

# *A Machine Learning Study Guide*



# Machine Learning Handbook

***The Definitive Guide***

*ICS5110, class of 2018/9*



**L-Università  
ta' Malta**





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# *Introduction*

This book explains popular Machine Learning terms. We focus to explain each term comprehensively, through the use of examples and diagrams. The description of each term is written by a student sitting in for ICS5110 APPLIED MACHINE LEARNING<sup>1</sup> at the University of Malta (class 2018/2019). This study-unit is part of the MSc. in AI offered by the Department of Artificial Intelligence, Faculty of ICT.

<sup>1</sup> <https://www.um.edu.mt/courses/studyunit/ICS5110>





# Activation Functions

Caterini (2018) defined artificial neural networks as “a model that would imitate the function of the human brain—a set of neurons joined together by a set of connections. Neurons, in this context, are composed of a weighted sum of their inputs followed by a nonlinear function, which is also known as an activation function.”

Activation functions are used in artificial neural networks to determine whether the output of the neuron should be considered further or ignored. If the activation function chooses to continue considering the output of a neuron, we say that the neuron has been activated. The output of the activation function is what is passed on to the subsequent layer in a multilayer neural network. To determine whether a neuron should be activated, the activation function takes the output of a neuron and transforms it into a value commonly bound to a specific range, typically from 0 to 1 or -1 to 1 depending on the which activation function is applied.

## Step Function

$$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases} \quad (1)$$

$$\frac{d}{d(x)}f(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ ? & \text{for } x = 0 \end{cases} \quad (2)$$

The Heavside step function, visualised in figure 1 and defined by equation 1, is one of the simplest activation functions that can be used in a neural network. This function returns 0 if the input of a node is less than a predetermined threshold (typically 0), or otherwise it returns 1 if the output of the node is greater than or equal to the threshold. This activation function was first used in a machine learning context by Rosenblatt (1957) in his seminal work describing the perceptron, the precursor to the modern day neural network.

Nowadays, the step function is seldom used in practice as it cannot be used to classify more than one class. Furthermore, since the derivative of this function is 0, as defined by equation 2, gradient descent algorithms are not be able to progressively update the weights of a network that makes use of this function (Snyman, 2005).

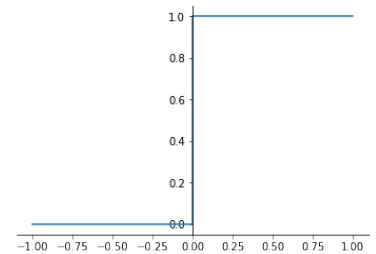


Figure 1: A graph of the step function.

### Linear Functions

$$f(x) = ax + b \quad (3)$$

$$\frac{d}{d(x)}f(x) = a \quad (4)$$

A linear activation function, is any function in the format of equation 3, where  $a, b \in \mathbb{R}$ . This function seeks to solve some of the shortcomings of the step function. The output produced by a linear activation function is proportional to the input. This property means that linear activation functions can be used for multi-class problems. However, linear functions can only be utilised on problems that are linearly separable and can also run into problems with gradient descent algorithms, as the derivative of a linear function is a constant, as seen in equation 4. Additionally, since the output of the linear function is not bound to any range, it could be susceptible to a common problem when training deep neural networks called the exploding gradient problem, which can make learning unstable (Goodfellow et al., 2016).

### Sigmoid Function

$$f(x) = \frac{1}{1 + e^{-x}} \quad (5)$$

$$\frac{d}{d(x)}f(x) = f(x)(1 - f(x)) \quad (6)$$

The sigmoid function or logistic function, visualised in figure 2 and represented by equation 5, is one of the most commonly used activation functions in neural networks, because of its simplicity and desirable properties. The use of this function in neural networks was first introduced by Rumelhart et al. (1986), in one of the most important papers in the field of machine learning, which described the back-propagation algorithm and the introduction of hidden layers, giving rise to modern day neural networks. The values produced by the sigmoid function are bound between 0 and 1, both not inclusive, which help manage the exploding gradient problem. The derivative of this function, represented by equation 6, produces a very steep gradient for a relatively small range of values, typically in the range of  $-2$  to  $2$ . This means that for most inputs that the function receives it will return values that are very close to either 0 or 1.

On the other hand, this last property makes the sigmoid function very susceptible to the vanishing gradient problem (Bengio et al., 1994). When observing the shape of the sigmoid function we see that towards the ends of the curve, the function becomes very unresponsive to changes in the input. In other words, the gradient of the function for large inputs becomes very close to 0. This can become very problematic for neural networks that are very deep in design, such as recurrent neural networks (RNNs). To address this

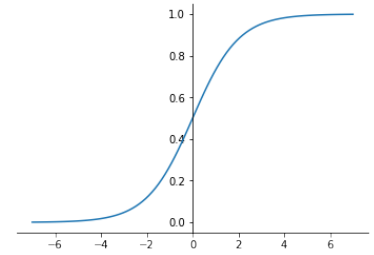


Figure 2: A graph of the sigmoid function.

problems in RNNs Long Short-Term Memory (LSTM) units were introduced as a variant of the traditional RNN architecture (Hochreiter and Schmidhuber, 1997).

### Hyperbolic Tangent

$$f(x) = \frac{(e^x - e^{-x})}{(e^x + e^{-x})} \quad (7)$$

$$\frac{d}{d(x)}f(x) = 1 - f(x)^2. \quad (8)$$

The hyperbolic tangent (tanh) function, visualised in figure 3 and represented by equation 7, is another common activation function that is sometimes used instead of sigmoid. The tanh function has the same characteristics of the sigmoid function mentioned above. In fact, when comparing figure 2 to figure 3 one can observe that the tanh function is simply a scaled and translated version of the sigmoid function. As a result of this scaling and translation, the tanh function has a steeper gradient towards the origin, and it returns values between -1 and 1. The derivative of the hyperbolic tangent function is represented by equation 8.

