Behind the Delays: Exploring Variations in Toronto's Transit Disruptions*

A Comparative Analysis of Delay Duration Across TTC Subway, Streetcar, and Bus Systems

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This paper examines the variations in delay durations across Toronto Transit Commission (TTC) transit modes—subway, streetcar, and bus using a Bayesian linear regression model. Our analysis shows that buses experience longer delays compared to subways and streetcars, with distinct patterns based on time of day. These findings are significant as they highlight specific areas where improvements can benefit daily commuters. For transit operators and city planners, this study provides practical findings to support decisions aimed at reducing congestion and enhancing reliability, ultimately improving the experience for passengers across Toronto's transit system.

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^{*}Code and data are available at: [https://github.com/ClaireUoft/Toronto_TTC_Transportation).

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1 Introduction

Urban transit systems play an important role in the daily lives of millions, influencing mobility, accessibility, and economic productivity in cities worldwide. However, transit delays can significantly impact passengers' schedules, urban congestion, and overall satisfaction with public transportation. In Toronto, the Toronto Transit Commission (TTC) operates a complex network of buses, streetcars, and subways, all of which are prone to delays caused by a variety of factors, including mechanical failures, traffic congestion, and operational disruptions. Despite the centrality of public transit to urban life, there is limited systematic analysis of how delay patterns vary across transit modes and the factors contributing to these delays.

This paper focuses on addressing this gap by analyzing the duration of delays across TTC transit modes—buses, streetcars, and subways—using a detailed dataset from Open Data Toronto. Specifically, it examines the effects of transit mode, time of day, and day of the week on delay duration. While previous studies primarily focused on historical data and specific transit types, their work examined delays across the TTC system over eight years but lacked the inclusion of all three transit modes (subway, streetcar, and bus) in a single extensive study for the most recent years.

To fill this gap, we employed a Bayesian linear regression model to estimate the relationship between delay duration and its predictors. The analysis incorporates over 80,000 delay events, harmonized across transit modes and cleaned for consistency. The results are buses experience the longest average delays, followed by streetcars and subways. Additionally, delays are longer during peak hours and vary by day of the week, with weekends exhibiting shorter delays on average.

These findings are significant for urban planners, transit operators, and policymakers, as they highlight key areas for intervention to improve transit reliability. Additionally, daily TTC users can use these findings to make informed decisions about when to leave their homes and which mode of transportation to take. Moreover, addressing these delays could enhance passenger satisfaction, reduce economic losses caused by tardiness, and increase the overall efficiency and reliability of Toronto's transit network. This study contributes to the broader field of urban transit research and findings into delay variability and providing an evidence-based foundation for targeted improvements in public transportation systems.

The remainder of this paper is structured as follows. Section 2 discusses the data used for this analysis, including key variables and sources, with particular attention to the quality metrics that affect polling accuracy. Section 3 outlines our modeling approach..., incorporating lessons learned from recent electoral cycles. Our predictions are under section of each model. Section 5 discusses the implications of our findings and suggests directions for future research. Finally, Section A evaluates methodology and survey copy.

2 Data

2.1 Overview

In this project, we used data from the opendatatoronto dataset created by (Gelfand 2022), alongside R (R Core Team 2023) and several important packages for analysis and visualization. tidyverse (Wickham et al. 2019) and dplyr (Wickham et al. 2023) supported data manipulation, while arrow (Richardson et al. 2024) handled large datasets efficiently. lubridate (Grolemund and Wickham 2011) managed date-time data, and ggplot2 (Wickham 2016) created visualizations. knitr (Xie 2023) and kableExtra (Zhu 2024) enhanced report formatting. Bayesian regression was conducted using rstanarm (Goodrich et al. 2022), with bayesplot (Gabry et al. 2019) for diagnostics, and modelsummary (Arel-Bundock 2022) generated summary tables. Together, these tools ensured efficient data processing, analysis, and presentation.

