Задание 1

$$\dot{x} = \cancelto{0}{\frac{\partial x}{\partial t}} + \frac{i}{\hbar}[\hat{H}, \hat{x}] = \frac{i}{2m\hbar}[\hat{p}^2, \hat{x}] = -\frac{i\hbar}{m}\partial_x = \frac{\hat{p}}{m}$$

Задание 2

$$|\varphi(t)\rangle = \hat{U}_t|\varphi_0\rangle$$

Из ДЗ 2 помним $\langle x_0\rangle = x_0$ $\quad\langle p_0\rangle = p_0$ $\quad\langle x_0^2\rangle = \frac{a^2}{2} + x_0^2$ $\quad\langle p_0\rangle = \frac{\hbar^2}{2a^2} + p_0^2$

$$\langle p\rangle = \langle\varphi_0|\hat{U}_t^\dagger\hat{p}\,\hat{U}_t|\varphi_0\rangle = \langle\varphi_0|\hat{p}|\varphi_0\rangle + \underbrace{\langle\varphi_0|\varepsilon e t|\varphi_0\rangle}_{\varepsilon e t\langle\varphi_0|\varphi_0\rangle} = \langle p_0\rangle + \varepsilon e t$$

$$\langle x\rangle = \langle\varphi_0|\hat{U}_t^\dagger\hat{x}\,\hat{U}_t|\varphi_0\rangle = \langle\varphi_0|\hat{x}|\varphi_0\rangle + \frac{\varepsilon e t^2}{m}$$

$$\langle\varphi|\hat{x}\hat{x}|\varphi\rangle = \langle\varphi_0|\hat{U}_t\hat{x}\hat{x}\hat{U}_t|\varphi_0\rangle$$

Так как $\hat{U}_t$ – унитарен $\Rightarrow$ $\hat{U}^\dagger\hat{U} = \hat{\mathbb{1}} = \hat{U}\hat{U}^\dagger$

$$\langle\varphi|\hat{x}\hat{x}|\varphi\rangle = \langle\varphi_0|\hat{U}_t^\dagger\hat{x}\hat{\mathbb{1}}\hat{x}\hat{U}_t|\varphi_0\rangle = \langle\varphi_0|\hat{U}_t^\dagger\hat{x}\hat{U}_t\hat{U}_t^\dagger\hat{x}\hat{U}_t|\varphi_0\rangle = \langle\varphi_0|x^2|\varphi_0\rangle + \frac{\varepsilon e t^2}{m}\langle\varphi_0|x|\varphi_0\rangle + \frac{\varepsilon^2 e^2 t^4}{4m^2} = \langle x_0^2\rangle + \frac{\varepsilon e t^2}{m}\langle x_0\rangle + \frac{\varepsilon^2 e^2 t^4}{4m^2}$$

$$\langle\varphi|\hat{p}^2|\varphi\rangle = \langle p_0^2\rangle + 2\varepsilon e t\langle p_0\rangle + \varepsilon^2 e^2 t^2$$

Задание 3

$$\hat{P}_+|\varphi(a)\rangle = \frac{1}{2}(\hat{\mathbb{1}} + \hat{I})|\varphi(a)\rangle = \frac{|\varphi(x)\rangle + |\varphi(-x)\rangle}{2}$$

Четна т.к.

$$◀ \ \frac{|\varphi(a)\rangle + |\varphi(-a)\rangle}{2}\ \Big|\cdot\hat{I} \Rightarrow \frac{|\varphi(-a)\rangle + |\varphi(a)\rangle}{2} = \frac{|\varphi(a)\rangle + |\varphi(-a)\rangle}{2} \ ■$$

$$\hat{P}_-|\varphi(a)\rangle = \frac{1}{2}(\hat{\mathbb{1}} - \hat{I})|\varphi(a)\rangle = \frac{|\varphi(x)\rangle - |\varphi(-x)\rangle}{2}$$

Доказательство нечетности аналогично.

Вспомним $\hat{I}^2 = \hat{\mathbb{1}}$ $\quad\hat{\mathbb{1}}^n = \hat{\mathbb{1}}$ $\quad\hat{\mathbb{1}}\hat{A} = \hat{A}\hat{\mathbb{1}} = \hat{A}$

$$\hat{P}_+\hat{P}_- = \frac{1}{4}(\hat{\mathbb{1}}^2 + \cancel{\hat{I}\hat{\mathbb{1}}} - \cancel{\hat{\mathbb{1}}\hat{I}} - \hat{I}^2) = \frac{1}{4}(\hat{\mathbb{1}} - \hat{\mathbb{1}}) = \hat{0}$$

$$\hat{P}_+ + \hat{P}_- = \frac{1}{2}(\hat{\mathbb{1}} + \hat{I} + \hat{\mathbb{1}} - \hat{I}) = \hat{\mathbb{1}}$$

$$\hat{P}_\pm^2 = \frac{1}{4}(\hat{\mathbb{1}}^2 \pm \hat{I}\hat{\mathbb{1}} \pm \hat{I}\hat{\mathbb{1}} + \hat{I}\hat{I}) = \frac{1}{2}(\hat{\mathbb{1}} \pm \hat{I})$$