Гамильтониан

$$i\hbar \frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

$$\triangleleft \; \frac{\partial \psi^*}{\partial t} = \frac{i}{\hbar}\hat{H}^*\psi = \frac{i}{\hbar}\int (\hat{H}\psi)^*\psi dx - \frac{i}{\hbar}\int \psi^*\hat{H}\psi dx = \frac{i}{\hbar}\int \psi^*(\hat{H}^\dagger - \hat{H})\psi dx = 0$$

$$\frac{\partial}{\partial t}\int |\psi|^2 dx = 0 \Rightarrow H^\dagger = \hat{H}$$

$$\| \int \frac{\partial \psi^*}{\partial t}\psi dx + \int \psi^* \frac{\partial \psi}{\partial t} dx$$

Производная оператора по времени

$$\mathring{A} \underset{\Delta t \to 0}{=} \frac{A(t+\Delta t) - A(t)}{\Delta t}$$

$\hat{\hat{A}}$ — хотим: $\langle \hat{\hat{A}} \rangle = \frac{d}{dt}\langle \hat{A} \rangle$

$$\frac{d}{dt}\langle \psi | \hat{A} | \psi \rangle = \langle \psi | \frac{\partial}{\partial t}\hat{A} | \psi \rangle + \langle \dot{\psi} | \hat{A} | \psi \rangle + \langle \psi | \hat{A} | \dot{\psi} \rangle = \langle \psi | \hat{\dot{A}} | \psi \rangle + \frac{i}{\hbar}\langle \psi | H^\dagger A | \psi \rangle - \frac{i}{\hbar} \ldots$$

по Шредингера

$$\hat{\hat{A}} = \frac{\partial A}{\partial t} + \frac{i}{\hbar}[H, A]$$

Если $\hat{A}(t)$ и $[H, A]$ то $\hat{\hat{A}} = 0$ => хорошее квантовое число

Импульс

$\hat{H}$ неменятся при трансляции системы

$\triangleleft$ Достаточно доказать что не меняется при достаточно малом $\delta\vec{r}$

$$\psi(\vec{r} + \delta\vec{r}) = \psi(\vec{r}) + \delta\vec{r}\,\nabla\psi(\vec{r}) + \ldots \to 0$$

$\hat{O}$ не меняет $\hat{H}$ если $[\hat{O}, \hat{H}] = 0$

$$[\hat{T}, \hat{H}] = 0 \Rightarrow [c\vec{\nabla}, \hat{H}] = 0$$

$$c\vec{\nabla} = \hat{p} \qquad c = -i\hbar$$

$i\hbar |\dot{\psi}\rangle = \hat{H}|\psi\rangle$ стат. ур. Шредингера

$\hat{H}$

$\hat{H} = \frac{\hat{p}^2}{2m}$

$|p\rangle = exp(\frac{i}{\hbar}\vec{p}\vec{x})$

$[\hat{H}, \hat{p}] = 0$

$-\frac{\hbar^2}{2m}\vec{\nabla}^2 exp(\frac{i}{\hbar}\vec{p}\vec{r}) = \underbrace{\frac{\vec{p}^2}{2m}}_{= E} exp(\frac{i}{\hbar}\vec{p}\vec{r})$

Семинар

$$\psi(\vec r+\delta\vec r) = \sum \nabla^n \psi(\vec r)\frac{\delta \vec r^{\,n}}{n!} \approx (1+\delta r\nabla)\psi = (1+\tfrac{i}{\hbar}\delta\hat r\hat p)$$

$$= \exp(\vec a\vec\nabla)\psi(\vec r) \Rightarrow \hat T_{\vec a} = \exp(\tfrac{i}{\hbar}\hat a\hat p)$$

$$\psi(\vec r+\vec a) = \int\frac{d^3p}{(2\pi\hbar)^{3/2}}\, b(\vec p)\exp\left(\frac{i\vec p\vec r}{\hbar}\right)$$

$$\psi(\vec r) = \int\frac{d^3p}{(2\pi\hbar)^{3/2}}\, a(\vec p)\exp\left(\frac{i\vec p\vec r}{\hbar}\right)$$

$$\Rightarrow b(\vec p) = \exp$$

① $\left[i\hbar\partial_x, \exp(\tfrac{i}{\hbar}ap_x)\right] = -a\exp(\tfrac{i}{\hbar}ap) + \cancel{i\hbar\exp(\tfrac{i}{\hbar}ap_x)\partial_x} - \cancel{i\hbar\exp(\tfrac{i}{\hbar}ap_x)\partial_x} = -a\exp(\tfrac{i}{\hbar}ap)$

② ...

Вывести соотнош. неопределенности

$$[\hat f, \hat g] = -i\hbar\hat C$$

$$\Delta\hat f = \sqrt{\langle(\hat f-\langle f\rangle)^2\rangle} = \sqrt{\langle \hat f_0^2\rangle}$$

$$\hat f_0 = \hat f - \langle\hat f\rangle$$

$$|\varphi\rangle = (\alpha\hat f_0 + i\hat g)|\psi\rangle$$

$$0 \le \langle\varphi|\varphi\rangle = \langle\psi|(\alpha\hat f_0 - i\hat g)(\alpha\hat f + i\hat g)|\psi\rangle = \langle\psi|(\alpha^2\hat f^2 + g_0^2 + i\alpha[\hat f_0,\hat g])|\psi\rangle = \alpha^2\langle\hat f_0^2\rangle + \langle\hat g_0^2\rangle + \alpha\hbar\langle\hat C\rangle \ge 0$$

$$\mathcal{D} \le 0$$

$$\hbar^2\langle C\rangle^2 - 4(\Delta f)^2(\Delta g)^2 \le 0$$

$\hat{A}, \hat{B}, \hat{\mathcal{L}}$

$[\hat{A}, \hat{B}] \neq 0 \quad [\hat{A}, \hat{\mathcal{L}}] = 0 \quad [\hat{B}, \hat{\mathcal{L}}] = 0$

доказать что есть вырожденные

$0 = (\hat{A}\hat{\mathcal{L}} - \hat{\mathcal{L}}\hat{A})\psi_i = \mathcal{L}_i \hat{A} \psi_i - \hat{\mathcal{L}} \hat{A} \psi_i = 0$