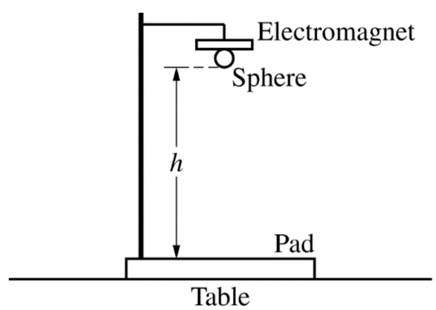
2018 Mechanics FRQ

Workthrough of the AP Physics C 2018 Mechanics FRQ questions (pertaining to this year's topics).

Problem 1



1. A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

(a) While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

(b) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

(c) Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.

(d) Using the straight line, calculate an experimental value for g.

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\ \text{m/s}^2$, $B = -0.524\ \text{m/s}$, and $C = +0.080\ \text{m}$. Vertically down is the positive direction.

(e) Using the student's equation above, do the following.

i. Derive an expression for the velocity of the sphere as a function of time.

ii. Calculate the new experimental value for g.

iii. Using $9.81\ \text{m/s}^2$ as the accepted value for g at this location, calculate the percent error for the value found in part (e)ii.

iv. Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

Part A: The student is probably calculating g by finding the time it akes the object to go a certain distance.

$\Delta x = v_0 t + \frac{1}{2}at^2$ <-- where a signifies g & $v_0 = 0$

Therefore, $g = \frac{2h}{t^2}$. Since t increases, g *decreases*, so the answer is Less Than.

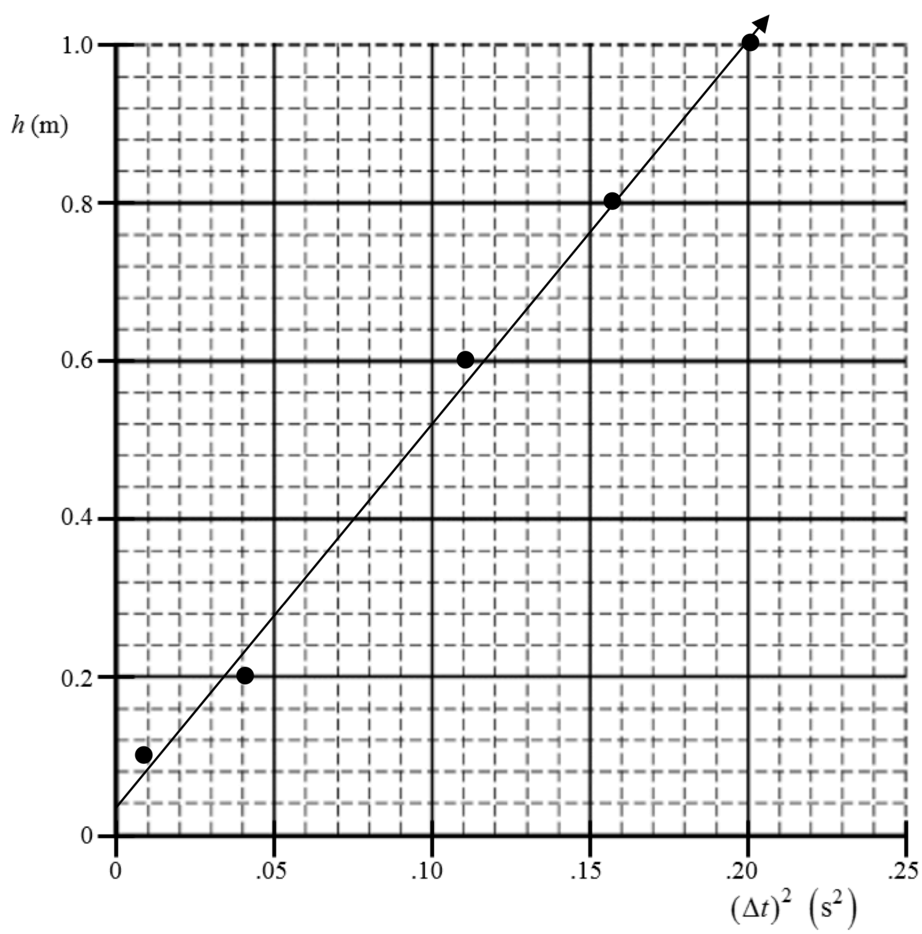
Part B:

