Exercise 7 – Dry convection (infinite Prandtl number)

The following dimensionless conservation equations describe convection in a fluid with high viscosity (e.g. Earth's mantle):

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \nabla^2 T \tag{1}$$

$$-\nabla^2 \omega = \mathrm{Ra} \cdot \frac{\partial T}{\partial x} \tag{2}$$

$$\nabla^2 \psi = -\omega \tag{3}$$

where ω is the vorticity, Ra is the Rayleigh number, and ψ is the stream function. Your task in this exercise is to write a Fortran program that solves these equations. For this purpose, combine the Poisson solver from Exercise 6 with the advection-diffusion model from Exercise 5.

1. Read in the control parameters from a namelist:
   - Number of grid points `nx` and `ny`.
   - Integration time `total_time`
   - Time step constants `a_adv` and `a_diff`
   - NEW: Convergence criterion for Poisson solver `max_err`.
   - NEW: Rayleigh number `Ra`.
   - NEW: Temperature initialization `T_ini_type`: `random` or `cosine`.
2. Initialize the variables:
   - Temperature `T` with random numbers or a cosine function.
     - Cosine e.g. $T(x) = 0.5 \cdot (1 + \cos(3\pi \cdot x/x_{max}))$
   - Stream function and vorticity `S=W=0` (only at the beginning)
   - Grid spacing `h=1./(ny-1.)`
3. Perform several time steps until the integration time is reached:
   - Compute $\mathrm{Ra} \cdot \partial T/\partial x$.
   - Determine ω from $\mathrm{Ra} \cdot \partial T/\partial x$ using the Poisson solver (equation 2).
   - Determine ψ from ω using the Poisson solver (equation 3).
   - Compute the wind velocities u and v from ψ.
   - Compute the time step from a_{adv}, a_{diff}, and the maximum wind speed in the model domain
   - Compute $\nabla^2 T^n_{i,j}$ and $\vec{v}_{i,j} \cdot \nabla T^n_{i,j}$ using the subroutines from Exercise 5.
   - Integrate forward in time:
     - $T^{n+1}_{i,j} = T^n_{i,j} + ...$ (equation 1, note without κ)
     - $t = t + \Delta t$

  - Boundary conditions:
    - T: as in Exercise 5 ($T = 1$ bottom, $T = 0$ top, $\partial T / \partial x = 0$ left and right).
    - ψ and ω: 0 at all boundaries
  - If the integration time has not yet been reached, perform the next time step.

4. Test the model with the following two namelists, and save the output in binary files.

```
&INPUTS
  nx=257
  ny=65
  a_diff=0.23
  a_adv=0.4
  total_time=0.1
  max_err=1.E-3
  Ra=1.E5
  T_ini_type='cosine'
/
```

```
&INPUTS
  nx=1025
  ny=65
  a_diff=0.23
  a_adv=0.4
  total_time=0.1
  max_err=1.E-3
  Ra=1.E5
  T_ini_type='random'
/
```

5. Plot the temperature fields with your favorite plotting tool.

Deadline: Please hand in your solutions (`.f90` files and plots of `T`) by **Tuesday, 14 May 2024, 23:59**.