Доказать что вариации по $S_m$, $S'_m$ и $S''_m$ равны.

1.

$$S_m = -m \int \left| g_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} \right|^{1/2} ds \tag{0.1}$$

$$S'_m = -m \int \left| g_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} \right| ds \tag{0.2}$$

Полагая что $\mathfrak{L} \not\to \infty$:

$$S_m = \int |\mathfrak{L}|^{1/2} ds \implies \delta S_m = \int \frac{\delta|\mathfrak{L}|}{|\mathfrak{L}|^{1/2}} ds = 0 \Leftrightarrow \delta S_m = \int \delta|\mathfrak{L}| ds = 0 \implies \delta|\mathfrak{L}| = 0 \tag{0.3}$$

$$S'_m = \int |\mathfrak{L}| ds \implies \delta S'_m = \int \delta|\mathfrak{L}| ds = 0 \implies \delta|\mathfrak{L}| = 0 \tag{0.4}$$

2.

$$S_m = -m \int \left| g_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} \right|^{1/2} ds \tag{0.5}$$

$$S''_m = -m \int \mathfrak{F} \left( g_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} \right) ds \tag{0.6}$$

$$S''_m = \int \mathfrak{F}(\mathfrak{L}) \implies \delta S''_m = \int \delta \mathfrak{F}(\mathfrak{L}) = \int \frac{\partial \mathfrak{F}}{\partial \mathfrak{L}} \delta \mathfrak{L} = 0 \tag{0.7}$$

Так как $\mathfrak{F}' \neq 0$ то:

$$\int \frac{\partial \mathfrak{F}}{\partial \mathfrak{L}} \delta \mathfrak{L} = 0 \Leftrightarrow \delta \mathfrak{L} = 0 \tag{0.8}$$

Так как $\delta$ это маленькое приращение то $\delta|\mathfrak{L}| = 0$ только в близи 0, а следовательно $\delta|\mathfrak{L}| = 0 \Leftrightarrow \delta\mathfrak{L} = 0$.