Задание 1

$\langle a| = a_1\langle\varphi_1| + a_0\langle\varphi_0| \qquad \langle b| = b_1\langle\varphi_1| + b_0\langle\varphi_0|$

$\langle a|b\rangle = \cancel{a_1 b_0\langle\varphi_1|\varphi_0\rangle}^{\,0} + b_1 a_1\langle\varphi_1|\varphi_1\rangle + \cancel{a_0 b_1\langle\varphi_0|\varphi_1\rangle}^{\,0} + a_0 b_0\langle\varphi_0|\varphi_0\rangle = b_1 a_1 + b_0 a_0 = 0$

$b_1 = \sin\frac{\Theta_b}{2} \qquad b_0 = \cos\frac{\Theta_b}{2}\exp(i\varphi_b)$ пусть для $|b\rangle$ $\Theta_b = 0 \quad \varphi_b = 0$

$a_1 = \sin\frac{\Theta}{2} \qquad a_0 = \cos\frac{\Theta}{2}\exp(i\varphi)$

$b_0 a_0 = 0 \Rightarrow \cos\frac{\Theta}{2}\exp(i\varphi) = 0 \Rightarrow \Theta = \pi \quad \varphi$ – любое $\blacksquare$

Задание 2

a) $\hat{H} = \begin{pmatrix} \mathcal{E}_0 & 0 \\ 0 & -\mathcal{E}_0 \end{pmatrix}$

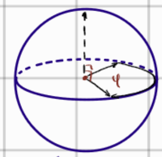
$|\iota(0)\rangle = |\iota_0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\mathcal{E}_+ = \mathcal{E}_0 \qquad \mathcal{E}_- = -\mathcal{E}_0$

$|\varphi_1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad |\varphi_2\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|\iota_0\rangle = \frac{1}{\sqrt{2}}|\varphi_1\rangle + \frac{1}{\sqrt{2}}|\varphi_2\rangle \Rightarrow |\iota\rangle = \frac{\exp\left(-\frac{i\mathcal{E}_0 t}{\hbar}\right)}{\sqrt{2}}|\varphi_1\rangle + \frac{\exp\left(\frac{i\mathcal{E}_0 t}{\hbar}\right)}{\sqrt{2}}|\varphi_2\rangle =$

$= \exp\left(-\frac{i\mathcal{E}_0 t}{\hbar}\right)\left[2^{-1/2}|\varphi_1\rangle + 2^{-1/2}\exp\left(\frac{2i\mathcal{E}_0 t}{\hbar}\right)|\varphi_2\rangle\right] \Rightarrow \theta = \pi/2 \qquad \varphi = \frac{2i\mathcal{E}_0 t}{\hbar}$



вращение

б) $\hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\sigma_1 = 1 \qquad \sigma_2 = -1$

$|\varphi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix} \qquad |\varphi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}$

$\langle\varphi_1| = \frac{1}{\sqrt{2}}(i, 1) \qquad \langle\varphi_2| = \frac{1}{\sqrt{2}}(-i, 1)$

$P(t) = |\langle\varphi_1|\iota\rangle|^2 = \frac{1}{4}\left|1 - i\exp\left(\frac{2i\mathcal{E}_0 t}{\hbar}\right)\right|^2 = \frac{1}{2}\left[\sin\left(\frac{2\mathcal{E}t}{\hbar}\right) + 1\right] \Rightarrow T_{min} = \frac{\pi\hbar}{4\mathcal{E}_0}$

$P(t) = |\langle\varphi_2|\iota\rangle|^2 = \frac{1}{4}\left|1 + i\exp\left(\frac{2i\mathcal{E}t}{\hbar}\right)\right|^2 = \frac{1}{2} - \frac{1}{2}\sin\left(\frac{2\mathcal{E}t}{\hbar}\right) \Rightarrow T_{min} = \frac{3\pi\hbar}{4\mathcal{E}_0}$

$\Rightarrow T = \frac{\pi\hbar}{4\mathcal{E}_0}$

c) $\langle\iota|\hat{\sigma}_y|\iota\rangle = \frac{1}{2}\left(\exp\left[-\frac{2i\mathcal{E}_0 t}{\hbar}\right], 1\right)\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} \exp\left[\frac{2i\mathcal{E}_0 t}{\hbar}\right] \\ 1 \end{pmatrix} = \frac{1}{2}\left(i, -i\exp\left[-\frac{2i\mathcal{E}_0 t}{\hbar}\right]\right)\begin{pmatrix} \exp\left[\frac{2i\mathcal{E}_0 t}{\hbar}\right] \\ 1 \end{pmatrix} = \frac{\exp\left(-\frac{2i\mathcal{E}_0 t}{\hbar}\right) - \exp\left(\frac{2i\mathcal{E}_0 t}{\hbar}\right)}{2i} = -\sin\left(\frac{2i\mathcal{E}_0 t}{\hbar}\right)$

d) $\exp\left(\frac{i\hat{H}t}{\hbar}\right)\hat{\sigma}_y\exp\left(\frac{i\hat{H}t}{\hbar}\right) = \hat{\sigma}_y + \frac{it}{\hbar}[\hat{H}, \hat{\sigma}_y] + \frac{it}{\hbar}\frac{1}{2}[\hat{H}, [\hat{H}, \hat{\sigma}]] + \ldots = \hat{\sigma}_y + \frac{it\mathcal{E}_0}{\hbar}\begin{pmatrix} 0 & -2i \\ -2i & 0 \end{pmatrix}$

$\langle\iota|\hat{\sigma}_y(t)|\iota\rangle = \langle\iota|\hat{\sigma}_y|\iota\rangle + \frac{it}{\hbar}(1\ 1)\begin{pmatrix} 0 & -2i \\ -2i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{4t\mathcal{E}}{\hbar}$

Задание 3

Из лекций $E_\pm = E \pm \sqrt{\mathcal{E}^2 d^2 + \Delta^2}$

$\alpha = -\cancel{\frac{\partial^2 E}{\partial\mathcal{E}^2}}^{\,0} \mp \frac{\partial\sqrt{\mathcal{E}^2 d^2 + \Delta^2}}{\partial\mathcal{E}} = \mp\frac{\Delta^2 d^2}{(d^2\mathcal{E}^2 + \Delta^2)^{3/2}}$