Ground Effect

Alessandro Chiarini, Andrea Rausa

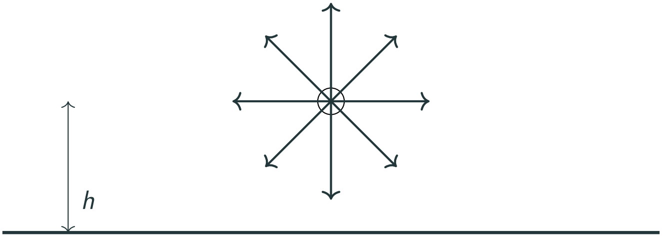
3 December 2021, Milano, Italy

Politecnico di Milano

Ground Effect

Method of images

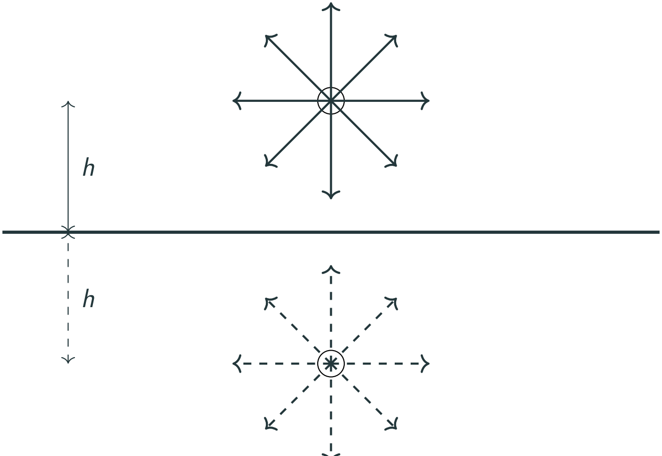
Consider for example a point-source of fluid placed at the position (0,h) (Cartesian coordinates) near a solid wall at $y = 0$.



Which is the velocity it induces? How can we consider the no-penetration boundary condition at the wall?

Method of images: source

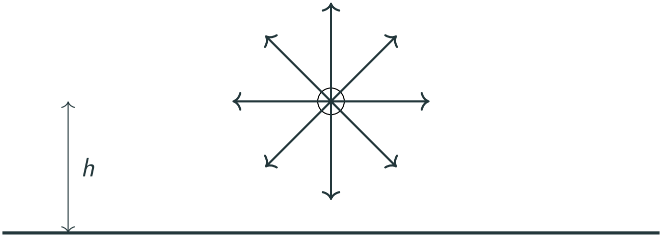
To cope with this problem we add a source of equal strength outside the boundary at $(0, -h)$. By symmetry, this source will produce an equal but opposite velocity field at $y = 0$, so that the boundary condition for the combined flow can be satisfied.



Method of images: source

Therefore the flow field in the (x, y) space is obtained by considering the contributions of both sources with intensity Q, i.e.

$$u(x, y) = Q \cdot (q_s(x, y) + q_s^m(x, y))$$



and you can easily demonstrate that $v(x, 0) = 0$. (Recall that you can only impose no-penetration boundary condition at the wall)

Example: No ground effect

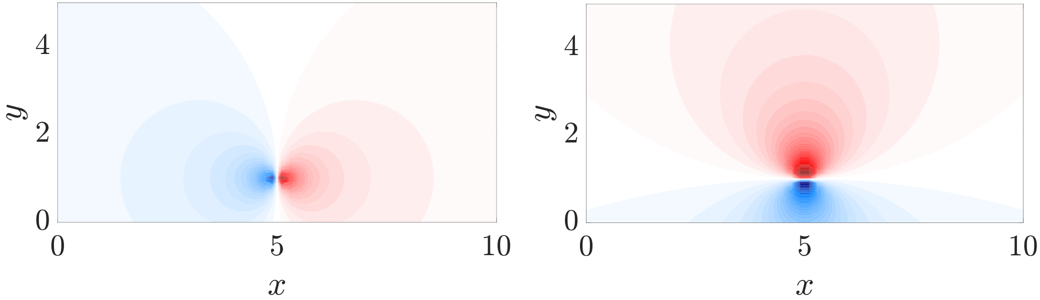


Figure 1: Left: U. Right: V.

