H31

1.

(1)

動滑車 I_1，定滑車 I_2 とすると

$$I_1 = \frac{1}{2} m_1 r_1^2$$

$$I_2 = \frac{1}{2} m_2 r_2^2$$

(2) $m_1 \ddot{x}_1 = m_1 g - T_1 - T_2$ ―①

$\frac{1}{2} m_1 r_1^2 \cdot \ddot{\theta}_1 = r_1 T_2 - r_1 T_1$ ―②

$\frac{1}{2} m_2 r_2^2 \cdot \ddot{\theta}_2 = r_2 T_2 - r_2 T_3$ ―③

$m_3 \ddot{x}_3 = m_3 g - T_3$ ―④

(3) $x_1 = -\frac{1}{2} x_3$

$\theta_1 = \frac{1}{2r_1} x_3$

$\theta_2 = -\frac{1}{r_2} x_3$

$-x_1 = r_1 \theta_1$

$\theta_1 = -\frac{x_1}{r_1} = \frac{1}{2r_1} x_3$

$x_3 = -r_2 \theta_2$

$\theta_2 = -\frac{x_3}{r_2}$

(4)

①〜④に(3)の式を微分して代入し，$m_2 = 2m_1$，$m_3 = 3m_1$ とすると

$$\begin{cases} -\frac{1}{2} m_1 \ddot{x}_3 = m_1 g - T_1 - T_2 \\ \frac{1}{2} m_1 r_1 \cdot \frac{1}{2r_1} \ddot{x}_3 = T_2 - T_1 \\ \frac{1}{2} 2m_1 r_2 \cdot \left(-\frac{1}{r_2} \ddot{x}_3\right) = T_2 - T_3 \\ 3m_1 \ddot{x}_3 = 3m_1 g - T_3 \end{cases}$$

$$\underbrace{\begin{pmatrix} -\frac{1}{2} m_1 & 1 & 1 & 0 \\ \frac{1}{4} m_1 & 1 & -1 & 0 \\ -m_1 & 0 & -1 & 1 \\ 3m_1 & 0 & 0 & 1 \end{pmatrix}}_{=:(a_1\ a_2\ a_3\ a_4) = A} \begin{pmatrix} \ddot{x}_3 \\ T_1 \\ T_2 \\ T_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 3 \end{pmatrix}}_{=:b} m_1 g$$

$$|A| = \begin{vmatrix} -\frac{1}{2}m_1 & 1 & 1 \\ \frac{1}{4}m_1 & 1 & -1 \\ -4m_1 & 0 & -1 \end{vmatrix} = \begin{vmatrix} -\frac{1}{2}m_1 & 1 & 1 \\ -\frac{1}{4}m_1 & 2 & 0 \\ -\frac{9}{2}m_1 & 1 & 0 \end{vmatrix} = -\frac{1}{4}m_1 + \frac{\overset{36}{\cancel{9}}}{4}m_1$$

$$= \frac{35}{4}m_1$$

$$|(\boldsymbol{a}_1\ \boldsymbol{b}\ \boldsymbol{a}_2\ \boldsymbol{a}_3)| = \begin{vmatrix} -\frac{1}{2}m_1 & 1 & 1 & 0 \\ \frac{1}{4}m_1 & 0 & -1 & 0 \\ -m_1 & 0 & -1 & 1 \\ 3m_1 & 3 & 0 & 1 \end{vmatrix} m_1 g = \begin{vmatrix} -\frac{1}{2} & 1 & 1 \\ \frac{1}{4} & 0 & -1 \\ -4 & -3 & -1 \end{vmatrix} m_1^2 g$$

$$= \begin{vmatrix} -\frac{1}{4} & 1 \\ -\frac{9}{2} & -2 \end{vmatrix} m_1^2 g = \left(\frac{1}{2} + \frac{9}{2}\right) m_1^2 g = 5m_1^2 g$$

$$|(\boldsymbol{a}_1\ \boldsymbol{a}_2\ \boldsymbol{b}\ \boldsymbol{a}_3)| = \begin{vmatrix} -\frac{1}{2} & 1 & 1 & 0 \\ \frac{1}{4} & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 3 & 0 & 3 & 1 \end{vmatrix} m_1^2 g = \begin{vmatrix} -\frac{1}{2} & 1 & 1 \\ \frac{1}{4} & 1 & 0 \\ -4 & 0 & -3 \end{vmatrix} m_1^2 g$$

$$= \left(\frac{\overset{6}{\cancel{3}}}{\cancel{2}\,4} + \frac{\overset{16}{\cancel{4}}}{4} + \frac{3}{4}\right) m_1^2 g = \frac{25}{4} m_1^2 g$$

$$|(\boldsymbol{a}_1\ \boldsymbol{a}_2\ \boldsymbol{a}_3\ \boldsymbol{b})| = \begin{vmatrix} -\frac{1}{2} & 1 & 1 & 1 \\ \frac{1}{4} & 1 & -1 & 0 \\ -1 & 0 & -1 & 0 \\ 3 & 0 & 0 & 3 \end{vmatrix} m_1^2 g = 3\begin{vmatrix} -\frac{\overset{6}{\cancel{3}}}{\cancel{2}\,4} & 1 & 1 \\ \frac{1}{4} & 1 & -1 \\ -1 & 0 & -1 \end{vmatrix} m_1^2 g$$

