Задание 1

$(1\ \ 0)^+ = \begin{pmatrix}1\\0\end{pmatrix}$

◀ $(1\ \ 0)\begin{pmatrix}1\\0\end{pmatrix}(1\ \ 0) = (1\ \ 0)[1+0] = (1\ \ 0)$ ■

Задание 2

$\begin{pmatrix}0\\1\\2\\3\end{pmatrix}^+ = \boxed{l^2 = 1+4+9=14} = \frac{1}{14}(0\ \ 1\ \ 2\ \ 3)$

$\begin{pmatrix}0\\1\\2\\3\end{pmatrix}\frac{1}{14}(0\ \ 1\ \ 2\ \ 3)\begin{pmatrix}0\\1\\2\\3\end{pmatrix} = \begin{pmatrix}0\\1\\2\\3\end{pmatrix}(1)$

Задание 3

$\begin{pmatrix}1&2&3\\0&-1&-2\end{pmatrix} \sim \begin{pmatrix}1&0&-1\\0&-1&-2\end{pmatrix} \sim \begin{pmatrix}1&0&-1\\0&1&2\end{pmatrix} = G$

$G = \begin{pmatrix}1&0&-1\\0&1&2\end{pmatrix}$ $\quad F = \begin{pmatrix}1&2\\0&-1\end{pmatrix}$ $\quad \det(F) = -1 \Rightarrow F^+ = F^{-1}$

$\begin{pmatrix}1&2\\0&-1\end{pmatrix}\begin{pmatrix}1&0&-1\\0&1&2\end{pmatrix} = \begin{pmatrix}1&2&-1+4\\0&-1&-2\end{pmatrix}$

$G^+ = \begin{pmatrix}1&0\\0&1\\-1&2\end{pmatrix}\left[\begin{pmatrix}1&0&-1\\0&1&2\end{pmatrix}\begin{pmatrix}1&0\\0&1\\-1&2\end{pmatrix}\right]^{-1} = \begin{pmatrix}1&0\\0&1\\-1&2\end{pmatrix}\begin{pmatrix}2&-2\\-2&5\end{pmatrix}^{-1} = \frac{1}{6}\begin{pmatrix}5&2\\2&2\\-1&2\end{pmatrix}$

$\left(\begin{array}{cc|cc}2&-2&1&0\\-2&5&0&1\end{array}\right) \sim \left(\begin{array}{cc|cc}1&-1&1/2&0\\0&3&1&1\end{array}\right) \sim \left(\begin{array}{cc|cc}1&0&5/6&1/3\\0&1&1/3&1/3\end{array}\right)$

$F^+ = F^{-1} = \begin{pmatrix}1&2\\0&-1\end{pmatrix}$

$A^+ = G^+F^+ = \frac{1}{6}\begin{pmatrix}5&2\\2&2\\-1&2\end{pmatrix}\begin{pmatrix}1&2\\0&-1\end{pmatrix} = \frac{1}{6}\begin{pmatrix}5&8\\2&2\\-1&-4\end{pmatrix}$

Задание 4

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \qquad rk(A) = 2$$

F

$$A \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad G$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$G^+ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \right]^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \\ 0 & 1/2 \end{pmatrix}$$

$$F^+ = \left[\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \right]^{-1} \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}^{t} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 & 1 & -1 \\ -1 & 1 & 2 \end{pmatrix}$$

$$A^+ = \frac{1}{6} \begin{pmatrix} 4 & 2 & -2 \\ -1 & 1 & 2 \\ -1 & 1 & 2 \end{pmatrix}$$

Задание 6

$$F = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \leftarrow i \qquad G = \begin{array}{c} \quad\quad j \\ \quad\quad \downarrow \\ (0 \dots 1 \dots 0) \end{array}$$

$$A = FG$$

$$\left. \begin{array}{l} F^+ = F^T \\ G^+ = G^T \end{array} \right| \Rightarrow A^+ = F^+ G^+$$