Задание 1

$$\dot{x} = \cancel{\frac{\partial x}{\partial t}}^{0} + \frac{i}{\hbar}\left[\hat{H}, \hat{x}\right] = \frac{i}{2m\hbar}\left[\hat{p}^2, \hat{x}\right] = -\frac{i\hbar}{m}\partial_x = \frac{\hat{p}}{m}$$

Задание 2

$$|\psi(t)\rangle = \hat{U}_t|\psi_0\rangle$$

Из ДЗ 2 помним $\langle x_0\rangle = x_0$ $\langle p_0\rangle = p_0$ $\langle x_0^2\rangle = \frac{a^2}{2} + x_0^2$ $\langle p_0\rangle = \frac{\hbar^2}{2a^2} + p_0^2$

$$\langle p\rangle = \langle\psi_0|\hat{U}_t^\dagger\hat{p}\hat{U}_t|\psi_0\rangle = \langle\psi_0|\hat{p}|\psi_0\rangle + \langle\psi_0|\mathcal{E}et|\psi_0\rangle = \langle p_0\rangle + \mathcal{E}et$$

$\mathcal{E}et\langle\psi_0|\psi_0\rangle$

$$\langle x\rangle = \langle\psi_0|\hat{U}_t^\dagger\hat{x}\hat{U}_t|\psi_0\rangle = \langle\psi_0|\hat{x}|\psi_0\rangle + \frac{\mathcal{E}et^2}{m}$$

$$\langle\psi|\hat{x}\hat{x}|\psi\rangle = \langle\psi_0|\hat{U}_t\hat{x}\hat{x}\hat{U}_t|\psi_0\rangle$$

Так как $\hat{U}_t$ – унитарен $\Rightarrow \hat{U}^\dagger\hat{U} = \hat{\mathbb{1}} = \hat{U}\hat{U}^\dagger$

$$\langle\psi|\hat{x}\hat{x}|\psi\rangle = \langle\psi_0|\hat{U}_t^\dagger\hat{x}\hat{\mathbb{1}}\hat{x}\hat{U}_t|\psi_0\rangle = \langle\psi_0|\hat{U}_t^\dagger\hat{x}\hat{U}_t\hat{U}_t^\dagger\hat{x}\hat{U}_t|\psi_0\rangle = \langle\psi_0|x^2|\psi_0\rangle + \frac{\mathcal{E}et}{m}\langle\psi_0|x|\psi_0\rangle + \frac{\mathcal{E}^2e^2t^2}{4m^2} = \langle x_0^2\rangle + \frac{\mathcal{E}et^2}{m}\langle x_0\rangle + \frac{\mathcal{E}^2e^2t^4}{4m^2}$$

$$\langle\psi|\hat{p}^2|\psi\rangle = \langle p_0^2\rangle + 2\mathcal{E}et\langle p_0\rangle + \mathcal{E}^2e^2t^2$$

Задание 3