Vertical: h, Horizontal: t^2 (can be the other way around too)

h	0.1	0.2	0.6	0.8	1
t	0.105	0.213	0.342	0.401	0.451
t^2	0.011	0.045	0.117	0.161	0.203

Part C:

The graph ends up looking like this:



Part D:

This can be hard because this tests your ability to draw straight lines and graphs.

The approximate slope $\frac{\Delta h}{\Delta(\Delta t)^2}$ is around 4.96 m/s^2. We know that the slope is half of gravity (from the equation in part A), and so we multiple this by two to get $g = 9.92\, m/s^2$ or something close to that.

Part E:

The student's equation is $y = At^2 + Bt + C$.

(i) We know the kinematic equation $\Delta y = \frac{1}{2}at^2 + v_0 t$ and that is basically the equation above (where it's actually $y = \frac{1}{2}at^2 + v_0 t + y_0$). That means $0.5a = 5.75 m/s^2$ and $v_0 = -0.524\, m/s$ and the initial height was 0.08m.

The velocity of the sphere as a function of time is found when we take the derivative of this equation, since $y' = v$ and so we get $y' = v = 2At + B$. We can substitute in the numbers (if you'd like, not required) to give us $v(t) = 11.5t - 0.524$.

(ii) The new experimental value of g is found when we use the equation $g = \frac{2h}{t^2}$ again and find that our new $v(t)$ equation's slope 11.5 is g. Another way we can figure this out us through:

$y = y_0 + v_1 t + \frac{1}{2}at^2 = At^2 + Bt + C$, which gives us $A = \frac{1}{2}a = \frac{1}{2}g$.

Therefore $g = 2A = (2)(5.75) = 11.5\ m/s^2$.

(iii) We now know that $g = 9.81$, so the percent error is $\frac{|e-o|}{o} \times 100\%$ and so we get $\frac{|11.5-9.81|}{9.81} \times 100 = 17.2\%$.

(iv) $h = y_0 = 1.4$ and $t = 0$. We know $a = 9.81$ and want to find v. We can use the kinematic equation $v^2 = v_0^2 + 2ay$, which gives us $v = \sqrt{-0.524^2 + (2 \times 11.5(1.40 - 0.08))} = 5.54$ m/s. There are lots of other ways to do this, as follows:

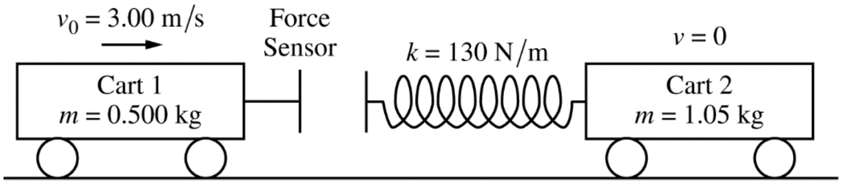
$y = At^2 + Bt + C \therefore v_1 = B = -0.524 \text{ m/s}$		
$a = 2A = 2 \times (5.75 \text{ m/s}^2) = 11.5 \text{ m/s}^2$		
$y_0 = C = 0.080 \text{ m}$		

$y = At^2 + Bt + C$		
$1.40 = (5.75 \text{ m/s}^2)t^2 + (-0.524 \text{ m/s})t + (0.080 \text{ m})$		
$5.75t^2 - 0.524t - 1.32 = 0$		
$t = 0.53 \text{ s or } -0.44 \text{ s} \therefore t = 0.53 \text{ s}$		

