$\boxed{1}$

(1)

$$\frac{dy}{dt} = \cos t - t\sin t \quad , \quad \frac{dx}{dt} = \cos t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{\cos t - t\sin t}{\cos t} = 1 - t \cdot \tan t$$

$$\therefore f'(x_0) = 1 - \frac{\pi}{4} \cdot \tan\frac{\pi}{4}$$

$$= 1 - \frac{\pi}{4}$$

また,

$$x_0 = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \quad , \quad y_0 = \frac{\pi}{4} \cdot \sin\frac{\pi}{4} = \frac{\pi}{4\sqrt{2}}$$

$$\therefore \frac{\pi}{4\sqrt{2}} = \left(1 - \frac{\pi}{4}\right) \cdot \frac{1}{\sqrt{2}} + b$$

$$\pi = 4 - \pi + 4\sqrt{2}b$$

$$\therefore b = \frac{\pi - 2}{2\sqrt{2}}$$

$$\therefore y = \left(1 - \frac{\pi}{4}\right)x + \frac{\pi - 2}{2\sqrt{2}}$$

(2) $$\frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$$

$$= \left(-\tan t - \frac{t}{\cos^2 t}\right) \cdot \frac{1}{\cos t}$$

$$\therefore f''(x_0) = \left(-\tan\frac{\pi}{4} - \frac{\frac{\pi}{4}}{\cos^2\frac{\pi}{4}}\right) \cdot \frac{1}{\cos\frac{\pi}{4}}$$

$$= \left(-1 - \frac{\frac{\pi}{4}}{\frac{1}{2}}\right) \cdot \frac{1}{\frac{1}{\sqrt{2}}}$$

$$= \left(-1 - \frac{\pi}{2}\right) \cdot \sqrt{2}$$

$$= -\sqrt{2} - \frac{\pi}{\sqrt{2}}$$

(3) $$\int_a^b y\,dx = \int_\alpha^\beta y(t)\frac{dx}{dt}\,dt$$

$$\therefore S = \int_0^{\frac{\pi}{2}} t\cos t \cdot \cos t \, dt$$
$$= \int_0^{\frac{\pi}{2}} t \, \frac{1+\cos 2t}{2} \, dt$$
$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (t + t\cos 2t) \, dt$$
$$= \frac{1}{2}\left[\frac{t^2}{2} + \frac{t}{2}\sin 2t + \frac{1}{4}\cos 2t\right]_0^{\frac{\pi}{2}}$$
$$= \frac{1}{2}\left(\frac{\pi^2}{8} - \frac{1}{4} - \frac{1}{4}\right)$$
$$= \frac{\pi^2}{16} - \frac{1}{4}$$

$T \quad t \quad \frac{1}{2}\sin 2t$

$- \quad 1 \quad -\frac{1}{4}\cos 2t$

$\boxed{2}$

(1) $$f'(x) = \frac{1}{2}e^{\frac{x}{2}} \cdot \sin\frac{\sqrt{3}}{2}x + e^{\frac{x}{2}} \cdot \frac{\sqrt{3}}{2}\cos\frac{\sqrt{3}}{2}x$$
$$= \frac{1}{2}e^{\frac{x}{2}}\left(\sin\frac{\sqrt{3}}{2}x + \sqrt{3}\cos\frac{\sqrt{3}}{2}x\right)$$
$$= \frac{1}{2}e^{\frac{x}{2}} \cdot 2\sin\left(\frac{\sqrt{3}}{2}x + \frac{\pi}{3}\right)$$
$$= e^{\frac{x}{2}}\sin\left(\frac{\sqrt{3}}{2}x + \frac{\pi}{3}\right)$$

$\sqrt{3} = r\sin\alpha$

$1 = r\cos\alpha$

$\therefore \tan\alpha = \sqrt{3}$

$\alpha = \tan^{-1}\sqrt{3}$

$= \frac{\pi}{3}$

$$f''(x) = \frac{1}{2}e^{\frac{x}{2}}\sin\left(\frac{\sqrt{3}}{2}x + \frac{\pi}{3}\right) + e^{\frac{x}{2}} \cdot \frac{\sqrt{3}}{2}\cos\left(\frac{\sqrt{3}}{2}x + \frac{\pi}{3}\right)$$
$$= \frac{1}{2}e^{\frac{x}{2}} \cdot 2\sin\left(\frac{\sqrt{3}}{2}x + \frac{2}{3}\pi\right)$$
$$= e^{\frac{x}{2}}\sin\left(\frac{\sqrt{3}}{2}x + \frac{2}{3}\pi\right)$$
$$\therefore f^{(n)}(x) = e^{\frac{x}{2}}\sin\left(\frac{\sqrt{3}}{2}x + \frac{\pi}{3}n\right)$$

(2) $$F(x) = \int e^{\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x \, dx$$

$$a(e^{ax}\sin bx)' = a^2e^{ax}\sin bx + ab\,e^{ax}\cos bx$$
$$-b(e^{ax}\cos bx)' = -bae^{ax}\cos bx + b^2e^{ax}\sin bx$$
$$(a^2+b^2)e^{ax}\sin bx = (ae^{ax}\sin bx - be^{ax}\cos bx)'$$

