R3

1.

(1) $I_G = \frac{2}{5}Mr^2$

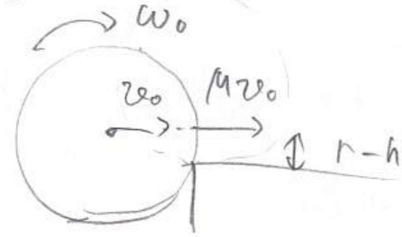
$I_P = I_G + Mr^2 = \frac{7}{5}Mr^2$

$$I_G = \int_0^{\pi}\int_0^{2\pi}\int_0^{r} \frac{M}{\frac{4}{3}\pi r^3} r^2\sin^2\varphi \cdot r^2\sin\varphi \, dr\, d\theta\, d\varphi$$

$$= 2\pi \cdot \frac{3M}{4\pi r^3} \cdot \left[\frac{r^5}{5}\right]_0^r \cdot 4 \cdot \frac{2}{3}$$

$$= \frac{2}{5}Mr^2$$

(2) $I_G \omega_0 + M v_0 (r-h) = I_P \omega$ — ①



(3)

段差を越えるための最小のωについて

$\frac{1}{2} I_P \omega^2 = Mgh$

$\omega^2 = 2Mgh \cdot \frac{5}{7Mr^2} = \frac{10gh}{7r^2}$

$\therefore \omega \geq \frac{1}{r}\sqrt{\frac{10gh}{7r^2}}$ — ②

$v = r\omega \quad \omega = \frac{v}{r}$

(4)

①で $\omega_0 = \frac{v_0}{r}$ とすると

$\frac{2}{5}Mr^2 \cdot \frac{v_0}{r} + Mv_0(r-h) = \frac{7}{5}Mr^2\omega$

$\therefore \omega = \frac{5}{7Mr^2}\left(\frac{2}{5}Mrv_0 + Mv_0(r-h)\right)$

$= \frac{5}{7r^2}\left(\frac{2}{5}r + r - h\right)v_0$

$= \frac{5}{7r^2}\left(\frac{7}{5}r - h\right)v_0$ — ③

これを②に代入.

$\frac{5}{7r^2}\left(\frac{7}{5}r - h\right)v_0 \geq \frac{1}{r}\sqrt{\frac{10gh}{7r^2}}$

$\therefore v_0 \geq \frac{1}{r}\sqrt{\frac{10gh}{7r^2}} \cdot \frac{1}{\frac{1}{r} - \frac{5h}{7r^2}} = \frac{1}{r}\sqrt{\frac{10gh}{7r^2}} \cdot \frac{7r^2}{7r-5h}$

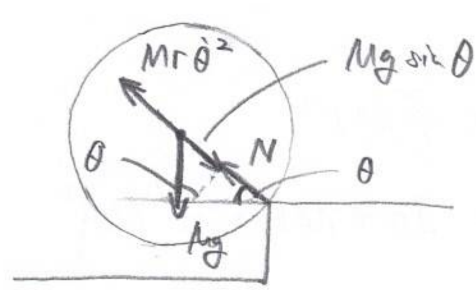
$= \frac{1}{7r-5h}\sqrt{\frac{10gh}{7r^2} \cdot 7^2 r^2}$

$= \frac{\sqrt{70gh}}{7r-5h}$

$\therefore v_0 \geq \frac{\sqrt{70gh}}{7r-5h}$

(5) $N + Mr\dot{\theta}^2 = Mg\sin\theta$

$\therefore\ N = Mg\sin\theta - Mr\dot{\theta}^2 \quad -③$



(6)

Nが最小値をとるのは明らかに衝突直後.

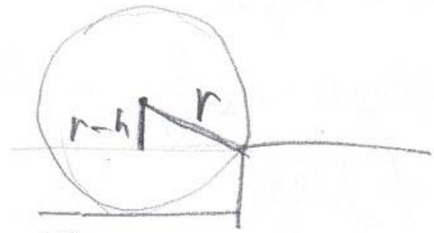
このとき

$\sin\theta = \frac{r-h}{r} = 1 - \frac{h}{r}$

$\dot{\theta}^2 = \omega^2$

より④に代入して

$N_{min} = Mg\left(1 - \frac{h}{r}\right) - Mr\omega^2 \quad -⑤$



(7)

$N_{min} \geq 0$であればよいので, ⑤に③を代入すると

$$Mr \cdot \left(\frac{5}{7r^2}\left(\frac{7}{5}r - h\right)v_0\right)^2 \geq Mg\left(1 - \frac{h}{r}\right)$$

$$= \left(\frac{1}{r} - \frac{5h}{7r^2}\right)v_0 = \frac{1}{r}\left(1 - \frac{5h}{7r}\right)v_0$$

$$\therefore\ r \cdot \frac{1}{r^2}\left(1 - \frac{5h}{7r}\right)^2 v_0^2 \geq g\left(1 - \frac{h}{r}\right)$$

$$v_0^2 \geq \frac{gr\left(1 - \frac{h}{r}\right)}{\left(1 - \frac{5h}{7r}\right)^2} = \frac{g(r-h)}{\left(1 - \frac{5h}{7r}\right)^2} = \frac{(7r)^2}{(7r-5h)^2} \cdot g(r-h)$$

$$\therefore\ v_0 \geq \frac{7r}{7r-5h}\sqrt{g(r-h)}$$

3.

(1) $E = \frac{1}{2} Nm\overline{v^2}$

(2) $P = \frac{Nm\overline{v^2}}{3L^3}$

(3)

(2) より

$$PL^3 = \frac{1}{3}Nm\overline{v^2} = \frac{2}{3}\cdot\frac{1}{2}Nm\overline{v^2} = \frac{2}{3}E$$

また，$PL^3 = nRT = \frac{N}{N_A}RT$ より

$$\frac{N}{N_A}RT = \frac{2}{3}E$$

$$\therefore T = \frac{2}{3}\frac{N_A}{NR}E \qquad \therefore T \propto E$$

(4)

(1) より

$$\overline{v^2} = \frac{2E}{Nm}$$

(3) より

$$= \frac{1}{Nm}\cdot\frac{3NRT}{N_A}$$

$$= \frac{3RT}{mN_A}$$

$$= \frac{3RT}{(\text{窒素の分子量})\times 10^{-3}}$$

∵ 分子量の単位は [g]

$$= \frac{3\cdot 8.3\cdot 280}{28\times 10^{-3}}$$

$$= 3\cdot 8.3\times 10^4$$

$$\therefore \overline{v} = \sqrt{3\cdot 8.3}\times 10^2$$

$$= \sqrt{24.9}\times 10^2$$

$$\approx 5\times 10^2$$

$$= 500\ \mathrm{m/s}$$

$$mv_x - (-mv_x) = 2mv_x$$

$$\frac{v_x t}{2L}\cdot 2mv_x = \frac{mv_x^2 t}{L}$$

$$F = \frac{Nm\overline{v_x^2}}{L} \qquad P = \frac{Nm\overline{v_x^2}}{L^3} =$$

$$PL^3 = Nm\overline{v_x^2} = \frac{1}{3}Nm\overline{v^2}$$

$$\therefore nRT = \frac{1}{3}Nm\overline{v^2}$$

$$= \frac{1}{3}\cdot nN_A\cdot 2\cdot\frac{1}{2}m\overline{v^2}$$