LeCun et al. (2012) analysed various factors that affect the performance of backpropagation, and suggested that tanh may be better suited than sigmoid as an activation function due to its symmetry about the origin, which is more likely to produce outputs that are on average close to zero, resulting in sparser activations. This means that not all nodes in the network need to be computed, leading to better performance. Glorot and Bengio (2010) studied in detail the effects of the sigmoid and tanh activation functions and noted how the sigmoid function in particular is not well suited for deep networks with random initialisation and go on to propose an alternative normalised initialisation scheme which produced better performance in their experiments.

### Rectified Linear Unit

$$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases} \quad (9)$$

$$\frac{d}{d(x)}f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases} \quad (10)$$

The Rectified Linear Unit (ReLU) function, visualised in figure 4 and represented by equation 9, returns 0 if the input of the function is negative, otherwise it outputs the value of the input itself. This function is non-linear in nature even though at first glance it may seem similar to an identity function. The ReLU function is becoming one of the more commonly used activation functions due to its simplicity, performance, and suitability to networks with many layers. Another

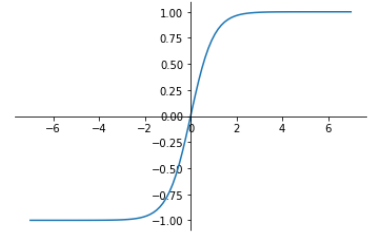


Figure 3: A graph of the hyperbolic tangent (tanh) function.

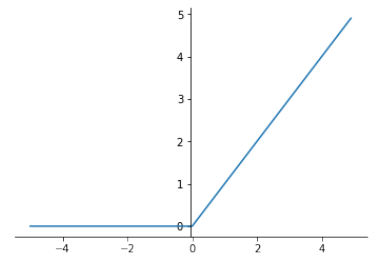


Figure 4: A graph of the ReLU function.

benefit of the ReLU function is that it produces sparse activations unlike many other commonly used functions such as the sigmoid.

The ReLU function has been used in many neural network models to improve their performance. [Nair and Hinton \(2010\)](#) use ReLU to improve the performance of Restricted Boltzmann Machines in object recognition. [Krizhevsky et al. \(2012\)](#) introduced a breakthrough Convolutional Neural Network (CNN) architecture called AlexNet, which pioneered the use of the ReLU activation function together with dropout layers to minimise over fitting in CNNs.

Unfortunately, because the gradient of the function for inputs that are negative is 0, as seen in equation 10, the ReLU function can still be susceptible to the vanishing gradient problem. To manage this problem a variant of the ReLU function, called Leaky ReLU is sometimes used. Rather than simply returning 0 for negative inputs, the leaky ReLU returns a very small value such as  $0.01x$ . [Maas et al. \(2013\)](#) compared the performance of Sigmoid, ReLU and Leaky ReLU functions and found that while the the performance of both the ReLU and Leaky ReLU functions was better than the performance achieved with the sigmoid function, the performance of the two ReLU functions was nearly identical.

# Confusion Matrix

A *confusion matrix* (CM), is a contingency table showing how well a model classifies categorical data. By convention (Sammur and Webb, 2017), the CM of an N-class model is an N×N matrix indexed by the true class in the row dimension and the predicted class in the column dimension (Table 1).

		Predicted Class	
		<i>spam</i>	$\neg$ <i>spam</i>
True Class	<i>spam</i>	10	1
	$\neg$ <i>spam</i>	2	100

Table 1: CM of a hypothetical binary classifier which predicts whether out-of-sample text objects are spam or not. In this example, 10 spam and 100 non-spam objects are classified correctly, whilst 1 spam and 2 non-spam objects are misclassified.

Even though CMs are commonly used to evaluate binary classifiers, they are not restricted to 2-class models (Martin and Jurafsky, 2018). A CM of a multi-class model would show the number of times the classes were predicted correctly and which classes were confused with each other (Table 2).

	<i>M&amp;M's</i>	<i>Skittles</i>	<i>Smarties</i>
<i>M&amp;M's</i>	34	3	8
<i>Skittles</i>	1	28	5
<i>Smarties</i>	2	4	22

Table 2: CM of a hypothetical sweets classifier. The main diagonal of the CM shows the number of correct predictions, whilst the remaining elements indicate how many sweets were misclassified.

The CM of the model  $h : X \mapsto C$  over the concept  $c : X \mapsto C$  using dataset  $S \subset X$  is formally defined as a matrix  $\Xi$  such that  $\Xi_{c,S}(h)[d_1, d_2] = |S_{h=d_1, c=d_2}|$  (Cichosz, 2014). The CM is constructed by incrementing the element corresponding to the true class *vis-a-vis* the predicted class for each object in the dataset (Algorithm ).

$\Xi \leftarrow 0$   $x \in S$   $d_1 \leftarrow c(x)$   $d_2 \leftarrow h(x)$   $\Xi_{d_1, d_2} \leftarrow \Xi_{d_1, d_2} + 1$  The CM is initialised to the zero matrix, and populated by iterating over all the objects  $x$  with corresponding true class  $d_1$  and predicted class  $d_2$  and incrementing the element  $(d_1, d_2)$  by 1 for each matching outcome.

In binary classification, the CM consists of 2 specially designated classes called the *positive* class and the *negative* class (Saito and Rehmsmeier, 2015). As indicated in Table 3, positive outcomes from the true class which are classified correctly are called *true positives* (TP), whilst misclassifications are called *false negatives* (FN). On the other hand, negative true class outcomes which are classified correctly are called *true negatives* (TN), and misclassifications are called *false*

positives (FP). In natural sciences, FP are called *Type I* errors and FN are known as *Type II* errors (Fielding and Bell, 1997).

	+ve	-ve
+ve	TP	FN
-ve	FP	TN

Table 3: CMs of binary classifiers have positive (+ve) and negative (-ve) classes, and elements called *true positives* (TP), *false positives* (FP), *true negatives* (TN) and *false negatives* (FN).

