University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MATA37 Winter 2025

Assignment # 4

You are may wish to work on this assignment prior to your tutorial during the week of Feb. **3rd**. You may ask questions about this assignment in that tutorial.

STUDY: Chapter 4 Sections: 4.5; 4.7 (OMIT Thm 4.35,).

HOMEWORK:

At the beginning of your TUTORIAL during the week of Feb. **10**th you may be asked to either submit the following "Homework" problems or write a quiz based on this assignment and/or related material from the lectures and textbook readings. All submissions to weekly evaluations will be done via crowdmark in your officially registered tutorial. This part of your assignment will count towards the 20% of your final mark, which is based on weekly assignments / quizzes.

Note, when using FTOC part I, you do not need to check that the continuity hypothesis is true before using the consequent. Everything that we give you to integrate will be continuous. You still only need to check your 'guessed/by inspection' antiderivative if the specific question's instructions require it.

1. Fill in the blanks in the "FTOC II" proof document (posted underneath the link to this assignment on Quercus - To be posted by end of day Mon. Feb. 3rd.).

2. Let $a, b \in \mathbb{R}$, $a < b$. Provide a complete and detailed proof of the following statement :

If f is continuous on $[a,b]$ and define $F(x) = \int_b^x f(t)dt$, any $x \in [a,b]$, then $F'_{-}(b) = f(b)$. That is that the left-hand derivative of F at $x = b$ equals $f(b)$.

Do not use FTOCI.

3. Explain what is mathematically wrong with the below argument (snippet of proof of FTOCII) :

   Let $a, b \in \mathbb{R},\ a < b$. Suppose that

   - f is continuous on $[a,b]$, and
   - Define $F(x) = \displaystyle\int_a^x f(t)dt$, any $x \in [a,b]$

   Let $x \in (a,b)$ be arbitrary. Then

$$
\begin{aligned}
F'(x) &= \lim h \to 0 \frac{F(x+h) - F(x)}{h} \text{,by def of } F' \\
&= \lim_{h \to 0} \frac{\int_a^{x+h} f(t)dt - \int_a^x f(t)dt}{h}, \text{ by def of } F \\
&= \lim_{h \to 0} \frac{\int_x^{x+h} f(t)dt}{h}, \text{ by FTOC I} \\
&= \lim_{h \to 0} \frac{1}{h} \int_x^{x+h} f(t)dt \\
&= \lim_{h \to 0} \frac{1}{(x+h) - x} \int_x^{x+h} f(t)dt
\end{aligned}
$$

4. Evaluate the following integrals. Make sure to appropriate justify your solutions. (Do NOT use any 'integration techniques' from Chapter 5). Only use material taught thus far in A37 W25 up to and including this assignment's STUDY section.

   (a) $\displaystyle\int_0^1 \frac{1}{\sin(t) + 1}\, dt$

   (b) $\displaystyle\int_{-1}^4 \left(|(u-2)(u+3)| + e^{4u}\right)\, du$

(c) $\displaystyle\int_{\frac{\pi}{6}}^{0} \frac{5}{1+9x^2}\,dx$

(d) $\displaystyle\int_{-2}^{2} f(x)dx$ where f be defined by $f(x) = \begin{cases} 4^{-x} & \text{, if } x \leq 0 \\ \sqrt{x}(x^2+1) & \text{, if } x > 0 \end{cases}$

5. Find the indicated derivatives of the following functions. Do NOT evaluate the integrals. Make sure to fully justify your work.

   (a) $\displaystyle Bob(x) = \int_{-\pi}^{x} \frac{1}{1+8^{t^2}}\,dt;\ Bob''(x)$

   (b) Let $x < 0$. $\displaystyle H(x) = \int_{x-10}^{4} \sin^3(5t)\,dt;\ H'(x)$

   (c) Define $\displaystyle g(x) = \int_{1}^{e^x} \frac{1-t^2}{1+t^4}\,dt;\ g'(x)$

   (d) $\displaystyle S(x) = \int_{3}^{\int_1^x \sin^3(t)dt} \frac{1}{1+t^2+\sin^6(t)}\,dt;\ S'(x)$

6. On what interval(s) is the function $\displaystyle f(x) = \int_{x}^{0} \frac{t^2}{t^2+t+2}\,dt$ concave down? Make sure to fully justify your solution. Do NOT evaluate the integral.

7. Let $x > 0$. Prove that the value of the following expression does <u>not</u> depend on x:

$$\int_0^x \frac{1}{1+t^4}\,dt + \frac{1}{3}\int_0^{\frac{1}{x^3}} \frac{1}{1+t^{\frac{4}{3}}}\,dt.$$

   Fully justify your argument. Do <u>not</u> evaluate any integrals.

We must accept finite disappointment, but never lose infinite hope. – *Martin Luther King Jr.*