Airfoils and wing-tail interaction

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Airfoils mutual interaction

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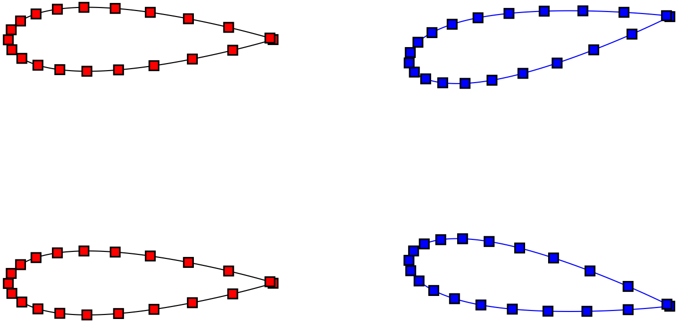
The Hess-Smith method can be extended to consider more than one airfoil:

- Airfoils in cascade (e.g. turbine blades cascade etc.)
- Single airfoil + flap
- Two airfoils in tandem

Airfoils mutual interaction

Which is the effect of changing α of the downstream airfoil on the aerodynamic loads of the upstream airfoil?



Hess-Smith: a brief recap

- Solid surface discretisation in N panels
- Uniform planar source and vortex distribution on each panel
- Sources strength varies for different panels, vortex intensity is the same
- $N + 1$ unknowns: N source strengths q_j and 1 vortex intensity γ
- It sets $\boldsymbol{u} \cdot \boldsymbol{n} = \boldsymbol{b}_n$ at the centre of every panel, i.e. at the control points: N equations
- It imposes the Kutta condition at the trailing edge: 1 equation
- It solves a linear system to obtain the unknowns q_j and γ
- It computes the pressure distribution and the aerodynamic loads

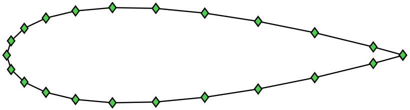
Hess-Smith: a brief recap

It is based on a distribution of sources and vortices on the surface body:

$$\phi = \phi_\infty + \phi_s + \phi_v$$

where:

- The vortex strength is constant over the whole body
- The source strength varies from panel to panel



Hess-Smith: a brief recap

It leads to the following linear system:

$$\begin{bmatrix} \boldsymbol{A}^s & \boldsymbol{a}^v \\ (\boldsymbol{c}^s)^T & c^v \end{bmatrix} \begin{bmatrix} \boldsymbol{q} \\ \gamma \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}^s \\ b^v \end{bmatrix}$$

$$A^s_{ij} = \boldsymbol{u}^s_j(x_{c,i}, y_{c,i}) \cdot \boldsymbol{n}_i \qquad a^v_i = \sum_{j=1}^{N} \boldsymbol{u}^v_j(x_{c,i}, y_{c,i}) \cdot \boldsymbol{n}_i$$

$$c^s_j = \boldsymbol{u}^s_j(x_{c,1}, y_{c,1}) \cdot \boldsymbol{\tau}_1 + \boldsymbol{u}^s_j(x_{c,N}, y_{c,N}) \cdot \boldsymbol{\tau}_N \qquad c^v = \sum_{j=1}^{N} \boldsymbol{u}^v_j(x_{c,1}, y_{c,1}) \cdot \boldsymbol{\tau_1} + \boldsymbol{u}^v_j(x_{c,N}, y_{c,N}) \cdot \boldsymbol{\tau_N}$$

$$b^s_i = -\boldsymbol{U}_\infty \cdot \boldsymbol{n}_i \qquad b^v = -\boldsymbol{U}_\infty(\boldsymbol{\tau}_1 + \boldsymbol{\tau}_N)$$

where $\boldsymbol{u}^s_j(x_{c,i}, y_{c,i})$ and $\boldsymbol{u}^v_j(x_{c,i}, y_{c,i})$ are the velocity a unitary constant source and vortex distribution on the $j-$th panel induced on the control point of the $i-$th panel:

Hess-Smith: extension to more than one airfoil

The extension to consider two airfoils is very simple.

- The total number of panels is $N_1 + N_2$.
- $N_1 + N_2 + 2$ unknowns: $N_1 + N_2$ sources strengths, $q_{1,j}$ and $q_{2,j}$, on each panels of the two airfoils and the vortices strength γ_1 and γ_2.
- $N_1 + N_2 + 2$ equations: $N_1 + N_2$ non penetration boundary conditions at the control points of each panel and 2 Kutta conditions on the two airfoils.
- The sources and vortices distribution on one airfoil produces a cross-influence also on the other airfoil.
- Pay attention to the reference system
- Once you consider 2 airfoils you can easily extend this to the case with more bodies.

