2002 Mech FRQs

Problem 1

Mech 1.

A crash test car of mass 1,000 kg moving at constant speed of 12 m/s collides completely inelastically with an object of mass M at time $t = 0$. The object was initially at rest. The speed v in m/s of the car-object system after the collision is given as a function of time t in seconds by the expression

$$v = \frac{8}{1 + 5t}.$$

(a) Calculate the mass M of the object.

(b) Assuming an initial position of $x = 0$, determine an expression for the position of the car-object system after the collision as a function of time t.

(c) Determine an expression for the resisting force on the car-object system after the collision as a function of time t.

(d) Determine the impulse delivered to the car-object system from $t = 0$ to $t = 2.0$ s.

Part A

We use conservation of momentum here. Inelastically means they stick together, so the equation is:

$(1000)(12) + (M)(0) = (1000 + M)(v)$, and $v = \frac{8}{1+5t}$.

M is therefore $= 1500(1 + 5t) - 1000$ kg. At t $= 0$, then M $=$ 500 kg.

Part B

$\frac{dx}{dt} = \frac{8}{1+5t}$ <- velocity v is the time derivative of position x

$x = \int \frac{8}{1+5t}\,dt$, and we substitute $u = 1 + 5t$ and $du = 5\,dt$ to get:

$x = \frac{8}{5}\int \frac{du}{u} = \frac{8}{5}\ln u + C = \frac{8}{5}\ln(1+5t) + C$. Since we know $x_0 = 0$, we get $C = 0$.

Therefore, $x = \frac{8}{5}\ln(1+5t)$.

Part C

We know $F = ma$ and $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{8}{1+5t}\right)$, and so $a = -\frac{40}{(1+5t)^2}$.

$F = (1500)(-\frac{40}{(1+5t)^2})$. It's important to remember the minus sign!

Now, $F = -\frac{60{,}000}{(1+5t)^2}$.

Alternate solution
For any indication that the force F is the time derivative of momentum p

$$F = \frac{dp}{dt}$$

For expressing the force in terms of the derivative of the velocity

$$F = m\frac{dv}{dt} = m\frac{d}{dt}\left(\frac{8}{1+5t}\right) = (1500)\left(-\frac{40}{(1+5t)^2}\right)$$

For the including the minus sign in the final expression for the force
For having the proper expression for the magnitude of the force

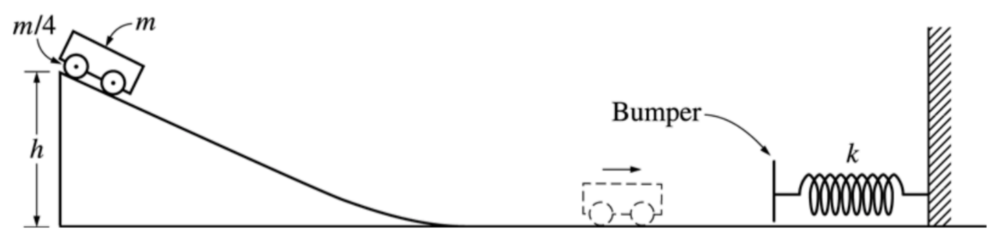
$$F = -\frac{60{,}000}{(1+5t)^2}$$

Part D

We want to find the input from t = 0 to t = 2. $J = \Delta p$, so the momentum at the two times can tell us what impulse is. (You can alternatively write $J = m\Delta v$).

$J = m(v_2 - v_0) = 1000(\frac{8}{11} - 8) = -10909$ Ns.

Problem 2



Mech 2.

The cart shown above is made of a block of mass m and four solid rubber tires each of mass $m/4$ and radius r. Each tire may be considered to be a disk. (A disk has rotational inertia $\frac{1}{2}ML^2$, where M is the mass and L is the radius of the disk.) The cart is released from rest and rolls without slipping from the top of an inclined plane of height h. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the total rotational inertia of all four tires.

(b) Determine the speed of the cart when it reaches the bottom of the incline.

(c) After rolling down the incline and across the horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant k. Determine the distance x_m the spring is compressed before the cart and bumper come to rest.

(d) Now assume that the bumper has a non-neglible mass. After the collision with the bumper, the spring is compressed to a maximum distance of about 90% of the value of x_m in part (c). Give a reasonable explanation for this decrease.

Part A

The wheels are the same so their I are the same. The $I = \frac{1}{2}MR^2$ and so we get their rotational intertia as $I = \frac{1}{8}mr^2$. Therefore the total I is $I_{tot} = 4I = \frac{1}{2}mr^2$.

