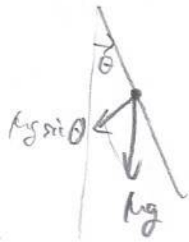
R4

1.

(1) $I = \frac{1}{3}ML^2$



$I = \int_0^L \frac{M}{L}x^2 dx = \frac{M}{L}\cdot\frac{1}{3}L^3 = \frac{1}{3}ML^2$

(2) $I\ddot{\theta} = -Mg\sin\theta \cdot \frac{L}{2}$

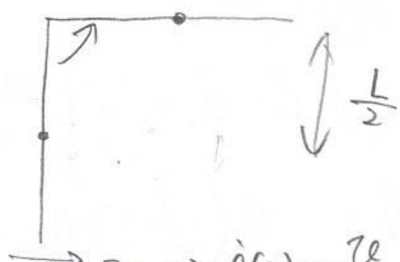
$\therefore \frac{1}{3}ML^2 \cdot \ddot{\theta} = -\frac{1}{2}MgL\sin\theta$

$\omega = \sqrt{\frac{3g}{2L}}$ $\qquad v = r\omega$, $\omega = \frac{v}{r}$

$T = 2\pi\sqrt{\frac{2L}{3g}}$

(3) $\ddot{\theta} = \frac{3}{ML^2}\cdot\left(-\frac{1}{2}MgL\cdot\theta\right) = -\frac{3g}{2L}\cdot\theta$

$\therefore T = 2\pi\sqrt{\frac{2L}{3g}}$



$v \Rightarrow \dot{\theta}(0) = \frac{v}{L}$

(4) $\frac{1}{2}I\dot{\theta}(0)^2 = Mg\cdot\frac{L}{2}$

$\therefore \frac{1}{2}\cdot\frac{1}{3}ML^2\cdot\frac{v^2}{L^2} = \frac{MgL}{2}$

$v^2 = 3gL$

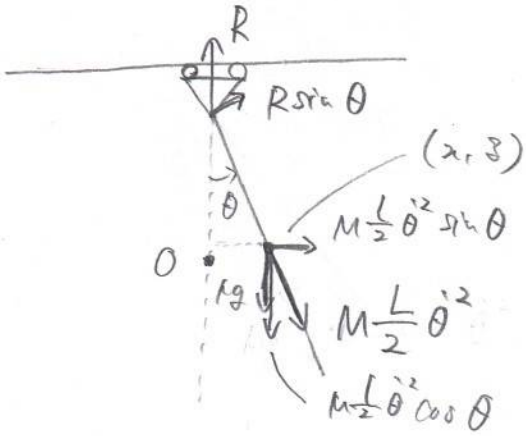
$\therefore v = \sqrt{3gL}$

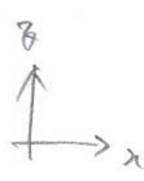
(5) $I' = \frac{1}{12}ML^2$

$I' = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{M}{L}x^2 dx$

$= 2\cdot\frac{M}{L}\cdot\frac{1}{3}\cdot\frac{L^3}{8} = \frac{1}{12}ML^2$

(6)





$$\begin{cases} M\ddot{x} = M\frac{L}{2}\dot{\theta}^2\sin\theta & \text{—①} \\ M\ddot{z} = R - M\frac{L}{2}\dot{\theta}^2\cos\theta - Mg & \text{—②} \\ I'\ddot{\theta} = -R\sin\theta\cdot\frac{L}{2} & \text{—③} \end{cases}$$

(7) $z = \frac{L}{2} - \frac{L}{2}\cos\theta = \frac{L}{2}(1-\cos\theta)$

$\therefore \dot{z} = \frac{L}{2}\sin\theta\cdot\dot{\theta}$

$\ddot{z} = \frac{L}{2}\cos\theta\cdot\dot{\theta}^2 + \frac{L}{2}\sin\theta\cdot\ddot{\theta}$

②に代入すると

$$\frac{ML}{2}\cos\theta\cdot\dot{\theta}^2 + \frac{ML}{2}\sin\theta\cdot\ddot{\theta} = R - \frac{ML}{2}\cos\theta\cdot\dot{\theta}^2 - Mg$$

$$\therefore R = \frac{ML}{2}\sin\theta\cdot\ddot{\theta} + ML\cos\theta\cdot\dot{\theta}^2 + Mg$$