Example: ground effect

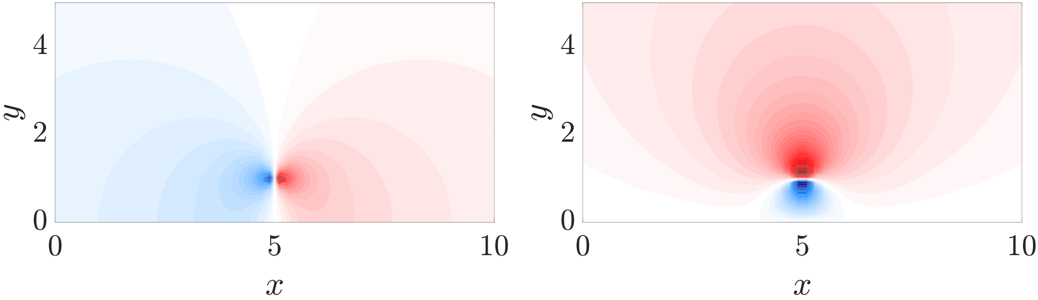
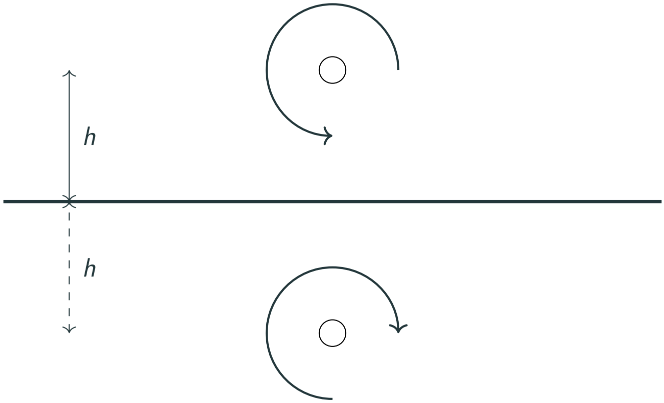


Figure 2: Left: U. Right: V.

Method of images: vortex

If we consider a vortex, to cope with this problem we add a vortex of opposite strength outside the boundary at $(0, -h)$. By symmetry, this source will produce an equal but opposite velocity field at $y = 0$, so that the boundary condition for the combined flow can be satisfied.



Example: No ground

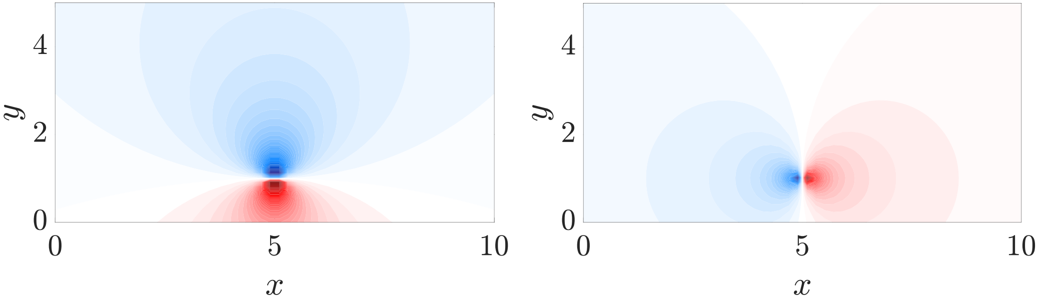


Figure 3: Left: U. Right: V.

Example: ground effect

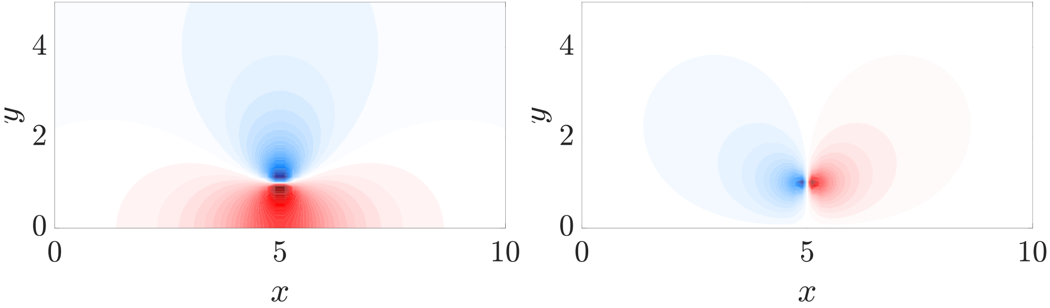


Figure 4: Left: U. Right: V.