The information presented in the CM can be used to evaluate the performance of different binary classifiers (Lu et al., 2004). A number of statistics (Equations 11-17) derived from the CM have been proposed in the literature (Deng et al., 2016) to gain a better understanding of what are the strengths and weaknesses of different classifiers. Caution should be exercised when interpreting metrics (Jeni et al., 2013), since the CM could be misleading if the data is imbalanced and an important subrange of the domain is underrepresented (Raeder et al., 2012). For instance, an albino zebra classifier which always returns negative will achieve high accuracy since albinism is a rare disorder.

These metrics are important in situations in which a particular type of misclassification, i.e. FP or FN, could have worse consequences than the other (Hassanien and Oliva, 2017). For example, FP are more tolerable than FN in classifiers which predict whether a patient has a disease. Both outcomes are undesirable, but in medical applications it is better to err on the side of caution since FN could be fatal.

*Accuracy* (ACC) is the proportion of correct predictions (Equation 11). It is a class-insensitive metric because it can give a high rating to a model which classifies majority class objects correctly but misclassifies interesting minority class objects (Branco et al., 2016). The other metrics should be preferred since they are more class-sensitive and give better indicators when the dataset is imbalanced.

$$ACC = \frac{|TP \cup TN|}{|TP \cup FP \cup TN \cup FN|} \quad (11)$$

*Negative predictive value* (NPV) is the ratio of the correct negative predictions from the total negative predictions (Equation 12).

$$NPV = \frac{|TN|}{|TN \cup FN|} \quad (12)$$

*True negative rate* (TNR), or *specificity*, is the ratio of the correct negative predictions from the total true negatives (Equation 13).

$$TNR = \frac{|TN|}{|TN \cup FP|} \quad (13)$$

*True positive rate* (TPR), also called *sensitivity* or *recall*, is the ratio of the correct positive predictions from the total true positives (Equation 14).

$$TPR = \frac{|TP|}{|TP \cup FN|} \quad (14)$$

Sensitivity and specificity can be combined into a single metric (Equation 15). These metrics are often used in domains in which minority classes are important (Kuhn and Johnson, 2013). For example, the sensitivity of a medical classifier (El-Dahshan et al., 2010) measures how many patients with the condition tested positive, and specificity measures how many did not have the condition and tested negative.

$$\text{Sensitivity} \times \text{Specificity} = \frac{|TP| \times |TN|}{|TP \cup FN| \times |TN \cup FP|} \quad (15)$$

Positive predictive value (PPV), or *precision*, is the ratio of the correct positive predictions from the total positive predictions (Equation 16). The difference between accuracy and precision is depicted in Figure 5.

$$\text{PPV} = \frac{|TP|}{|TP \cup FP|} \quad (16)$$

Precision and recall are borrowed from the discipline of *information extraction* (Sokolova and Lapalme, 2009). A composite metric called *F-score*, *F1-score*, or *F-measure* (Equation 17) can be derived by finding their harmonic mean (Kelleher et al., 2015).

$$\text{F-score} = 2 \times \frac{\text{PPV} \times \text{TPR}}{\text{PPV} + \text{TPR}} \quad (17)$$

The complements of ACC, NPV, TNR, TPR and PPV are called, respectively, *error rate*, *false omission rate*, *false positive rate*, *false negative rate* and *false discovery rate*.

The metrics can be adapted for evaluating multi-class models by decomposing an N-class CM into 2-class CMs, and evaluating them individually (Stager et al., 2006). The literature describes two methods for decomposing this kind of CM. In the *1-vs-1* approach, 2-class CMs are constructed for each pairwise class as shown in Table 4.

+ve	-ve
M&M's	{Skittles, Smarties}
Skittles	{M&M's, Smarties}
Smarties	{M&M's, Skittles}

In the *1-vs-rest* approach, 2-class CMs are constructed for each class and the remaining classes combined together as shown in Table 5.

+ve	-ve
M&M's	Skittles $\cup$ Smarties
Skittles	M&M's $\cup$ Smarties
Smarties	Skittles $\cup$ M&M's

Using all metrics could be counterproductive due to information redundancy, but none of the metrics is enough on its own (Ma and Cukic, 2007). For instance, recall is class-sensitive but it would give a perfect score to an inept model which simply returns the positive

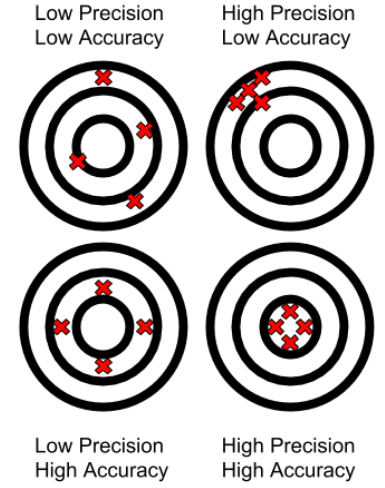


Figure 5: Accuracy vs Precision.

Table 4: 2-class CMs derived from the classes in Table 2. The +ve classes are paired separately with each -ve class.

Table 5: 2-class CMs derived through decomposition of the 3-class CM from Table 2 using the 1-vs-rest approach.

class. Thus, the best approach is to evaluate with complementary pairs (Gu et al., 2009) such as sensitivity *vs* specificity, or precision *vs* recall; or a combined measure such as the F-score.

Taking into account the above, CMs are suitable for visualising, evaluating, and comparing the performance of binary or multi-class classifiers. They should be used in conjunction with metrics such as the F-measure to avoid bias, especially if the dataset is unbalanced. For further details on the theoretical aspects of CMs and for practical examples in R refer to (Cichosz, 2014); for examples in Python refer to (Müller et al., 2016).

The following example is motivated by the samples in the *Scikit-Learn* documentation and the work of (Géron, 2017). The models in Figure 6 were trained on the *wines* dataset included with Scikit-Learn.