2.2 Measurement

Table 1: Preview of Cleaned Dataset for Bus, Subway, and Streetcar (One Example Each)

Date	Time	Day	Transit_mode	Line	Location	Incident	Min Delay
2024-01-01	02:08:00	Monday	Bus	89	KEELE AND GLENLAKE	Vision	10
2024-01-01	02:45:00	Monday	Streetcar	505	DUNDAS AND MCCAUL	Security	10
2024-01-01	02:00:00	Monday	Subway	1	SHEPPARD STATION	N/A	0

The measurement process refers to how real-world transit delays—such as a bus stuck in traffic, a subway experiencing a mechanical failure, or a streetcar held by emergency services—are captured and represented as entries in a dataset. Each entry corresponds to a distinct delay event recorded by the Toronto Transit Commission (TTC).

Delay Duration (Min Delay):

This variable represents the length of a transit delay, measured in minutes. It is derived from the difference between scheduled and actual event resolution times, as recorded by the TTC's operational systems. These durations are logged for each incident and standardized in the dataset as numerical values.

Transit Mode (Transit_mode):

This categorical variable identifies the mode of transit where the delay occurred: subway, streetcar, or bus. The transit mode is directly logged as part of the operational data.

Time (Time):

The timestamp of the delay event, recorded in the dataset in the format HH:MM:SS, specifies when the delay occurred. This variable is used to analyze temporal patterns in delays, such as peak hours.

Day (Day):

The day of the week when the delay occurred (e.g., Monday, Tuesday). This variable is derived from the event date and shows the potential weekly trends in delays.

Line (Line):

This variable represents the transit line or route affected by the delay (e.g., Line 1 for subway or specific route numbers for buses and streetcars). It is directly recorded in the TTC's data logs.

Location (Location):

The specific location or station where the delay occurred is captured in this variable, allowing for spatial analysis of delay patterns.

Incident (Incident):

This categorical variable captures the cause of the delay, such as "Mechanical," "Security," or "Collision." Raw incident descriptions are grouped into broader categories to ensure consistency across all transit modes.

2.3 Data Cleaning

The transit delay data was prepared for analysis by processing raw datasets for buses, street-cars, and subways. Key steps included:

1. Adding a Transit Mode Column:

A Transit_mode column was added to differentiate between buses, streetcars, and subways.

2. Removing Unnecessary Columns:

Irrelevant columns were dropped, retaining only essential variables like Date, Time, Day, Transit_mode, Line, Location, Incident, and Min Delay.

3. Standardizing Subway Line Identifiers:

Subway lines were recoded into consistent and meaningful labels (e.g., "YU" to "1").

4. Harmonizing Column Names and Categories:

Column names were standardized across datasets, and incidents were grouped under broader categories (e.g., "Collision - TTC Involved" and "Collision - TTC" were both labeled "Collision").

5. Handling Missing and Mismatched Values:

Invalid or missing data (e.g., incidents labeled "999") were reassigned as "N/A".

6. Combining Datasets:

All three datasets were merged into a unified dataset with consistent structure and formatting.

Subsampling the Dataset:

To address computational challenges with the original dataset of over 80,000 observations, a random subsample of 0.1% was selected. This ensured the subset remained representative of delay events across transit modes while enabling efficient model fitting without compromising analytical integrity.

Histogram of Delays:

Figure 1 Delays were capped at 45 minutes to focus on typical delay durations and exclude extreme values that could distort the distribution.

Boxplot of Delays:

Figure 2 The interquartile range (IQR) method was used to remove extreme outliers. The upper bound was calculated as 1.5 * IQR above the third quartile, ensuring accurate representation of central tendencies and variability in delay durations.

2.4 Outcome variables

2.4.1 Delay Duration (Min Delay)

The main outcome variable in this study is the duration of delays, measured in minutes (Min Delay). This variable quantifies the length of each delay experienced by the TTC's different transit modes (subway, streetcar, bus). The primary goal of the analysis is to determine which transit mode exhibits the highest average delay duration.

To visualize the distribution of delay durations, a histogram Figure 1 is presented to provide an overview of the range of delay durations for each transit mode. This figure highlights the spread and common ranges of delay times, allowing us to observe whether there are particularly long delays for any given mode.