Example: vortex + source, No ground

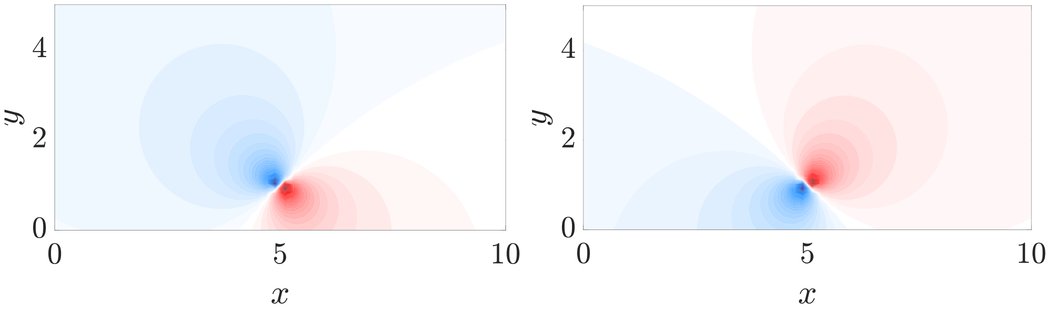


Figure 5: Left: U. Right: V.

Example: vortex + source, ground

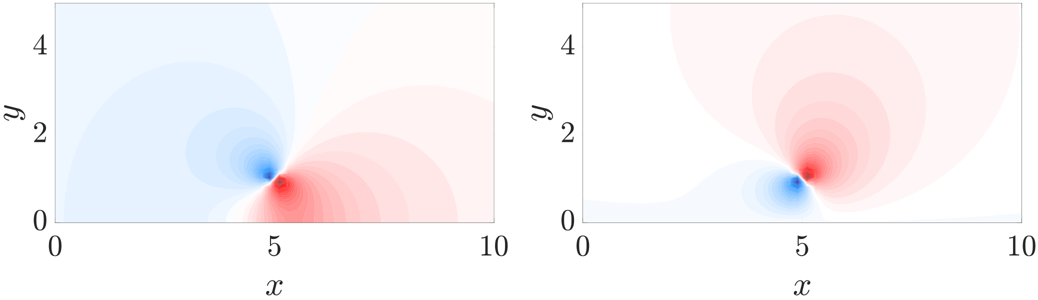


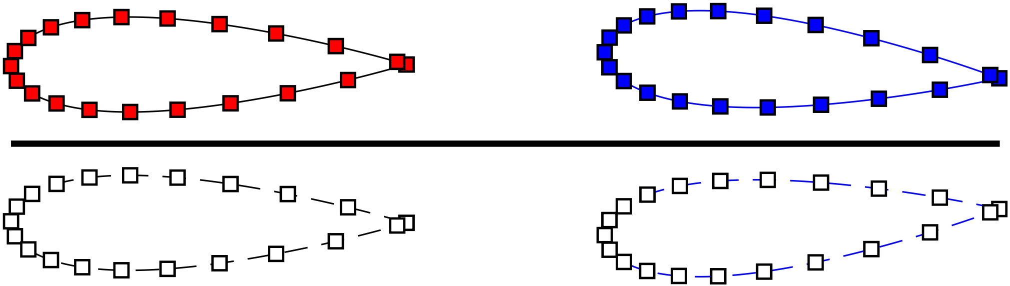
Figure 6: Left: U. Right: V.

For airfoil(s)

We impose the method of images on the Hess-smith method.

Method of images + Hess-Smith

We add image airfoils outside the domain for $y < 0$ with same distribution of sources and opposite distribution of vortices.



Method of images + Hess-Smith

The overall potential is:

$$\phi = \phi_\infty + \underbrace{\phi_s + \phi_v}_{\text{real airfoils}} + \underbrace{\phi_s^m + \phi_v^m}_{\text{mirror airfoils}}$$

where

- The vortex strength in the image airfoil is the opposite of the vortex strength in the real airfoil
- The source strength of each panel of the mirror airfoil is the same of the source strength on the corresponding panel of the real airfoil

Image method + Hess-Smith

Therefore, the velocity of the flow (considering one airfoil) is expressed as

$$\boldsymbol{u}(x,y) = \underbrace{\boldsymbol{U}_\infty}_{\text{Uniform flow}} + \underbrace{\sum_{j=1}^{N} \boldsymbol{u}_j^s(x,y) q_j}_{\text{real airfoil}} + \underbrace{\gamma \sum_{j=1}^{N} \boldsymbol{u}_j^v(x,y)}_{\text{real airfoil}} + \underbrace{\sum_{j=1}^{N} \boldsymbol{u}_{m,j}^s(x,y) q_j}_{\text{mirror airfoil}} - \underbrace{\gamma \sum_{j=1}^{N} \boldsymbol{u}_{m,j}^v(x,y)}_{\text{mirror airfoil}}$$

