

$$\frac{d\vec{y}}{dt} + \hat{\Gamma} \vec{y} = \vec{\varphi}$$

Матричная функция Грина

$$\vec{y}(t) = \int d\tau \hat{G}(t-\tau) \vec{\varphi} \quad \left| \Rightarrow \left(\frac{d}{dt} + \hat{\Gamma} \right) \int d\tau \left(\frac{d}{d\tau} + \hat{\Gamma} \right) \hat{G}(t-\tau) \vec{\varphi}(\tau) = \vec{\varphi}(t) \right.$$

$$\frac{d}{dt} \hat{G}(t) + \hat{\Gamma} \hat{G} = \delta(t) \mathbb{1}$$

$$\hat{G}(t) = \Theta(t) \exp(-\hat{\Gamma} t)$$

$$\frac{d}{dt} \left\{ \Theta(t) \exp[-\hat{\Gamma} t] \right\} =$$

$$= \delta(t) \exp[-\hat{\Gamma} t] + \Theta(t) \frac{d}{dt} \exp[-\hat{\Gamma} t] =$$

$$= \frac{d}{dt} \exp[-\hat{\Gamma} t] \quad \dots$$

$$\exp(\hat{A}) = \sum_n \frac{1}{n!} \hat{A}^n$$

$$\exp(\hat{A} + \hat{B}) = \boxed{[\hat{A}, \hat{B}] = 0} = \exp(\hat{A}) \exp(\hat{B})$$

$$= \delta(t) \exp[-\hat{\Gamma} t] + \Theta(t) \hat{\Gamma} \exp[-\hat{\Gamma} t]$$

$$\begin{aligned} \text{Пусть } \hat{a} : \hat{a} \hat{\Gamma} = \gamma \hat{a} \\ \vec{y} = \sum_i y_i \vec{a}_i \end{aligned} \quad \left| \Rightarrow \begin{cases} \frac{dy_1}{dt} + \lambda_1 y_1 = \varphi_1 \\ \dots \\ \frac{dy_n}{dt} + \lambda_n y_n = \varphi_n \end{cases} \right.$$

$$\hat{\Gamma} = \sum_i \lambda_i \vec{a}_i \vec{b}_i^T \quad \Rightarrow \quad \hat{\Gamma}^n = \sum_i \lambda_i^n \vec{a}_i \vec{b}_i^T = \hat{A} \hat{\Lambda} \hat{B}$$

сод. строка

$$\hat{G}(t) = \Theta(t) \sum_i \exp(\lambda_i t) \vec{a}_i \vec{b}_i^T = \Theta(t) \hat{A} \exp(-t \hat{\Lambda}) \hat{B}$$

Если $\hat{\Gamma}$ - вырождена то

$$\hat{\Gamma} = \hat{A} \hat{\Lambda} \hat{B}$$

$$\hat{\Lambda} = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda' \end{pmatrix}$$

$$\exp[-\hat{\Lambda} t] = \exp \left[- \begin{pmatrix} \lambda & 0 \\ 0 & \lambda' \end{pmatrix} t + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} t \right] = \sum_{m=0}^{\infty} \frac{1}{m!} (\hat{A}^m + m \hat{A}^{m-1} \hat{N}) = \exp(\hat{A}) + \sum_{m=0}^{\infty} \frac{\hat{A}^{m-1}}{(m-1)!} \hat{N} = \exp \left[-\lambda t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \left[1 - t \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right]$$

Если $\gamma = \gamma(t)$

$$\dot{x} + \gamma(t)x = \varphi$$

$$\dot{G}(t, \tau) + \gamma(t) G(t, \tau) = \delta(t - \tau)$$

$$\begin{aligned} t < \tau \quad G = 0 \quad t = \tau + \varepsilon \quad G = 1 \\ \frac{dG}{G} = \gamma dt \end{aligned} \quad \left| \Rightarrow G = \exp \left[- \int_{-\tau}^t dt \gamma(t) \right] \right.$$

$$\frac{d\vec{y}}{dt} - \hat{\Gamma}(t) \vec{y} = \vec{q}$$

$$G = \Theta(t-\tau) T \exp \left[- \int_{\tau}^t dt_1 \hat{\Gamma}(t_1) \right]$$

$$\mathbb{1} - \int_{\tau}^{t_1} dt_1 \hat{\Gamma}(t_1) \rightarrow \int_{\tau}^{t_1} dt_2 \int_{\tau}^{t_1} dt_1 \Gamma(t_1) \Gamma(t_2) - \int_{\tau}^{t_1} \int_{\tau}^{t_2} \int_{\tau}^{t_3} dt_3 dt_2 dt_1 \Gamma(t_1) \Gamma(t_2) \Gamma(t_3) + \dots$$

Семинар

$$\left[\frac{d^4}{dt^4} + 4 \frac{d^2}{dt^2} + 3 \right] x = \varphi(t) \Rightarrow$$

$$\begin{cases} x_1 = x \\ x_2 = \dot{x} \\ x_3 = \ddot{x} \\ x_4 = \dddot{x} \end{cases} \Rightarrow \frac{d}{dt} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -4 & 0 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \varphi(t) \end{pmatrix}$$

$$\Gamma \Rightarrow \begin{vmatrix} -\lambda & 1 \\ 4 & -\lambda \end{vmatrix} = \lambda^2 - 4 \Rightarrow \lambda_{1,2} = \pm 2$$

$$\begin{aligned} \lambda_1 = 2 &\Rightarrow \begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow -2x_1 + 1x_2 = 0 \Rightarrow x_1 = \frac{1}{2}x_2 \Rightarrow h_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ \lambda_2 = -2 &\Rightarrow \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow 2x_1 + 1x_2 = 0 \Rightarrow x_1 = -\frac{1}{2}x_2 \Rightarrow h_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \end{aligned} \Rightarrow \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & -1 \end{pmatrix}$$

$$\begin{aligned} \lambda_1 = 2 &\Rightarrow (x_1, x_2) \begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix} \Rightarrow \xi_1 = (2, 1) \\ \lambda_2 = -2 &\Rightarrow (x_1, x_2) \begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix} \Rightarrow \xi_2 = (2, -1) \end{aligned} \Rightarrow \begin{pmatrix} 1 & -1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$G(t) = \mathcal{L}^{-1}(t) \sum \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & -1 \end{pmatrix} \begin{pmatrix} e^{-2t} & 0 \\ 0 & e^{2t} \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{2} \\ 1 & -\frac{1}{2} \end{pmatrix}$$

D3

$$\Gamma = \begin{pmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{pmatrix} \quad G = ?$$

$$\det(\Gamma - \lambda \mathbb{I}) = (\lambda - \lambda) \Rightarrow \lambda_{1,2,3,4} = \lambda$$

$$x(t) = \exp \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{pmatrix} t \left(\mathbb{I} - \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} t + \frac{t^2}{2} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \frac{t^3}{6} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right)$$