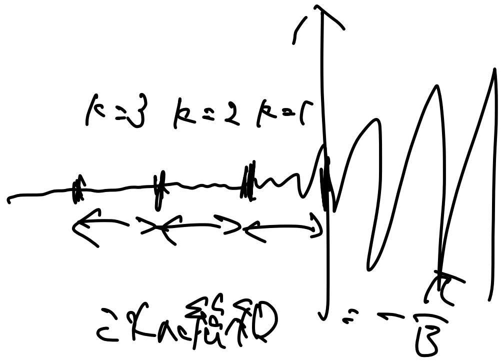
$$\therefore \int e^{ax}\sin bx\,dx = \frac{1}{a^2+b^2}\left(ae^{ax}\sin bx - be^{ax}\cos bx\right)$$

$$\therefore F(x) = \frac{e^{\frac{x}{2}}}{\frac{1}{4}+\frac{3}{4}}\left(\frac{1}{2}\sin\frac{\sqrt{3}}{2}x - \frac{\sqrt{3}}{2}\cos\frac{\sqrt{3}}{2}x\right)+C$$

$$= \frac{1}{2}e^{\frac{x}{2}}\left(\sin\frac{\sqrt{3}}{2}x - \sqrt{3}\cos\frac{\sqrt{3}}{2}x\right)+C$$

よって，$f(x)$ の原始関数の1つは

$$F(x) = \frac{1}{2}e^{\frac{x}{2}}\left(\sin\frac{\sqrt{3}}{2}x - \sqrt{3}\cos\frac{\sqrt{3}}{2}x\right)$$



(3) $\displaystyle\int_{-\infty}^{0} f(x)\,dx$

$$= \lim_{\beta\to\infty}\sum_{k=1}^{\infty}\left|\int_{-\frac{2}{\sqrt{3}}\pi k}^{-\frac{2}{\sqrt{3}}\pi(k-1)} e^{\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x\,dx\right|$$

ここで，

$$\int_{-\frac{2}{\sqrt{3}}\pi k}^{-\frac{2}{\sqrt{3}}\pi(k-1)} e^{\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x\,dx$$

$$= \left[\frac{1}{2}e^{\frac{x}{2}}\left(\sin\frac{\sqrt{3}}{2}x - \sqrt{3}\cos\frac{\sqrt{3}}{2}x\right)\right]_{-\frac{2}{\sqrt{3}}\pi k}^{-\frac{2}{\sqrt{3}}\pi(k-1)}$$

$$= \frac{1}{2}e^{-\frac{1}{\sqrt{3}}\pi(k-1)}\cdot(-\sqrt{3})\cdot(-1)^{k-1} - \frac{1}{2}e^{-\frac{1}{\sqrt{3}}\pi k}\cdot(-\sqrt{3})\cdot(-1)^{k}$$

$$= \frac{\sqrt{3}}{2}(-1)^k e^{-\frac{1}{\sqrt{3}}\pi(k-1)}\left(1 - e^{-\frac{\pi}{\sqrt{3}}}\right)$$

$$\therefore \left|\int_{-\frac{2}{\sqrt{3}}\pi k}^{-\frac{2}{\sqrt{3}}\pi(k-1)} e^{\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x\,dx\right| = \frac{\sqrt{3}}{2}e^{-\frac{1}{\sqrt{3}}\pi(k-1)}\left(1+e^{-\frac{1}{\sqrt{3}}\pi}\right)$$

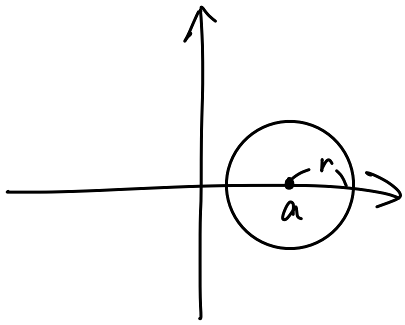
$$\therefore \lim_{\beta\to\infty}\sum_{k=1}^{\infty}\left|\int_{-\frac{2}{\sqrt{3}}\pi k}^{-\frac{2}{\sqrt{3}}\pi(k-1)} e^{\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x\,dx\right|$$

$$= \lim_{\beta\to\infty}\frac{\sqrt{3}}{2}\left(1+e^{-\frac{1}{\sqrt{3}}\pi}\right)\cdot\frac{1-\left(e^{-\frac{1}{\sqrt{3}}\pi}\right)^{\beta}}{1-e^{-\frac{1}{\sqrt{3}}\pi}}$$

$$= \frac{\sqrt{3}}{2}\,\frac{1+e^{-\frac{1}{\sqrt{3}}\pi}}{1-e^{-\frac{1}{\sqrt{3}}\pi}}$$

[3]