Therefore the unknowns are again the N source strengths $\boldsymbol{q}$ + 1 the vortex strength γ.
The $N+1$ unknowns are evaluated as usual by imposing

$$\begin{cases} \boldsymbol{u} \cdot \boldsymbol{n} = \boldsymbol{b}_n \text{ at the control point of every panel of the real airfoil} \\ \text{Kutta condition at the trailing edge} \end{cases}$$

Image method + Hess-Smith

Therefore the linear system that has to be solved is modified:

$$\begin{bmatrix} \boldsymbol{A}^s & \boldsymbol{a}^v \\ (\boldsymbol{c}^s)^T & c^v \end{bmatrix} \begin{bmatrix} \boldsymbol{q} \\ \gamma \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}^s \\ b^v \end{bmatrix}$$

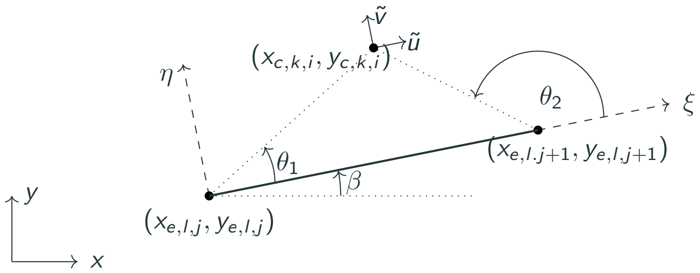
$$A^s_{ij} = \left(\boldsymbol{u}^s_j(x_{c,i}, y_{c,i}) + \boldsymbol{u}^s_{m,j}(x_{c,i}, y_{c,i})\right) \cdot \boldsymbol{n}_i \qquad a^v_i = \left(\sum_{j=1}^{N} \boldsymbol{u}^v_j(x_{c,i}, y_{c,i}) - \sum_{j=1}^{N} \boldsymbol{u}^v_{m,j}(x_{c,i}, y_{c,i})\right) \cdot \boldsymbol{n}_i$$

$$c^s_j = \left(\boldsymbol{u}^s_j(x_{c,1}, y_{c,1}) + \boldsymbol{u}^s_{m,j}(x_{c,1}, y_{c,1})\right) \cdot \boldsymbol{\tau}_1 + \left(\boldsymbol{u}^s_j(x_{c,N}, y_{c,N}) + \boldsymbol{u}^s_{m,j}(x_{c,N}, y_{c,N})\right) \cdot \boldsymbol{\tau}_N$$

$$c^v = \sum_{j=1}^{N} \left(\boldsymbol{u}^v_j(x_{c,1}, y_{c,1}) \cdot \boldsymbol{\tau_1} + \boldsymbol{u}^v_j(x_{c,N}, y_{c,N}) \cdot \boldsymbol{\tau_N}\right) - \left[\sum_{j=1}^{N} \left(\boldsymbol{u}^v_{m,j}(x_{c,1}, y_{c,1}) \cdot \boldsymbol{\tau_1} + \boldsymbol{u}^v_{m,j}(x_{c,N}, y_{c,N}) \cdot \boldsymbol{\tau_N}\right)\right]$$

Image method + Hess-Smith

$\boldsymbol{u}^s_j(x_{c,i}, y_{c,i})$, $\boldsymbol{u}^s_{m,j}(x_{c,i}, y_{c,i})$ ($\boldsymbol{u}^v_j(x_{c,i}, y_{c,i})$, $\boldsymbol{u}^v_{m,j}(x_{c,i}, y_{c,i})$) refer to the velocity a unitary constant source (vortex) distribution on the $j-$th panel of the real and image (with the m subscript) airfoils induce on the $i-$th panel control point of real airfoil.



The non penetration boundary condition must be imposed only on the control points of the real airfoil.

Image method + Hess-Smith

This is easily extended to the case of more than one airfoil!