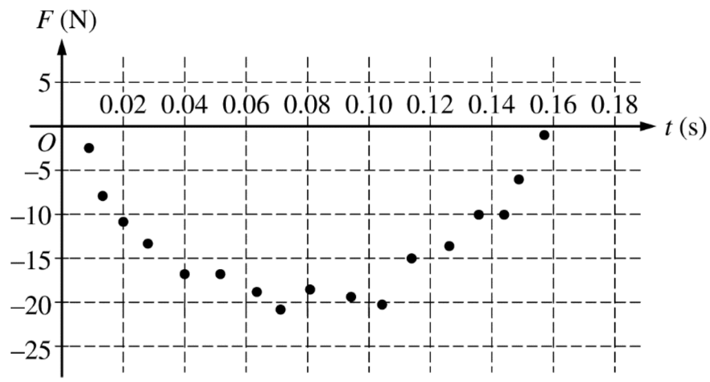
$v(t) = (11.5 \text{ m/s}^2)(0.53 \text{ s}) - 0.524 \text{ m/s}$		
$v = 5.54 \text{ m/s}$		

$U_i + K_i = U_f + K_f$		
$mgh + \frac{1}{2}mv_1^2 = \frac{1}{2}mv^2$		
$v = \sqrt{2(11.5)(1.40 - 0.080) + (-0.524)^2} = 5.54 \text{ m/s}$		

Problem 2



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00\ \text{m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130\ \text{N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - (500\ \text{N/s})t$.



This problem seems to require some knowledge about oscillations, so if there is a part that needs one, we'll skip it!

(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

Part A:

$J = \Delta p$ <-- impulse-momentum principle

$J = \int F\,dt = \int 3200t^2 - 500t\,dt = \frac{3200t^3}{3} + \frac{500t^2}{2}$ from t = 0 to t = 0.16;

That gives us a value of $J = -2.03\,Ns$.

Part B:

(i) We know that momentum is conserved and we know the Δp.

$J = m_1(v_{1f} - v_{1i})$, therefore $v_{1f} = \frac{J}{m_1} + v_{1f} = \frac{-2.03}{0.5} + 3 = -1.06$ m/s.

(ii) cart 1 moves left because the velocity is negative, and therefore to the left.

Part C:

(i) We also use the momentum-impulse principle and get:

$-J = -m_1(v_{1f} - v_{1i}) = m_2 v_{2f}$, this gives us $v_{2f} = \frac{-J}{m_2}$

$v_{2f} = \frac{2.03}{1.05} = 1.93$ m/s.

Alternate solution
For using the conservation of momentum to calculate the speed of cart 2 after the collision
$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \therefore v_{2f} = \frac{m_1\left(v_{1i} - v_{1f}\right)}{m_2}$
For correct substitution of the answer from part (b)(i)
$v_{2f} = \frac{(0.500\ \text{kg})((3.00\ \text{m/s}) - (-1.06\ \text{m/s}))}{(1.05\ \text{kg})} = 1.93\ \text{m/s}$

(ii) An elastic collision is where the two objects separate afterwards and go in opposite directions, as opposed to an inelastic collision where they stick together. In an elastic collution KE is conserved, and we can check it.

$K_i = K_f \to \frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$

$(0.5)(3)^2 = (0.5)(-1.06)^2 + (1.05)(1.93)^2 \to 4.50 \approx 4.47.$

Part D:

(i) Now, we finally use the *conservation of momentum*! $m_1 v_{1i} = (m_1 + m_2)v_{cm}$

Therefore, the velocity of the center of mass is $v_{cm} = \frac{m_1 v_{1i}}{m_1 + m_2} = 0.97$ m/s.

This is a slightly different conservation of momentum equation but I'm sure you understand what it's doing.

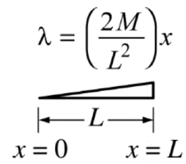
(ii) Elastic Potential Energy: $PE_{spr} = \frac{1}{2}kx^2$.

