ROTATIONAL MOTION

ANGULAR MOTION

$\theta = \frac{s}{r}, \quad \omega = \frac{v}{r}, \quad \alpha = \frac{a_t}{r}, \quad \alpha_t = r\sqrt{\alpha^2 + \omega^4}, \quad a_c = \omega^2 r$ <-- centripetal acceleration

<u>*Rolling constraint*</u>: $v = r\omega$, and $v_{CM} = \frac{2\pi R}{T}$

Kinetic Rolling Energy: $\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

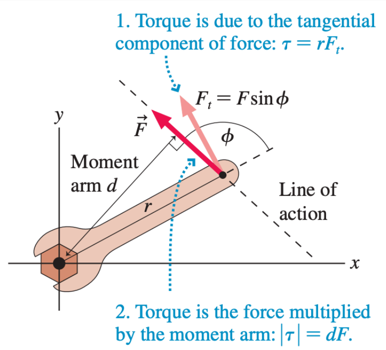
Torque

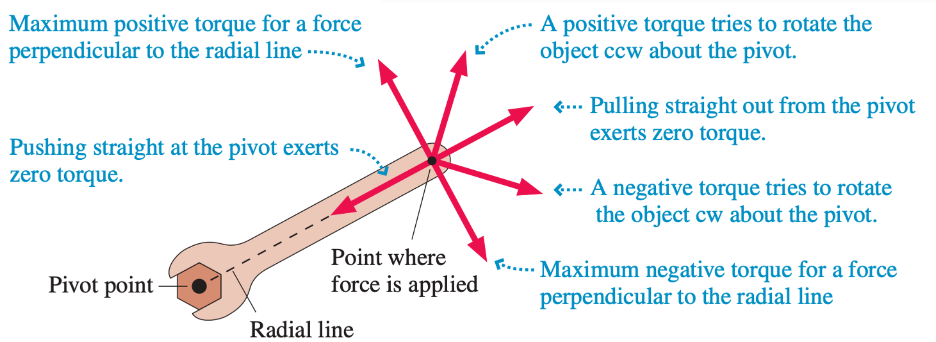
Torque: $\tau = r \times F = rF\sin\theta = I\alpha$

gravitational torque: $\tau_{grav} = -Mgx_{cm}$

There is the constant Torque Model --> for objects in which net torque is constant:

- Model object as a rigid body with constant angular acceleration
- **Newton's Second Law**: $\tau = I\alpha$





EXAMPLE 12.8 **Applying a torque**

Luis uses a 20-cm-long wrench to turn a nut. The wrench handle is tilted 30° above the horizontal, and Luis pulls straight down on the end with a force of 100 N. How much torque does Luis exert on the nut?

VISUALIZE **FIGURE 12.21** shows the situation. The angle is a negative $\phi = -120°$ because it is *clockwise* from the radial line.

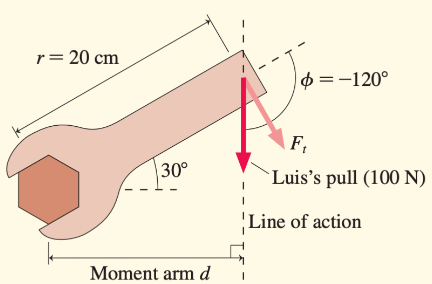


FIGURE 12.21 A wrench being used to turn a nut.

SOLVE The tangential component of the force is

$$F_t = F\sin\phi = -86.6\text{ N}$$

According to our sign convention, F_t is negative because it points in a cw direction. The torque, from Equation 12.21, is

$$\tau = rF_t = (0.20\text{ m})(-86.6\text{ N}) = -17\text{ Nm}$$

Alternatively, Figure 12.21 has drawn the *line of action* by extending the force vector forward and backward. The *moment arm*, the distance between the pivot point and the line of action, is

$$d = r\sin(60°) = 0.17\text{ m}$$

Inserting the moment arm in Equation 12.22 gives

$$|\tau| = dF = (0.17\text{ m})(100\text{ N}) = 17\text{ Nm}$$

The torque acts to give a cw rotation, so we insert a minus sign to end up with

$$\tau = -17\text{ Nm}$$

ASSESS The largest possible torque, if Luis pulled perpendicular to the 20-cm-long wrench, would have a magnitude of 20 Nm. Pulling at an angle will reduce this, so 17 Nm is a reasonable answer.

Rotation About a Center of Mass

$x_{CM} = \frac{1}{M}\Sigma m_i x_i = \frac{m_1 x_1 + m_2 x_2 \ldots}{m_1 + m_2 \ldots}$

The center of mass is the mass-weighted center of the object.

You can also find it through **integration** of the x and y directions.

Moment of Inertia

$I = \Sigma mr^2$, the sum of all particles = $\int r^2 \; dm$

Parallel Axis Theorem: $I = I_{CM} + Mh^2$

Value	Equation
Solid Sphere	$\frac{2}{5}mr^2$
Rod Center	$\frac{1}{12}ml^2$
Rod End	$\frac{1}{3}ml^2$
Hollow Hoop (through axis)	mr^2
Solid Disk (through axis)	$\frac{1}{2}mr^2$
Hollow Sphere	$\frac{2}{3}mr^2$

Downhill Race

1. Particle
2. Solid Sphere
3. Solid Cylinder
4. Circular Hoop

The smaller the number in front the longer it takes.

ANGULAR MOMENTUM

$L = I\omega = r \times p \sin\theta = rmv \sin\theta$

It is always perpendicular to the plane

$\tau = \frac{dL}{dt}$, the total L is always conserved

Static Equilibrium: an extended object that is completely stationary

$\theta = \tan^{-1}\left(\frac{t}{2h}\right)$

ROTATIONAL ENERGY

$K_{rot} = \frac{1}{2}m_1 v_2^2 + \frac{1}{2}m_2 v_2^2 + \ldots$

An object's moment of inertia depends on the axis of rotation.

$$K_{rot} = \frac{1}{2}I\omega^2$$