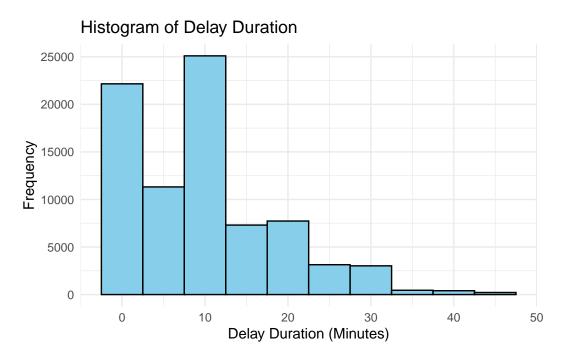


Figure 1: Histogram of Delay Duration (Filtered at 45 Minutes)

Lastly, Figure 2 displays a boxplot comparing delay durations across subway, streetcar, and bus transit modes. The boxplot shows the median delays and potential outliers that may suggest unusual or extreme events, while showing variability in delay times across the different transit modes.

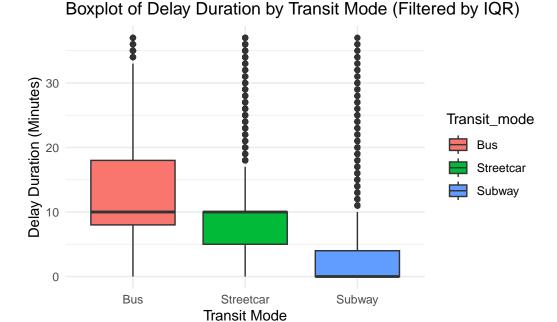


Figure 2: Boxplot of Delay Duration by Transit Mode (Filtered by IQR)

2.5 Predictor variables

2.5.1 Transit Mode

The main predictor variable is Transit Mode, which includes three categories: subway, street-car, and bus. Each transit mode has distinct characteristics, service areas, and operational challenges that may contribute to variations in delay duration.

To examine this relationship, Figure 3 plots the average delay duration for each transit mode. This chart helps us visualize which transit mode experiences the longest delays on average and illustrates the extent to which each mode contributes to the overall delay picture.

2.5.2 Time of Day

Another key predictor is Time of Day shown in Figure 4. This variable plays a crucial role in understanding how congestion and operational challenges fluctuate throughout the day. For example, delays might be more frequent and prolonged during peak hours (morning and evening rush periods) due to increased ridership.

Figure 4 shows a line plot representing average delay duration across different hours of the day. This visualization allows us to determine the times at which each transit mode is most

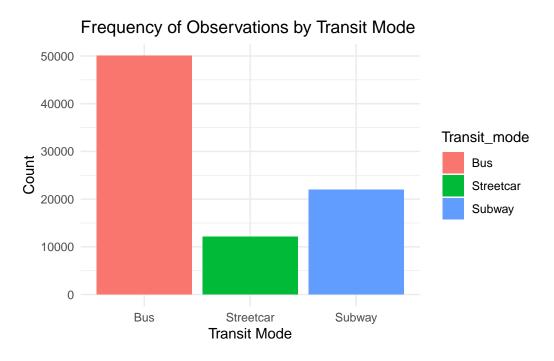


Figure 3: Average Delay Duration by Transit Mode

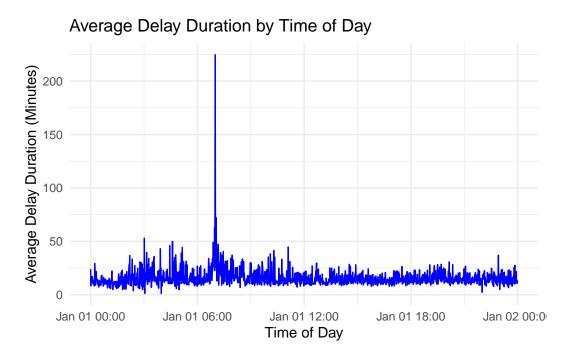


Figure 4: Average Delay by Time of Day

vulnerable to delays, and whether certain transit modes are disproportionately affected during rush hours.

