Moist convection

- So far: Rayleigh-Bénard convection
  - Dry atmosphere
  - No heat sources except at the boundaries

- Exercise 9: Rainy-Bénard convection
  - Moist atmosphere
  - Additional prognostic equation for water vapor
  - Condensation produces latent heat which influences temperature

New prognostic equations (dimensional)

Water vapor

$$\frac{\partial q}{\partial t} + \vec{v} \cdot \nabla q = \kappa_q \nabla^2 q - C$$

- q: Specific humidity
- $\vec{v}$: Velocity vector (u,v)
- κ_q: Diffusion coefficient for water vapor
- C: Condensation

Temperature

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa \nabla^2 T - \frac{g}{c_p} v + \frac{L}{c_p} C$$

Dry adiabatic lapse rate   Latent heating

- T: Temperature
- κ: Diffusion coefficient for temperature
- g: Gravitational acceleration
- c_p: Heat capacity of dry air at constant pressure
- L: Latent heat of vaporization

Condensation

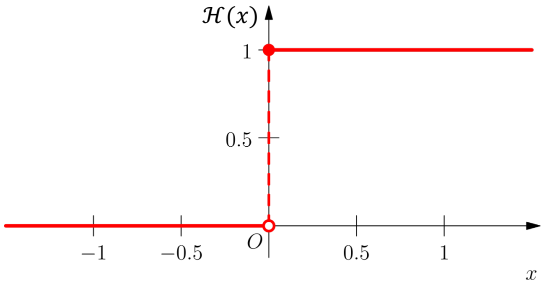
Saturation adjustment: q is reduced to its saturated value and excess vapor is converted to cloud liquid water:

$$C = \frac{q - q_s}{\tau} \mathcal{H}(q - q_s)$$

Saturation specific humidity follows Clausius-Clapeyron relation:

$$q_s \approx q_0 \cdot e^{\alpha T}$$

q: Specific humidity
q_s: Saturation specific humidity
τ: Time scale for condensation
$\mathcal{H}$: Heaviside step function



q_0: Reference specific humidity
α: Scaling factor ($\alpha = L/(R_v T_0^2)$)
T: Temperature
T_0: Reference temperature
R_v: Gas constant for water vapor

Nondimensionalize

new

$$\tilde{x} = \frac{x}{H} \qquad \tilde{T} = \frac{T}{\Delta T} \qquad \tilde{t} = \frac{t}{H^2/\kappa} \qquad \tilde{u} = \frac{u}{\kappa/H} \qquad \tilde{q} = \frac{q}{q_0}$$

$$\tilde{\tau} = \frac{\tau}{H^2/\kappa} \qquad \tilde{v} = \frac{v}{\kappa/H} \qquad \tilde{q}_s = \frac{q_s}{q_0}$$

H: Height of the model domain
ΔT: Temperature difference
κ: Diffusion coefficient
q_0: Reference specific humidity

Water vapor equation

$$\frac{q_0\kappa}{H^2}\left(\frac{\partial q}{\partial t}+\vec{v}\cdot\nabla q\right)=\frac{q_0\kappa_q}{H^2}\nabla^2 q-\frac{q_0\kappa}{H^2}\cdot\frac{q-q_s}{\tau}\mathcal{H}(q-q_s)\quad\Bigg|\cdot\frac{H^2}{q_0\kappa}$$

$$\frac{\partial q}{\partial t}+\vec{v}\cdot\nabla q=\underbrace{\frac{\kappa_q}{\kappa}}_{S_m}\nabla^2 q-\frac{q-q_s}{\tau}\mathcal{H}(q-q_s)$$

Temperature equation

$$\frac{\kappa \Delta T}{H^2}\left(\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T\right) = \frac{\kappa \Delta T}{H^2} \nabla^2 T - \frac{g\kappa}{c_p H} v + \frac{L q_0 \kappa}{c_p H^2} \cdot \frac{q - q_s}{\tau} \mathcal{H}(q - q_s) \quad \Bigg| \cdot \frac{H^2}{\kappa \Delta T}$$

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \nabla^2 T - \underbrace{\frac{gH}{c_p \Delta T}}_{\Gamma} v + \underbrace{\frac{L q_0}{c_p \Delta T}}_{\lambda} \cdot \frac{q - q_s}{\tau} \mathcal{H}(q - q_s)$$

New prognostic equations (dimensionless)

Water vapor

$$\frac{\partial q}{\partial t} + \vec{v} \cdot \nabla q = S_m \nabla^2 q - \frac{q - q_s}{\tau} \mathcal{H}(q - q_s)$$

Temperature

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \nabla^2 T - \Gamma v + \lambda \frac{q - q_s}{\tau} \mathcal{H}(q - q_s)$$

Saturation specific humidity

$$q_s = e^{\alpha T}$$

New dimensionless numbers

q: Specific humidity

$\vec{v}$: Velocity vector (u,v)

S_m: Ratio of diffusivities ($S_m = \kappa_q / \kappa = 1.33$)

κ: Diffusion coefficient for temperature

κ_q: Diffusion coefficient for water vapor

q_s: Saturation specific humidity

τ: Time scale for condensation

$\mathcal{H}$: Heaviside step function

T: Temperature

Γ: Lapse rate ($\Gamma = gH/(c_p \Delta T)$)

λ: Latent heating ($\lambda = Lq_0/(c_p \Delta T)$)

α: Scaling factor for saturation specific humidity ($\alpha = L\Delta T/(R_v T_0^2)$)

Exercise 9

Adapt your Fortran program from Exercise 8 such that it solves the (dimensionless) equations for the advection and condensation of water vapor:

(1) $$\frac{\partial q}{\partial t} + \vec{v} \cdot \nabla q = S_m \nabla^2 q - \frac{q - q_s}{\tau} \mathcal{H}(q - q_s)$$

(2) $$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \nabla^2 T - \Gamma v + \lambda \frac{q - q_s}{\tau} \mathcal{H}(q - q_s)$$

Exercise 9

1. Read in the input variables from a namelist:
   - Number of grid points `nx` and `ny`
   - Integration time `total_time`
   - Time step constants `a_adv` and `a_diff`
   - Convergence criterion for Poisson solver `max_err`
   - Rayleigh number `Ra`
   - Temperature initialization `T_ini_type`: `random` or `cosine`
   - Prandtl number `Pr`
   - NEW: Exponential scaling factor for saturation specific humidity `alpha`
   - NEW: Dry adiabatic lapse rate `gamma`
   - NEW: Latent heating rate `lambda`
   - NEW: Time scale for condensation `tau`

Exercise 9

2. Initialize the variables:
   - Temperature `T` with random numbers or a cosine function
   - Stream function and vorticity `S=W=0` (only at the beginning)
   - Specific humidity `Q` with `0.8*Qsat` (80% relative humidity)
   - Grid spacing `h=1./(ny-1.)`

3. Perform several time steps until the integration time is reached:
   - Determine ψ from ω using the Poisson solver (Equation 2)
   - Compute the wind speeds u and v from ψ
   - Compute the time step from a_adv, a_diff and the maximum wind speed in the model domain: $\Delta t = \min\left(a_{diff} \cdot \frac{h^2}{\max(1,\mathrm{Pr})}, a_{adv} \cdot \frac{h}{v_{max}}\right)$
   - Compute $\mathrm{Ra} \cdot \partial T/dx$

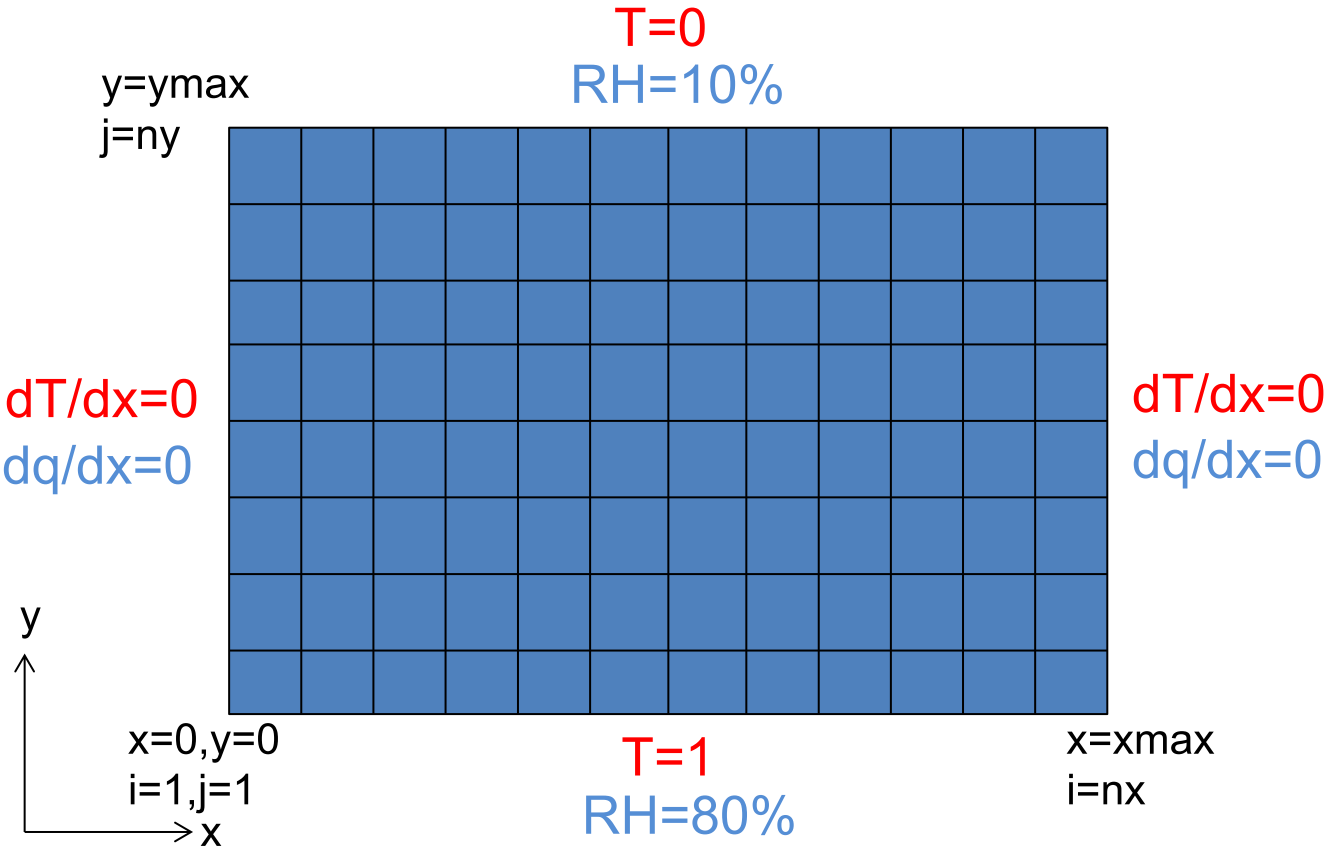
Exercise 9

- Compute the advection and diffusion terms of temperature, vorticity, and specific humidity, and integrate forward in time (no condensation yet):
    - $q_{i,j}^{n+1} = q_{i,j}^{n} +$ ... (Equation 1)
    - $T_{i,j}^{n+1} = T_{i,j}^{n} +$ ... (Equation 2)
    - $\omega_{i,j}^{n+1} = \omega_{i,j}^{n} +$ ... (Exercise 8)
    - $t = t + \Delta t$
- Diagnose condensation:
    - $C = \frac{q-q_s}{\tau}\mathcal{H}(q - q_s)$
    - $T_{i,j}^{n+1} = T_{i,j}^{n+1} + \lambda C$
    - $q_{i,j}^{n+1} = q_{i,j}^{n+1} - C$

Exercise 9

ψ and ω: 0 at all boundaries

Boundary conditions for T and q:



Exercise 9

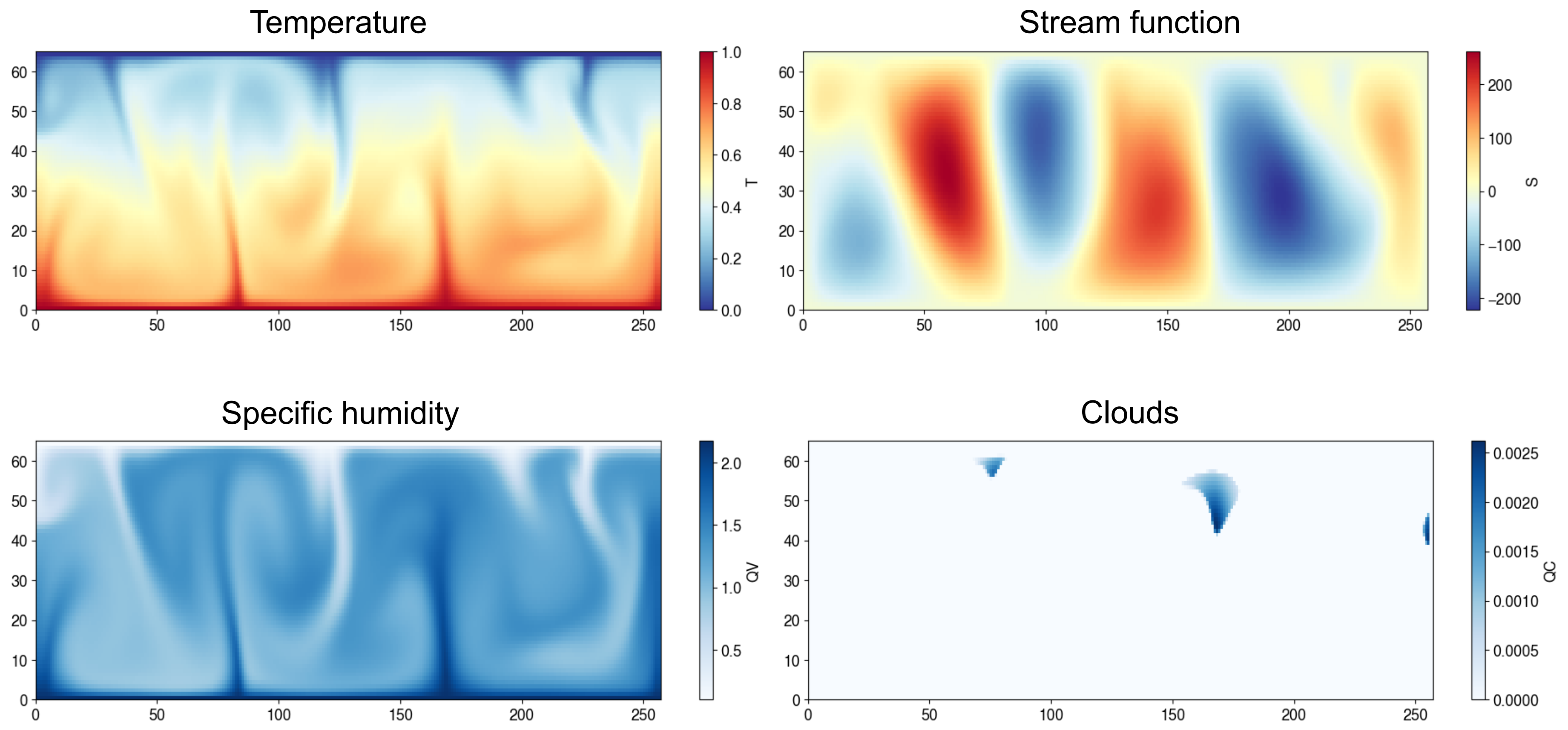
4. Write out the variables T, q, and C (optionally also ψ and ω) to a NetCDF file and plot them with your favorite plotting tool.

