

Computer Systems Principles

Data Representation in C

Today

- Data representation in C
- Conversion between data types
 - Type casting explicit and implicit
 - Sign extension
 - Overflows
- Bit Manipulation
 - bitwise *and*, *or*, *exclusive-or*, and *not*

Ariane 5

Exploded 37 seconds after lift-off with cargo worth 500 million



Why..

- Computed horizontal velocity as floating-point number
- **Converted** to 16-bit integer
- Worked for Ariane 4
- **Overflowed for Ariane 5**

Representation Matters!

- Why do we care so much about bits and how to manipulate them?
- Bits are important for low-level systems programming tasks
 - representing sets in compiler analyses,
 - accessing systems resources,
 - processing, reading, and writing in terms of streams of bits (such as processing packets in network programming),
 - cryptography (encoding or decoding data with complex bit-manipulation), etc.

What do these binary sequences represent?

- 1101
 - unsigned integer: 11 ?
 - signed integer: -5?
- 01000001
 - unsigned integer: 65?
 - character A?
- C data types: unsigned integers, signed integers, characters,floats, doubles.

Signed
0
1
2
3
4
5
6
7
-8
-7
-6
-5
-4
-3
-2
-1



Bits
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111



UnSigned
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Representation Matters!

- No self-identifying data
 - Looking at a sequence of bits doesn't tell you what they mean
 - Could be signed, unsigned integer
 - Could be floating-point number
 - Could be part of a string
- The machine interprets what those bits mean!

A simple C program..1

```
# include <stdio.h> // This is needed to run the
                    // printf() function

int main()
{
    // displays the content inside the quotes
    printf("C Programming is Fun!");
    return 0;
}
```


A simple C program..2

```
# include <stdio.h> // This is needed to run the
                    // printf() function

int main()
{
    int var = 10;
    int c = 69;
    // displays the content inside the quotes
    printf("Number = %d",var);
    printf("Character of ASCII value 69: %c",c);
    printf("Character of ASCII value 69: %d",c);
    return 0;
}
```

A simple C program..3

```
# include <stdio.h> // This is needed to run printf()
int main()
{
    int a;
    short int b;
    unsigned int c;
    char d;
    // size-of displays the size of the data type
    printf("Size of int=%d bytes\n",sizeof(a));
    printf("Size of short int=%d bytes\n",sizeof(b));
    printf("Size of unsigned int=%d bytes\n",sizeof(c));
    printf("Size of char=%d bytes\n",sizeof(d));
    return 0;
}
```

Data types in C

`int x;`

“typically reflecting the natural size of integers on the host machine” [K&R]

- first IBM PC: `int` [16bits]
- today’s PC: `int` [32bit]
(even on 64-bit PCs – but be careful!)

Data types in C (for gcc)

C Data Type	Typical 32-bit	Intel IA 32	x86-64
<code>char</code>	1	1	1
<code>short</code>	2	2	2
<code>int</code>	4	4	4
<code>long</code>	4	4	8
<code>long long</code>	8	8	8
<code>float</code>	4	4	4
<code>double</code>	8	8	8
<code>long double</code>	8	10/12	10/16
<code>pointer</code>	4	4	8

Code Portability?

Notice that `long` and `pointer` data types are different on different processors (and maybe compilers).

Casting Signed to Unsigned

C allows conversions from **Signed** (two's complement) to **Unsigned**.

```
short int          x = 15213;  
unsigned short int ux = (unsigned short) x;  
short int          y = -15213;  
unsigned short int uy = (unsigned short) y;
```

Resulting Value

- **No change in bit representation!**
- Non-negative values unchanged
- Negative values change into (large) positive values

Signed vs. Unsigned in C

- Declaration for two signed and unsigned integers

- ```
int tx, ty; // signed
unsigned ux, uy; // unsigned
```