2.5.3 Day of the Week

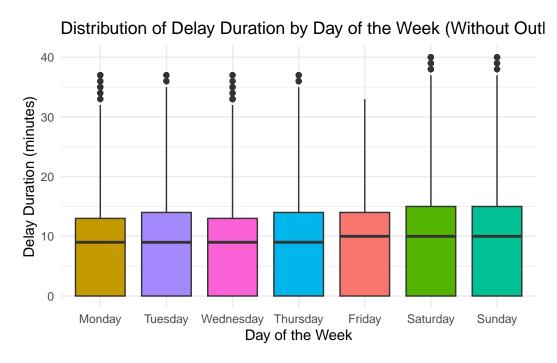


Figure 5: Delay Duration by Day of the Week

The Day of the Week is an important predictor that may influence delays due to varying passenger demand and maintenance schedules. Weekdays typically see different patterns compared to weekends, with increased rider demand and therefore greater potential for delays.

Figure 5 is a bar chart that shows the average delay duration for each day of the week, grouped by transit mode. It allows us to observe any weekly trends in delay duration, such as consistent weekday delays or weekend-specific issues. The predictor variables, including transit mode, time of day, and day of the week, help in assessing the specific conditions that affect the delay duration.

3 Model

The Bayesian linear regression model is designed to predict the duration of delays experienced by different TTC transit modes (subway, streetcar, or bus) based on various predictors such as transit mode, time of day, and day of the week. Background details and diagnostics are included in Appendix B.

3.1 Model Set-Up

Define y_i as the duration of the delay (in minutes) for a given transit event. Let β_i represent the transit mode (subway, streetcar, or bus), and γ_i and δ_i represent the time and day of the week, respectively. The generative model is specified as follows:

$$y_i|\mu_i, \sigma \sim \text{Normal}(\mu_i, \sigma)$$
 (1)

$$\mu_i = \alpha + \beta_i + \gamma_i + \delta_i \tag{2}$$

$$\alpha \sim \text{Normal}(0, 2.5)$$
 (3)

$$\beta \sim \text{Normal}(0, 2.5)$$
 (4)

$$\gamma \sim \text{Normal}(0, 2.5)$$
 (5)

$$\delta \sim \text{Normal}(0, 2.5)$$
 (6)

$$\sigma \sim \text{Exponential}(1)$$
 (7)

Here:

- y_i is the observed delay duration for a specific transit event.
- α is the baseline average delay duration.
- β_i captures the effect of transit mode (subway, streetcar, or bus).
- γ_i captures the effect of the time of day (e.g., peak or off-peak).
- δ_i captures the effect of the day of the week.
- σ is the standard deviation, representing unexplained variability in y_i .

This Bayesian model allows for estimating the contributions of transit mode, time, and day to delay duration while accounting for uncertainty in the data.

We run the model in R (R Core Team 2023) using the rstanarm package of Goodrich et al. (2022). We use the default priors from rstanarm.

3.2 Model justification

The specified Bayesian generative model is designed to estimate how transit mode, time of day, and day of the week contribute to delay duration. The choice of a Bayesian framework allows for explicit modeling of uncertainty in the data and provides posterior distributions for all parameters of interest.

We expect that the transit mode (β_i) will have a significant impact on the delay duration (y_i) , as different modes (subway, streetcar, and bus) are subject to varying operational challenges and infrastructure constraints. For instance, subways might experience shorter average delays due to dedicated tracks, while buses could encounter higher delays due to traffic congestion.

The time of day (γ_i) is hypothesized to influence delays, with peak hours (e.g., rush hour) likely contributing to longer delays due to higher demand and operational stress. Similarly, the day of the week (δ_i) may play a role, as weekends often have reduced schedules and lower demand, potentially resulting in shorter delays.

The prior distributions for the parameters are chosen to be weakly informative, specifically Normal(0, 2.5) for α , β , γ , and δ , reflecting prior beliefs that the effects are likely centered around zero but allowing for moderate deviations. The standard deviation (σ) is modeled with an Exponential(1) prior, which is appropriate for ensuring positive values while discouraging overly large variances in unexplained delay durations.

