[1]

(1)
$$\begin{cases} 2x + 2y + az = b \\ 4x + 3y = b \\ 2x + 3z = c - 1 \end{cases}$$

$$\begin{pmatrix} 0 & 1 & -2 & 1 \\ 2 & 2 & a & b \\ 4 & 3 & 0 & b \\ 2 & 1 & 1 & c \end{pmatrix} \to \begin{pmatrix} 2 & 2 & a & b \\ 0 & 1 & -2 & 1 \\ 0 & -1 & -2a & -b \\ 0 & -1 & 1-a & c-b \end{pmatrix}$$

$$\to \begin{pmatrix} 2 & 2 & a & b \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -2(a+1) & 1-b \\ 0 & 0 & -(a+1) & c-b+1 \end{pmatrix}$$

$$\to \begin{pmatrix} 2 & 2 & a & b \\ 0 & 1 & -2 & 1 \\ 0 & 0 & a+1 & b-c-1 \\ 0 & 0 & 0 & b-2c-1 \end{pmatrix}$$

解がただ1つ存在するとき

$$a \neq -1, \quad b = 2c + 1$$

(2)

—— a 直線になっているとき、

$$a = -1, \quad b - c - 1 = 0, \quad b - 2c - 1 = 0$$

$$\begin{aligned} b - c &= 1 \\ -)\ b - 2c &= 1 \\ \hline c &= 0 \\ b &= 1 \end{aligned}$$

$$\therefore a = -1, \quad b = 1, \quad c = 0$$

$$\begin{pmatrix} 2 & 2 & -1 & 1 \\ 0 & 1 & -2 & 1 \end{pmatrix} \to \begin{pmatrix} 2 & 0 & 3 & -1 \\ & 1 & -2 & 1 \end{pmatrix}$$

$$\begin{aligned} 2x + 3z &= -1 \\ y - 2z &= 1 \end{aligned}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = t \begin{pmatrix} -\frac{3}{2} \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$$

$$= t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$$

$$\frac{x+\frac{1}{2}}{-3} = \frac{y-1}{4} = \frac{z}{2}$$

$\boxed{2}$

$$|A| = \begin{vmatrix} 1+\lambda_1 & \lambda_1 & \cdots & \lambda_1 \\ \lambda_2 & 1+\lambda_2 & \cdots & \lambda_2 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_n & \lambda_n & \cdots & 1+\lambda_n \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \lambda_1 & \lambda_1 & \cdots & \lambda_1 \\ -1 & 1+\lambda_2 & \lambda_2 & \cdots & \lambda_2 \\ 0 & \lambda_3 & 1+\lambda_3 & \cdots & \lambda_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \lambda_n & \lambda_n & \cdots & 1+\lambda_n \end{vmatrix}$$

$$= \begin{vmatrix} 1+\lambda_1+\lambda_2 & \lambda_1+\lambda_2 & \cdots & \lambda_1+\lambda_2 \\ \lambda_3 & 1+\lambda_3 & \cdots & \lambda_3 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_n & \lambda_n & \cdots & 1+\lambda_n \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \lambda_1+\lambda_2 \\ -1 & \\ \vdots & \\ 0 & \end{vmatrix}$$

$$\therefore |A| = 1 + \lambda_2 + \cdots + \lambda_n$$

$$\begin{vmatrix} \lambda_1 & \lambda_1 & \cdots & \lambda_1 \\ \lambda_2 & 1+\lambda_2 & \cdots & \lambda_2 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_n & \lambda_n & \cdots & 1+\lambda_n \end{vmatrix} = \begin{vmatrix} 0 & \lambda_1 & \cdots & \lambda_1 \\ -1 & 1+\lambda_2 & \cdots & \lambda_2 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \lambda_n & \cdots & 1+\lambda_n \end{vmatrix}$$

$$= \begin{vmatrix} \lambda_1 & \lambda_1 & \cdots & \lambda_1 \\ \lambda_3 & 1+\lambda_3 & \cdots & \lambda_3 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_n & \lambda_n & \cdots & \lambda_n \end{vmatrix}$$

$$= \lambda_1$$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \frac{1}{1 + \lambda_2 + \cdots \lambda_n} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{pmatrix}$$

$\boxed{3}$

(1) 行基本変形は，その行基本変形に対応するある行列を左からかけることに相当。

$$\begin{pmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & \ddots & \vdots \\ q_{n1} & \cdots & q_{nn} \end{pmatrix} \left(\begin{array}{ccc|ccc} a_{11} & \cdots & a_{1n} & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} & 0 & \cdots & 1 \end{array} \right)$$

$$= \left(\begin{array}{ccc|ccc} q\Sigma a & \cdots & q\Sigma a & q\Sigma e & \cdots & q\Sigma e \\ & & \vdots & & & \vdots \end{array} \right)$$

$$\xrightarrow{n次} Q[A|E] = [B|P]$$

とすると，

$$\begin{cases} QA = B \\ QE = Q = P \end{cases}$$

$$\therefore PA = B$$

$$\therefore B = PA$$

(2) (2)

(1) より $B = PA$. P, A は正方行列なので

$$|B| = |P||A| \qquad \therefore |A| = \frac{|B|}{|P|}$$

ここで

$$|B| = \begin{vmatrix} 2 & 1 & -1 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -2 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1 & -2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{vmatrix} = 2 \cdot (-2) = -4$$

$$|P| = \begin{vmatrix} 1 & 0 & 0 & 0 \\ -5 & -2 & 0 & 0 \\ 1 & \frac{1}{3} & 1 & 0 \\ 4 & -1 & 1 & 3 \end{vmatrix} = 3 \begin{vmatrix} 1 & 0 & 0 \\ -5 & -2 & 0 \\ 1 & \frac{1}{3} & 1 \end{vmatrix} = 3 \begin{vmatrix} 1 & 0 \\ -5 & -2 \end{vmatrix} = -6$$

$$\therefore |A| = \frac{2}{3}$$

(b) $Ax = {}^t(1\ 0\ 1\ 0\ 1)$

$PAx = P\,{}^t(1\ 0\ 1\ 0\ 1)$

$\therefore\ Bx = P\,{}^t(1\ 0\ 1\ 0\ 1)$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 1 & -5 & -2 & 0 & 0 \\ \frac{2}{3} & 1 & \frac{1}{3} & 1 & 0 \\ 1 & 4 & -1 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \\ 3 \end{pmatrix}$$

$$\therefore \begin{cases} 2x_1 + x_2 - x_3 + x_4 - x_5 = 1 \\ x_2 - 2x_3 - x_4 + x_5 = 2 \\ x_3 + x_4 - x_5 = -1 \\ -2x_4 + x_5 = 1 \\ x_5 = 3 \end{cases}$$

$-2\ \ -1\ \ 3$

$2 - 1 + 1 - 3 = -1$

$$\therefore\ x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$$