- Explicit casting between signed & unsigned

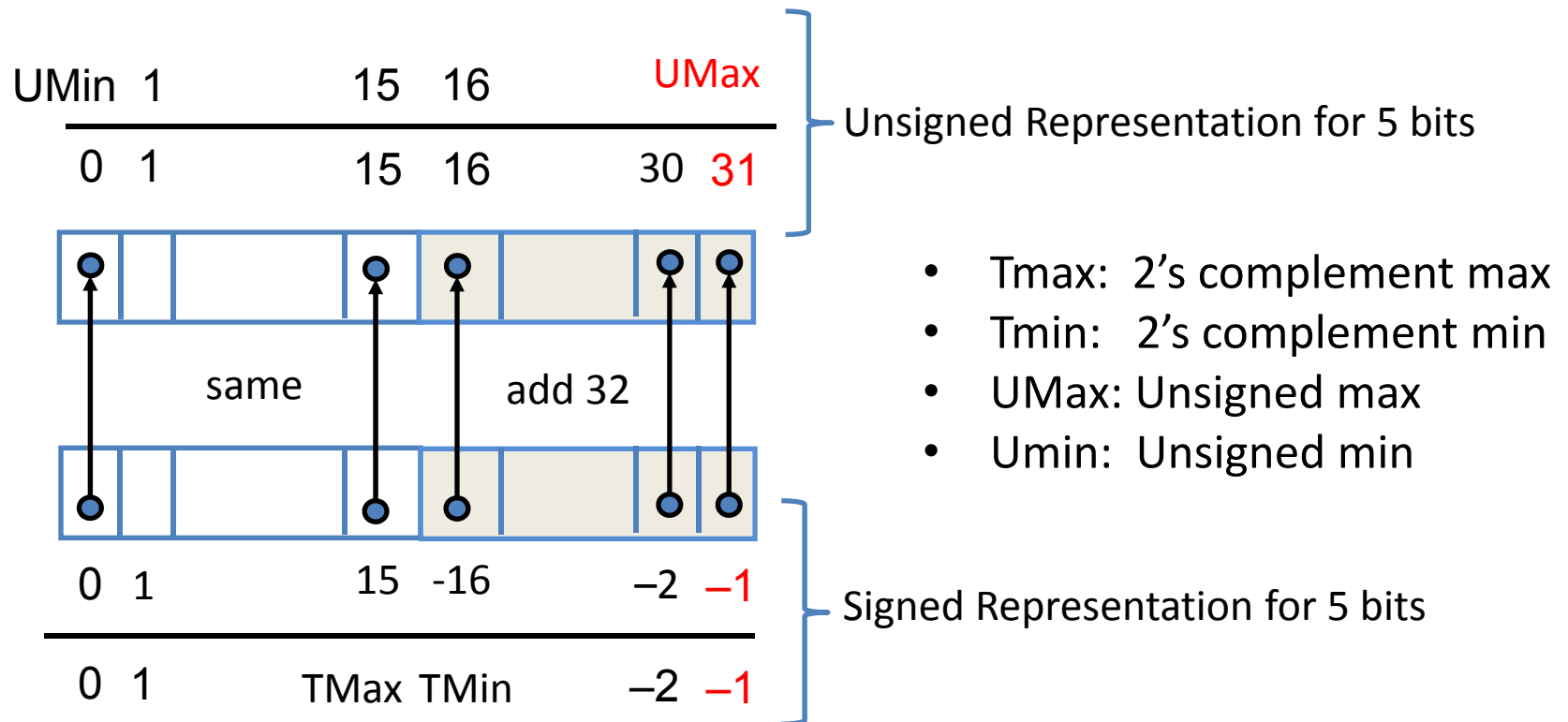
- ```
tx = (int) ux;
uy = (unsigned) ty;
```

- Implicit casting also occurs via assignments and procedure calls

- ```
tx = ux;
uy = ty;
```

# Explanation of Casting Surprises

- 2's Complement to Unsigned: (Same number of bits)
  - A small negative value maps to a large positive value!
  - (e.g., Signed representation of -1 maps to unsigned representation of 31! (5-bit integers))



# Expanding Bit representation : Sign Extension

- Converting from smaller to larger integer data type
- C automatically performs sign- or zero- extension