Figure 6: Decision boundary learned by a linear and non-linear binary classifier.

	Linear	Non-Linear
Accuracy	0.72	0.78
Specificity	0.77	0.77
Sensitivity	0.70	0.78
Precision	0.84	0.86
F-score	0.76	0.82

Table 6: Statistics derived from the CMs in Figure 7.

As it can be deduced from Figure 6, the decision boundary of the non-linear model is a better fit than the linear model. The CMs in Figure 7 also show that non-linear model performs better with a higher TP, and consequently lower TN. The biggest advantage of the non-linear model is the higher sensitivity resulting in a better F-score.

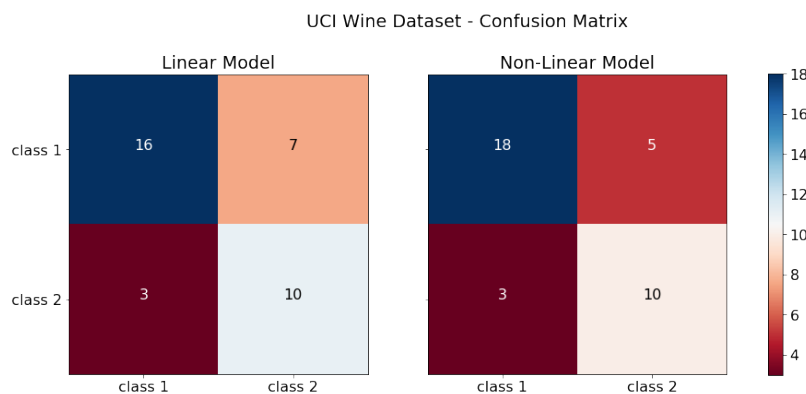


Figure 7: The linear classifier has 16 TP, 10 TN, 7 FN and 3 FP, whilst the non-linear classifier has 18 TP, 10 TN, 5 FN and 3 FP.



# Cross-Validation

Cross-validation (CV) is an estimation method used on supervised learning algorithms to assess their ability to predict the output of unseen data (Varma and Simon, 2006; Kohavi, 1995). Supervised learning algorithms are computational tasks like classification or regression, that learn an input-output function based on a set of samples. Such samples are also known as the labeled training data where each example consists of an input vector and its correct output value. After the training phase, a supervised learning algorithm should be able to use the inferred function in order to map new input unseen instances, known as testing data, to their correct output values (Caruana and Niculescu-Mizil, 2006). When the algorithm incorporates supervised feature selection, cross-validation should always be done external to the selection (feature-selection performed within every CV iteration) so as to ensure the test data remains unseen, reducing bias (Ambroise and McLachlan, 2002; Hastie et al., 2001). Therefore, cross-validation, also known as out-of-sample testing, tests the function's ability to generalize to unseen situations (Varma and Simon, 2006; Kohavi, 1995).

Cross-validation has two types of approaches, being i) the exhaustive cross validation approach which divides all the original samples in every possible way, forming training and test sets to train and test the model, and ii) the non-exhaustive cross validation approach which does not consider all the possible ways of splitting the original samples (Arlot et al., 2010).

The above mentioned approaches are further divided into different cross-validation methods, as explained below.

## *Exhaustive cross-validation*

### *Leave-p-out (LpO)*

This method takes  $p$  samples from the data set as the test set and keeps the remaining as the training set, as shown in Fig. 9a. This is repeated for every combination of test and training set formed from the original data set and the average error is obtained. Therefore, this method trains and tests the algorithm  $\binom{n}{p}$  times when the number of samples in the original data set is  $n$ , becoming inapplicable when  $p > 1$  (Arlot et al., 2010).

### Leave-one-out (LOO)

This method is a specific case of the LpO method having  $p = 1$ . It requires less computation efforts than LpO since the process is only repeated  $n_{choose1} = n$  times, however might still be inapplicable for large values of  $n$  (Arlot et al., 2010).

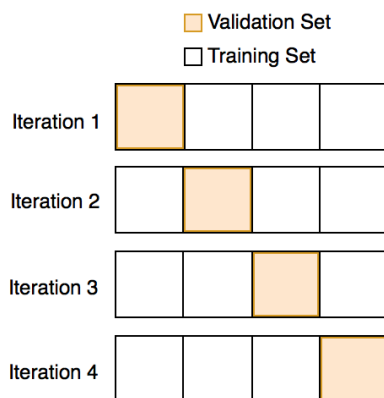
### Non-exhaustive cross-validation

#### Holdout method

This method randomly splits the original data set into two sets being the training set and the test set. Usually, the test set is smaller than the training set so that the algorithm has more data to train on. This method involves a single run and so must be used carefully to avoid misleading results. It is therefore sometimes not considered a CV method (Kohavi, 1995).

#### $k$ -fold

This method randomly splits the original data set into  $k$  equally sized subsets, as shown in Fig. 10. The function is then trained and validated  $k$  times, each time taking a different subset as the test data and the remaining  $(k - 1)$  subsets as the training data, using each of the  $k$  subsets as the test set once. The  $k$  results are averaged to produce a single estimation. Stratified  $k$ -fold cross validation is a refinement of the  $k$ -fold method, which splits the original samples into equally sized and distributed subsets, having the same proportions of the different target labels (Kohavi, 1995).



#### Repeated random sub-sampling

This method is also known as the Monte Carlo CV. It splits the data set randomly with replacement into training and test subsets using some predefined split percentage, for every run. Therefore, this generates new training and test data for each run but the test data

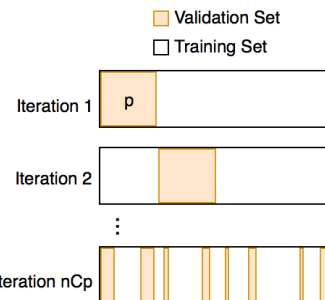


Figure 8: Leave-p-Out Exhaustive Cross Validation

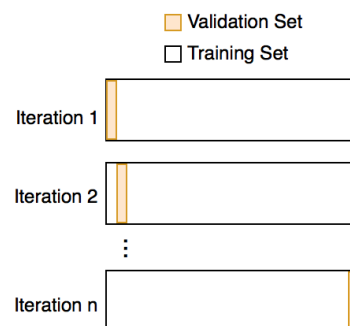


Figure 9: Leave-One-Out Exhaustive Cross Validation

Figure 10:  $k$ -Fold Cross Validation where  $k=4$

of the different runs might contain repeated samples, unlike that of  $k$ -fold (Xu and Liang, 2001).

