Empirical Investigation of Rotational Inertia from Dynamic Measurement

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Abstract

The objective of this lab was to determine the rotational inertia of a tri-disk wheel from a dynamic measurement and compare it to a theoretical value calculated from the mass and dimensions of the wheel. Experimental trials were carried out by measuring the drop time of a mass hanging from a string wound around one of the three sub-wheels of the apparatus. Using these measurements for calculations, a least-squares linear regression was preformed of the torque due to the string vs acceleration divided by radius which accounted for 98.7% of the data variance. The experimental value for the moment of inertia of the entire wheel, 109±6 [kg m^2], was the slope of this model, and contained the theoretical value of 114.7±0.8 [kg m^2]. Due to the high confidence of the model and the agreement of the two values for inertia, the results provide a reliable representation of the moment of inertia.

Procedures

The following procedure was executed to experimentally determine the moment of inertia for a tri-disk wheel. The wheel used consisted of three uniform disks with varying radii rotating around the same principal axis. Trials were conducted as follows: a string with a loop on the end was hooked onto one of the disks and round about it, and then a mass was attached to the other end of the string, such that the string could apply a torque to the wheel to rotate the wheel. To standardize the drop height, the system was adjusted so that the bottom of the mass aligned with the bottom of the tabletop. The same person then started a timer while letting go of the disk, allowing the hanging mass to accelerate downward, and then stopped the timer when the hanging mass hit the floor. Measurements for the radius of the wheel, drop height, mass of the hanging mass, and fall time were recorded. For each radius and mass combination, three trials were recorded, and an average of the drop time was taken. A total of six pairings were tested overall, two different mass values for each radius size — care was taken to ensure the masses were not too small or too large as to result in erroneous results. The torque from the string and the acceleration divided by the radius were calculated from the experimental time values and other known quantities, and then used to perform a least-squared linear regression, the resulting slope being the value of the moment of inertia. Theoretical values for the moment of inertia of the tri-disk wheel and its sub-wheels were also calculated using the mass and dimensions of the tri-disk wheel.

Analysis

Exploration 1

Before experimental data was collected, the moment of the inertia of the wheel and individual sub-wheels was calculated using measurements and uncertainties provided by the lab manual (see Fig. 1). The total mass of the wheel was given as ***M* = 4.92 ± 0.02 [kg]**.

Figure 1: Wheel dimensions of three disks *from lab manual*

L_1 (cm)	L_2 (cm)	L_3 (cm)	D_1 (cm)	D_2 (cm)	D_3 (cm)
2.53 ± .01	2.58 ± .01	2.54 ± .01	7.64 ± .01	15.33 ± .02	5.08 ± .01

Note: *L* refers to depth of the sub-wheel and *D* refers to the diameter of the sub-wheel.

The moment of the inertia for the wheel is the sum the individual moments of inertia of each uniform disk, sub-wheel, which can be calculated with the mass and radius of the disk (see Fig. 2, 'Moment of Inertia of Uniform Disk').

Figure 2: General moment of inertia equations *from lab manual*

Formula for Moment of Inertia

$$I = \sum_{i=1}^{n} m_i r_i^2 = \int r^2 dm$$

Moment of Inertia of Uniform Disk

$$I_{disk} = \frac{1}{2} M R^2$$

Moment of Inertia of Wheel

$$I_{wheel} = \frac{1}{2}(M_1 R_1^2 + M_2 R_2^2 + M_3 R_3^2)$$

Because the mass of each disk was unknown due to the construction of the wheel, the individual masses were instead expressed as a product of *density* ρ and *volume* $V_i = \pi \left(\frac{D_i}{2}\right)^2 L_i$ such that $M_i = \rho V_i$. As the tri-wheel had a uniform density, density could be found by dividing total mass by the total volume of the apparatus (see Fig. 3, 'Density'). Substituting ρV_i for M_i in the 'Moment of Inertia of Wheel' equation (see Fig. 2) and simplifying the expression, an equation (see Fig. 3, 'Moment of Inertia of Wheel Calculation') was found. This equation in terms of *M,* *L,* and *D* was then used for the actual calculation of the inertia of the wheel in kilograms meters squared.

Figure 3: Using density for moment of inertia calculation *from lab manual*

Density

$$\rho = \frac{M}{V} = \frac{4M}{\pi \sum_i L_i D_i^2}$$

Moment of Inertia of Wheel Calculation

$$I_{wheel} = \frac{1}{32}\rho\pi \sum_i L_i D_i^4 = \frac{M}{8}\frac{\sum_i L_i D_i^4}{\sum_i L_i D_i^2}$$

The calculated moment of inertia of the entire wheel was 114.7 ± 0.8 [kg m^2]. Uncertainty (see Fig. 4) was found using the upper-lower bound method to propagate the uncertainty in the dimension measurements found in Fig. 1. The moments of inertia of the sub-wheels (see Fig. 5) were calculated with the equation for the inertia of disks using the radius, as well as density and volume to find the mass of the disks.