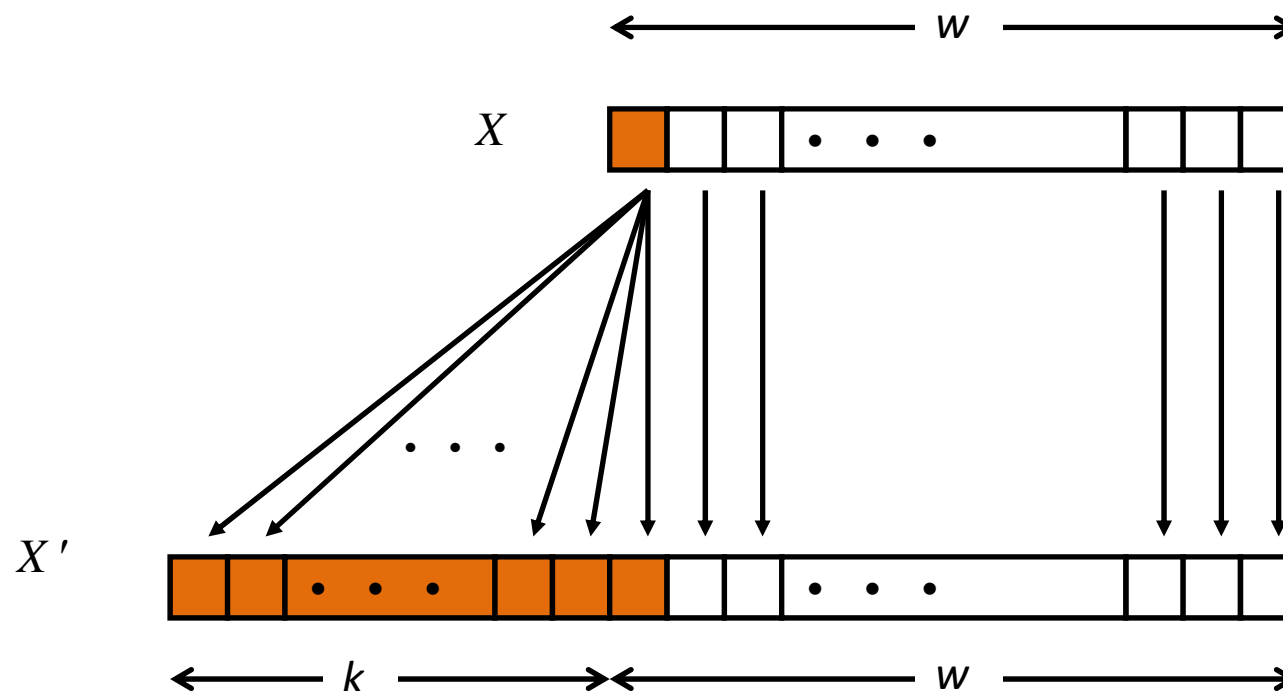
```
short int sx = -12345;
int x = sx;
unsigned short int usx = sx;
unsigned int ux = usx;
```

| Variables | Value  | Hexadecimal representation | Binary representation               |
|-----------|--------|----------------------------|-------------------------------------|
| sx        | -12345 | cf c7                      | 11001111 11000111                   |
| usx       | 53191  | cf c7                      | 11001111 11000111                   |
| x         | -12345 | ff ff cf c7                | 11111111 11111111 11001111 11000111 |
| ux        | 53191  | 00 00 cf c7                | 00000000 00000000 11001111 11000111 |



# Expanding Bit representation : Sign Extension

- Given  $w$ -bit signed integer  $x$ :
  - Convert it to  $w+k$ -bit integer with same value
  - Make  $k$  copies of sign bit:  $X' = \underbrace{x_{w-1}, \dots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2}, \dots, x_0$