Example: No ground

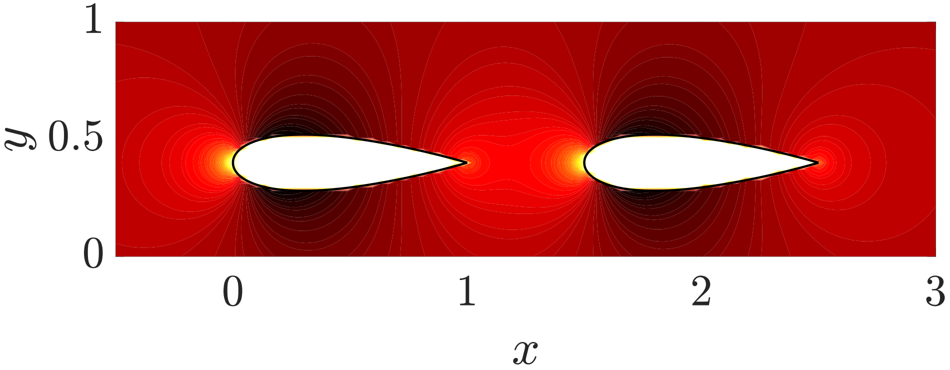


Figure 7: U

Example: No ground

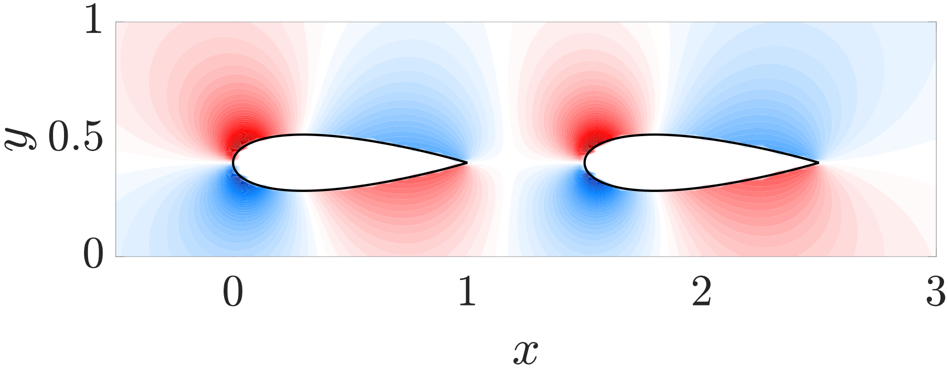


Figure 8: *V*

Example: ground

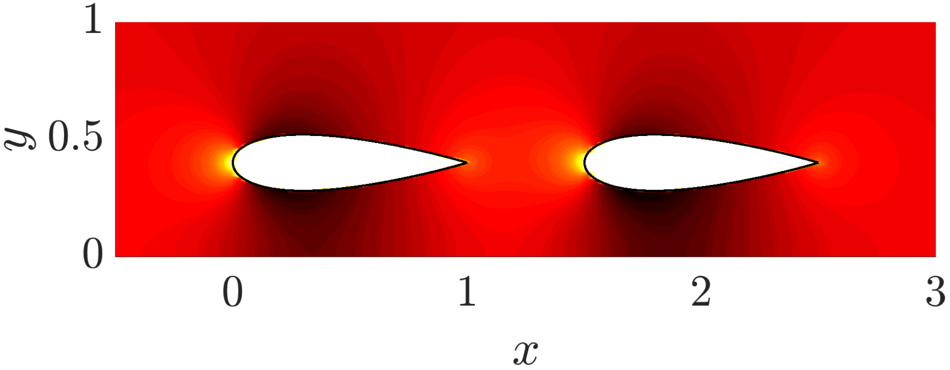


Figure 9: *U*

Example: ground

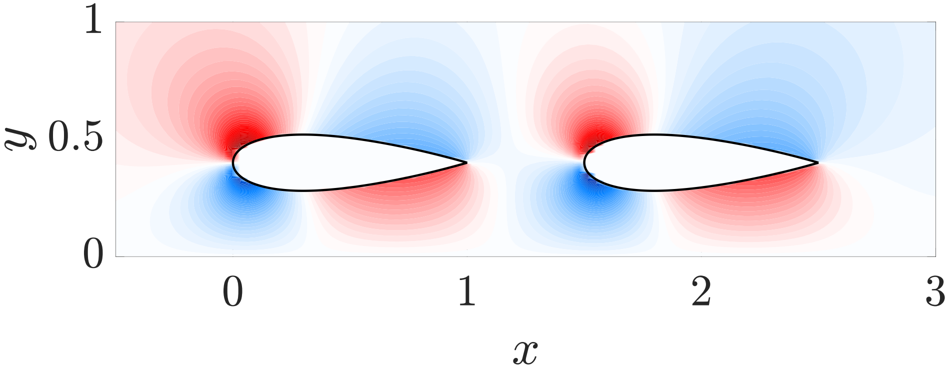


Figure 10: V

Example

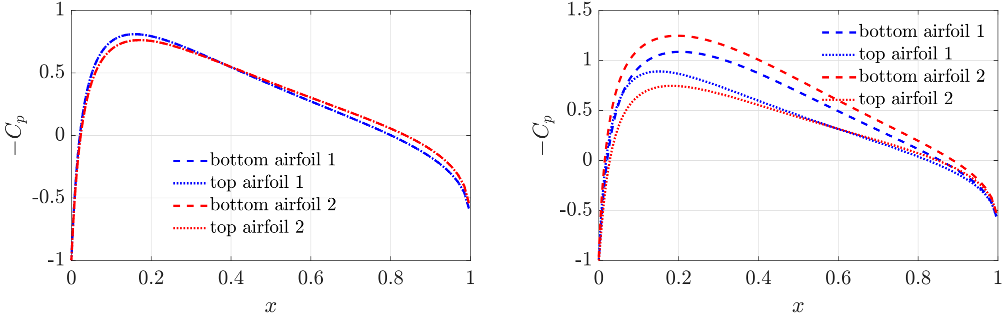
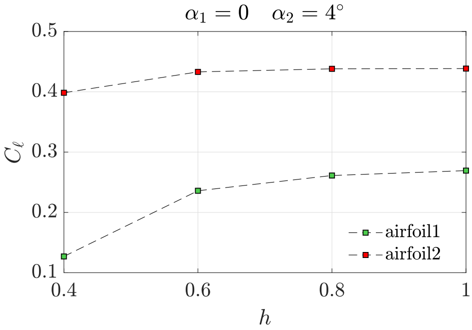