Hess-Smith: extension to more than one airfoil

It is based on a distribution of sources and vortices on the surface body:

$$\phi = \phi_\infty + \phi_{1,s} + \phi_{1,v} + \phi_{2,s} + \phi_{2,v}$$

where:

- The vortex strengths γ_1 and γ_2 are constant over the two bodies
- The source strengths $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ on the two bodies vary from panel to panel

Hess-Smith: extension to more than one airfoil

Therefore, the velocity of the flow field is expressed as:

$$\boldsymbol{u}(x,y) = \underbrace{\boldsymbol{U}_\infty}_{\text{Uniform flow}} + \underbrace{\sum_{j=1}^{N_1} \boldsymbol{u}^s_{1,j}(x,y) q_{1,j}}_{\text{Airfoil 1}} + \underbrace{\gamma_1 \sum_{j=1}^{N_1} \boldsymbol{u}^v_{1,j}(x,y)}_{\text{Airfoil 1}} + \underbrace{\sum_{j=1}^{N_2} \boldsymbol{u}^s_{2,j}(x,y) q_{2,j}}_{\text{Airfoil 2}} + \underbrace{\gamma_2 \sum_{j=1}^{N_2} \boldsymbol{u}^v_{2,j}(x,y)}_{\text{Airfoil 2}}$$

and to evaluate the $2N_1 + 2N_2 + 1$ unknowns we impose

$$\begin{cases} \boldsymbol{u} \cdot \boldsymbol{n} = \boldsymbol{b}_n & \text{at the control point of every panel} \\ \text{Kutta condition} \end{cases}$$

Hess-Smith: extension to more than one airfoil

It leads to the following linear system:

$$\begin{bmatrix} \boldsymbol{A}^s_{11} & \boldsymbol{A}^s_{12} & \boldsymbol{a}^v_{11} & \boldsymbol{a}^v_{12} \\ \boldsymbol{A}^s_{21} & \boldsymbol{A}^s_{22} & \boldsymbol{a}^v_{21} & \boldsymbol{a}^v_{22} \\ (\boldsymbol{c}^s_{11})^T & (\boldsymbol{c}^s_{12})^T & c^v_{11} & c^v_{12} \\ (\boldsymbol{c}^s_{21})^T & (\boldsymbol{c}^s_{22})^T & c^v_{21} & c^v_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_1 \\ \boldsymbol{q}_2 \\ \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}^s_1 \\ \boldsymbol{b}^s_2 \\ b^v_1 \\ b^v_2 \end{bmatrix}$$

$$A^s_{kl,ij} = \boldsymbol{u}^s_{l,j}(x_{c,k,i}, y_{c,k,i}) \cdot \boldsymbol{n}_{k,i} \qquad c^s_{kl,j} = \boldsymbol{u}^s_{l,j}(x_{c,k,1}, y_{c,k,1}) \cdot \boldsymbol{\tau}_{k,1} + \boldsymbol{u}^s_{l,j}(x_{c,k,N}, y_{c,k,N_k}) \cdot \boldsymbol{\tau}_{k,N_k}$$

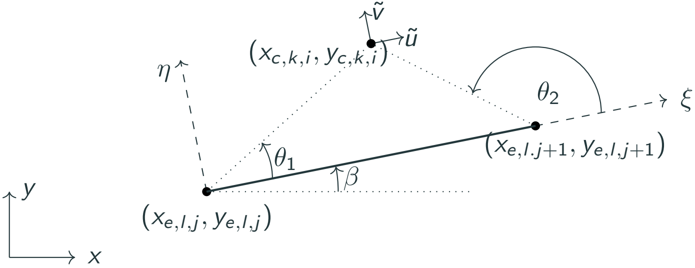
$$a^v_{kl,i} = \sum_{j=1}^{N_l} \boldsymbol{u}^v_{l,j}(x_{c,k,i}, y_{c,k,i}) \cdot \boldsymbol{n}_{k,i} \qquad c^v_{kl} = \sum_{j=1}^{N_l} \boldsymbol{u}^v_{l,j}(x_{c,k,1}, y_{c,k,1}) \cdot \boldsymbol{\tau}_{k,1} + \boldsymbol{u}^v_{l,j}(x_{c,k,N}, y_{c,k,N_k}) \cdot \boldsymbol{\tau}_{k,N_k}$$

$$b^s_{k,i} = -\boldsymbol{U}_\infty \cdot \boldsymbol{n}_{k,i} \qquad b^v_k = -\boldsymbol{U}_\infty(\boldsymbol{\tau}_{k,1} + \boldsymbol{\tau}_{k,N_k})$$

$k = 1, 2$ and $l = 1, 2$ refer to the airfoils.