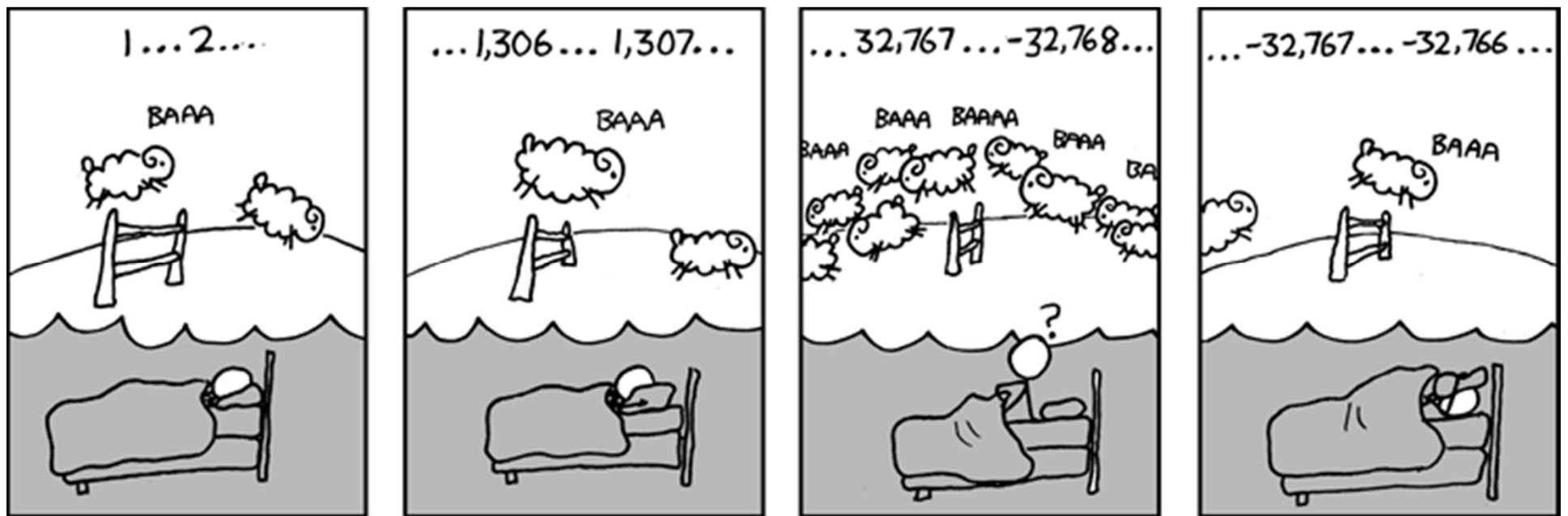
# Expanding, Truncating: Basic Rules

- Expanding (e.g., `short int` to `int`)
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected results
- Truncating (e.g., `unsigned short` to `short`)
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small numbers can yield unexpected behavior

# Casting Surprises: Expression evaluation

- If there is a mix of unsigned and signed in a single expression, **signed values are implicitly cast to unsigned!**
- Including comparison operations `<`, `>`, `==`, `<=`, `>=`
- E.g.:  $W = 32$  TMIN = **-2,147,483,648** ( $2^{31}$ ) TMAX = **2,147,483,647** ( $2^{31}-1$ )

| Constant-1    | Relation | Constant-2        | Evaluation |
|---------------|----------|-------------------|------------|
| 0             | ==       | 0u                | unsigned   |
| -1            | <        | 0                 | signed     |
| -1            | >        | 0u                | unsigned   |
| 2147483647    | >        | -2147483648       | signed     |
| 2147483647u   | <        | -2147483648       | unsigned   |
| -1            | >        | -2                | signed     |
| (unsigned) -1 | >        | -2                | unsigned   |
| 2147483647    | <        | 2147483648u       | unsigned   |
| 2147483647    | >        | (int) 2147483648u | signed     |



Is this dynamic ram??

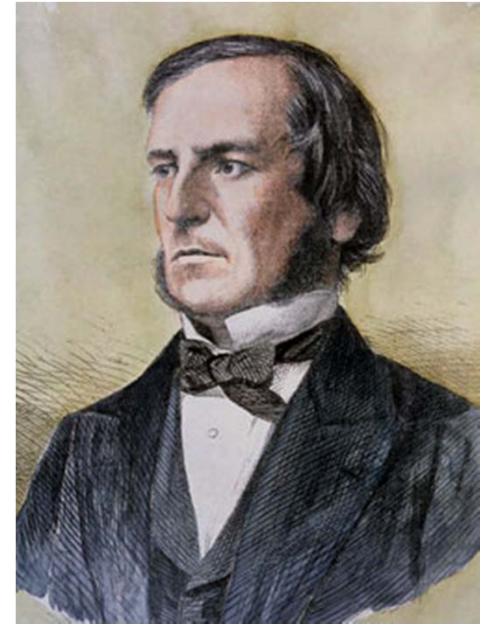
Source: <http://xkcd.com/571/>

# Bit-Manipulations

Boolean Algebra:

- Developed by George Boole in the 19<sup>th</sup> Century and applied to Digital Systems by Claude Shannon

| Operators | Operator Definition  |
|-----------|----------------------|
| &         | Bitwise AND          |
|           | Bitwise OR           |
| ^         | Bitwise exclusive OR |
| ~         | Bitwise complement   |
| <<        | Shift left           |
| >>        | Shift right          |



