Technical Report

Efficient, Proximity-Preserving Node Overlap Removal

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Chapter 1

Subject presentation and state of the art

A graph is a data structure encoding information with the use of nodes and edges (which are binary relations between nodes).

Graph drawing aims to represent a given information as a graph, generally through a "node-link" layout, letting nodes and edges be displayed.

Most of the layout algorithms consider nodes as points, but some need to let appear additional information as labels. For example, London subway maps would be useless without the indication of the stations on the lines.

This could lead to an overlap of some nodes. That must be avoided, as it clearly confuses the understanding of the graph.

Moreover, as we generally consider that the original layout contains significant information, an other parameter to deal with is to maintain the "global shape" of the initial representation.

This "global shape" can be seen as "preserving the proximity relations between nodes", "preserving the orthogonal ordering of nodes"(see [16]) or "preserving the relative positions of nodes by limiting the vertices displacement"(see [7]), and a choice between these criteria has to be made.

The easiest approach is to "scale" the layout until no overlaps occur. This method has the advantage to preserve the global shape of the layout, but the area of the graph can become very inconvenient. That is why a compromise between the preservation of the "shape of the graph" and a minimization of the total area has to be found.

Different algorithms have been devised to answer the problem, each of

them focusing on a different "global shape" definition. We consider here only algorithms dealing with the overlap removal as a post-processing step (after the graph has been drawn).

Spring-electrical model : Eades [4] and Fruchterman and Reingold [5], recentely adapted by various authors ([8], [15])

Stress model [11]

Voronoi cluster busting algo [7]

VPSC [3]

PRISM [6]

Since PRISM :

ePRISM : PRISM on overlap of edges [9]

Work on Wordle (`http://www.wordle.net/`) : Mani-Wordle [13] and RWordle [17]

More details about the force-directed drawing algorithms can be found in [12].

Our project consists in understanding the algorithm PRISM proposed by Gansner and Hu in [6] and in analyzing the feasability of its implementation as a plugin for the Tulip software [1].

Chapter 2

PRISM algorithm

2.1 Formal notations

2.2 Description of the algorithm

The PRISM algorithm focuses on two main constraints for the final layout of the graph. First, the area taken by the layout must be minimal. The second constraint is to preserve the global "shape" of the original layout by maintaining all proximity relations between the nodes.

The PRISM algorithm runs in two main steps ; in a first step, it removes iteratively the overlaps between near nodes of the given graph G. Then it finds the non-near overlaping nodes and removes these overlaps as well.

We consider for this algorithm that a node i has a certain width w_i and height h_i, thus forming a rectangle containing the label, likely to cause overlaps.

2.2.1 Overlap removal between near nodes

Use of a proximity graph - Delaunay Triangulation

To find easily the overlaps between near nodes of the graph G, it will efficient to work on a proximity graph of G. Such a graph will also guarantee the preservation of the proximity relations during the different stages of the algorithm.

A *proximity graph* is a graph in which two vertices are connected by an edge if (and only if) they satisfy a given geometrical property (a survey on

proximity graphs can be found in [10]).

The *Delaunay triangulation* (DT) (named after the work of Delaunay [2]) of a graph G is a triangulation of the graph such that none of the circumscribed circles of the triangles in DT(G) contains a vertex. This particular triangulation also maximises the minimum angle of the triangles found. The Delaunay triangulation of G, as a triangulation, is also a planar graph, and has thus at most $3n - 6$ edges (if $|V(G)| = n$), which is a very practical parameter for the algorithm.

The nearest neighbors of a vertex $v \in V(G)$ tend to form triangles with v and in particular, the closest neighbor of v has an edge with v in DT(G), as the nearest neighbor graph of G is a subgraph of DT(G) (see [10]).

In the PRISM algorithm, we consider that the near nodes in G are connected by an edge in the Delaunay triangulation of G.

Thus, the algorithm's first goal is to remove overlaps along the edges of the Delaunay triangulation of G.

Ideal edge length

The idea is to find the "ideal length" of the Delaunay triangulation edges : the "ideal length" of an edge is such that the two edge ends have no overlap.

In order to do that, we calculate an *overlap factor* f_{ij} for each edge (i, j) $(i, j \in V(G))$ of the Delaunay triangulation of G :

$$f_{ij} = \max(\min(\frac{w_i/2 + w_j/2}{x_i - x_j}, \frac{h_i/2 + h_j/2}{y_i - y_j}), 1)$$

where (x_i, y_i) are the coordinates of vertex i, w_i its width and h_i its height.

If two nodes i and j have no overlap, then $f_{ij} = 1$. If i and j do overlap, then that overlap can be removed by expanding the edge (i, j) by the overlap factor found f_{ij}.

Thus, the "ideal length" of an edge of the Delaunay triangulation is $l_{ij} = f_{ij}||p_i - p_j||$, where p_i is the initial set of coordinates of a node i.

We now want to find coordinates for the nodes of the initial graph such that the edges length in $DT(G)$ are close to their ideal length.

Proximity stress model

Finding this new set of coordinates means minimizing the following sum :

$$\sum_{i,j \in E(DT(G))} w_{ij}(||p_i - p_j|| - l_{ij})^2$$

where l_{ij} is the overlap factor and w_{ij} is a classic weighting factor, used to equalize the contributions to the total sum from the different edges.