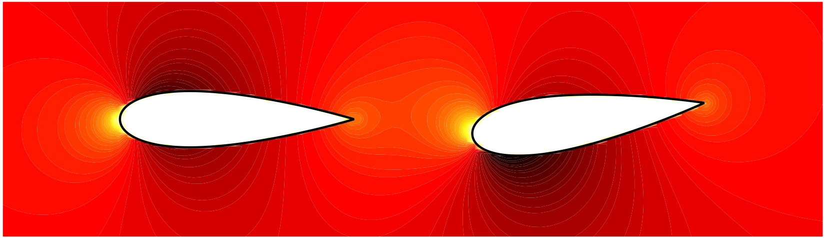
Hess-Smith: extension to more than one airfoil

$\boldsymbol{u}^s_{l,j}(x_{c,k,i}, y_{c,k,i})$ ($\boldsymbol{u}^v_{l,j}(x_{c,k,i}, y_{c,k,i})$) refer to the velocity a unitary constant source (vortex) distribution on the $j-$th panel of the $l-$th airfoil induces on the $i-$th panel control point of the $k-$th airfoil.



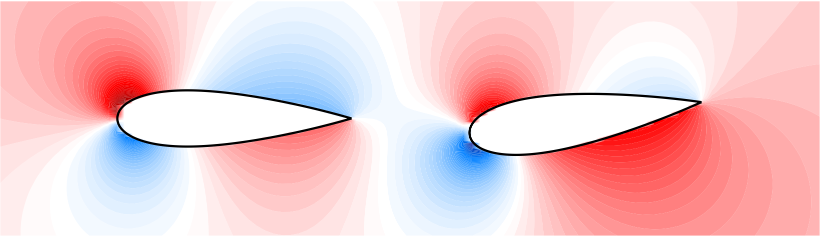
Example: Two NACA 0024 in Tandem

$\alpha_1 = 4°$, $\alpha_2 = -4°$, $x_{12} = 1.5$, $y_{12} = 0$



Example: Two NACA 0024 in Tandem

$\alpha_1 = 4^\circ,\ \alpha_2 = -4^\circ,\ x_{12} = 1.5,\ y_{12} = 0$



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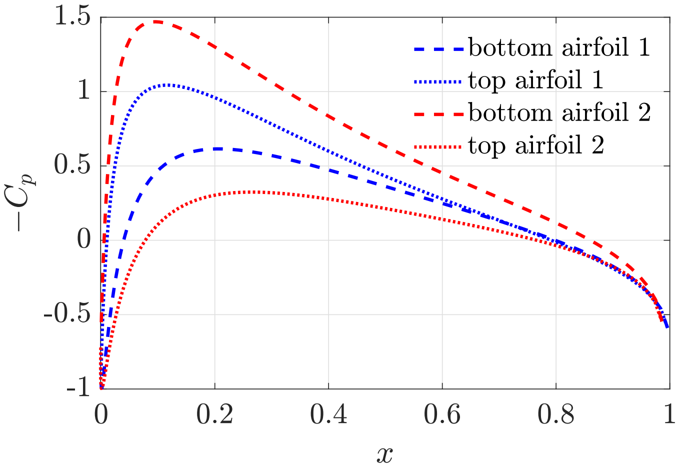

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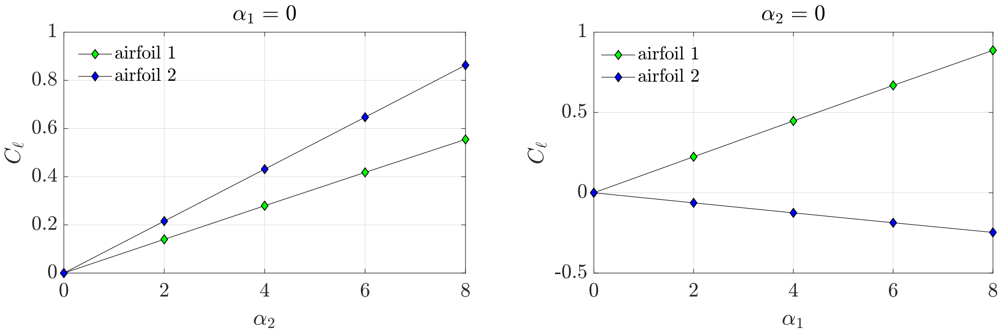


Why does it happen? How does the C_p distribution change compared to single airfoil configuration?