3.3 Assumptions and Limitations

The analysis assumes a linear relationship between delay duration and predictors (transit mode, time of day, and day of the week), with delays following a normal distribution. Independence between delay events is assumed, though real-world factors like cascading delays might violate this. While the IQR method removes extreme outliers, it may exclude rare but valid events, and the model explains only a small fraction of the variation ($R^2 = 0.033$), leaving many influencing factors unaccounted for. Simplistic representations of time and day fail to capture interactions or seasonal trends, and external influences like weather or events are not considered. Additionally, data inconsistencies, such as missing values and aggregated incident types, may impact accuracy, limiting the model's ability to fully explain delay dynamics.

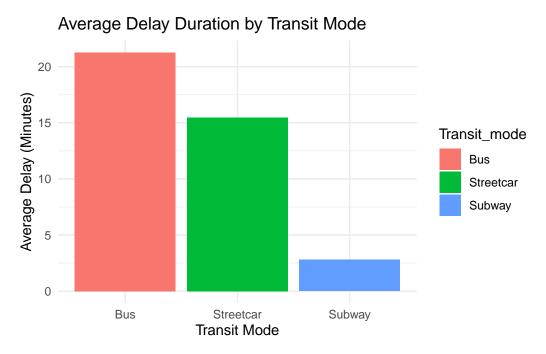
4 Results

These statistics indicate that, on average, TTC streetcars experienced a delay duration of approximately 15.7 minutes, while buses faced an average delay of 12.5 minutes, and subways experienced the shortest average delay of 9.3 minutes. The total number of delay events recorded was 50,039 for buses, 21,979 for subways, and 12,107 for streetcars.

Table 2: Summary statistics of delay events for TTC transit modes

Transit_mode	Average Delay Duration (Minutes)	Count of Events
Bus	21.273167	50039
Subway	2.806497	21979
Streetcar	15.476666	12107

The bar plot visually confirms that streetcars have consistently higher average delay durations compared to buses and subways, aligning with the numerical analysis."



The model summary Table 3 results indicate that, compared to buses, streetcars and subways have shorter delays, with average reductions of 6.8 and 18.8 minutes, respectively. Delays also vary by day of the week, with Tuesday and Wednesday showing slightly shorter delays. However, the model explains only about 3% of the variance in delay duration ($R^2 = 0.033$),

and the RMSE of 44.88 suggests moderate prediction error. While transit mode and day of the week have some influence, much of the variation in delay duration remains unexplained.

The Figure 6 graph indicates that bus face the highest average delay durations, peaking at over 40 minutes in the early morning. Streetcar have moderate delays throughout the day, averaging between 15-20 minutes, while subways consistently experience the lowest delays, averaging below 10 minutes. The significant early morning peak for streetcars suggests this as a important period for intervention to reduce delays.

Predicted Average Delay Duration by Time of Day for Each Transit Mode Bus Streetcar Subway Time of Day (Hourly)

Figure 6: Predicting average delay duration by time of day for each transit mode

5 Discussion

5.1 Understanding Transit Delays Across Modes

This study aimed to analyze delay durations across Toronto Transit Commission (TTC) transit modes—subway, streetcar, and bus—using data from 2024. The findings shows significant differences in delay patterns, with buses experiencing the longest delays on average, followed by streetcars and subways. These results align with expectations given the shared-road infrastructure of buses and streetcars, which are more exposed to traffic congestion and external disruptions. Subways, operating on dedicated tracks, exhibit fewer and shorter delays, highlighting the reliability of grade-separated transit systems.