Part B

We use conservation of energy $(E_{top} = E_{bottom})$: $mgh + 4(\frac{1}{4}mgh) = KE_{trans} + KE_{rot}$, where $M = m + \frac{m}{4} = \frac{5}{4}m$.

$2mgh = \frac{1}{2}(2m)v^2 + \frac{1}{2}(\frac{1}{2}mr^2)\omega$, and we know $\omega = \frac{v}{r}$. and therefore $K_{bottom} = \frac{5}{4}mv^2$.

Therefore, $v = \sqrt{\frac{8}{5}gh}$.

Part C

One again, we know that energy is conserved. (It might be an inelastic collision but the mass of the bumper is negligible and its KE is negligible and there is no loss of energy.)

$U_{spr} = \frac{1}{2}kx_m^2$ <-- at maximum compresison

We can either set this U to be equation to KE or PE at the top, but we get the same answer:

$\frac{1}{2}kx_m^2 = 2mgh$ or $\frac{1}{2}kx_m^2 = \frac{5}{4}Mv^2$

Therefore, we get $x_m = 2\sqrt{\frac{mgh}{k}}$.

Part D

The spring is compressed less because there is a dissipation of energy (most likely thermal) when the cart collides with the bumper. Since the bumper has mass this is an inelastic collision and therefore KE is not conserved.

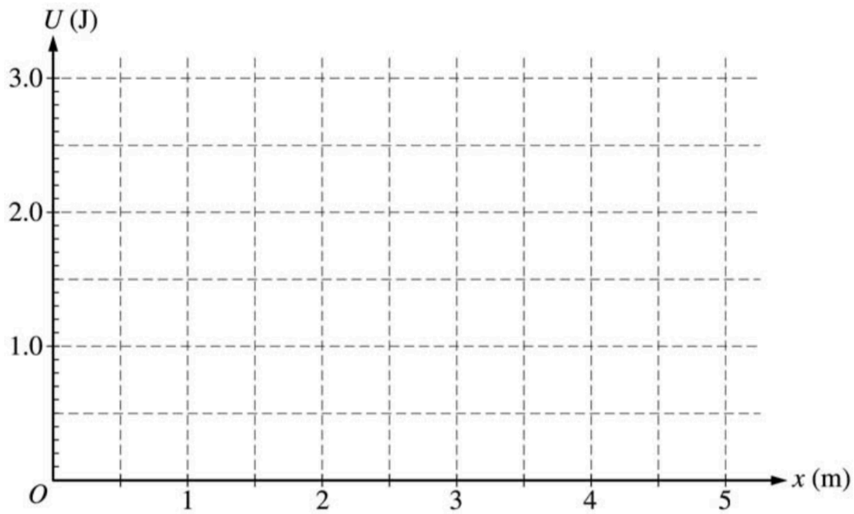
> *From guidelines: For an explanation that discusses the inelastic collision with a loss of mechanical energy or a reduced velocity resulting in a smaller compression. The discussion should have been correctly stated. If there were any incorrect statements, then 1 point was subtracted. Points were neither added or subtracted for irrelevant statements, such as references to friction.*

Problem 3

Mech 3.

An object of mass 0.5 kg experiences a force that is associated with the potential energy function $U(x) = \frac{4.0}{2.0 + x}$, where U is in joules and x is in meters.

(a) On the axes below, sketch the graph of $U(x)$ versus x.



(b) Determine the force associated with the potential energy function given above.

(c) Suppose that the object is released from rest at the origin. Determine the speed of the particle at $x = 2$ m.

In the laboratory, you are given a glider of mass 0.5 kg on an air track. The glider is acted on by the force determined in part (b). Your goal is to determine experimentally the validity of your theoretical calculation in part (c).

(d) From the list below, select the additional equipment you will need from the laboratory to do your experiment by checking the line next to each item. If you need more than one of an item, place the number you need on the line.