*"Laws of Thought"*  
image source: Wikipedia

# Bit-Manipulations

Boolean Algebra:

- Encode “True” as 1 and “False” as 0

Not ( $\sim A$ )

| $\sim$ |   |
|--------|---|
| 0      | 1 |
| 1      | 0 |

And ( $A \& B$ )

| $\&$ | 0 | 1 |
|------|---|---|
| 0    | 0 | 0 |
| 1    | 0 | 1 |

Or ( $A | B$ )

| $ $ | 0 | 1 |
|-----|---|---|
| 0   | 0 | 1 |
| 1   | 1 | 1 |

Xor  $A \wedge B$

| $\wedge$ | 0 | 1 |
|----------|---|---|
| 0        | 0 | 1 |
| 1        | 1 | 0 |

# Bit-Manipulations

Boolean operations are applied bitwise on the bit sequences (i.e., by columns)

Not ( $\sim A$ )

$$\begin{array}{r} \sim 1\ 0\ 1\ 0 \\ \hline 0\ 1\ 0\ 1 \end{array}$$

And ( $A \& B$ )

$$\begin{array}{r} 0\ 1\ 1\ 0 \\ \& 1\ 0\ 1\ 0 \\ \hline 0\ 0\ 1\ 0 \end{array}$$

Or ( $A|B$ )

$$\begin{array}{r} 0\ 1\ 1\ 0 \\ | 1\ 0\ 1\ 0 \\ \hline 1\ 1\ 1\ 0 \end{array}$$

Xor  $A^{\wedge} B$

$$\begin{array}{r} 0\ 1\ 1\ 0 \\ ^{\wedge} 1\ 0\ 1\ 0 \\ \hline 1\ 1\ 0\ 0 \end{array}$$

# Bit Manipulations

Boolean algebra obeys some of the properties of integer algebra.. **but not all!**

| Boolean                               | Boolean                     | Integer                    |
|---------------------------------------|-----------------------------|----------------------------|
| Sum and product identities            | $A   0 = A$<br>$A \& 1 = A$ | $A + 0 = A$<br>$A * 1 = A$ |
| Zero is product annihilator           | $A \& 0 = 0$                | $A * 0 = 0$                |
| Cancellation of negation              | $\sim(\sim A) = A$          | $-(-A) = A$                |
| Laws of Complements                   | $A   \sim A = 1$            | $A + -A \neq 1$            |
| Every element has an additive inverse | $A   \sim A \neq 0$         | $A + -A = 0$               |



# Bit Manipulations: shift operators

- Left shift:  $x \ll k$  : Shift bit-vector  $x$  left  $k$  positions
  - Throw away extra bits on the left
  - Fill with 0's on the right.
  - $10\underline{01\ 0001} \ll 2 = \underline{0100\ 01}00$
  - $x \ll k$  is equivalent to  $x * 2^k$

# Bit Manipulations: shift operators

- Right shift:  $x \gg k$  : Shift bit-vector  $x$  right  $k$  positions.
  - Throw away extra bits on the right
  - $x \gg k$  corresponds to  $x/2^k$  for rounded down.

TWO KINDS:

- Logical Shift: Fill with 0's on the left. Applies to C *unsigned*.
  - $\underline{1001}0001 \gg 3 = 000\underline{10010}$ 
    - $\underline{1001}0001 \gg 3$  in decimal :  $145 / 2^3 = 18.125$
    - $000\underline{10010}$  in decimal : 18
- Arithmetic Shift : Replicate with most significant bit on the left.
  - Copies the sign bit – applies to C *signed* numbers
  - Arithmetic shift is equivalent to logical shift for positive numbers
  - $\underline{1001}0001 \gg 3 = \underline{1111}0010$ 
    - $\underline{1001}0001 \gg 3$  in decimal:  $(-111) / 2^3 = -13.875$
    - $\underline{1111}0010$  in decimal: -14

# Comparison with shifting in Java

C:

- Has *signed* and *unsigned* types
- >> operates according to the *type* of the operand

Java:

