DSTI

FSML2 A24 Exam

Before you start this exam, make sure you have read the recommendations containing most common students' mistakes. Also keep in mind that since generative AI appeared, the percentage of students failing the exam has been multiplied by 3. You are strongly advised to refer to your FSML2 classes instead of trying to interpret some more or less relevant answer from a generative AI. You are much more intelligent than any machine.

Make sure your answers are properly justified, any notation introduced by you is clearly defined.

For this exam, you are expected to provide: A single pdf file containing all your answers, numeric results, graphics ; One or multiple files containing your code in its original form (R, Rmd or other).

Exercise A Let X and Y be two independent random variables with exponential distributions of parameters λ and μ

1. Compute the joint density $f_{X,Y}(x,y)$?
2. Compute $P(X \leq Y)$.
3. Let $Z = Max(X,Y)$. Compute the density of Z.

Exercise B Let θ be a positive number.

Let f be the following function:

$$f(x) = \begin{cases} \frac{1}{4\theta^2}x & \text{if } x \in [0, 2\theta] \\ \frac{1}{2\theta} & \text{if } x \in]2\theta, 3\theta] \end{cases}$$

1. Prove that f is a density function.
2. Let X be a random variable with density f. Compute the expectation and variance of X.
3. Find an estimator $\hat{\theta}$ of θ using the method of moments.
4. Define a confidence interval for θ.
5. Simulate 1000 observations of X.
6. Show with a graph that your answer is correct.

Exercise C Let X and Y be two independent standard Gaussian random variables.

α a positive real number

Let ϵ be a random variable, independent from X and Y taking values $-\alpha$ and α with same probability $1/2$. Let $Z = \epsilon X$.

1. Prove that Z is a Gaussian random variable.
2. Determine the distribution of the random vector $U = (Y, Y - Z, Y + Z)$.
3. Given a sample of Z of size n, determine an unbiased estimator of α^2 and derive a confidence interval for α^2.
4. Compute this interval numerically for the attached data for a confidence interval of level 95%.