All of the above cross-validation methods are used to check whether the model has been overfitted or underfitted and hence estimating the model's ability of fitting to independent data. Such ability is measured using quantitative metrics appropriate for the model and data (Kohavi, 1995; Arlot et al., 2010). In the case of classification problems, the misclassification error rate is usually used whilst for regression problems, the mean squared error (MSE) is usually used. MSE is represented by Eq. 18, where  $n$  is the total number of test samples,  $Y_i$  is the true value of the  $i^{th}$  instance and  $\hat{Y}_i$  is the predicted value of the  $i^{th}$  instance.

$$MSE = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \quad (18)$$

Underfitting is when the model has a low degree (e.g.  $y = x$ , where the degree is 1) and so is not flexible enough to fit the data making the model have a low variance and high bias (Baumann, 2003), as seen in Fig. 12a. Variance is the model's dependence on the training data and bias is model's assumption about the shape of the data (Arlot et al., 2010). On the other hand, as seen in Fig. 12b, overfitting is when the model has a too high degree (e.g.  $y = x^{30}$ , where the degree is 30) causing it to exactly fit the data as well as the noise and so lacks the ability to generalize (Baumann, 2003), making the model have a high variance. Cross-validation helps reduce this bias and variance since it uses most of the data for both fitting and testing and so helps the model learn the actual relationship within the data. This makes cross-validation a good technique for models to acquire a good bias-variance tradeoff (Arlot et al., 2010).

As stated in (Kohavi, 1995), the LOO method gives a 0% accuracy on the test set when the number of target labels are equal to the number of instances in the dataset. It is shown that the  $k$ -fold CV method gives much better results, due to its lower variance, especially when  $k = 10, 20$ . Furthermore, R. Kohavi et al. state that the best accuracy is achieved when using the stratified cross-validation method, since this has the least bias.

Therefore, let's take an example using the stratified  $k$ -fold cross-validation method with  $k = 10$ . Let's say that we are trying to solve age group classification, using eight non-overlapping age groups being 0-5, 6-10, 11-20, 21-30, 31-40, 41-50, 51-60, and 61+. We are using the FG-NET labelled data set, which contains around 1000 images of individuals aged between 0 and 69. Before we can start training our model (e.g. CNN), we must divide our data set into training and test subsets and this is where cross validation comes in. Therefore, we start by taking the 1000 images of our data set and splitting them according to their target class. Let us assume we have an equal amount of 125 (1000/8) images per class<sup>2</sup>. As depicted in Fig. 13, we can now start forming our 10 folds by taking 10% of each age-group bucket, randomly without replacement. Hence, we will

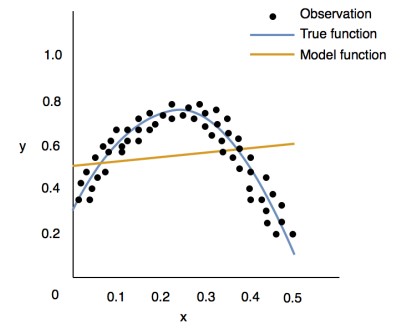


Figure 11: Model Underfitting

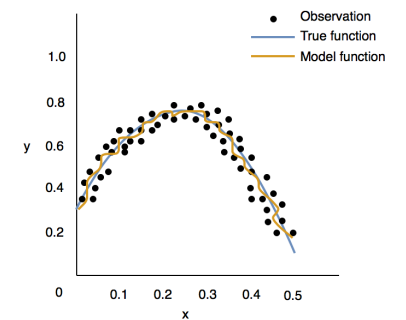


Figure 12: Model Overfitting

<sup>2</sup> Down-sampling or up-sampling are common techniques used when there is an unequal amount of samples for the different classes.

end up with 10 subsets of 100 images that are equally distributed along all age-groups. With these subsets, we can estimate our model's accuracy with a lower bias-variance tradeoff. Since we are using 10-fold CV, we will train and test our model 10 times. For the first iteration, we shall use subset 1 as the validation set and subsets 2 to 10 as the training set, for the second iteration we use subset 2 as the test set and subsets 1 plus 3 to 10 as our training set, and so on (as shown in Fig. 10). For each iteration we use the misclassification error rate to obtain an accuracy value and we finally average the 10 accuracy rates to obtain the global accuracy of our model when solving age group classification, given the FG-NET data set. Hence, we have now estimated the prediction error of the model and have an idea of how well our model performs in solving such a problem. It is important to note that cross-validation is *just* an estimation method and when using our model in real-life applications we do not apply CV but rather train our model with all the data we have.



Figure 13: Stratified 10-fold cross-validation on 1000 labelled images of 8 different classes

As concluded by [Varma and Simon \(2006\)](#), cross-validation is well implemented when everything is taken place within every CV iteration (including preprocessing, feature-selection, learning new algorithm parameter values, etc.), and the least bias can be achieved when using nested CV methods.

