the factors in better way
pd.DataFrame(V, columns=['Factor 1', 'Factor 2'],index=df.columns)
```

```
eig_val_arr :  [1642.95125211 1272.5513268 ]
lambda 1 :  [[1642.95125211    0.        ]
 [   0.         1272.5513268 ]]
============================  V   ===============================
[[ 25.7364257   17.4015057 ]
 [ 12.41686423  15.01631854]
 [ 10.1069656    5.07516886]
 [-17.72187007  13.81970914]
 [ 13.80502848  -9.55794098]
 [ 10.37464186 -15.62180803]
 [ 10.58214011 -13.86045751]]
```

Out[256]:

	Factor 1	Factor 2
cost	25.736426	17.401506
size	12.416864	15.016319
alcohol	10.106966	5.075169
reputat	-17.721870	13.819709
color	13.805028	-9.557941
aroma	10.374642	-15.621808
taste	10.582140	-13.860458

EFA: Solving for W

$$W = (ZY^T)(YY^T)^{-1}$$

$$\downarrow$$

$$W = (N\hat{V}^T)(N\hat{C})^{-1}$$

$$\downarrow$$

$$W = \hat{V}^T \hat{C}^{-1}$$

In [257]:
```
C1_inv = np.linalg.inv(C1)
W = V.T@C1_inv
print(W.shape)
print(W)
```

```
(2, 7)
[[ 0.01566475  0.00755766  0.00615171 -0.01078661  0.00840258  0.00631464
   0.00644093]
 [ 0.0136745   0.01180017  0.00398818  0.01085984 -0.00751085 -0.01227597
  -0.01089187]]
```

$$z = \left(\hat{V}^T \hat{C}^{-1}\right)\tilde{x}$$

Dimensionality reduction transformation

$$z_n = W\tilde{x}_n$$

Dimensionality Reduction Transformation Relation

In [258]:
```
z = W@x_n
z1 = z.T
print(z1.shape)

#df1=pd.DataFrame(z1)
#sns.pairplot(df1)
#plt.show()
```

```
(19, 2)
```

In [259]:
```
z1
```

Out[259]:
```
matrix([[-1.17190741,  0.76613586],
        [ 2.10816381,  1.21028941],
        [ 0.73240728,  0.5963717 ],
        [ 0.18837171, -1.72648882],
        [-1.10806557,  0.70036646],
        [-0.10024158,  0.55751336],
        [-0.5725751 , -0.48435819],
        [-0.08276991, -0.62956604],
        [ 0.59257047, -1.37333168],
        [-1.17190741,  0.76613586],
        [ 2.10816381,  1.21028941],
        [ 0.73240728,  0.5963717 ],
        [ 0.18837171, -1.72648882],
        [-1.10806557,  0.70036646],
        [-0.10024158,  0.55751336],
        [-0.5725751 , -0.48435819],
        [-0.08276991, -0.62956604],
        [ 0.59257047, -1.37333168],
        [-1.17190741,  0.76613586]])
```

In [260]:
```
print(df)
```

```
    cost  size  alcohol  reputat  color  aroma  taste
0     10    15       20       85     40     30     50
1    100    70       50       30     75     60     80
2     65    30       35       80     80     60     90
3      0     0       20       30     80     90    100
4     10    25       10      100     50     40     60
5     25    35       30       40     45     30     65
6      5    10       15       65     50     65     85
7     20     5       10       40     60     50     95
8     15    10       25       30     95     80    100
9     10    15       20       85     40     30     50
10   100    70       50       30     75     60     80
11    65    30       35       80     80     60     90
12     0     0       20       30     80     90    100
13    10    25       10      100     50     40     60
14    25    35       30       40     45     30     65
15     5    10       15       65     50     65     85
16    20     5       10       40     60     50     95
17    15    10       25       30     95     80    100
18    10    15       20       85     40     30     50
```

In [ ]: