Exercise 8 – Dry convection (finite Prandtl number)

Adapt your Fortran program from Exercise 7 such that it solves the conservation equations for a fluid with a finite Prandtl number (e.g., the atmosphere):

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial\omega}{\partial t}+\vec{v}\cdot\nabla\omega\right)=\nabla^2\omega+\mathrm{Ra}\frac{\partial T}{\partial x} \tag{1}$$

$$\nabla^2\psi=-\omega \tag{2}$$

$$\frac{\partial T}{\partial t}+\vec{v}\cdot\nabla T=\nabla^2 T \tag{3}$$

where Pr is the Prandtl number, ω is the vorticity, Ra is the Rayleigh number, and ψ is the stream function.

1. Read in the control parameters from a namelist:
   - Number of grid points `nx` and `ny`.
   - Integration time `total_time`
   - Time step constants `a_adv` and `a_diff`
   - Convergence criterion for Poisson solver `max_err`
   - Rayleigh number `Ra`
   - Temperature initialization `T_ini_type`: `random` or `cosine`
   - NEW: Prandtl number `Pr`.
2. Initialize the variables:
   - Temperature `T` with random numbers or a cosine function.
   - Stream function and vorticity `S=W=0` (only at the beginning)
   - Grid spacing `h=1./(ny-1.)`
3. Perform several time steps until the integration time is reached:
   - Determine ψ from ω using the Poisson solver (equation 2).
   - Compute the wind speeds u and v from ψ.
   - Compute the time step: $\Delta t=\min(a_{diff}\cdot\frac{h^2}{\max(1.,\mathrm{Pr})}, a_{adv}\cdot\frac{h}{v_{max}})$
   - Compute $\mathrm{Ra}\cdot\partial T/\partial x$
   - Determine $\nabla^2 T^n_{i,j}$ and $\vec{v}_{i,j}\cdot\nabla T^n_{i,j}$ and $\nabla^2\omega^n_{i,j}$ and $\vec{v}_{i,j}\cdot\nabla\omega^n_{i,j}$ using the subroutines from Exercise 5.
   - Integrate forward in time:
     - $\omega^{n+1}_{i,j}=\omega^n_{i,j}+\ldots$ (equation 1)
     - $T^{n+1}_{i,j}=T^n_{i,j}+\ldots$ (equation 3)
     - $t=t+\Delta t$

- Boundary conditions:
  - T: as in Exercises 5 and 7 ($T = 1$ bottom, $T = 0$ top, $\partial T/\partial x = 0$ left and right).
  - ψ and ω: 0 at all boundaries.
- If the integration time has not yet been reached, perform the next time step.

4. Write a makefile and compile the program with `make`.
5. Test the program with different Prandtl numbers (between 0.001 and 10), and plot the resulting temperature fields.

Example namelist

```
&INPUTS
  Pr=0.1
  nx=257
  ny=65
  a_diff=0.15
  a_adv=0.4
  total_time=0.1
  max_err=1.E-3
  Ra=1.E6
  T_ini_type='cosine'
/
```

Deadline: Please hand in your solutions (`.f90` files and plots of `T`) by **Tuesday, 21 May 2024, 23:59**.