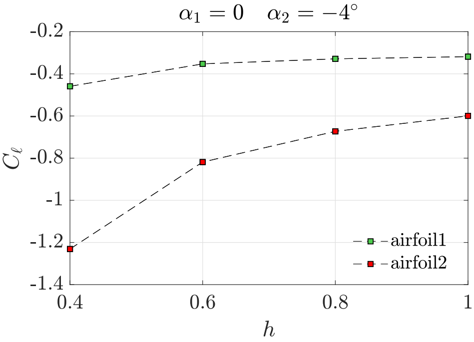


Figure 11: Left: No ground. Right: ground

Example



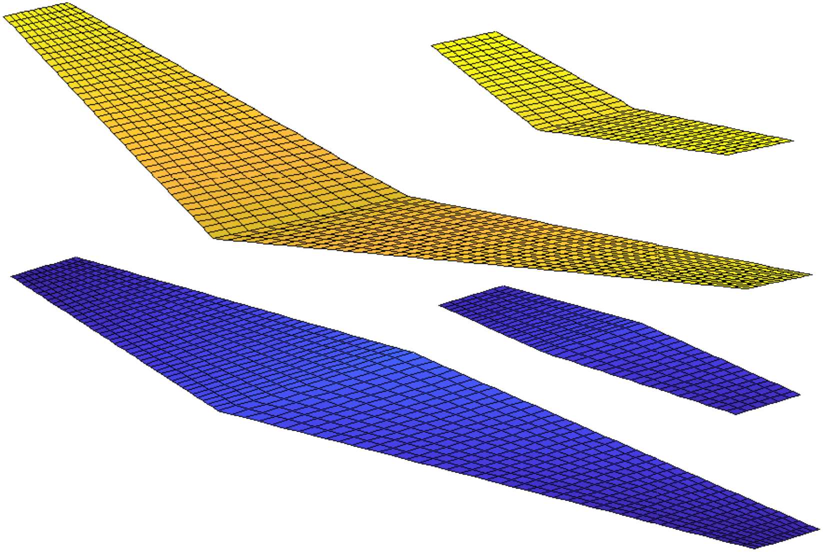
$\alpha_1 = 0 \quad \alpha_2 = 4^\circ$
0.5
0.4
0.3
0.2
0.1
C_ℓ
0.4
0.6
0.8
1
h
airfoil1
airfoil2



$\alpha_1 = 0 \quad \alpha_2 = -4^\circ$
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
C_ℓ
0.4
0.6
0.8
1
h
airfoil1
airfoil2

