University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MATA37 Winter 2025

Assignment # 2

You are may wish to work on this assignment prior to your tutorial during the week of Jan. 20th. You may ask questions about this assignment in that tutorial.

STUDY: Chapter 4, Sections: 4.3 (OMIT Thm 4.13 - we can always derive these results by definition).

HOMEWORK:

At the beginning of your TUTORIAL during the week of Jan. 27th you may be asked to either submit the following "Homework" problems or write a quiz based on this assignment and/or related material from the lectures and textbook readings. All submissions to weekly evaluations will be done via crowdmark in your officially registered tutorial. This part of your assignment will count towards the 20% of your final mark, which is based on weekly assignments / quizzes.

1. Evaluate each of the following. (You need to decide what 'method' to use : geometrically; analytically; integration properties; etc.). Make sure to fully justify your answers.

   (a) $\displaystyle\int_{-1}^{0} (x-1)(1+x+x^2)\,dx.$

   (b) $\displaystyle\int_{-6}^{0} f(x)\,dx$ where $f(x) = \begin{cases} 1 & \text{if } x < -6 \\ 1+\sqrt{-x^2-25-10x} & \text{if } -6 \le x \le -5 \\ x & \text{if } x > -5. \end{cases}$

(c) $\displaystyle\int_2^8 (|x| + g(x))\ dx$ if g is continuous everywhere and $\displaystyle\int_0^{100} g(x)\ dx = \pi$, $\displaystyle\int_2^0 g(x)\ dx = 1$, and $\displaystyle\int_8^{100} g(x)\ dx = 99$.

2. Express

$$\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{1 + \frac{3k}{n}} \cdot \frac{3}{n}$$

as a definite integral with

(a) $\triangle x = \frac{1}{n}$;

(b) $a = 0$;

(c) $a = 1$.

Hint : use "reverse engineering" to write this limit as a definite integral for each of the requirement (a) - (c). Make sure to fully justify your solutions.

3. The following are additional definite integral properties.

(a) Prove each of following properties using ONLY the Riemann definition of the definite integral. Make sure to fully justify your work.

i. Let $a, b \in \mathbb{R},\ a < b$.
If f and g are integrable on $[a, b]$ and $f(x) \geq g(x)$ on $[a, b]$ then $\displaystyle\int_a^b f(x)\ dx \geq \int_a^b g(x)\ dx$.

ii. (This is called the Integral Inequality)

Let $a, b \in \mathbb{R},\ a < b$.
If f is integrable on $[a, b]$ and $\exists m, M \in \mathbb{R}\ \forall x \in [a, b]\ \ m \leq f(x) \leq M$ then $m(b-a) \leq \displaystyle\int_a^b f(x)\ dx \leq M(b-a)$.

(b) (not graded) Provide a sample diagram to illustrate the Integral Inequality property. That is, carefully draw and label an *example* picture to illustrate the this particular property.

4. Let $a, b \in \mathbb{R}, a < b$.

   (a) Prove : if f is continuous on $[a, b]$ then

   $$\left| \int_a^b f(x)dx \right| \leq \int_a^b |f(x)|dx.$$

   (b) Suppose that f is continuous on $[a, b]$. Prove that

   $$\left| \int_a^b f(x)\cos(x)\ dx \right| \leq \int_a^b |f(x)|\ dx.$$

Inspiration exists, but it has to find you working. – *Picasso*