# Online Learning Algorithms

## Introduction

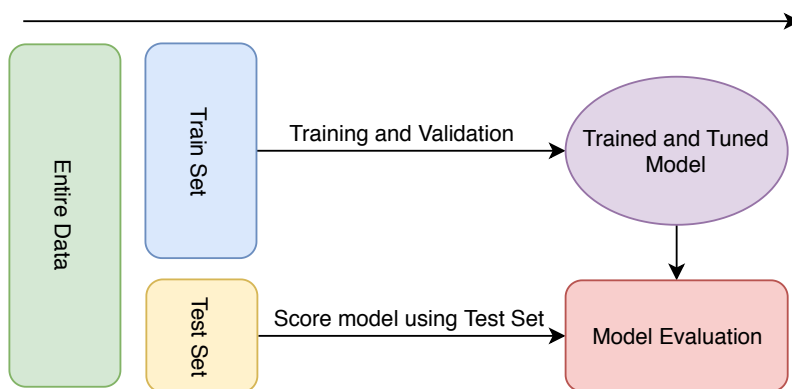


Figure 14: The traditional batch train-test machine learning approach workflow.

In the traditional machine learning approach depicted in Figure 14, we usually have some historical data to train an algorithm on for predicting some future events (Oza, 2005). However, since most data environments are dynamic and will change, the trained model eventually becomes outdated. To tackle this, we usually automate model re-training based on a timeframe (i.e. weekly or daily basis). Although this helps with keeping the model up-to-date, this is still not enough. Even if we consider model re-training on a daily basis, the model would still be at least one day late.

Furthermore, as Pagels (2018) argued, no matter how good a specific model is, it would always be an imperfect representation of the problem. Moreover, to have the best prediction for today, we cannot rely on a model with knowledge about yesterday only. Enter online learning algorithms, a family of techniques that are modelled to consume as much data available (one sample at a time), as fast as possible, while continuously learning and adapting different learning parameters. In the following sections, we will dive deeper into the subject of online learning, and its particular usage.

## Why use Online Learning?

While being highly adaptable to dynamic underlying data structures since they make no statistical assumptions on the distribution of the

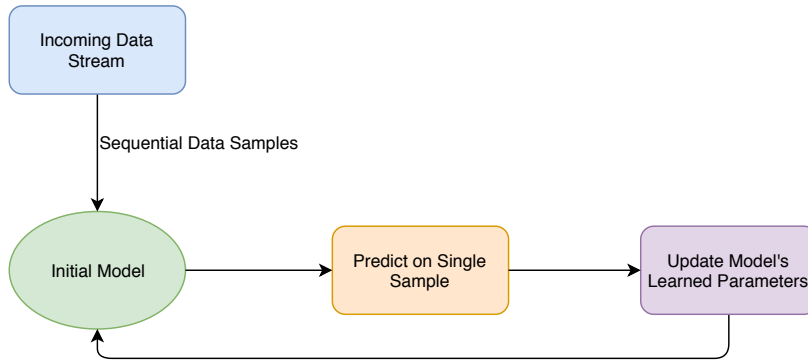


Figure 15: Basic workflow architecture of online learning.

data (Hoi et al., 2014), online learning techniques are also highly data efficient. Since online learning algorithms are only updated using the most recent data samples in the stream (as illustrated in figure 15), such data samples are no longer stored or needed once the algorithm has passed over them, maintaining a much smaller data storage (Oza, 2005). Such algorithms are also very fast since only a single pass on a smaller data set is made, in contrast to the standard approach where the optimisation function needs multiple iterations over the entire dataset. Thus, as argued by Hoi et al. (2018), online learning algorithms scale much better than the traditional approach.

As aforementioned, in offline machine learning, we load an entire dataset in memory, process it, then train a specific model, and then deploy the model into production. However, as more and more data is being generated, especially with the bright spotlight on Big Data, this methodology is proving to be more and more tedious. Some data sets are too large to fit into memory, even with distributed computing measures in place. Thus, online learning can drastically help in this scenario due to its small data storage property, especially when considered as online distributed computing and out-of-core computation (Zhang et al., 2017), which is a huge plus.

To further extract the important usability of online learning, let us, as an analogy, consider the case of an online news portal where news articles are custom and shown to the users based on which categories that respective user usually tends to click. Pretending that a terrible disaster is happening or has happened on one specific day, and the government issues a 24-hour emergency evacuation; therefore, the majority of the user-base would start clicking on this news more and more. With the traditional batch approach, even with a re-training time of 24 hours, the system would fail to push this article to users who typically do not click on domestic affairs articles (i.e. users only interested in sports or entertainment). As a result, the same data content structure will be assumed by the algorithm even though there was a drastic change of events.

In addition to this, given the same batch algorithm, after re-training in the following day, it would now start to suggest this article to a high percentage of the user-base, which by this time, such news might

no longer be relevant or applicable.

Another small application resides in the online advertising domain. With different events and occasions happening every day, especially unscheduled or unforeseeable events which go viral, ads must stay relevant all the time to ensure the highest click-rate probability, and thus, must always synchronise to the affairs of the physical world. Ads must be intelligent enough to be aware of the hidden data distribution to adapt to data morphism.

In both examples, a traditional static model will fail due to being too slow to react to the dynamic underlying relationships present in the data. This problem is more formally known as concept drift (Schlimmer and Granger, 1986). The following section will further explain the notion of Concept Drift.

### Concept Drift

Concept Drift occurs when the hidden context of the data changes. For instance, weather predictions are highly dependant on the season (the context), and as the seasons change so does the weather (Widmer and Kubat, 1996). As highlighted by Krawczyk and Cano (2018); Gama et al. (2014), based on the distribution drift speed and severity, concept drift can be of four types as depicted in Figure 16:

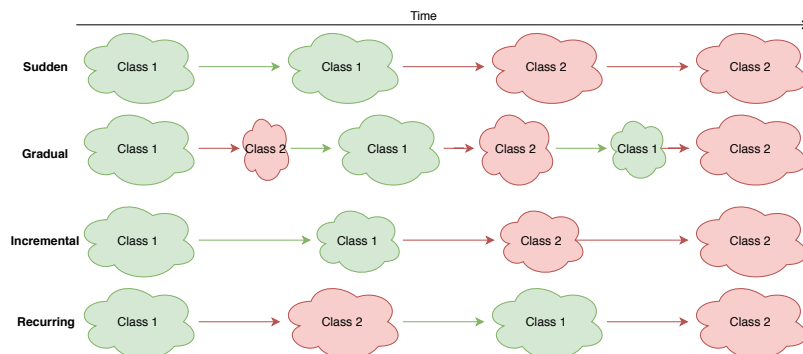


Figure 16: The four types of Concept Drift.