Figure 4: Moment of inertia of wheel *from lab notebook*

Moment of Inertia of Entire Wheel [kg m^2]	**I = 114.7 ± 0.8**	
Uncertainty	MoI [kg m^2]	|ΔMoI| [kg m^2]
Upper Bound*	115.4	0.7
Lower Bound*	113.9	0.8

Figure 5: Moments of inertia of sub-wheels *from lab notebook*

Sub-wheel	Contribution to Moment of Inertia [kg m^2]
Small wheel (radius = 2.54 cm)	1.27
Medium wheel (radius = 3.82 cm)	6.47
Large wheel (radius = 7.67 cm)	106.9

Exploration 2

Exploration 2 concerns the calculations preformed with experimental data. The time values in Fig. 6 were measured as described in the ‘Procedures’ section of this report. The uncertainty in the time measurements ΔTavg was the found by taking the maximum of the distances of the time measurements from the average.

Figure 6: Data from experimental trials *from lab notebook*

Data			Applied Torque	time data				
Wheel	radius (cm)	mass (kg)	mg*R product	time1	time2	time3	Tavg (s)	ΔTavg (s)
large	7.665	0.1	751.17	2.18	2.16	1.96	2.10	0.14
large	7.665	0.2	1502.34	1.38	1.38	1.35	1.37	0.02
medium	3.82	0.2	748.72	2.88	2.73	2.96	2.86	0.13
medium	3.82	0.25	935.9	2.56	2.48	2.51	2.52	0.04
small	2.54	0.5	1244.6	2.5	2.45	2.48	2.48	0.03
small	2.54	0.4	995.68	2.88	2.85	2.98	2.90	0.08

Calculating acceleration

Because $distance = d = v_0 t + \frac{1}{2} a t^2$ and initial velocity is zero for the experimental trials, acceleration can be calculated as follows: $a = 2\frac{d}{t^2}$ (see Fig. 7, ‘acceleration results). The upper and lower bounds of acceleration were calculated using the upper and lower bounds of the average time to propagate the uncertainty.

Calculating the plotting data and error bars

The net force on the system was equal to mass times acceleration and also equal to gravitational force minus tension force: $F_{net} = mg - T = ma$. The torque of the string is equal to the tension force of the string times the radius of the disk, which solving for tension force from the previous equation is $\tau_{\mathrm{string}} = \mathrm{TR} = \mathrm{m(g-a)R}$. Knowing that $\tau_{net} = \tau_{string} - \tau_{friction} = I\alpha$, the torque from the string can be re-expressed as $\tau_{\mathrm{string}} = \mathrm{m(g-a)R} = \mathrm{I}\frac{\mathrm{a}}{\mathrm{R}} + \tau_{\mathrm{friction}}$. This is equation is the basis of the following plot and linear regression. The y-data points are calculated to be the torque of the string, *m(g-a)R* (see Fig. 7, ‘y’).The x-data points are calculated to be acceleration divided by radius, *a/R* (see Fig. 7, ‘x’). Then, the slope of the linear regression will be equivalent to the moment of inertia *I* and the y-intercept will be equivalent to the torque due to friction. The error for the x and y plotting data was calculated using the upper-lower bound method and the upper/lower bounds of acceleration. The lower and upper bars were the distance between the bound and the middle value (see Fig. 8). The uncertainties for y and x respectively were the maximum of the lower bar and the upper bar.

Figure 7: Acceleration and plotting data calculations *from lab notebook*

necessary calculation parameters		acceleration results			Plotting Data	
		a (units)	a UB (units)	a LB (units)	y	x
drop d (cm) =	73	33.11	38.27	28.90	725.79381	4.31918799
drop uncertainity (cm) =	0.5	77.79	80.66	75.05	1383.09124	10.1484464
g (cm/s^2) =	980.00	17.89	19.77	16.26	735.051304	4.68349821
		23.05	23.97	22.18	913.885702	6.03445564
		23.80	24.56	23.08	1214.37116	9.37095864
		17.32	18.44	16.29	978.082427	6.81907298

Figure 8: Error bars calculations *from lab notebook*

Error Bars for Y					Error Bars for X				
y upper	y lower	y upper bar	y lower bar	Δy	x upper	x lower	x lower bar	x upper bar	Δx
729.02	721.84	3.23	3.95	3.95	4.99	3.77	0.67	0.55	0.67
1387.29	1378.69	4.20	4.40	4.40	10.52	9.79	0.37	0.36	0.37
736.30	733.61	1.25	1.44	1.44	5.18	4.26	0.49	0.43	0.49
914.72	913.01	0.83	0.87	0.87	6.27	5.81	0.24	0.23	0.24
1215.29	1213.41	0.92	0.96	0.96	9.67	9.09	0.30	0.29	0.30
979.13	976.94	1.05	1.14	1.14	7.26	6.41	0.44	0.41	0.44