Table 3: Summary of model results evaluating the relationship between transit mode and delay duration

model1 (Intercept) 25.051 Transit_modeStreetcar -6.799 Transit_modeSubway -18.849 Time 0.000
$ \begin{array}{lll} {\rm Transit_modeStreetcar} & -6.799 \\ {\rm Transit_modeSubway} & -18.849 \\ {\rm Time} & 0.000 \\ \end{array} $
-18.849 Time 0.000
Time 0.000
TO 1.6 1
DayMonday -1.043
DaySaturday -2.160
DaySunday -1.572
DayThursday -1.161
DayTuesday -3.601
DayWednesday -3.712
Num.Obs. 8412
R2 0.033
R2 Adj. 0.031
Log.Lik43 938.441
ELPD -43986.3
ELPD s.e. 622.5
LOOIC 87 972.7
LOOIC s.e. 1245.0
WAIC 87 985.6
RMSE 44.88

5.2 Temporal and Weekly Patterns in Delays

The analysis also identified temporal trends, with peak hours contributing to longer delays across all modes. This reflects the increased strain on transit networks during rush hours, driven by higher ridership and demand. Additionally, delays were shorter on weekends, likely due to reduced traffic congestion and lower service frequency. These findings underscore the importance of tailoring operational strategies to address the unique challenges of peak periods.

5.3 Computational Constraints and Data Subsampling

While the original dataset contained over 80,000 observations, computational challenges necessitated subsampling to 0.1% of the data. This approach ensured the analysis remained feasible without compromising the representativeness of the sample. However, this step may have excluded rare but significant delay events, potentially influencing the findings.

5.4 Implications for Policy and Practice

These findings have practical applications for transit planning and policy. Prioritizing interventions for buses and streetcars, such as dedicated lanes or signal prioritization, could reduce delays significantly. Understanding temporal patterns can guide resource allocation, with more staff and services deployed during peak hours. For daily TTC users, understanding delay variability can assist in better trip planning and mode selection.

5.5 Limitations and next steps

The model accounted for only 3% of the variation in delay durations ($R^2 = 0.033$), suggesting the need to include additional factors, such as weather, traffic incidents, and ridership levels, in future analyses. Temporal variables were modeled independently, without exploring interactions or seasonal effects, which could further refine the results. Missing or inconsistent entries (e.g., incidents labeled as "N/A") may have introduced biases, while the IQR method for outlier removal may have excluded rare but valid delay events. Additionally, future research should expand on these findings by incorporating additional predictors, such as weather and traffic data, to enhance model accuracy. Analyzing interactions between variables, such as the combined effects of time of day and transit mode, is also suggested. Lastly, exploring the impact of infrastructure improvements, such as bus rapid transit (BRT) or streetcar signal prioritization, on delay reduction is recommended.

A Appendix

A.1 Ideal Methodology and Survey for Research on TTC Transit Delays

What is the population, frame, and sample?

- Population: All TTC transit riders who use subway, streetcar, and bus services in Toronto.
- Frame: Riders present at various TTC stations, stops, and on-board vehicles over a representative period of time. This can be supplemented by targeting potential riders through online platforms like TTC's website and social media pages.
- Sample: A subset of TTC riders across the three transit modes. A stratified random sampling approach will be used to ensure proportional representation from subway, streetcar, and bus users.

How is the sample recruited?

• The sample will be recruited through a mix of in-person surveys at key subway stations, streetcar stops, and bus terminals, as well as online surveys promoted through TTC's social media and website. Participation incentives, such as small prizes or free ride vouchers, will be used to encourage greater engagement.

Sampling Approach:

• Stratified Random Sampling: The entire population is divided into three strata based on the type of transit mode used (subway, streetcar, bus). Random samples from each stratum will be selected to ensure proportional representation.

Trade-offs:

- Pros: Ensures that each transit mode is adequately represented in the sample, providing a more accurate comparison across the modes.
- Cons: This approach can be more complex and costly compared to simple random sampling, and may require more logistical effort to ensure each stratum is properly represented.

How is non-response handled?

• To minimize non-response, participants will receive multiple invitations to participate through different means (e.g., follow-up emails for online surveys, re-approaching riders in-person). Statistical weighting will also be used to adjust for non-response based on known demographics.

What is good and bad about the questionnaire?