Method of images + Weissinger

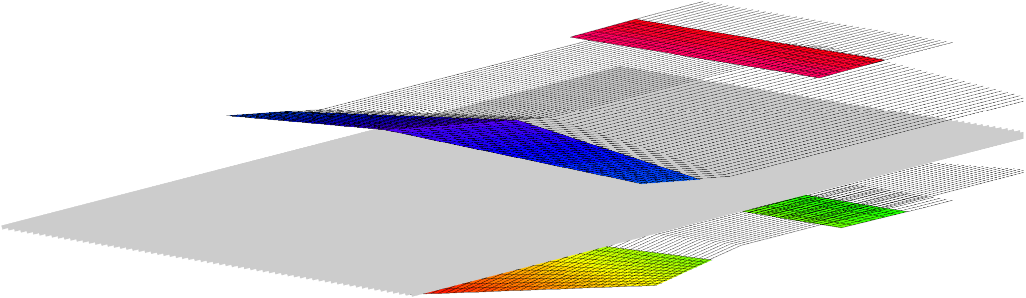
Method of images + Weissinger



Method of images + Weissinger

- We add a mirror wing with same panelisation as the real wing
- A horse vortex of circulation $-\Gamma_j$ is associated to the $j-$th panel of the mirror wing, where Γ_j is the circulation of the horse vortex associated to the correspondent panel in the real wing
- The overall velocity field is given by the combination of both the contribution of the real and mirror wings added to the uniform flow.
- Therefore the unknown are still $2M \times N$ (i.e. the number of the panels in the real wing)
- The unknowns Γ_j are determined by imposing the no-penetration boundary condition on the control points of all the real wing's panels.

Method of images + Weissinger



Method of images + Weissinger

The velocity field (considering only one wing) is expressed as:

$$\boldsymbol{u}(x,y) = \underbrace{\boldsymbol{U}_\infty}_{\text{Uniform flow}} + \underbrace{\sum_{j=1}^{2M\times N} \Gamma_j \boldsymbol{q}_j(x,y)}_{\text{Real wing}} - \underbrace{\sum_{j=1}^{2M\times N} \Gamma_j \boldsymbol{q}_{m,j}(x,y)}_{\text{Mirror wing}}.$$

Therefore the unknowns are again the $2M \times N$ circulations Γ_j associated to the wing's panels. They are obtained by imposing:

$$\boldsymbol{u}(x_{c,i}, y_{c,i}) \cdot \boldsymbol{n}_i = \boldsymbol{b}_i \quad i = 1, .., 2M \times N.$$

Method of images + Weissinger

It leads to the following linear system:

$$A\Gamma = \boldsymbol{b}$$

where:

$$A_{ij} = \left(\boldsymbol{q_j}(x_{c,i}, y_{c,i}) - \boldsymbol{q}_{m,j}(x_{c,i}, y_{c,i})\right) \cdot \boldsymbol{n}_i \qquad \boldsymbol{b}_i = -\boldsymbol{U}_\infty \cdot \boldsymbol{n}_i$$

Recall that (see slides of the previous lesson):

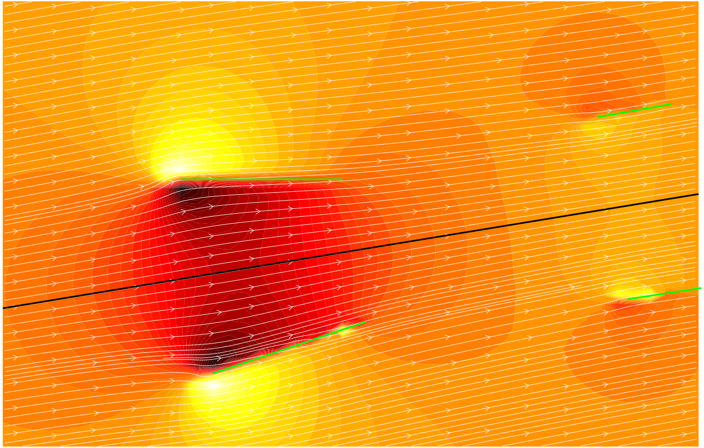
$$\boldsymbol{q}_j(x_{c,i}, y_{c,i}) = \frac{1}{4\pi}\boldsymbol{r}_0 \cdot \left(\frac{\boldsymbol{r}_1}{|\boldsymbol{r}_1|} - \frac{\boldsymbol{r}_2}{|\boldsymbol{r}_2|}\right) \frac{\boldsymbol{r}_1 \times \boldsymbol{r}_2}{|\boldsymbol{r}_1 \times \boldsymbol{r}_2|^2}$$

Method of images + Weissinger

It can be easily expanded to the cases with more than one wing

Method of images + Weissinger

It works also in the 3D case!