- Has only *signed* types
- >> is arithmetic shift, >>> is logical shift

# Bit Manipulations in C

```
include <stdio.h> // This is needed to run printf()
function
int main()
{
 int a;
 short int b;
 unsigned int c;
 char d;
 // size-of displays the size of the data type
 printf("Size of int=%d bytes\n",sizeof(a));
 printf("Size of short int=%d bytes\n",sizeof(b));
 printf("Size of unsigned int=%d bytes\n",sizeof(c));
 printf("Size of char=%d bytes\n",sizeof(d));
 return 0;
}
```

# iClicker Question

Compute this *arithmetic* right shift:

1001 0001 >> 2

- a) 1111 0010
- b) 1110 0100
- c) 1110 0101
- d) 0010 0100

# Ariane 5

Exploded 37 seconds after lift-off with cargo worth 500 million



Why..

- Computed horizontal velocity as floating-point number
- **Converted** to 16-bit integer
- Worked for Ariane 4
- **Overflowed for Ariane 5**

# Ariane 5: Spot the problem..

```
L_M_BV_32 := TBD.T_ENTIER_32S ((1.0/C_M_LSB_BV) * G_M_INFO_DERIVE(T_ALG.E_BV));
```

```
if L_M_BV_32 > 32767 then
```

```
 P_M_DERIVE(T_ALG.E_BV) := 16#7ffff#;
```

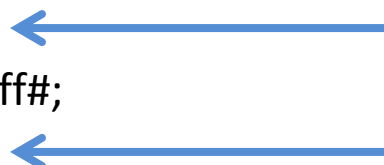
```
elsif L_M_BV_32 < -32768 then
```

```
 P_M_DERIVE(T_ALG.E_BV) := 16#8000#;
```

```
else
```

```
 P_M_DERIVE(T_ALG.E_BV) := UC_16S_EN_16NS(TBD.T_ENTER_16S(L_M_BV_32));
```

```
end if;
```



Checks for integer range for vertical velocity of rocket

```
P_M_DERIVE(T_ALG.E_BH) :=
UC_16S_EN_16NS (TDB.T_ENTIER_16S ((1.0/C_M_LSB_BH) *
G_M_INFO_DERIVE(T_ALG.E_BH)));
```

Horizontal velocity..



# Ariane 5: Problem Fix

```
L_M_BV_32 := TBD.T_ENTIER_32S ((1.0/C_M_LSB_BV) * G_M_INFO_DERIVE(T_ALG.E_BV));
```

```
if L_M_BV_32 > 32767 then
```

```
 P_M_DERIVE(T_ALG.E_BV) := 16#7ffff#;
```

```
elsif L_M_BV_32 < -32768 then
```

```
 P_M_DERIVE(T_ALG.E_BV) := 16#8000#;
```

```
else
```

```
 P_M_DERIVE(T_ALG.E_BV) := UC_16S_EN_16NS(TBD.T_ENTER_16S(L_M_BV_32));
```

```
end if;
```



Checks for integer range for vertical velocity of rocket

```
L_M_BH_32 := TBD.T_ENTIER_32S ((1.0/C_M_LSB_BV) * G_M_INFO_DERIVE(T_ALG.E_BH));
```

```
if L_M_BH_32 > 32767 then
```

```
 P_M_DERIVE(T_ALG.E_BH) := 16#7ffff#;
```

```
elsif L_M_BH_32 < -32768 then
```

```
 P_M_DERIVE(T_ALG.E_BH) := 16#8000#;
```

```
else
```

```
 P_M_DERIVE(T_ALG.E_BH) := UC_16S_EN_16NS(TBD.T_ENTER_16S(L_M_BH_32));
```

```
end if;
```



Checks for integer range for horizontal velocity of rocket!



# Summary

- Bit representation and manipulation is extremely useful in a wide variety of applications like compiler analyses, network programming, cryptography and many more!
- The **same binary sequence** can be used to represent ASCII characters, unsigned binary, and two's complement integers. Their **interpretation** is based on the context in which they are defined!
- C has different **data types** to store integers and floating point numbers that have **different memory sizes** on different operating systems.
- **Typecasting** operations between two different data types can be explicit or implicit.
  - Casting surprises when changing between data types can **change the numeric value**.
  - Casting surprises also occur if we **use arithmetic and relational operators** on two different data types.
- Boolean algebra includes {not, and, or and x-or} operations and left and right shifts.
  - Not to be confused with conditional operators !
- Next class we will cover more programming in C!