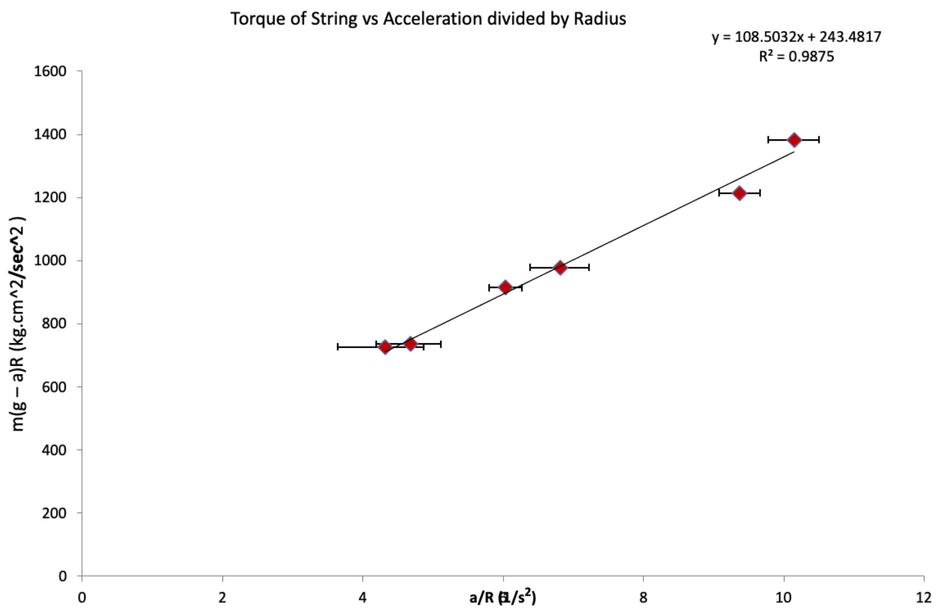
A least-squared linear regression of the x and y plotting data was carried out using the excel LINEST function (see Fig. 9). The slope of 109 ± 6 [kg m^2] is equivalent to the moment of inertia. The y-intercept of 243 ± 44 [Nm] is equal to the torque due to friction.

Figure 9: LINEST (y = m(g-a)R, x = a/R) *from lab notebook*

LINEST			
slope =	108.503156	243.481717	= intercept
U(slope) =	6.10472317	44.1845045	= U(intercept)
R² =	0.98749618	32.8676439	
	315.902304	4	
	341263.577	4321.12806	

Figure 10: Plot of 'Torque of String vs Acceleration divided by Radius' with least-squared regression line and error bars *from lab notebook*

N. B. The vertical error bars were too small to appear at this scale.



Discussion

The results for this lab were reasonable and consistent. Looking at the calculated values of inertia for the sub-wheels, the largest disk had the largest contribution to the moment of inertia and the smallest disk had the smallest contribution, as expected. The inertia values increased with radius as the general equation for inertia describes. This makes sense because the larger wheel had more mass and mass that was farther from the axis of rotation, making the disk more resistant to changes in motion. For the moment of inertia of the entire wheel, the theoretical value of inertia, I = 114.7±0.8 [kg m^2], fell within one standard error of the experimental value of inertia, I = 109±6 [kg m^2]. This experimental value was found from the slope and slope uncertainty of the least-squares linear regression of torque from the string vs acceleration divided by radius. The least-squared linear regression had an R-squared value of 0.987, meaning 98.7% of the data variance could be explained by the model. The combination of the literature value falling within one standard error of the experimental results and the high level of linearity of the model suggests a high level of confidence in the results of this lab. The y-intercept of 243 ± 44 [Nm] from the linear regression is equivalent to the torque due to friction and is a reasonable value for this torque; the value is positive and a similar order of magnitude as the string torque but smaller. Overall, allowing for provided uncertainties and limited precision, the results of this lab provide an accurate model of the inertia of the wheel.

There were multiple sources of potential error within the lab. One source of error was that the wheel apparatus used was lopsided with a poorly attached axle. Fixing the apparatus so that the tri-wheel was only able to spin around its axis and had no other degrees of freedom would significantly reduce this source of error. Beyond the uncertainty of the fall times due to limited precision from manual timing, there was also error in these measurements because the person stopping the timer at the end of the hanging mass' fall was not level with the ground but looking on from above. This source of error could be reduced by having the hanging mass fall onto a sensor that would automatically stop the time when hit.