Задание 2

$$\hat{A} = \begin{pmatrix} 2 & 3 & 4 \\ -1 & -2 & -3 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & 3 & 4 \\ 0 & -1/2 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \hat{A} = \begin{pmatrix} 2 & 3 & 4 \\ 0 & -1/2 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1/2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\hat{A} = \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 4 \\ 0 & -1/2 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{cases} \mathcal{L}y = b \\ Ux = y \end{cases}$$

$$\left(\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 0 & -1/2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 3/2 & 2 & 1/2 & 0 & 0 \\ 0 & -1/2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 3 & 1 \\ 0 & 1 & 0 & 0 & -2 & -2 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{array}\right) = U^{-1}$$

$$\mathcal{L}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$x = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

Задание 3

$$\hat{A} = \begin{pmatrix} 1 & 1+i & 0 \\ 1-i & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\det\left(\hat{A} - \lambda \hat{\mathbb{1}}\right) = 0 \quad \Rightarrow \quad \lambda_1 = 3 \quad \lambda_2 = 3 \quad \lambda_3 = 0$$

$$\lambda_1: \quad \begin{pmatrix} -2 & 1+i & 0 \\ 1-i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} x = 0 \Rightarrow \begin{cases} -2x_1 + (1+i)x_2 = 0 \\ -(1-i)x_1 - x_2 = 0 \end{cases} \Rightarrow x_1 = -\frac{x_2}{1-i} \Rightarrow h_1 = \begin{pmatrix} 1 \\ 1-i \\ 0 \end{pmatrix} \Rightarrow h_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1-i \\ 0 \end{pmatrix}$$

$$\lambda_2: \quad h_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda_3: \quad \begin{pmatrix} 1 & 1+i & 0 \\ 1-i & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} x = 0 \Rightarrow x_1 + (1+i)x_2 = 0 \Rightarrow x_1 = \frac{x_2}{1+i} \Rightarrow h_3 = \begin{pmatrix} 1 \\ 1+i \\ 0 \end{pmatrix} \Rightarrow h_3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1+i \\ 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1-i & 0 & 1+i \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 3 & & 0 \\ & 3 & \\ 0 & & 0 \end{pmatrix} \begin{pmatrix} 1 & 1+i & 0 \\ 0 & 0 & 1 \\ 1 & 1-i & 0 \end{pmatrix}$$

Задание 4

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{pmatrix} = A \quad \Rightarrow \quad A^{+}A = \begin{pmatrix} 14 & 14 \\ 14 & 14 \end{pmatrix} \Rightarrow \lambda_1 = 28 \quad \lambda_2 = 0 \quad \lambda_3 = 0$$

① $\hat{\Sigma} = \begin{pmatrix} 2\sqrt{7} & 0 \\ 0 & 0 \end{pmatrix}$

② $\begin{pmatrix} 14 & 14 & 0 \\ 14 & 14 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow h_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad h_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad h_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

③ $U_1 = \frac{1}{2\sqrt{14}} \begin{pmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 3 & 3 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$$U_2 = \begin{pmatrix} -3/\sqrt{10} \\ 0 \\ 1/\sqrt{10} \end{pmatrix}$$

$$U_3 = \begin{pmatrix} -1/\sqrt{35} \\ \sqrt{5/7} \\ -3/\sqrt{35} \end{pmatrix}$$

Задание 5

а) $U\Sigma V^\dagger = U\Sigma \mathbb{1} V^\dagger = \underbrace{U\Sigma U^\dagger}_{S}\underbrace{UV^\dagger}_{U'}$

б) $U^\dagger S U = \Sigma$

$U^\dagger U' = V^\dagger$

$UU^\dagger = \mathbb{1}$