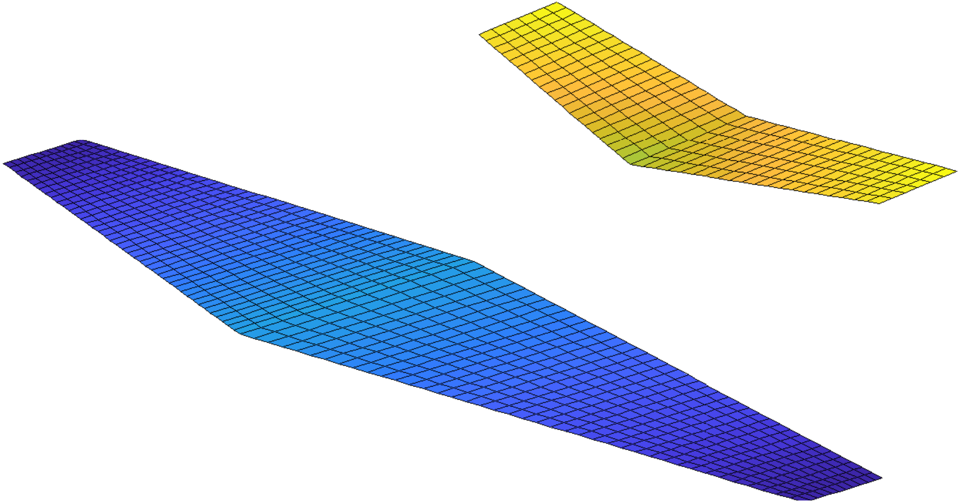
Wing/tail plane interaction

Wing/tail plane interaction

The Weissinger method can be extended to consider the tail plane. For example, it affects:

- Stability derivatives
- Overall Aerodynamic loads

Wing/tail plane interaction



Weissinger method: a brief recap

- $2M \times N$ panels
- A horse vortex of intensity Γ_k is associated to each panel
- The velocity induced by each vortex is evaluated using the Biot-Savart law
- The $2M \times N$ Γ_k unknowns are evaluated imposing the no-penetration boundary condition on each panel
- The $2D$ lift distribution along the wing span is obtained using the Kutta-Jukowsky theorem
- Only the wake vortices contribute to the evaluation of the induced angle of incidence
- It predicts both the lift and the induced drag

Weissinger method: a brief recap

The velocity field is expressed as:

$$\boldsymbol{u}(x, y) = \underbrace{\boldsymbol{U}_\infty}_{\text{Uniform flow}} + \underbrace{\sum_{j=1}^{2M \times N} \Gamma_j \boldsymbol{q}_j(x, y)}_{\text{Wing horse vortices cntribution}}$$

and the $2M \times N$ unknowns are obtained by imposing:

$$\boldsymbol{u}(x_{c,i}, y_{c,i}) \cdot \boldsymbol{n}_i = \boldsymbol{b}_i \quad i = 1, .., 2M \times N.$$

Weissinger method: a brief recap

It leads to the following linear system:

