University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MATA37 Winter 2025

Assignment # 5

You are may wish to work on this assignment prior to your tutorial during the week of Feb. 10th. You may ask questions about this assignment in that tutorial.

STUDY: Chapter 5: Section 5.1 (OMIT Thm 5.4 - we never mix variables;*if we perform a u-subst. to a definite integral then our u integrand* ***must*** *have corresponding u integration limits if we keep our integral in definite form*); Section 5.2.

HOMEWORK:

At the beginning of your TUTORIAL during the week of Feb. 24th you may be asked to either submit the following "Homework" problems or write a quiz based on this assignment and/or related material from the lectures and textbook readings. All submissions to weekly evaluations will be done via crowdmark in your officially registered tutorial. This part of your assignment will count towards the 20% of your final mark, which is based on weekly assignments / quizzes.

UTSC Reading week runs Tues. Feb. 18th - 22nd. No lectures, tutorials or regular weekly office hours run during reading week.

1. (not graded) Let $a \in \mathbb{R}$. Suppose that f is continuous on $[-a, a]$. Prove the following statements. Use **only** the subst. rule and integration properties. Do NOT use FTOC I.

   (a) If f is an even function on $[-a, a]$ then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$.

(b) If f is an odd function on $[-a, a]$ then $\int_{-a}^{a} f(x)dx = 0$.

2. (a) Watch the example video: https://play.library.utoronto.ca/watch/73a834ad8f835f0ae5107f

   (b) Evaluate the following integrals using any of our 3 integration techniques. Namely, Chap. 4 methods, u-sub, int by parts.

      i. $\int_{-1}^{1} \ln(x+2)dx$.
      ii. $\int \frac{\ln(x)}{x\sqrt{x}}dx$.
      iii. $\int_{0}^{1} (x^2+2x+1)^4(x+1)\ dx$.
      iv. $\int x^2 \ln(x)dx$.
      v. $\int e^{3x}\ sin(4x)\ dx$.
      vi. $\int \sec^3(2x)\tan(2x)dx$.
      vii. $\int_{-1}^{1} (\tan(x) + \frac{x^{\frac{1}{3}}}{(1+x^2)^7} - x^{17}\cos(x))\ \mathrm{d}x$
      viii. $\int x\sec^2(x)\ dx$.
      ix. $\int \frac{dx}{1+e^x}$.
      x. $\int (1-\cos(x))^5 \sin^5(x)\ dx$.
      xi. $\int_{0}^{4} \frac{\sqrt{x}}{1+\sqrt{x}}dx$.
      xii. $\int_{0}^{1} t\sqrt{1+t^2}dt$.
      xiii. $\int \sqrt{4+\sqrt{x}}\ dx$.

3. Let f:$[0,1]$->$\mathbb{R}$ be a strictly increasing function that has continuous first order derivatives (i.e. continuously differentiable) such that $f(0) = 1$ and $f(1) = 2$. Prove

$$\int_{0}^{1} f(x)\ dx + \int_{1}^{2} f^{-1}(x)\ dx = 2$$

where $f^{-1}(x)$ is the inverse function of f.

4. (not graded) Let $a, b \in \mathbb{R}$, $a < b$. Let $\ell, m, n, p \in \mathbb{Z}^{\geq 0}$. A **trigonometric integral** is an integral of the form :

- $\displaystyle\int \cos^m(x)\sin^n(x)dx$ or $\displaystyle\int_a^b \cos^m(x)\sin^n(x)dx$, or
- $\displaystyle\int \sec^p(x)\tan^\ell(x)dx$ or $\displaystyle\int_a^b \sec^p(x)\tan^\ell(x)dx$

Which of the following are trigonometric integrals and which are not? Justify.

(a) $\displaystyle\int (\cot^3(3x) + 1)\ dt$

(b) $\displaystyle\int_0^1 \sin^2(3x)\ dx$

(c) $\displaystyle\int \frac{\tan(x) + \sin(x)}{\sec(x)}\ dx$

(d) $\displaystyle\int \cos(u)\cos(4u)\ du$

Useful trig identities to know :

- $\cos^2(A) + \sin^2(A) = 1$
- $1 + \tan^2(A) = \sec^2(A)$
- $\sin^2(A) = \frac{1-\cos(2A)}{2}$
- $\cos^2(A) = \frac{1+\cos(2A)}{2}$
- $\sin(2A) = 2\cos(A)\sin(A)$.

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5. Trigonometric integrals can be solved with either identities and algebra Chap 4. methods, u-sub, or integration by parts. Apply our already established integration strategies (SEE Integration Module - OMIT trigonometric substitution and partial fraction decomposition for now) to evaluate the following trigonometric integrals:

(a) $\displaystyle\int \tan^3(x)\ dx$

(b) $\int \sin^2(x)\cos^2(x)\ dx$

(c) $\int \tan^4(u)\sec^6(u)\ du$

(d) $\int \tan^2(x)\sec(x)\ dx$

Do. Or do not. There is no try. – ***Yoda***