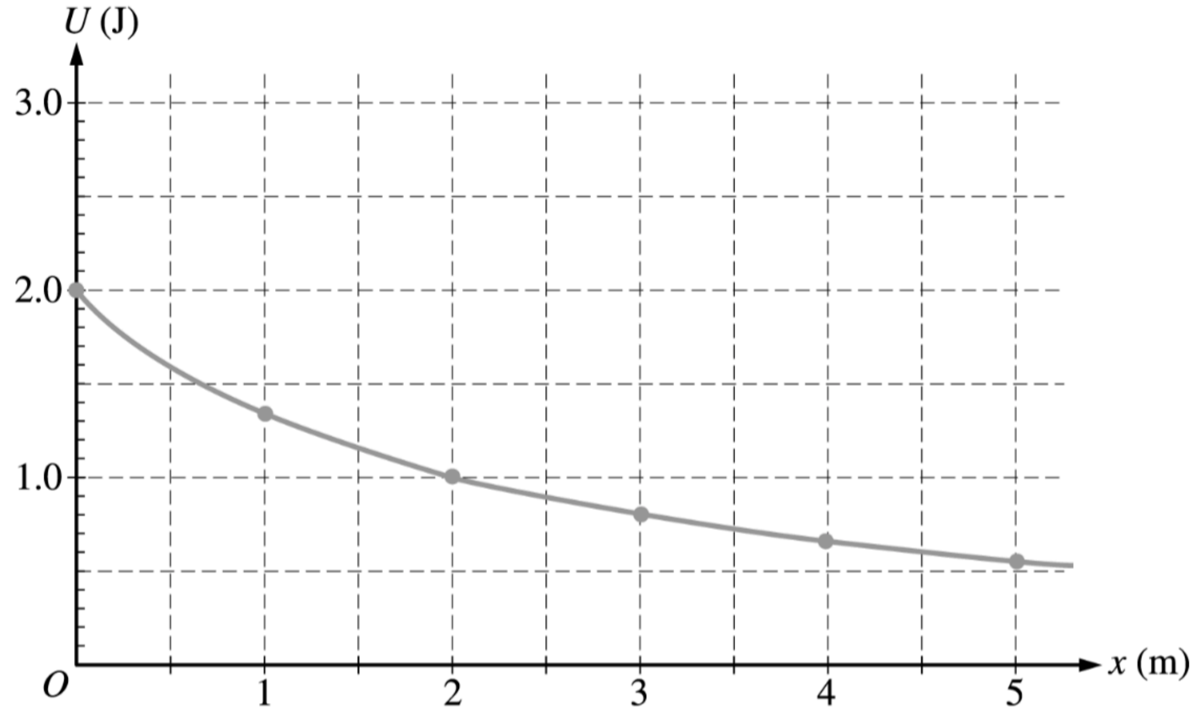
___ Meterstick ___ Stopwatch ___ Photogate timer ___ String ___ Spring

___ Balance ___ Wood block ___ Set of objects of different masses

(e) Briefly outline the procedure you will use, being explicit about what measurements you need to make in order to determine the speed. You may include a labeled diagram of your setup if it will clarify your procedure.

Part A

The graph looks like this:



Make sure that you have the correct shape: **concave upward** and **monotonically decreasing**.

y-intercept of $U(0) = 2J$. and U at $x = 5$ is within the range $0.5 \leq U(5) \leq 0.75$ J.

Part B

$F = ma$, and $PE = \frac{4}{2+x}$. We can find F by finding $F = -\frac{dU}{dx}$ and therefore $F = \frac{4}{(2+x)^2}$ (using the power rule and whatnot).

Part C

At x = 2, the PE is equal to 1 J. Since there is conservation of energy, we have $PE_0 + KE_0 = PE_2 + KE_2$ and since it starts from rest, $KE_0 = 0$. Therefore, $2 = 1 + \frac{1}{2}(0.5)v^2$ and therefore $v = 2$ m/s.

Part D/E

This is more to the dicretion of the reader and how you answer it.

(d) 2 points

For indicating items of equipment consistent with the procedure described in part (e) (at least two of the items if more than two were used) — 2 points

Note: If part (e) was not attempted, only 1 point maximum was awarded. Unreasonable indications, such as all the items being checked, were not awarded any points.

(e) 3 points

For a complete description of any correct procedure. — 3 points

Partial credit was awarded for less complete descriptions. The following were common examples. Other examples, though rarely cited, could receive partial or full credit.

1. Using photogates

   Place the photogates near $x = 2$ m and a small distance apart (such as a glider length). Measure the distance between the photogates. Measure the time the glider takes to travel between the photogates. Obtain the speed from distance/time.

   Note: No points were given if the distance measured was from 0 to 2 m and the time to travel 2 m was used.

2. Using a spring

   The spring constant k of the spring must be known, or if not, then measured. Set up the spring at $x = 2$ m so that it is compressed when struck by the glider. Measure the distance of maximum compression x_m. The velocity can then be determined from the equation $\frac{1}{2}kx_m = \frac{1}{2}mv^2$.

3. Treating the glider as a projectile

   Adjust the starting point so that $x = 2$ m is at end of the track. Thus the glider leaves the track at this point and becomes a projectile. The height of the track determines the time interval t that the glider is in the air. The horizontal distance x from the end of the track to the point where the glider hits the ground is measured and then the velocity is computed from x/t.

Please don't just check every material and hope that some of them are right.

Also, if you're using a photogate timer, make sure the two are right next to each other to calculated instantaneous velocity and not the total velocity.