

Exercise 7 – Dry convection (infinite Prandtl number)

The following dimensionless conservation equations describe convection in a fluid with high viscosity (e.g. Earth's mantle):

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \nabla^2 T \quad (1)$$

$$-\nabla^2 \omega = \text{Ra} \cdot \frac{\partial T}{\partial x} \quad (2)$$

$$\nabla^2 \psi = -\omega \quad (3)$$

where ω is the vorticity, Ra is the Rayleigh number, and ψ is the stream function. Your task in this exercise is to write a Fortran program that solves these equations. For this purpose, combine the Poisson solver from Exercise 6 with the advection-diffusion model from Exercise 5.

1. Read in the control parameters from a namelist:

- Number of grid points `nx` and `ny`.
- Integration time `total_time`
- Time step constants `a_adv` and `a_diff`
- **NEW**: Convergence criterion for Poisson solver `max_err`.
- **NEW**: Rayleigh number `Ra`.
- **NEW**: Temperature initialization `T_ini_type`: `random` or `cosine`.

2. Initialize the variables:

- Temperature `T` with random numbers or a cosine function.
 - Cosine e.g. $T(x) = 0.5 \cdot (1 + \cos(3\pi \cdot x/x_{max}))$
- Stream function and vorticity `S=W=0` (only at the beginning)
- Grid spacing `h=1./(ny-1.)`

3. Perform several time steps until the integration time is reached:

- Compute $\text{Ra} \cdot \partial T / \partial x$.
- Determine ω from $\text{Ra} \cdot \partial T / \partial x$ using the Poisson solver (equation 2).
- Determine ψ from ω using the Poisson solver (equation 3).
- Compute the wind velocities u and v from ψ .
- Compute the time step from a_{adv} , a_{diff} , and the maximum wind speed in the model domain
- Compute $\nabla^2 T_{i,j}^n$ and $\vec{v}_{i,j} \cdot \nabla T_{i,j}^n$ using the subroutines from Exercise 5.
- Integrate forward in time:
 - $T_{i,j}^{n+1} = T_{i,j}^n + \dots$ (equation 1, note without κ)
 - $t = t + \Delta t$

- Boundary conditions:
 - T : as in Exercise 5 ($T = 1$ bottom, $T = 0$ top, $\partial T / \partial x = 0$ left and right).
 - ψ and ω : 0 at all boundaries
 - If the integration time has not yet been reached, perform the next time step.
4. Test the model with the following two namelists, and save the output in binary files.

```
1 &INPUTS
2   nx=257
3   ny=65
4   a_diff=0.23
5   a_adv=0.4
6   total_time=0.1
7   max_err=1.E-3
8   Ra=1.E5
9   T_ini_type='cosine'
10 /
```

```
1 &INPUTS
2   nx=1025
3   ny=65
4   a_diff=0.23
5   a_adv=0.4
6   total_time=0.1
7   max_err=1.E-3
8   Ra=1.E5
9   T_ini_type='random'
10 /
```

5. Plot the temperature fields with your favorite plotting tool.

Deadline: Please hand in your solutions (. f90 files and plots of T) by **Tuesday, 14 May 2024, 23:59**.