これを③に代入

$$\frac{1}{12}ML^2\cdot\ddot{\theta} = -\left(\frac{ML}{2}\sin\theta\cdot\ddot{\theta} + ML\cos\theta\cdot\dot{\theta}^2 + Mg\right)\sin\theta\cdot\frac{L}{2}$$

$$\ddot{\theta} = -6\left(\frac{1}{2}\sin\theta\cdot\ddot{\theta} + \cos\theta\cdot\dot{\theta}^2 + \frac{g}{L}\right)\sin\theta$$

$$= -3\sin^2\theta\cdot\ddot{\theta} - 6\sin\theta\cos\theta\cdot\dot{\theta}^2 - \frac{6g}{L}\sin\theta$$

$$\therefore \ddot{\theta} = -\frac{6\sin\theta\cos\theta\cdot\dot{\theta}^2 + \frac{6g}{L}\sin\theta}{1+3\sin^2\theta}$$

(8) $\ddot{\theta} \approx -\frac{6g}{L}\theta$

$\omega = \sqrt{\frac{6g}{L}}$

$\therefore T = 2\pi\sqrt{\frac{L}{6g}}$

$T = 2\pi\sqrt{\frac{L}{6g}}$

3.

I.

$$m v_x - (-m v_x) = 2m v_x \qquad k_B = \frac{R}{N_A}$$

(ア) $\frac{v_x t}{2L}$

(イ) $2m v_x \cdot \frac{v_x t}{2L} = \frac{m v_x^2 t}{L}$

$$F = N \cdot \frac{m\overline{v_x^2}}{L} = \frac{Nm\overline{v_x^2}}{L}$$

$$p = \frac{Nm\overline{v_x^2}}{L^3}$$

(ウ) 体積

(エ) $\frac{1}{2} k_B T$

(オ) $\frac{3}{2} k_B T$

(カ) 2

(キ) $\frac{5}{2} k_B T$

(ク) $\frac{5}{2} R$

$$\overline{v_x^2} = \frac{1}{3}\overline{v^2} \qquad pV = Nm\overline{v_x^2}$$

$$\frac{1}{2}m\frac{1}{3}\overline{v^2} = \frac{1}{2}k_B T \qquad \frac{1}{2}RT = N_A \frac{1}{2} m\overline{v_x^2}$$

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T \qquad \frac{1}{2}m\overline{v_x^2} = \frac{1}{2}\frac{R}{N_A}T$$

$$U = \frac{5}{2}k_B T \cdot N_A = \frac{5}{2}RT$$

II.

(1) $T_H = \frac{P_A V_1}{nR}$

(2) $\Delta U = 0$

(3) $Q_{AB} = W_{AB} = \int_{A\to B} p\,dV$

$$= \int_{V_1}^{V_2} \frac{nRT_H}{V} dV$$

$$= nRT_H \ln\frac{V_2}{V_1} \quad \text{— 吸収}$$

$$= P_A V_1 \ln\frac{V_2}{V_1}$$

$$\frac{nRT_H \ln\frac{V_2}{V_1} - nRT_L \ln\frac{V_2}{V_1}}{nRT_H \ln\frac{V_2}{V_1} + \frac{3}{2}nR(T_H - T_L)}$$

(4) $Q_{BC} = \frac{3}{2}nR(T_L - T_H)$ — 放出

$Q_{CD} = nRT_L \ln\frac{V_1}{V_2}$ — 放出

$Q_{DA} = \frac{3}{2}nR(T_H - T_L)$ — 吸収

(5) $\eta = \frac{Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}}{Q_{AB} + Q_{DA}} = \frac{(T_H - T_L)\ln\frac{V_2}{V_1}}{T_H \ln\frac{V_2}{V_1} + \frac{3}{2}(T_H - T_L)}$

(6) $\eta' = \frac{Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}}{Q_{AB}} = \frac{(T_H - T_L)\ln\frac{V_2}{V_1}}{T_H \ln\frac{V_2}{V_1}} = 1 - \frac{T_L}{T_H}$