1. **Sudden:** The data distribution is immediately changed to a different class.
2. **Gradual:** The data distribution gradually transitions by having varying proportions of the different classes mixing together over time, until it completely changes to the new class.
3. **Incremental:** The data distribution slowly morphs from one class to another.
4. **Recurring:** The data distribution periodically transitions between previous classes.

### *Dealing with Concept Drift*

Thus, to combat this, as the concept drifts, so must the model's transition function that maps the inputs to the outputs. Due to the constant model updates performed through online learning (sample by sample), the transition function would be dynamic and adapts to the changing distribution (Gama et al., 2014; Hoi et al., 2018; Lane and Brodley, 1998). In addition to this, another approach is to have a sliding window that shifts with the data stream. The purpose of this window, as discussed by Wozniak (2011), is to keep a set of instances that offer the best representation of the present data distribution. As newer data samples arrive in the stream, the window slides towards more recent instances, resulting in the exclusion of the oldest samples from the window. Online learning achieves this window technique through the 'forgetting rate' which sets how fast older data is discarded to make room for newer instances.

### *Forgetting Rate*

Even though the design for most online learning algorithms is for fast execution speeds and thus adapted from less complex algorithms, implementation challenges are also present. As argued by Gepperth and Hammer (2016), this leads us to one of the most significant problems in online learning, Catastrophic Interference. The latter happens when the model abruptly forgets knowledge learnt for previous data. Most online learning algorithms have a forgetting rate parameter. This parameter allows the user to decide the speed at which the learning algorithm forgets old data; thus, how much data to retain. Moreover, the correct calibration of this rate is essential and challenging to perfect since a high value would result in catastrophic interference, while a lower value would result in the algorithm not adapting to the incoming samples in the stream. In addition to this, good initialisations are critical in this approach to steer away from slow convergence.

### *Conclusion*

In this chapter, we introduced and discussed the sub-field of online learning algorithms concerning machine learning. Online learning is a highly useful tool that allows us to take machine learning to a whole other level by solving problems that otherwise would seem to be out of our technical ability. With the exponential importance for Big Data analytics, online learning arms us with the capabilities to process high-velocity data while also being fast to adapt to frequent changes in the data due to the ever-increasing data velocity.



# *Sample Selection Bias*

In traditional statistics, the algorithms assume that the the data samples are being drawn in line with the same distribution, and different classes and values of data should appear with roughly the same frequency that they actually occur in the real world. However, this is rarely the case and results in the data becoming biased, meaning that the method of sample collection favours a particular type of data, skewing the distribution of the values (Cuddeback et al., 2004).

There could be a number of reasons as to why this bias occurs (Tommasi et al., 2017), such as:

1. It might be costly or unpractical to collect certain data.
  - For example, measuring spending habits of large groups of people would be cumbersome, and these people may not necessarily give accurate information.
2. The entity collecting the data only has access to data belonging to a particular class.
  - For example, if a doctor only has access to patients that are sick, a large portion of the sample would represent sick patients, whereas in relation to the total population the number of sick people would be relatively much smaller.
3. Confirmation bias, whereby people tend to recall only examples that confirm their existing beliefs.
4. Incorrect sampling techniques (Marshall, 1996), such as sampling from the top of a list instead of randomly
5. Sampling using results generated from another process.
  - For example, if a system is being trained on detecting fraudulent transactions, and some of these transactions were classified incorrectly by another process, then the model will be trained on false data.

An interesting example that tends to happen within Maltese populations is that related with politics. When a political party passes an online poll regarding a particular issue, the results always tend to be in its favour. This stems from the fact that even though the poll is open to the general public, the outreach of the poll would be much more inclined to reach people within the party - they would

have subscribed to social media and news pages, and would regularly follow or check up on their articles and news.

### *False Information created by Selection Bias*

Having selection bias within a dataset can create false information which does not exist in the actual population, and can lead to inaccurate estimates of the relationships between variables (Cuddeback et al., 2004).

For example, assume a college accepts students that either have high math skills or high science skills. Therefore if a student in this college does not have high math skills, then he must instead have high science skills. This means that a negative correlation between maths and science skills has been created, which does not represent the actual population.

### *How To Prevent Selection Bias*

When collecting a sample of data, one should be attentive in order to prevent selection bias occurring in the dataset. This step of preventing bias is important as not only could it skew the results, but might end up rendering the analysis and conclusions gained from the dataset as useless.

### *Stratified Sampling*

This process involves splitting the population into sub-populations before selecting, in order to ensure that an adequate number from each group is selected. For example, splitting a population into age categories and selecting a number from each category (Krautenbacher et al., 2017)

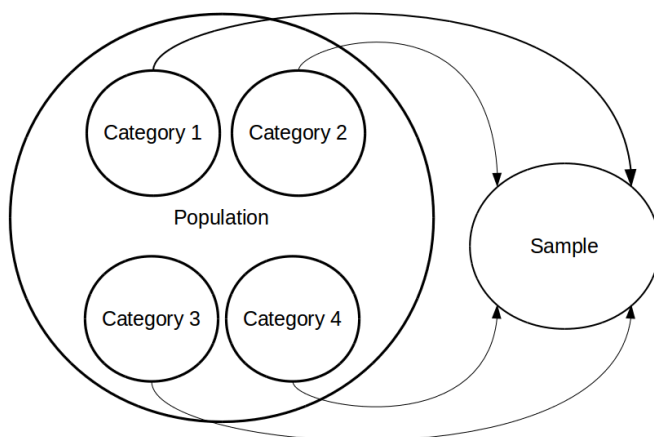


Figure 17: Stratified sampling: splitting the population into categories and selecting accordingly

### *Not Self Selecting*

Draw from a sample that is not self selecting. If the sample is self-selected, people who are willing to volunteer may share similar characteristics (such as be more outgoing and extroverted), which will create a bias.

