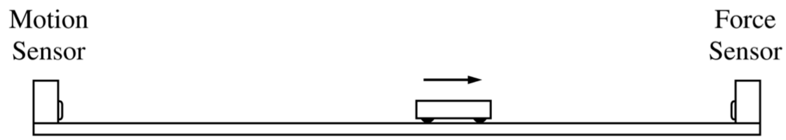
2001 Mechanics FRQ

Questions

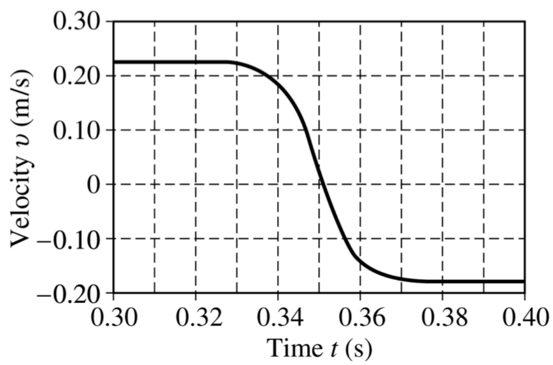
Solution/Guidelines

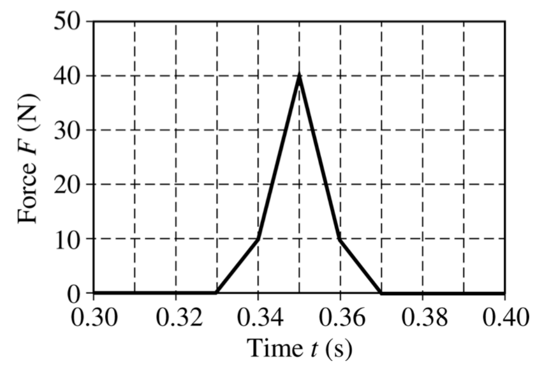
Problem 1



Mech 1.

A motion sensor and a force sensor record the motion of a cart along a track, as shown above. The cart is given a push so that it moves toward the force sensor and then collides with it. The two sensors record the values shown in the following graphs.





(a) Determine the cart's average acceleration between $t = 0.33$ s and $t = 0.37$ s.

(b) Determine the magnitude of the change in the cart's momentum during the collision.

(c) Determine the mass of the cart.

(d) Determine the energy lost in the collision between the force sensor and the cart.

Part A

Average acceleration is the slope of the velocity graph from point t = 0.33 to t = 0.37. The coordinates are (0.33, 0.22) and (0.37, -0.18). Therefore, the slope is (0.22-(-0.18))/(0.33-0.37) = -10. The average acceleration is −10 m/s^2.

The average acceleration is the change in velocity divided by the time interval

For correct subtraction to find the time interval **1 point**

$\Delta t = t_f - t_i = 0.37 - 0.33 = 0.04$ s

From graph: $v_i = 0.22$ m/s

For getting v_i in the range $0.2 < v_i \leq 0.25$ m/s **1 point**

From graph: $v_f = -0.18$ m/s

For getting v_f in the range $-0.15 \geq v_f > -0.20$ m/s **1 point**

For getting Δv consistent with the student's values of v_i and v_f, including subtracting in the correct direction **1 point**

$\Delta v = v_f - v_i = -0.18\ \text{m/s} - 0.22\ \text{m/s} = -0.40\ \text{m/s}$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-0.4\ \text{m/s}}{0.04\,\text{s}}$$

For correct substitution of values in the above equation **1 point**

$\bar{a} = -10\ \text{m/s}^2$

For showing deceleration (e.g., with a minus sign) **1 point**

Part B

The change is momentum is impulse: $J = \Delta p$. Therefore, the area under the Ft graph is the change in momentum.

$$J = 2(0.01 \times 10) + (0.02 \times 10) + (0.02 \times 30 \times 0.5) \quad = \quad 0.6 \quad \frac{\text{kg m}}{s}.$$

MAKE SURE TO HAVE THE RIGHT `UNITS`.

