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- GROUP 5 -

INVERSE & TRANSPOSE

matrix

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matrix TRANSPOSE

the **transpose** of an *m x n* matrix A is the *n x m* matrix B, defined by $b_{ji} = a_{ij}$

for i = 1,...,n and j = 1,...,m

the transpose of A is **denoted by A^T**

EXAMPLES

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 2 & 1 \\ 4 & 3 & 2 \\ 1 & 2 & 5 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -3 & 4 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

4 algebraic rules

FOR TRANSPOSES

1. $(A^T)^T = A$
2. $(\alpha A)^T = \alpha A^T$
3. $(A + B)^T = A^T + B^T$
4. $(AB)^T = B^T A^T$

matrix

SYMMETRIC

An *nxn* matrix is said to be symmetric if $\mathbf{A}^T = \mathbf{A}$

EXAMPLES

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 5 \\ 4 & 5 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & -2 \\ 2 & -2 & -3 \end{bmatrix}$$

matrix IDENTITY

$$I_1 = [1]$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

identity matrix of size *n* is the *nxn* square matrix with ones on the main diagonal and zeros elsewhere

identity matrix is usually denoted by ***I***

IA = AI = I for any matrix A

matrix
INVERSION

An *nxn* matrix A is said to be nonsingular or invertible if there exists a matrix B such that

AB = BA = I

The matrix B is said to be the inverse of A
We can denote B as $\mathbf{A^{-1}}$

Two inversion rules:

$$(\boldsymbol{AB})^{-1} = \boldsymbol{B}^{-1}\boldsymbol{A}^{-1}$$
$$(\boldsymbol{A}+\boldsymbol{B})^{-1} \neq \boldsymbol{A}^{-1} + \boldsymbol{B}^{-1}$$

EXAMPLE

$$\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -\frac{1}{10} & \frac{2}{5} \\ \frac{3}{10} & -\frac{1}{5} \end{bmatrix}$$

are inverses of each other, since

$$\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{10} & \frac{2}{5} \\ \frac{3}{10} & -\frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

TO FIND THE INVERSE MATRIX

DETERMINANT & ADJOINT

If A is a 2x2 matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\mathrm{adj}(\mathbf{A}) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The formula to find the inverse of a matrix is

$$A^{-1} = \frac{1}{\det(A)} \mathrm{adj}(A)$$

TO FIND THE INVERSE MATRIX

DETERMINANT & ADJOINT

The formula to find the inverse of a matrix is

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

If A is a 3x3 matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$\det(\mathbf{A}) = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

$$\operatorname{adj}(\mathbf{A}) = \begin{bmatrix} +\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & +\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & +\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ +\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & +\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}$$

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Thank you

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