Гамильтониан

$i\hbar \frac{\partial}{\partial t}\Psi = \hat{H}\Psi$

$\triangleleft\ \frac{\partial \Psi^*}{\partial t} = \frac{i}{\hbar}\hat{H}^*\Psi = \frac{i}{\hbar}\int (\hat{H}\Psi)^*\Psi dx - \frac{i}{\hbar}\int \Psi^*\hat{H}\Psi dx = \frac{i}{\hbar}\int \Psi^*(\hat{H}^+ - \hat{H})\Psi dx = 0$

$\frac{\partial}{\partial t}\int |\Psi|^2 dx = 0 \Rightarrow H^+ = \hat{H}$

$\quad '' \int \frac{\partial \Psi^*}{\partial t}\Psi dx + \int \Psi^* \frac{\partial \Psi}{\partial t} dx$

Производная оператора по времени

$\mathring{A} \underset{\Delta t \to 0}{=} \frac{A(t+\Delta t) - A(t)}{\Delta t}$

$\hat{\mathring{A}}$ – хотим: $\langle \hat{\mathring{A}} \rangle = \frac{d}{dt}\langle \hat{A} \rangle$

$\frac{d}{dt}\langle \psi | \hat{A} | \psi \rangle = \langle \psi | \frac{\partial}{\partial t}\hat{A} | \psi \rangle + \langle \dot{\psi} | \hat{A} | \psi \rangle + \langle \psi | \hat{A} | \dot{\psi} \rangle = \langle \psi | \hat{\mathring{A}} | \psi \rangle + \frac{i}{\hbar}\langle \psi | H^+ A | \psi \rangle - \frac{i}{\ } \ldots$

по Шредингеру

$\hat{\mathring{A}} = \frac{\partial A}{\partial t} + \frac{i}{\hbar}[H, A]$

Если $\hat{A}(t)$ и $[H, A]$ то $\hat{\mathring{A}} = 0 \Rightarrow$ хорошее квантовое число

Импульс

$\hat{H}$ неменятся при трансляции системы

$\triangleleft$ Достаточно доказать что не меняется при достаточно малом $\delta \vec{r}$

$\psi(\vec{r} + \delta\vec{r}) = \psi(\vec{r}) + \delta\vec{r}\,\nabla\psi(\vec{r}) + \underbrace{\ldots}_{0}$

$\hat{O}$ не менает $\hat{H}$ если $[\hat{O}, \hat{H}] = 0$

$[\hat{T}, \hat{H}] = 0 \Rightarrow [C\vec{\nabla}, \hat{H}] = 0$

$C\vec{\nabla} = \hat{p} \qquad C = -i\hbar$

$i\hbar\,|\dot{\psi}\rangle = \hat{H}|\psi\rangle$ стат. ур. Шредингера

$\hat{H}$

$$\hat{H} = \frac{\hat{p}^2}{2m}$$

$$|p\rangle = \exp\left(\tfrac{i}{\hbar}\vec{p}\vec{x}\right)$$

$$[\hat{H}, \hat{p}] = 0$$

$$-\frac{\hbar^2}{2m}\vec{\nabla}^2 \exp\left(\tfrac{i}{\hbar}\vec{p}\vec{r}\right) = \underset{\substack{\parallel \\ E}}{\frac{\vec{p}^{\,2}}{2m}} \exp\left(\tfrac{i}{\hbar}\vec{p}\vec{r}\right)$$

Семинар

$$\psi(\vec{r}+\delta\vec{r}) = \sum \nabla^n \psi(\vec{r}) \frac{\delta \vec{r}^{\,n}}{n!} \approx (1+\delta r \nabla)\psi = \left(1+\frac{i}{\hbar}\delta\vec{r}\hat{p}\right)$$

$$= \exp(\vec{a}\vec{\nabla})\psi(\vec{r}) \Rightarrow \hat{T}_{\vec{a}} = \exp\left(\tfrac{i}{\hbar}\hat{a}\hat{p}\right)$$

$$\psi(\vec{r}+\vec{a}) = \int \frac{d^3p}{(2\pi\hbar)^{3/2}} b(\vec{p}) \exp\left(\frac{i\vec{p}\vec{r}}{\hbar}\right)$$

$$\psi(\vec{r}) = \int \frac{d^3p}{(2\pi\hbar)^{3/2}} a(\vec{p}) \exp\left(\frac{i\vec{p}\vec{r}}{\hbar}\right)$$

$\Rightarrow b(\vec{p}) = \exp$

① $$\left[i\hbar\partial_x, \exp\left(\tfrac{i}{\hbar}ap_x\right)\right] = -a\exp\left(\tfrac{i}{\hbar}ap\right) + \cancel{i\hbar\exp\left(\tfrac{i}{\hbar}ap_x\right)}\partial_x - \cancel{i\hbar\exp\left(\tfrac{i}{\hbar}ap_x\right)}\partial_x = -a\exp\left(\tfrac{i}{\hbar}ap\right)$$

② ...

Доказывал у доски пункты ②, ⑤

Вывести соотнош. неопределенности

$$[\hat{f}, \hat{g}] = -i\hbar\hat{C}$$

$$\Delta\hat{f} = \sqrt{\langle(\hat{f}-\langle f\rangle)^2\rangle} = \sqrt{\langle\hat{f}_0^2\rangle}$$

$$\hat{f}_0 = \hat{f} - \langle\hat{f}\rangle$$

$$|\varphi\rangle = (\alpha\hat{f}_0 + i\hat{g})|\psi\rangle$$

$$0 \le \langle\varphi|\varphi\rangle = \langle\psi|(\alpha\hat{f}_0 - i\hat{g})(\alpha\hat{f} + i\hat{g})|\psi\rangle = \langle\psi|(\alpha^2\hat{f}^2 + g_0^2 + i\alpha[\hat{f}_0,\hat{g}])|\psi\rangle = \alpha^2\langle\hat{f}_0^2\rangle + \langle\hat{g}_0^2\rangle + \alpha\hbar\langle\hat{C}\rangle \ge 0$$

$$D \le 0$$

$$\hbar^2\langle C\rangle^2 - 4(\Delta f)^2(\Delta g)^2 \le 0$$

$\hat{A}, \hat{B}, \hat{\mathcal{L}}$

$[\hat{A}, \hat{B}] \neq 0 \quad [\hat{A}, \hat{\mathcal{L}}] = 0 \quad [\hat{B}, \hat{\mathcal{L}}] = 0$

доказать что есть вырожденные

$0 = (\hat{A}\hat{\mathcal{L}} - \hat{\mathcal{L}}\hat{A})\psi_i = \mathcal{L}_i \hat{A}\psi_i - \hat{\mathcal{L}}\hat{A}\psi_i = 0$