Exercise 9 – Moist convection

Adapt your Fortran program from Exercise 8 such that it solves the (dimensionless) equations for the advection and condensation of water vapor:

$$\frac{\partial q}{\partial t} + \vec{v} \cdot \nabla q = S_m \nabla^2 q - \frac{q - q_s}{\tau} \mathcal{H}(q - q_s) \tag{1}$$

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \nabla^2 T - \Gamma v + \lambda \frac{q - q_s}{\tau} \mathcal{H}(q - q_s) \tag{2}$$

where q is specific humidity, q_s is saturation specific humidity, S_m is the ratio of diffusivities ($S_m = \kappa_q / \kappa \approx 1.33$), Γ is the dry adiabatic lapse rate, λ is latent heating, τ is the time scale for condensation, and $\mathcal{H}$ is the Heaviside step function:

$$\mathcal{H}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases} \tag{3}$$

The saturation specific humidity is a function of temperature, with the slope determined by the scaling factor α:

$$q_s = \exp(\alpha T) \tag{4}$$

1. Read in the control parameters from a namelist:
   - Number of grid points `nx` and `ny`
   - Integration time `total_time`
   - Time step constants `a_adv` and `a_diff`
   - Convergence criterion for Poisson solver `max_err`
   - Rayleigh number `Ra`
   - Temperature initialization `T_ini_type`: `random` or `cosine`
   - Prandtl number `Pr`
   - NEW: Exponential scaling factor for saturation specific humidity `alpha`
   - NEW: Dry adiabatic lapse rate `gamma`
   - NEW: Latent heating `lambda`
   - NEW: Time scale for condensation `tau`
2. Initialize the variables:
   - Temperature `T` with random numbers or a cosine function
   - Stream function and vorticity `S=W=0` (only at the beginning)
   - Specific humidity `Q` with `0.8*Qsat` (80% relative humidity)
   - Grid spacing `h=1./(ny-1.)`

3. Perform several time steps until the integration time is reached:

   - Determine ψ from ω using the Poisson solver (Exercise 8).
   - Compute the wind speeds u and v from ψ.
   - Compute the time step: $\Delta t = \min(a_{diff} \cdot \frac{h^2}{\max(1,\mathrm{Pr})}, a_{adv} \cdot \frac{h}{v_{max}})$
   - Compute $\mathrm{Ra} \cdot \partial T/\partial x$
   - Compute the advection and diffusion terms of temperature, vorticity, and specific humidity, and integrate forward in time (no condensation yet):
     - $T_{i,j}^{n+1} = T_{i,j}^{n} + \ldots$ (equation 2)
     - $\omega_{i,j}^{n+1} = \omega_{i,j}^{n} + \ldots$ (exercise 8)
     - $q_{i,j}^{n+1} = q_{i,j}^{n} + \ldots$ (equation 1)
     - $t = t + \Delta t$
   - Diagnose condensation:
     - $C = \frac{q-q_s}{\tau}\mathcal{H}(q-q_s)$
     - $T_{i,j}^{n+1} = T_{i,j}^{n+1} + \lambda C$
     - $q_{i,j}^{n+1} = q_{i,j}^{n+1} - C$
   - Boundary conditions:
     - T: as in Exercises 5, 7, and 8 ($T = 1$ bottom, $T = 0$ top, $\partial T/\partial x = 0$ left and right).
     - ψ and ω: 0 at all boundaries.
     - q: 80% relative humidity at the bottom, 10% relative humidity at the top, $\partial q/\partial x = 0$ left and right.

4. Write out the variables T, q, and C (optionally also ψ and ω) to a NetCDF file and plot them with your favorite plotting tool.

Example namelist

```
&INPUTS
  Pr=0.7
  nx=257
  ny=65
  a_diff=0.15
  a_adv=0.4
  total_time=0.1
  max_err=1.E-3
  Ra=1.E7
  T_ini_type='cosine'
  alpha=1.0
  gamma=0.5
  lambda=0.2
  tau=2.0
/
```

Deadline: Please hand in your solutions (`.f90` files and plots) by **Tuesday, 11 June 2024, 23:59**.