Example namelist:

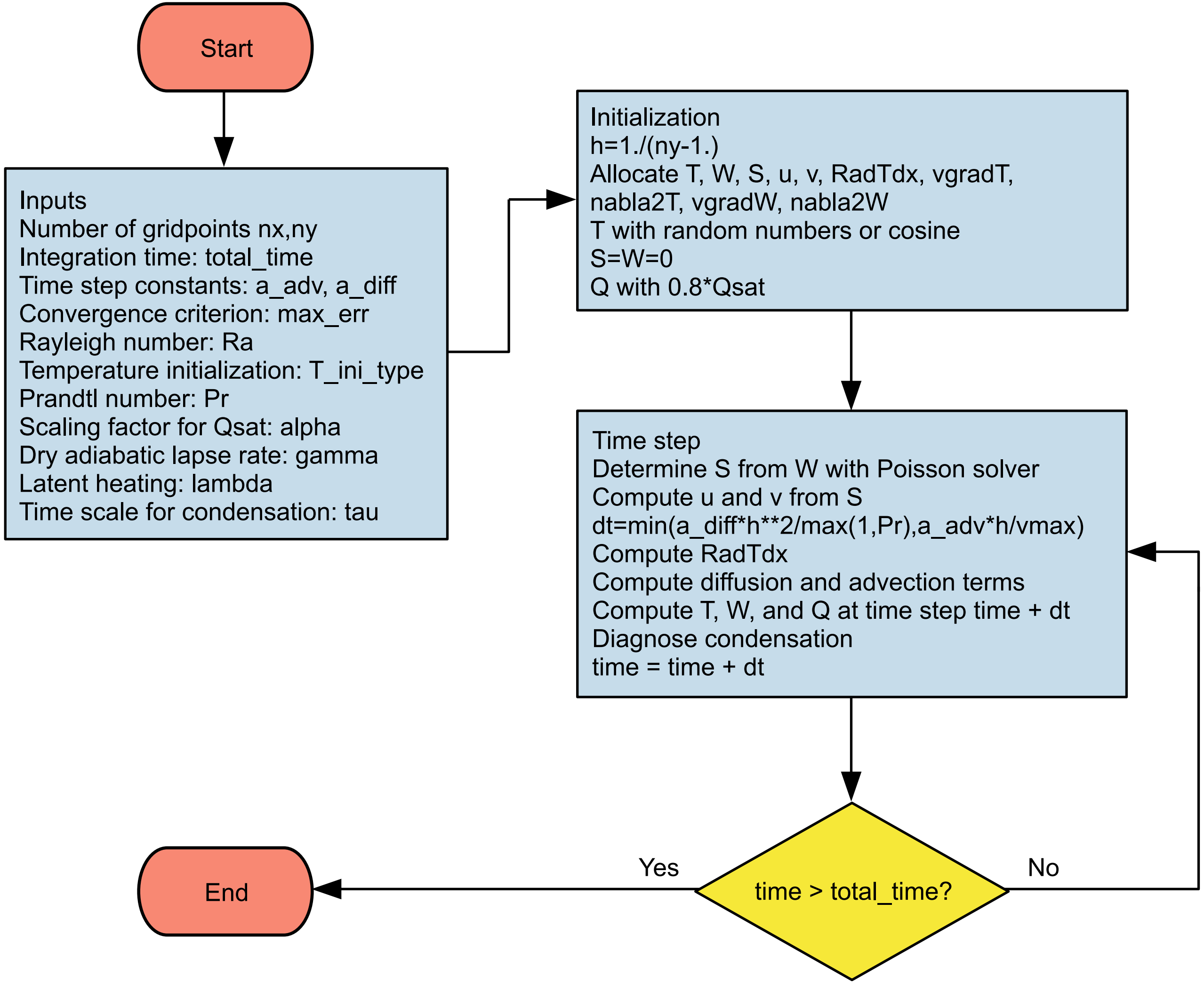
```
&INPUTS
  Pr=0.7
  nx=257
  ny=65
  a_diff=0.15
  a_adv=0.4
  total_time=0.1
  max_err=1.E-3
  Ra=1.E7
  T_ini_type='cosine'
  alpha=1.0
  gamma=0.5
  lambda=0.2
  tau=2.0
/
```

Resulting temperature, flow, and humidity fields

Time=0.01



Flowchart for Exercise 9



Exercise 9

Deadline:

- Please hand in your solutions (.f90 files and plots) no later than Tuesday, **11 June 2024, 23:59**.

Questions?

- Email me (marina.duetsch@univie.ac.at)
  Or pass by my office (UZA II, 2G551).