(1)



$$V = \supset - \sqsupset$$

(2) $x = a \pm \sqrt{r^2 - y^2}$

$$\therefore V = \int_{-r}^{r} \pi \left(a + \sqrt{r^2 - y^2}\right)^2 dy - \int_{-r}^{r} \pi \left(a - \sqrt{r^2 - y^2}\right)^2 dy$$

$$= \pi \int_{-r}^{r} \left(a^2 + 2a\sqrt{r^2 - y^2} + (r^2 - y^2)\right) - \left(a^2 - 2a\sqrt{r^2 - y^2} + (r^2 - y^2)\right) dy$$

$$= \pi \int_{-r}^{r} 4a\sqrt{r^2 - y^2}\, dy$$

$y = r\sin\theta$ と置換

$\frac{dy}{d\theta} = r\cos\theta$

y	$-r \to r$
θ	$-\frac{\pi}{2} \to \frac{\pi}{2}$

$\sin\theta = -1 \to \theta = -\frac{\pi}{2}$

$\sin\theta = 1 \to \theta = \frac{\pi}{2}$

$$\therefore V = 4\pi a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r\sqrt{1 - \sin^2\theta} \cdot r\cos\theta\, d\theta$$

$$= 4\pi a r^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

$$= 8\pi a r^2 \int_{0}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

$$= 8\pi a r^2 \cdot \frac{1}{2} \cdot \frac{\pi}{2}$$

$$= 2\pi^2 a r^2$$

[4]

(1) $g(x) = \ln\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right) - \ln\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)$

$$= \frac{-\frac{1}{2}\sin\frac{x}{2} + \frac{1}{2}\cos\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}} - \frac{-\frac{1}{2}\sin\frac{x}{2} - \frac{1}{2}\cos\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}$$

$$= \frac{1}{2}\left(\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}+\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right)$$

$$= \frac{1}{2}\left(\frac{2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right)$$

$$= \frac{1}{\cos x}$$

$$\cos\frac{x}{2}\cos\frac{x}{2}-\sin\frac{x}{2}\sin\frac{x}{2}$$
$$=\cos\left(\frac{x}{2}+\frac{x}{2}\right)$$
$$=\cos x$$

(2)

$$f'(x) = -\frac{\sin x}{\cos x} = -\tan x$$

$$L = \int_0^{\frac{\pi}{3}} \sqrt{1+(f'(x))^2}\,dx$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{1+\tan^2 x}\,dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{1}{\cos x}\,dx$$

$$= \int_0^{\frac{\pi}{3}} g'(x)\,dx$$

$$= \left[\ln\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)\right]_0^{\frac{\pi}{3}}$$

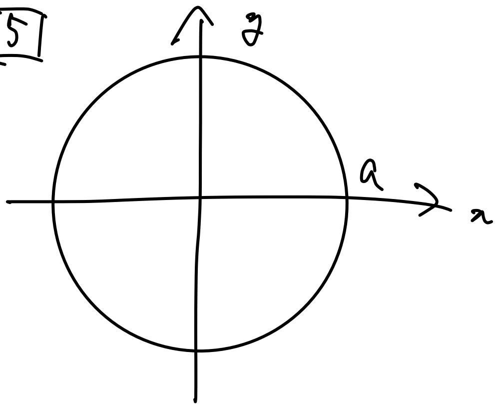
$$= \ln\left(\frac{\frac{\sqrt{3}}{2}+\frac{1}{2}}{\frac{\sqrt{3}}{2}-\frac{1}{2}}\right) - \ln\left(\frac{1}{1}\right)$$

$$= \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$$

$$= \ln\left(\frac{3+2\sqrt{3}+1}{2}\right)$$

$$= \ln\left(2+\sqrt{3}\right)$$

5



$$y^2 = a^2 - x^2$$

$$y = \pm\sqrt{a^2 - x^2}$$

$$\frac{dy}{dx} = \frac{-\cancel{2}x}{\cancel{2}\sqrt{a^2-x^2}} \qquad \left(\frac{dy}{dx}\right)^2 = \frac{x^2}{a^2-x^2}$$

$$1+\left(\frac{dy}{dx}\right)^2 = \frac{a^2}{a^2-x^2}$$

$$V = 2\int_0^a \pi y^2 \, dx$$

$$= 2\pi \int_0^a (a^2 - x^2)\, dx$$

$$= 2\pi \left[a^2 x - \frac{x^3}{3} \right]_0^a$$

$$= 2\pi \left(a^3 - \frac{a^3}{3} \right)$$

$$= \frac{4}{3}\pi a^3$$

$$S = 2\int_0^a 2\pi \cdot |y| \cdot \sqrt{(dx)^2 + (dy)^2}$$

$$= 4\pi \int_0^a \sqrt{a^2 - x^2} \cdot \frac{a}{\sqrt{a^2-x^2}}\, dx$$

$$= 4\pi a \left[x \right]_0^a$$

$$= 4\pi a^2$$

$$y = \cos^{-1} x$$

$$\cos y = x$$

$$-\sin y = \frac{dx}{dy}$$

$$\frac{dy}{dx} = -\frac{1}{\sin y}$$

$$= -\frac{1}{\sqrt{1-\cos^2 y}}$$

$$= -\frac{1}{\sqrt{1-x^2}}$$