$KE_i = KE_f + PE_{spr} \to \frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}(m_1 + m_2)v_{cm}^2 + PE_{spr}$

$\to PE_{spr} = \frac{1}{2}(m_1 v_{1i}^2 - (m_1 + m_2)v_{cm}^2)$. Plugging in the numbers, we get 1.52 J.

Alternate Solution:
For correctly determining the magnitude of the maximum force exerted between the carts
Set $\frac{dF}{dt} = 0 = 6400t - 500 \therefore 6400t = 500 \therefore t = 0.078$ s
$F_{MAX} = 3200(0.078)^2 - 500(0.078) = -19.5$
$\lvert F_{MAX} \rvert = 19.5$ N
Note: Can estimate from the graph, and accept the range of 20 N to 21 N.
For calculating the maximum compression of the spring
$F_{MAX} = -kx_{MAX}$
$x_{MAX} = -\frac{F_{MAX}}{k} = -\frac{(-19.5\text{ N})}{(130\text{ N/m})} = 0.15\text{ m}$
Note: If estimating from the graph, accept the range from 0.15 m to 0.16 m.
For substituting into an equation for the elastic potential energy at maximum spring compression
$U_S = \frac{1}{2}kx_{MAX}^2 = \left(\frac{1}{2}\right)(130\text{ N/m})(0.15\text{ m})^2$
$U_S = 1.46$ J
Note: If estimating from the graph, accept range from 1.46 J to 1.70 J.

Problem 3



3. A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

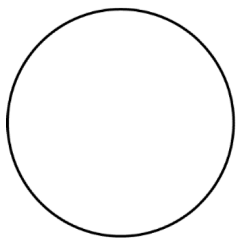


Figure 1

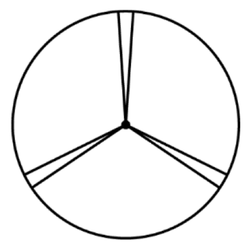


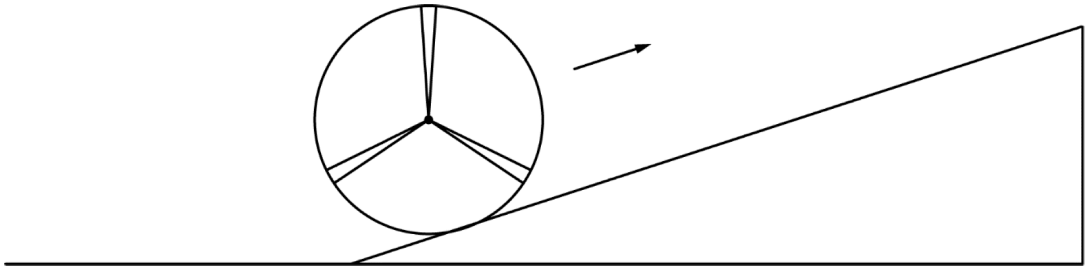
Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

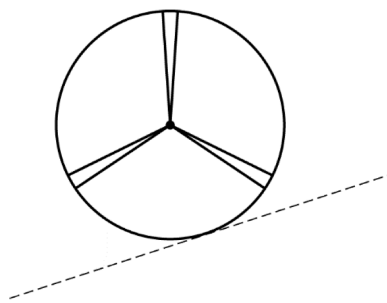
(c) Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.



The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



ii. Justify your choice for the direction of each of the forces drawn in part (d)i.

(e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

Part A:

$I = \int r^2\, dm$ -> $= \int x^2\, dm$

$m = \int \lambda\, dx = \int \frac{2M}{L^2} x\, dx$, so $dm = \frac{2M}{L^2} x\, dx$

$I = \int x^2 \times \frac{2M}{L^2} x\, dx = \frac{2M}{L^2} \int x^3\, dx$. Simplifying we get $2M/L^2 \left[\frac{x^4}{4}\right]_{x=0}^{x=L} = ML^2/2$.