$$A\Gamma = \boldsymbol{b}$$

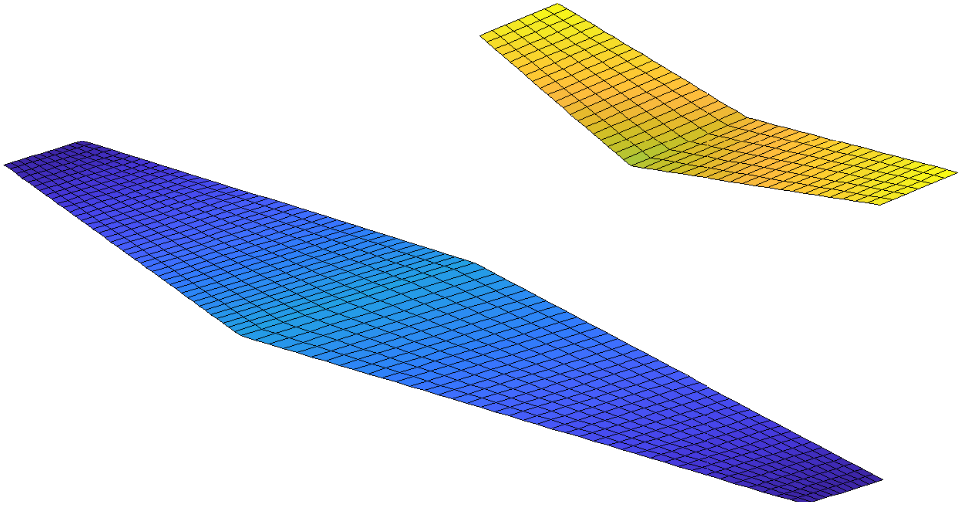
where:

$$A_{ij} = \boldsymbol{q_j}(x_{c,i}, y_{c,i}) \cdot \boldsymbol{n}_i \qquad \boldsymbol{b}_i = -\boldsymbol{U}_\infty \cdot \boldsymbol{n}_i$$

Recall that (see slides of the previous lesson):

$$\boldsymbol{q}_j(x_{c,i}, y_{c,i}) = \frac{1}{4\pi}\boldsymbol{r}_0 \cdot \left(\frac{\boldsymbol{r}_1}{|\boldsymbol{r}_1|} - \frac{\boldsymbol{r}_2}{|\boldsymbol{r}_2|}\right)\frac{\boldsymbol{r}_1 \times \boldsymbol{r}_2}{|\boldsymbol{r}_1 \times \boldsymbol{r}_2|^2}$$

Weissinger method: extension to consider also the tail plane



Weissinger method: extension to consider also the tail plane

- The total number of panel is $2M_w \times N_w + 2M_t \times N_t$
- Horse vortices are also associated to the tail plane panels
- $2M_w \times N_w + 2M_t \times N_t$ unknowns: $\mathbf{\Gamma}^w$ and $\mathbf{\Gamma}^t$
- The unknowns are evaluated by imposing no-penetration boundary condition on all the panels' control points
- For the evaluation of α_i also the contribution of the tail plane wake is considered

Weissinger method: extension to consider also the tail plane

The velocity field is expressed as:

$$\boldsymbol{u}(x,y) = \underbrace{\boldsymbol{U}_\infty}_{\text{Uniform flow}} + \underbrace{\sum_{j=1}^{2M_w \times N_w} \Gamma_j^w \boldsymbol{q}_j^w(x,y)}_{\text{Wing contribution}} + \underbrace{\sum_{j=1}^{2M_t \times N_t} \Gamma_j^t \boldsymbol{q}_j^t(x,y)}_{\text{Tail plane contribution}}$$

and the $2M_w \times N_w + 2M_t \times N_t$ unknowns are obtained by imposing:

$$\begin{cases} \boldsymbol{u}(x_{c,i}^w, y_{c,i}^w) = \boldsymbol{b}_i^w & i = 1,..,2M_w \times N_w \\ \boldsymbol{u}(x_{c,i}^t, y_{c,i}^t) = \boldsymbol{b}_i^t & i = 1,..,2M_t \times N_t \end{cases}$$

Weissinger method: extension to consider also the tail plane

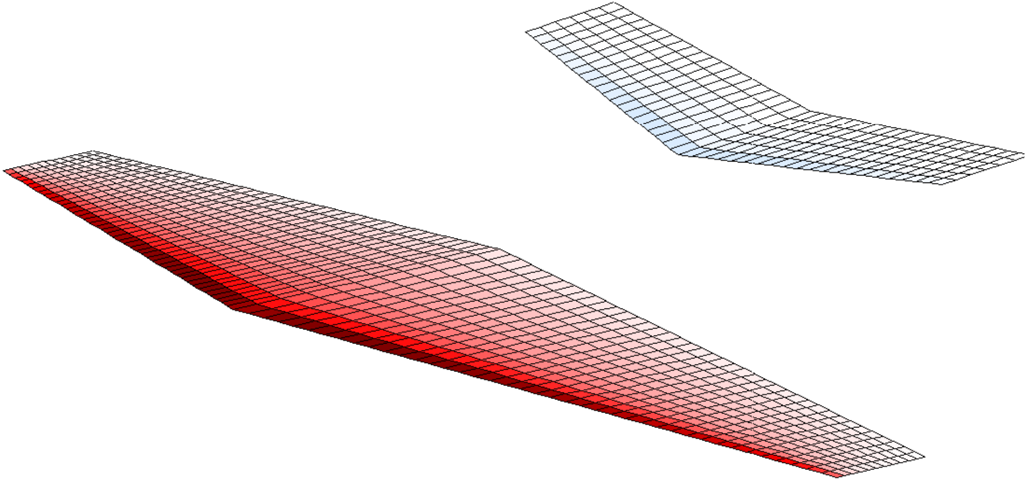
It leads to the following linear system:

