Задание 1

$$\dot{x} = \cancel{\frac{\partial x}{\partial t}}^{0} + \frac{i}{\hbar}[\hat{H}, \hat{x}] = \frac{i}{2m\hbar}[\hat{p}^2, \hat{x}] = -\frac{i\hbar}{m}\partial_x = \frac{\hat{p}}{m}$$

Задание 2

$$|\varphi(t)\rangle = \hat{U}_t|\varphi_0\rangle$$

Из ДЗ 2 помним $\langle x_0\rangle = x_0$ $\langle p_0\rangle = p_0$ $\langle x_0^2\rangle = \frac{a^2}{2} + x_0^2$ $\langle p_0^2\rangle = \frac{\hbar^2}{2a^2} + p_0^2$ $\langle\varphi|\hat{x}\hat{p} + \hat{p}\hat{x}|\varphi\rangle = 2\langle x_0\rangle\langle p_0\rangle$

$$\langle p\rangle = \langle\varphi_0|\hat{U}_t^+\hat{p}\hat{U}_t|\varphi_0\rangle = \langle\varphi_0|\hat{p}|\varphi_0\rangle + \langle\varphi_0|\mathcal{E}et|\varphi_0\rangle = \langle p_0\rangle + \mathcal{E}et$$

$$\langle x\rangle = \langle\varphi_0|\hat{U}_t^+\hat{x}\hat{U}_t|\varphi_0\rangle = \langle\varphi_0|\hat{x}|\varphi_0\rangle + \frac{t}{m}\langle\varphi_0|\hat{p}|\varphi_0\rangle + \frac{\mathcal{E}et^2}{m} = \langle x_0\rangle + \frac{t}{m}\langle p_0\rangle + \frac{\mathcal{E}et^2}{2m}$$

$$\langle\varphi|\hat{x}\hat{x}|\varphi\rangle = \langle\varphi_0|\hat{U}_t\hat{x}\hat{x}\hat{U}_t|\varphi_0\rangle$$

Так как $\hat{U}_t$ – унитарен $\Rightarrow$ $\hat{U}^+\hat{U} = \hat{1} = \hat{U}\hat{U}^+$ $\hat{x}\hat{f}(t) = \hat{f}(t)\hat{x}$ $\hat{p}\hat{f}(t) = f(t)\hat{p}$ $\hat{p}\hat{x} = \hat{x}\hat{p} - i\hbar$

$$\langle\varphi|\hat{x}\hat{x}|\varphi\rangle = \langle\varphi_0|\hat{U}_t^+\hat{x}\hat{1}\hat{x}\hat{U}_t|\varphi_0\rangle = \langle\varphi_0|\hat{U}_t^+\hat{x}\hat{U}_t\hat{U}_t^+\hat{x}\hat{U}_t|\varphi_0\rangle = \langle\varphi_0|\left(\hat{x} + \frac{t\hat{p}}{m} + \frac{Eet^2}{2m}\right)^2|\varphi_0\rangle = \langle\varphi_0|\left(\hat{x} + \frac{t\hat{p}}{m}\right)^2 + \frac{Eet^2}{m}\left(\hat{x} + \frac{t\hat{p}}{m}\right) + \frac{E^2e^2t^4}{4m}|\varphi_0\rangle = \langle x_0^2\rangle + \frac{t}{m}\langle\varphi_0|\hat{x}\hat{p} + \hat{p}\hat{x}|\varphi_0\rangle + \frac{t^2}{m^2}\langle p_0^2\rangle +$$

$$\frac{Eet^2}{m}\left(\langle x_0\rangle + \frac{t}{m}\langle p_0\rangle\right) + \frac{E^2e^2t^4}{4m^2} = \langle x_0^2\rangle + \frac{2t}{m}x_0p_0 + \frac{t^2}{m^2}\langle p_0^2\rangle + \frac{Eet^2}{m}\left(\langle x_0\rangle + \frac{t}{m}\langle p_0\rangle\right) + \frac{E^2e^2t^4}{4m^2}$$

$$\delta_x = \frac{a^2}{2} + \frac{\hbar^2t^2}{2m^2a^4}$$

$$\langle\varphi|\hat{p}^2|\varphi\rangle = \langle\varphi|\left(\frac{Eet}{m} + \hat{p}\right)^2|\varphi\rangle = \boxed{\frac{Eet}{m}\hat{p} = \hat{p}\frac{Eet}{m}} = \frac{E^2e^2t^2}{m^2} + 2\frac{Eet}{m}\langle p_0\rangle + \langle p_0^2\rangle$$

$$\delta_p = \frac{\hbar^2}{2a^2}$$

Задание 3

$$\hat{P}_+|\varphi(a)\rangle = \frac{1}{2}(\hat{1} + \hat{I})|\varphi(a)\rangle = \frac{|\varphi(x)\rangle + |\varphi(-x)\rangle}{2}$$

Четна т.к.

$$\blacktriangleleft\ \frac{|\varphi(a)\rangle + |\varphi(-a)\rangle}{2}\ \Big|\cdot\hat{I} \Rightarrow \frac{|\varphi(-a)\rangle + |\varphi(a)\rangle}{2} = \frac{|\varphi(a)\rangle + |\varphi(-a)\rangle}{2}\ \blacksquare$$

$$\hat{P}_-|\varphi(a)\rangle = \frac{1}{2}(\hat{1} - \hat{I})|\varphi(a)\rangle = \frac{|\varphi(x)\rangle - |\varphi(-x)\rangle}{2}$$

Доказательство нечетности аналогично.

Вспомним $\hat{I}^2 = \hat{1}$ $\hat{1}^n = \hat{1}$ $\hat{1}\hat{A} = \hat{A}\hat{1} = \hat{A}$

$$\hat{P}_+\hat{P}_- = \frac{1}{4}(\hat{1}^2 + \cancel{\hat{I}\hat{1}} - \cancel{\hat{1}\hat{I}} - \hat{I}^2) = \frac{1}{4}(\hat{1} - \hat{1}) = \hat{0}$$

$$\hat{P}_+ + \hat{P}_- = \frac{1}{2}(\hat{1} + \hat{I} + \hat{1} - \hat{I}) = \hat{1}$$

$$\hat{P}_\pm^2 = \frac{1}{4}(\hat{1}^2 \pm \hat{I}\hat{1} \pm \hat{I}\hat{1} + \hat{I}\hat{I}) = \frac{1}{2}(\hat{1} \pm \hat{I})$$