> *I got 0.7 kg m/s for my answer, but apparently 0.6 is the right one!*

Alternate method:

Expressing the change in momentum in terms of the average force:

$\Delta p = \overline{F}\Delta t$

For some method of using the graph to find the average force for the four non-zero intervals such as indicating that the area is equivalent to 6 boxes each with a height of 10 N, so that $\overline{F} = 60/4 = 15$ N **1 point**

$\Delta p = (15\ \text{N})(0.04\ \text{s}) = 0.6\ \text{N}\cdot\text{s}$ or $\Delta p = 0.6\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$

For correct numerical value of 0.6 **1 point**

For correct units **1 point**

Part C

$F = ma$, $p = mv$, and since we know $\Delta p = m\Delta v$, we can find m using $m = \frac{\Delta p}{\Delta v} = \frac{0.6Ns}{0.4m/s} = 1.5$ kg.

Alternate solutions:

Expressing the average force in terms of the average acceleration:

$\overline{F} = m\overline{a}$

For correct equation ($F = ma$ also accepted) **1 point**

For correct substitution of values consistent with those obtained above **1 point**

$m = \frac{\overline{F}}{\overline{a}} = \frac{15\ \text{N}}{10\ \text{m/s}^2} = 1.5\ \text{kg}$

Part D

$\Delta E = E_f - E_i$, and we know this is kinetic energy: $KE = \frac{1}{2}mv^2$. Plugging in the values, we get:

$\Delta E = \frac{1}{2}(1.5)(0.22)^2 - \frac{1}{2}(1.5)(-0.18)^2 = 0.012$ J.

Problem 2

Mech 2.

An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass $M_J = 1.90 \times 10^{27}$ kg and radius $R_J = 7.14 \times 10^7$ m.

(a) If the radius of the planned orbit is R, use Newton's laws to show each of the following.

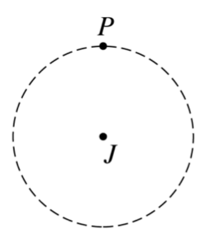
i. The orbital speed of the planned satellite is given by $\upsilon = \sqrt{\frac{GM_J}{R}}$.

ii. The period of the orbit is given by $T = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$.

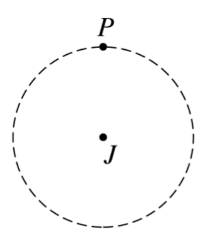
(b) The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^4 s. Determine the required orbital radius in meters.

(c) Suppose that the injection of the satellite into orbit is less than perfect. For an injection velocity that differs from the desired value in each of the following ways, sketch the resulting orbit on the figure. (J is the center of Jupiter, the dashed circle is the desired orbit, and P is the injection point.) Also, describe the resulting orbit qualitatively but specifically.

i. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly faster than the correct speed for a circular orbit of that radius.

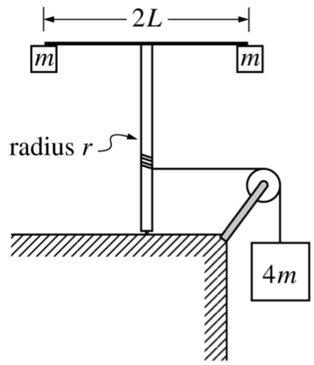


ii. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly slower than the correct speed for a circular orbit of that radius.



Since this problem involves gravitation and oscillation, we can safely skip this problem because the 2020 exam doesn't cover these topics.

Problem 3



Experiment A

Mech 3.

A light string that is attached to a large block of mass $4m$ passes over a pulley with negligible rotational inertia and is wrapped around a vertical pole of radius r, as shown in Experiment A above. The system is released from rest, and as the block descends the string unwinds and the vertical pole with its attached apparatus rotates. The apparatus consists of a horizontal rod of length $2L$, with a small block of mass m attached at each end. The rotational inertia of the pole and the rod are negligible.