$$\begin{bmatrix} \boldsymbol{A}_{ww} & \boldsymbol{A}_{tw} \\ \boldsymbol{A}_{wt} & \boldsymbol{A}_{tt} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Gamma}^{w} \\ \boldsymbol{\Gamma}^{t} \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}^{w} \\ \boldsymbol{b}^{t} \end{bmatrix}$$

where:

$$\begin{aligned}
A_{ww,ij} &= \boldsymbol{q}_j^w(x_{c,i}^w, y_{c,i}^w) \cdot \boldsymbol{n}_i^w \quad i = 1,..,2M_w \times N_w \;\; \& \;\; j = 1,..,2M_w \times N_w \\
A_{tw,ij} &= \boldsymbol{q}_j^t(x_{c,i}^w, y_{c,i}^w) \cdot \boldsymbol{n}_i^w \quad i = 1,..,2M_w \times N_w \;\; \& \;\; j = 1,..,2M_t \times N_t \\
A_{wt,ij} &= \boldsymbol{q}_j^w(x_{c,i}^t, y_{c,i}^t) \cdot \boldsymbol{n}_i^t \quad i = 1,..,2M_t \times N_t \;\; \& \;\; j = 1,..,2M_w \times N_w \\
A_{tt,ij} &= \boldsymbol{q}_j^t(x_{c,i}^t, y_{c,i}^t) \cdot \boldsymbol{n}_i^t \quad i = 1,..,2M_t \times N_t \;\; \& \;\; j = 1,..,2M_t \times N_t \\
b_i^w &= -\boldsymbol{U}_\infty \cdot \boldsymbol{n}_i^w \quad i = 1,..,2M_w \times N_w \\
b_i^t &= -\boldsymbol{U}_\infty \cdot \boldsymbol{n}_i^t \quad i = 1,..,2M_t \times N_t
\end{aligned}$$

Wing/tail plane interaction: circulation

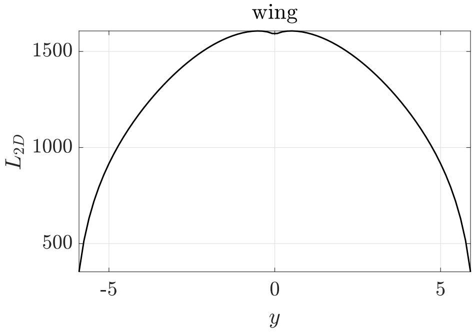


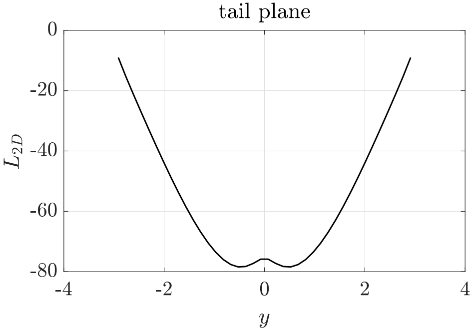
Wing/tail plane interaction: aerodynamic loads

For the computation of the aerodynamic loads the procedure is as for the single wing configuration, but in this case both the contributions of both the wing and the tail-plane have to be considered.

PS: to evaluate the induced angle of incidence remember to use only the wing's and tail's wake vortices contribution.

Wing/tail plane interaction: aerodynamic loads





Example: effect of the tilt angle

$$d_w = -5^\circ \quad b_{w,span} = 12 \quad c_{w,root} = 3 \quad \frac{c_{w,root}}{c_{w,tip}} = 3 \quad f_w = 15^\circ \quad z_t = 0.5 \quad x_t = 5$$

$$d_t = 5^\circ \quad b_{t,span} = 6 \quad c_{t,root} = 1.5 \quad \frac{c_{t,root}}{c_{t,tip}} = 1.5 \quad f_t = 10^\circ$$