$$\frac{d\vec{y}}{dt} + \hat{\Gamma}\vec{y} = \vec{\varphi}$$

Матричная функция Грина

$$\vec{y}(t) = \int d\tau\, \hat{G}(t-\tau)\vec{\varphi}$$

$$\frac{d}{dt}\hat{G}(t) + \hat{\Gamma}\hat{G} = \delta(t)\mathbb{1}$$

$$\Rightarrow \left(\frac{d}{dt} + \hat{\Gamma}\right)\int dt \left(\frac{d}{dt} + \hat{\Gamma}\right)\hat{G}(t) + \vec{\varphi}(\tau) = \vec{\varphi}(t)$$

$$\hat{G}(t) = \Theta(t)\exp(-\hat{\Gamma}t)$$

$$\exp(\hat{A}) = \sum_n \frac{1}{n!}\hat{A}^n$$

$$\exp(\hat{A}+\hat{B}) = \boxed{[\hat{A},\hat{B}] = 0} = \exp(\hat{A})\exp(\hat{B})$$

$$\frac{d}{dt}\left\{\Theta(t)\exp[-\hat{\Gamma}(t)]\right\} =$$

$$= \delta(t)\exp[-\hat{\Gamma}t] + \Theta(t)\frac{d}{dt}\exp[-\hat{\Gamma}t] =$$

$$= \boxed{\frac{d}{dt}\exp[-\hat{\Gamma}t] \quad \ldots} = \delta(t)\exp[-\hat{\Gamma}t] + \Theta(t)\hat{\Gamma}\exp[-\hat{\Gamma}t]$$

Пусть $\vec{a}$: $\vec{a}\hat{\Gamma} = \gamma\vec{a}$

$$\vec{y} = \sum_i y_i \vec{a}_i$$

$$\Rightarrow \begin{cases} \frac{dy_1}{dt} + \lambda_1 y_1 = \varphi_1 \\ \ldots \\ \frac{dy_n}{dt} + \lambda_n y_n = \varphi_n \end{cases}$$

$$\hat{\Gamma} = \sum_i \lambda_i \vec{a}_i \underbrace{\vec{b}_i^T}_{\text{соб. строка}} \Rightarrow \hat{\Gamma}^n = \sum_i \lambda_i^n \vec{a}_i \vec{b}_i^T = \hat{A}\hat{\Lambda}\hat{B}$$

$$\hat{G}(t) = \Theta(t)\sum_i \exp(\lambda_i t)\vec{a}_i\vec{b}_i^T = \Theta(t)\hat{A}\exp(-t\hat{\Lambda})\hat{B}$$

Если $\hat{\Gamma}$ - вырождена то

$$\hat{\Gamma} = \hat{A}\hat{\Lambda}\hat{B}$$

$$\hat{\Lambda} = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$$

$$\exp[-\hat{\Lambda}t] = \exp\left[-\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}t + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}t\right] = \sum_{m=0}^{\infty}\frac{1}{m!}\left(\hat{A}^m + m\hat{A}^{m-1}\hat{N}\right) = \exp(\hat{A}) + \sum_{m=0}^{\infty}\frac{\hat{A}^{m-1}}{(m-1)!}\hat{N} = \exp\left[-\lambda t\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\left[1 - t\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right]$$

Если $\gamma = \gamma(t)$

$$\dot{x} + \gamma(t)x = \varphi$$

$$\dot{G}(t,\tau) + \gamma(t)G(t,\tau) = \delta(t-\tau)$$

$t < \tau \quad G = 0 \qquad t = \tau + \varepsilon \quad G = 1$

$$\frac{dG}{G} = \gamma\, dt$$

$$\Rightarrow G = \exp\left[-\int_{-\tau}^{t} dt\, \gamma(t)\right]$$

$$\frac{d\vec{y}}{dt} - \hat{\Gamma}(t)\,\vec{y} = \varphi$$

$$G = \Theta(t-\tau)\ T\exp\left[-\int_{\tau}^{t} dt_1\, \hat{\Gamma}(t_1)\right]$$

$$\shortparallel$$

$$\mathbb{1} - \int_{\tau}^{t_1} dt_1\, \hat{\Gamma}(t_1) + \int_{\tau}^{t_1}\int_{\tau}^{t_2} dt_2\, dt_1\, \Gamma(t_1)\, \Gamma(t_2) - \int_{\tau}^{t_1}\int_{\tau}^{t_2}\int_{\tau}^{t_3} dt_3\, dt_2\, dt_1\, \Gamma(t_1)\Gamma(t_2)\Gamma(t_3) + \ldots$$

Семинар

$$\left[\frac{d^4}{dt^4} + 4\frac{d^2}{dt^2} + 3\right] x = \varphi(t) \quad =>$$

$x_1 = x$

$x_2 = \dot{x}$

$x_3 = \ddot{x}$

$x_4 = \dddot{x}$

$=>$

$$\frac{d}{dt} x_4 + 4x_2 + 3x_1 = \varphi(t) => \dot{x}_4 - 4x_3 - 3x_1 = \varphi(t)$$

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -4 & 0 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \varphi(t) \end{pmatrix}$$

$$\hat{\Gamma} => \begin{vmatrix} -\lambda & 1 \\ 4 & -\lambda \end{vmatrix} = \lambda^2 - 4 => \lambda_{1,2} = \pm 2$$

$$\lambda_1 = 2 => \begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 => -2x_1 + 1x_2 = 0 => x_1 = \frac{1}{2}x_2 => h_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\lambda_2 = -2 => \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 => 2x + 1x_1 = 0 => x_1 = -\frac{1}{2}x_2 => h_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$=> \begin{pmatrix} 1/2 & 1/2 \\ 1 & -1 \end{pmatrix}$$

$$\lambda_1 = 2 => (x_1\ x_2)\begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix} => \xi_1 = (2, 1)$$

$$\lambda = -2 => (x_1\ x_2)\begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix} => \xi_2 = (2, -1)$$

$$=> \begin{pmatrix} 1 & -1 \\ 1/2 & 1/2 \end{pmatrix}$$

$$G(t) = \Theta(t) \sum \begin{pmatrix} 1/2 & 1/2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} e^{-2t} & 0 \\ 0 & e^{2t} \end{pmatrix}\begin{pmatrix} 1 & 1/2 \\ 1 & -1/2 \end{pmatrix}$$

D3

$$\Gamma = \begin{pmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{pmatrix} \qquad G - ?$$

$$\det(\Gamma - \iota \mathbb{1}) = (\lambda - \iota) => \iota_{1234} = \lambda$$

$$x(t) = \exp\left(\begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{pmatrix} t\right)\left(\mathbb{1} - \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} t + \frac{t^2}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \frac{t^3}{6}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\right)$$