But there are some situations where keeping l_{ij} is not a good idea (see figure)

We thus want to avoid removing the "big" overlaps in one iteration only. So we have to replace l_{ij} by $\min(l_{ij}, s)$, where $s > 1$ will be a limiting factor. The authors found that $s = 1.5$ worked well.

We now want to minimize :

$$\sum_{i,j \in E(DT(G))} w_{ij}(||p_i - p_j|| - \min(l_{ij}, s))^2$$

This type of sum is called a "stress function", in analogy with the well studied *stress model* introduced by Kruskal in [14] and applied to graph drawing by Kamada and Kawai in [11]. The above sum is called the *proximity stress model*.

The minimization of the sum gives new positions for the nodes of G.

Iteration and termination

This first phase provides us with a new layout of the graph G, in which the nodes positions are given according to the previous minimization of the proximity stress model.

This layout may still contain overlaps. We must thus iterate the construction of the Delaunay triangulation, the computation of the overlap factors, the minimization of the proximity stress model and the move of the nodes until no more overlaps occur along the edges of the Delaunay triangulation of the graph.

The process of the first stage makes clear that no overlaps can appear : the distance between nodes can only be increasing and the proximity graph on which we are calculating, as a triangulation, is a rigid graph.

But the authors do not explain formally the reason of the termination of this phase : the stress function could always be smaller but never reach a local minimum. Moreover, the number of iterations needed during the first phase is not explicited or bounded in the article.

The author's implementation of PRISM contains a threshold : if the gain in the minimization of the stress function is smaller, the first phase of the algorithm ends.

2.2.2 Overlap removal between non-near nodes

The first step removes the overlap between ends of edges of the Delaunay triangulation of the graph. But some overlaps can be caused by nodes not being near, and thus not generating an edge in the proximity graph. These overlaps can not have been removed by the first stage of the PRISM algorithm.

To find these still overlapping nodes, we have to use a scan-line algorithm.

Scan-line algorithm

A scan-line algorithm is a algorithm which will consider all the points of a layout.

We can not use a algorithm which uses the graph properties because the vertices do not know the positions of the others vertices, so we can not find the overlaps by using the vertices properties.

For all the ordinate's points we consider all the points in the abscissa. If there is an overlap at a point, we add the edge (between the two vertices overlapping) in the Delaunay triangulation.

It is interesting to note that one of the opponent algorithms, VPSC, only uses a scan-line algorithm to remove all overlaps (mettre lien)

Overlap removal

The second stage uses the same processus as the first one, only adding the overlapping edges found by the scan-line algorithm to the Delaunay triangulation before the calculation of the overlap factors and the resolution of the proximity stress model.

This stage ends when no more overlaps are found by the scan-line algorithm.

Algorithm 1: PRISM

Input: p_i^0 : coordinates of each vertex
width w_i and height h_i of each vertex ($i = 1, 2, ..., |V|$)

repeat
  G_{DT} : proximity graph of G by Delaunay triangulation
  for *all edges of* G_{DT} **do**
    Compute the overlap factor
  $\{p_i\}$: solution of the proximity stress model
  $p_i^0 = p_i$
until no more overlaps along edges of G_{DT};
repeat
  G_{DT} : proximity graph of G by Delaunay triangulation
  Find overlaps in G through a scan-line algorithm
  Add the overlapping edges to G_{DT}
  for *all edges of* G_{DT} **do**
    Compute the overlap factor
  $\{p_i\}$: solution of the proximity stress model
  $p_i^0 = p_i$
until no more overlaps found by the scan-line algorithm;

2.2.3 Dissimilarity metrics

To be able to compare different graph layouts, the authors propose three dissimilarity metrics : the area taken by the layout, a metric based on the edge length ratio in the proximity graphs of the initial and final layouts, and a metric measuring the vertices displacement between the initial and the final layouts.

Area

As said before, it is easy to remove all the overlaps by extending all the edges of the initial layout. But the final layout can be extremely large and thus unreadable, so we want to keep an area as small as possible.

Edge length ratio

We first calculate the ratio between the edge lengths of the proximity graphs of the original layout and the final one. The metric is then defined as the normalized standard deviation to the mean ratio found.

This metric has to be as small as possible to minimize the changes made to the edges length during the algorithm.

The edge length ratio has to be calculated on a rigid graph (as the Delaunay triangulations) to be meaningful : two layouts of the same graph can be completely different if the graph is not rigid. (exemple ?)

Vertices displacement

→ expliquer comment on fait : matrice de rotation, translation, ajustement de taille, addition de tout ça

2.3 Complexity

→ comparaison avec VPSC

Chapter 3

Implementation within Tulip

3.1 The Tulip framework

Tulip presentation [1]
    Tulip node structure (problems with label size)
    Solution : forcing node size and labels

3.2 Resolution of the stress model

Stress majorization

Not possible natively in Tulip
    Too many dependencies in GraphViz (python script)
    Use of Eigen possible ?

Newton-Raphson method - Kamada & Kawai

Implementation done : pseudo-code, explications
    Tests and time

3.3 Scan-line algorithm

Implementation details

3.4 Tests and results

Tests and time of the Kamada and Kawai solution

Chapter 4

Conclusion

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