$$= 3\begin{vmatrix} -\frac{7}{4} & 2 \\ -1 & -1 \end{vmatrix} m_1^2 g = 3\left(\frac{7}{4} + \frac{\overset{8}{\cancel{2}}}{4}\right) = \frac{45}{4} m_1^2 g$$

$$T_1 = \frac{5m_1^2 g}{\overset{7}{\cancel{\frac{35}{4}}}m_1} = \frac{4}{7}m_1 g$$

$$T_2 = \frac{\overset{5}{\cancel{\frac{25}{4}}}m_1^2 g}{\overset{7}{\cancel{\frac{35}{4}}}m_1} = \frac{5}{7}m_1 g$$

$$T_3 = \frac{\overset{9}{\cancel{\frac{45}{4}}}m_1^2 g}{\overset{7}{\cancel{\frac{35}{4}}}m_1} = \frac{9}{7}m_1 g$$

(5)

$$|(\mathbf{b}\ \mathbf{a}_2\ \mathbf{a}_3\ \mathbf{a}_4)| = \begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 3 & 0 & 0 & 1 \end{vmatrix} m_1 g = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ -3 & 0 & -1 \end{vmatrix} m_1 g$$

$$= (-1+3+3)\, m_1 g = 5 m_1 g$$

$$\therefore \ddot{x}_3 = \frac{\overset{1}{\cancel{5}} m_1 g}{\frac{\overset{7}{\cancel{35}}}{4} m_1} = \frac{4}{7} g$$

$$\therefore x_3 = \frac{1}{2} \ddot{x}_3 t^2 = \frac{2}{7} g t^2$$

3.

(1)

ピストンと壁面への衝突1回あたりの力積は

$$m v_{ix} - (-m v_{0x}) = 2m v_{0x}$$

単位時間あたりの衝突回数は

$$\frac{v_{0x}}{2l_0}$$

よって単位時間あたりの力積は

$$2m v_{0x} \cdot \frac{v_{0x}}{2l_0} = \frac{m v_{0x}^2}{l_0}$$

$$\frac{1}{2} Nm \langle v^2 \rangle_0 = \frac{3}{2} P_0 l_0^3$$

(2)

壁面におよぼす力は力積に等しい。また、ピストンの面積はl_0^2なので

$$P_0 = N \cdot \frac{m \langle v_x^2 \rangle_0}{l_0} \cdot \frac{1}{l_0^2} = \frac{1}{3} \frac{Nm \langle v^2 \rangle_0}{l_0^3}$$

(3)

気体の状態方程式より

$$k_B = \frac{R}{N_A} \quad k_B N_A = R$$

$$nRT_0 = \frac{1}{3} Nm \langle v^2 \rangle_0$$

$$\therefore T_0 = \frac{1}{3} \frac{N}{n} \frac{1}{R} m \langle v^2 \rangle_0 = \frac{1}{3} \frac{N_A}{R} m \langle v^2 \rangle_0 = \frac{m \langle v^2 \rangle_0}{3k_B}$$

(4)

$l = 2l_0$のときの温度T'は$TV^{\frac{2}{3}} =$一定より

$$T' (2l_0^3)^{\frac{2}{3}} = T_0 (l_0^3)^{\frac{2}{3}}$$

$$nRT_0 = P_0 l_0^3$$

$$\therefore T' = \frac{T_0}{2^{\frac{2}{3}}} = 2^{-\frac{2}{3}} T_0$$

$$\therefore \Delta U = \frac{3}{2} nR (T' - T_0) = \frac{3}{2} nR \left(2^{-\frac{2}{3}} - 1\right) T_0$$

$$= \frac{3}{2} \left(2^{-\frac{2}{3}} - 1\right) P_0 l_0^3$$

(5)

$l_0 \to 2l_0$の操作は断熱変化なので、ΔUは気体にかかる仕事になる。

よって、(4)の状態の運動エネルギーの総和は$l = l_0$のときの運動エネルギーの総和とΔUの和になる。よって、

$$\frac{1}{2} Nm \langle v^2 \rangle = \frac{1}{2} Nm \langle v^2 \rangle_0 + \Delta U = \frac{1}{2} Nm \langle v^2 \rangle_0 + \frac{3}{2} \left(2^{-\frac{2}{3}} - 1\right) \cdot \frac{1}{3} Nm \langle v^2 \rangle_0$$

$$= 2^{-\frac{2}{3}} Nm \langle v^2 \rangle_0$$

$$\therefore \langle v^2 \rangle = 2 \cdot 2^{-\frac{2}{3}} \langle v^2 \rangle_0 = 2^{\frac{1}{3}} \langle v^2 \rangle_0 = \sqrt[3]{2} \langle v^2 \rangle_0$$