Method of images + Weissinger

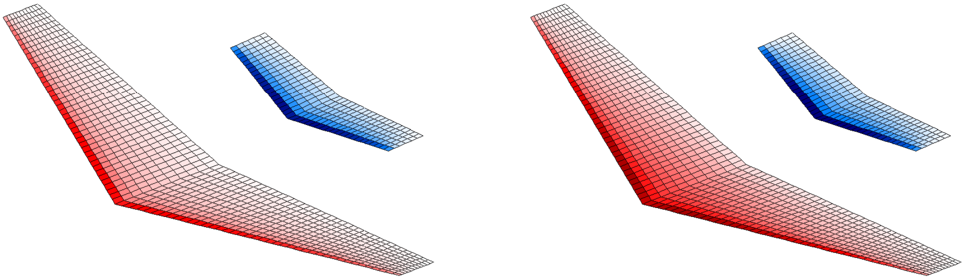


Figure 12: Γ distribution. $d = 5^\circ$, $\alpha = 1^\circ$, $b_{span} = 15$, $c_{root} = 3$, $c_{root}/c_{tip} = 3$, $f = 15^\circ$, $z_t = 0.5$, $x_t = 5$, $c_{root,t} = 1.5$, $c_{root,t}/c_{tip,t} = 1.5$, $b_{span,t} = 6$, $f_t = 10^\circ$, $d_t = 5^\circ$, $\epsilon_t = 3^\circ$. Left: no ground. Right: ground $h = 0.4$.

Method of images + Weissinger

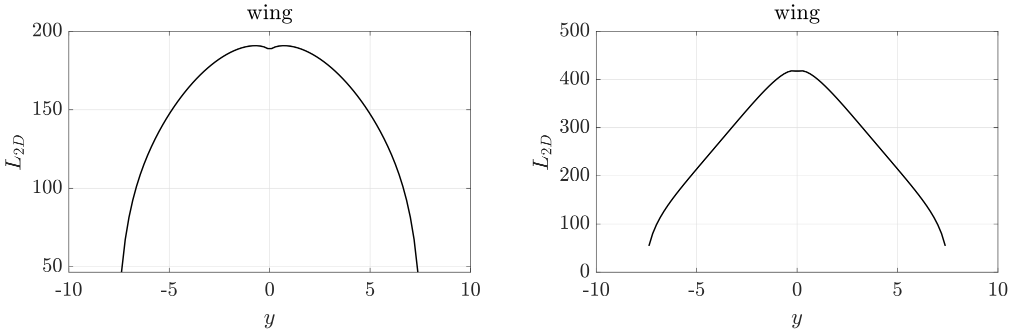
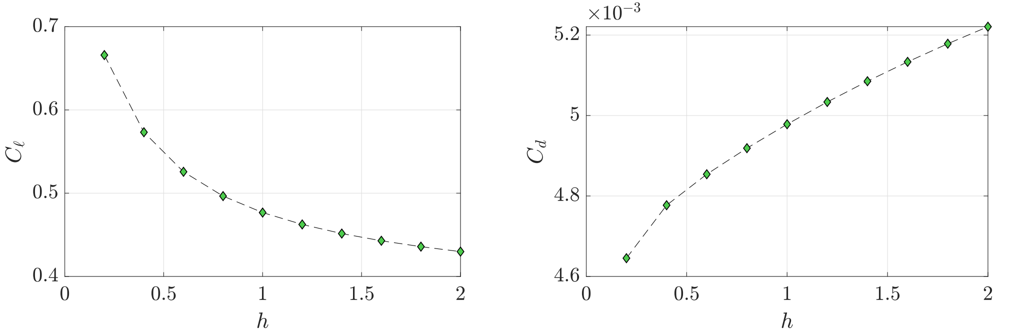


Figure 13: Left: no ground. Right: ground $h = 0.4$.

Which is the effect of the ground on the wing performances?

Method of images + Weissinger

$d = 5^\circ$, $\alpha = 1^\circ$, $b_{span} = 15$, $c_{root} = 3$, $c_{root}/c_{tip} = 3$, $f = 15^\circ$, $z_t = 0.5$, $x_t = 5$, $c_{root,t} = 1.5$, $c_{root,t}/c_{tip,t} = 1.5$, $b_{span,t} = 6$, $f_t = 10^\circ$, $d_t = 5^\circ$, $\epsilon_t = -1^\circ$.



Note that the C_ℓ has a different dependence on h compared to the $2D$ case ...