### *Analyze Dropouts*

Determine factors affecting drop outs and establish that no differences exist between participants and non-participants. If those refusing to participate all belong to a certain category, this will create a bias and results need to be corrected to account for these differences (Alonso et al., 2006).

### *Sufficient Sample Size*

Ensure a large enough sample size. If the sample size is too small, it will make it more difficult to create an accurate representation.

### *Detecting Selection Bias within a Dataset*

Once data has been collected, or when working on a dataset gained from a different source, it is important to determine whether any sample selection bias exists within the dataset before performing analysis or drawing any conclusions. The processes mentioned below are some methods which can be useful in the detecting such bias.

### *Comparing Distributions*

If it is possible to capture information regarding distribution about the whole population, one may then compare this with the sample population. If the distribution is drastically different, it will indicate that a sample selection bias is present.

As a simple example, if the total population contains 50% males and females, whilst your sample contains 75% males and 25% females, it will indicate a selection bias which will skew the results.

Techniques exist to allow the comparison of distributions and measuring the difference, such as Bayesian Analysis, Kolmogorov-Smirnov test or Chi-Squared test. (Griffin et al., 2013)

### *Two Step Estimator*

This method, as defined by Heckman (1979), comprises the use of two multiple regression models:

- One model is used to examine the interest of the study.
- The other regression model is used to detect selection bias and to statistically correct the substantive model. The independent variable could be set to represent participation. The dependent variable could represent different related statistics.

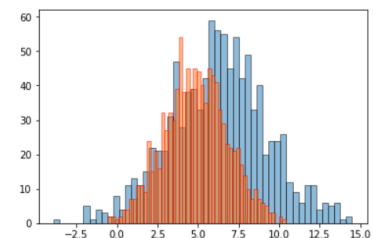


Figure 18: Comparing Distributions

### Two Step Estimator Example

Imagine a survey aiming to collect information about European citizens quality of life. This would include factors such as job status, salary, level of education, country GDP, amongst others.

The first model would simply use these factors to create a model determining each citizens overall quality of life.

The second model would then measure relationships between participation and other variables. If for example a positive correlation exists between level of education and participation, then a bias in the study has occurred whereby those with a lower level of education are being underrepresented.

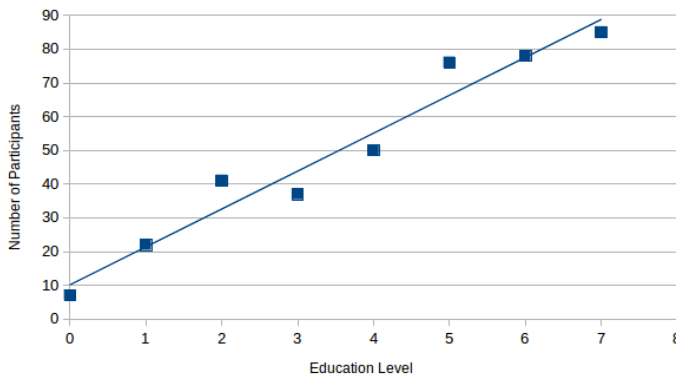


Figure 19: Two Step Estimator: Using regression models to determine correlations between participants and non-participants

### How to Deal With Selection Bias

If presented with a dataset which contains selection bias but which cannot be changed or modified due to a number of reasons, then adequate measures should be taken to either mitigate the bias or account for it in the results.

### Post-stratification

In an attempt to make the results more representative of the total population, higher weightings are given to the lower class. If say a sample contains 25% women, but the general population has 50%, then you could adjust the estimates by assigning women in the sample higher weights (Holt and Smith, 1979).

As explained by Cortes et al. (2008), the generalization error  $R$  on the newly weighted sample is defined as follows:

$$R(h) = \sum_{i=1}^m w_i c(h, z_i) \quad (19)$$

Where  $m$  is the number of samples,  $h$  is the error value,  $w$  is the weight,  $c$  is the cost function and  $z$  is the sample value.

### *Synthetic Minority Oversampling Technique (SMOTE)*

Another method of dealing with imbalanced classes is that of generating synthetic data that closely resembles the underrepresented class. In cases where the majority class greatly outweighs the minority class, having a larger dataset that more closely resembles the minority class will balance the results generated.

The way this process works is that new samples are created by finding the midpoint between the line segments joining the existing samples within the minority class.

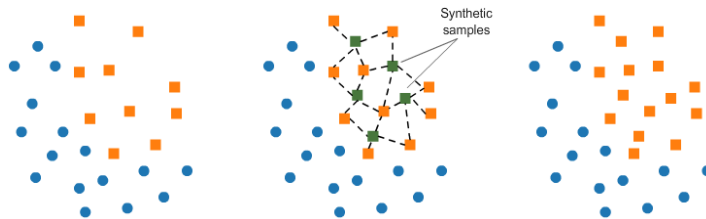


Figure 20: SMOTE: generating synthetic samples of the minority class

SMOTE may also create certain drawbacks. Since the synthetic results are generated on a small set of existing features, it may create an overfitting scenario which might not be able to correctly classify the true outlier nature of the minority class. SMOTE might also skew the distribution of the dataset which could affect certain analysis. The large addition of additional samples could also slow down learning speed. (Weiss et al., 2007)

Studies seem to have shown that using SMOTE techniques to over-represent the minority class, as opposed to under-representing the majority class by selecting fewer, tends to perform better (Chawla et al., 2002).

### *Applicability Disclosure*

Giving full disclosure: If it is not possible to eliminate the selection bias from the dataset, then it is important to fully disclose who exactly the results and knowledge gained from the data are applicable to.



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