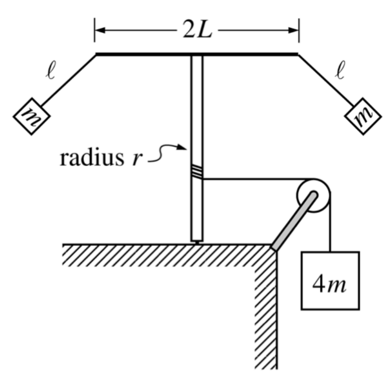
(a) Determine the rotational inertia of the rod-and-block apparatus attached to the top of the pole.

(b) Determine the downward acceleration of the large block.

(c) When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare with the value $4mgD$? Check the appropriate space below.

____ Greater than $4mgD$ ____ Equal to $4mgD$ ____ Less than $4mgD$

Justify your answer.



Experiment B

The system is now reset. The string is rewound around the pole to bring the large block back to its original location. The small blocks are detached from the rod and then suspended from each end of the rod, using strings of length ℓ. The system is again released from rest so that as the large block descends and the apparatus rotates, the small blocks swing outward, as shown in Experiment B above. This time the downward acceleration of the block decreases with time after the system is released.

(d) When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare to that in part (c) ? Check the appropriate space below.

____ Greater ____ Equal ____ Less

Justify your answer.

Part A

> ***INCORRECT***: *Rotational inertia is I, and for a horizontal rod about the center, the equation is* $\frac{1}{12}ML^2$. *The mass is* $2m$ *and* $L = 2L$. *Therefore,* $I = \frac{2}{3}ML^2$.

We know $I = \Sigma mr^2$ and $I = mL^2 + mL^2$, and therefore $I = 2mL^2$. We're summing the inertias. WHY!?

Part B

The large block has a few forces acting on it. There is the force of gravity and the force of the tension in the strong. $F = ma$.

$4mg - T = 4ma$, and we need to use torque: $\tau = I\alpha = Tr$, and so $T = \frac{I\alpha}{r}$. Plugging it back in we get $T = \frac{I\alpha}{r} = 4mg - 4ma$.

$\frac{2mL^2\alpha}{r} = 4mg - 4ma$ (substituting from part A). $\alpha = \frac{a}{r}$, so $a = \frac{2g^2}{L^2+2r^2}$.

For a correct formula for the rotational inertia **1 point**

$I = \sum mr^2$

For a sum containing a term of the form mL^2 (may include extra incorrect terms, but the point was not awarded if the expression does not contain an mL^2 term) **1 point**

$I = mL^2 + mL^2$

For the correct answer **1 point**

$I = 2mL^2$

3. (b) **6 points**

For a correct expression of Newton's 2nd law **1 point**

$F = ma$

For correct substitutions into Newton's law **1 point**

$4mg - T = 4ma$

For a correct formula for torque **1 point**

$\tau = I\alpha$ or Tr

$I\alpha = Tr$

$$T = \frac{I\alpha}{r}$$

From Newton's 2nd law equation above:

$T = 4mg - 4ma$

Substituting into the torque equation:

$$\frac{I\alpha}{r} = 4mg - 4ma$$

For substituting the expression for I from part (a) into Newton's law **1 point**

$$\frac{2mL^2\alpha}{r} = 4mg - 4ma$$

For the expression $\alpha = a/r$ **1 point**

Substituting this expression into the previous equation:

$$\frac{2mL^2 a}{r^2} = 4mg - 4ma$$

For the correct answer **1 point**

$$a = \frac{2gr^2}{L^2 + 2r^2}$$

Part C

$4mgD$ is the potential energy of the large block (mgh). There is more PE at the start, and it gradually gets converted to KE. Therefore, it should be **EQUAL TO 4mgD** because *the kinetic energy gained by the two smaller blocks comes from the decrease in PE of the 4m block*. In other words, total energy is conservered.

Part D

Now, it should be less because the small blocks are also gainig PE, and since the total energy is still *4mgD*, the KE is less than what it was in part C.