Part B:

There are three rods, and each rod has a moment of inertia of $\frac{ML^2}{2}$.

$I = 3I_{rod} + I_{hoop} = 3(ML^2/2) + ML^2$, therefore $I = \frac{5}{2}ML^2$. We add the rod and hoop inertias to find the inertia of the whole system.

> *The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.*

Part C:

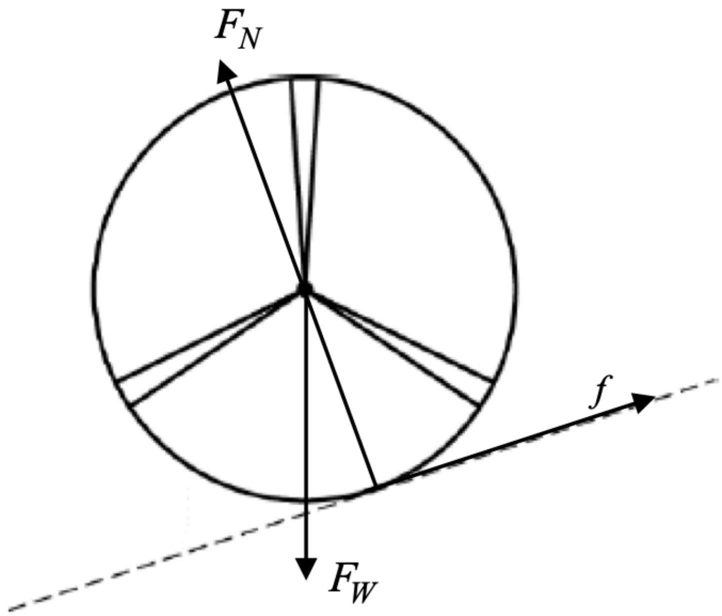
> ***Incorrect***: *We can use the rotational equation $\omega = \omega_0 t + \frac{1}{2}\alpha t^2$.*
>
> $F = ma_t = (3M)\alpha L$ *==> therefore* $\alpha = F/(3ML)$.
>
> $\omega_0 = (\omega - \frac{1}{2}\alpha t^2)/t = \omega - \frac{1}{2}\frac{F}{3ML}t^2$.

$\tau\Delta t = I(\omega - \omega_0)$, so $Fr\Delta t = I\omega$ (Using `torque`!).

$\omega = \frac{Fr\Delta t}{I} = \frac{FLt}{I} = \frac{FLt}{\frac{5}{2}ML^2}$. Therefore, the answer is $\frac{2Ft}{5ML}$.

Part D:

(i) The forces are a downward gravity, normal force, and that's it!



For drawing the weight of the hoop-rod system in the correct direction		1 point
For drawing the normal force in the correct direction		1 point
For drawing the frictional force in the correct direction		1 point
Note: A maximum of two points can be earned if there are any extraneous vectors or any vector has an incorrect point of exertion.		

(ii) The force of gravity (or weight) is downwards because it always points downards. The normal force is perpendicular to the surface, and the frictional force prevents the wheel from slipping (without *f* the wheel would slip downwards, so f points upwards).

For a correct justification of the direction of the forces in part (d)(i)		1 point
Example: The normal force is always perpendicular to the surface, so it will be directed up and to the left. The gravitational force is always vertically downward. The friction will be opposite of the direction of rotation; therefore, it is directed up the incline.		

Part E:

If we're looking for a change in height, it can be expressed using PE. This is using `conservation of Energy`!

$K_1 = U_{g2} \rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH$

Therefore, we find that $H = \frac{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}{mg}$, and we substitute $v = L\omega$ because $v = r\omega$ and $r = L$ in this situation.

So, $H = \frac{\frac{1}{2}(m_{tot}L^2 + I_{tot})\omega^2}{m_{tot}g}$. Replacing the $m_{tot} = 3M + M$, we get $H = \frac{(4ML^2 + I_{tot})\omega^2}{8Mg}$.