• The survey will be pilot tested with a small group of riders to identify any ambiguous or confusing questions. Pilot testing is essential for improving survey quality, as it allows researchers to identify and address issues before full deployment (Centra et al. 1991). Questions are designed to be clear, conversational, and relevant to respondents. Cognitive load will be considered by varying question types and placing simpler questions before more complex ones. Multiple-choice questions will be used for questions with specific options, while open-ended questions will be used to collect detailed feedback. In addition, the survey introduction will specify the purpose, contact information of the surveyors, the confidentiality of responses, and ethics clearance if applicable.

A.2 Idealed Survey

A sample survey can be found at : https://forms.gle/WLkXZTSNGCxoMQdM7 The survey questions demo are also detailed below.

A.3 Survey Demo

Background and Purpose of the Study

- I am conducting a survey to understand transit delays on the TTC.

Participation Involvement

- You are being asked to complete a short survey about your experiences with TTC transit delays. This survey will take approximately 5-7 minutes to complete.

Confidentiality

- Your responses will be completely confidential. No individual respondent will be identifiable in any of the published data, and all identifying information will be removed before analysis.

Usage of Data

- The results of this survey will be summarized and may be used in reports or presentations to better understand and address transit delays. If you consent, your data will be included in the calculation of summary statistics for this research.

Withdrawal from the Study

- Participation is completely voluntary. You may choose to withdraw your data from the study up to one month after completing the survey, by notifying the researchers via email.

Contact Information

- If you have questions or concerns, please contact the research team at claire.chang@mail.utoronto.ca

Informed Consent

- 1. After carefully reading the informed consent document, please indicate below whether you consent to have your anonymized responses included in the research study?
- Yes, I consent to have my anonymized survey responses used in this research.
- No, I do not want to have my data used, but I would still like to complete the survey.

2.	Demographics
•	Which age group do you belong to?
	- Under 18
	-18-29
	-30-49
	-50-64
	-65+
•	How often do you use the TTC?
	- Daily
	- 2-3 times a week
	- Once a week
	- Rarely
3.	Transit Use
	Transit Use Which transit mode do you primarily use?
	Which transit mode do you primarily use?
	Which transit mode do you primarily use? — Subway
•	Which transit mode do you primarily use? - Subway - Streetcar
•	Which transit mode do you primarily use? - Subway - Streetcar - Bus
•	Which transit mode do you primarily use? - Subway - Streetcar - Bus Do you use multiple transit modes during a single trip?
•	Which transit mode do you primarily use? - Subway - Streetcar - Bus Do you use multiple transit modes during a single trip? - Yes
•	Which transit mode do you primarily use? - Subway - Streetcar - Bus Do you use multiple transit modes during a single trip? - Yes - No
•	Which transit mode do you primarily use? - Subway - Streetcar - Bus Do you use multiple transit modes during a single trip? - Yes - No Delay Experience

- Frequently (at least once a week)
- Occasionally (at least once a month)
- Rarely (less than once a month)
- Never
- Which transit mode do you experience the most delays with?
 - Subway
 - Streetcar
 - Bus
- Please estimate the average delay time for the mode you experience the most delays with:
 - Less than 5 minutes
 - 5-10 minutes
 - -10-20 minutes
 - More than 20 minutes

5. Impact of Delays

- How do delays impact your travel plans? (Select all that apply)
 - Late arrival to work/school
 - Missed appointments
 - Increased stress
 - No major impact
- How do you usually cope with delays? (Select all that apply)
 - Wait patiently
 - Switch to another mode (e.g., take a bus if the subway is delayed)
 - Cancel or change plans
 - Other (please specify)

6. Open-Ended Question

• Please describe any specific issues you have faced with delays on TTC transit modes. How do these experiences affect your perception of TTC services?

7. Suggestions

 What changes would you suggest to reduce delays or improve your experience with TTC services?

8. Conclusion

• Thank you for your time and valuable feedback! Your responses will contribute to a better understanding of TTC transit delays and how to address them effectively.

B Model details

B.1 Diagnostics

Figure 7a is a trace Plot that shows the sampled values of each parameter from the MCMC chains over the iterations. Ideally, these chains should overlap well and display consistent movement across the same regions. From the trace plot, the chains for all parameters appear to have converged and are overlapping consistently, without any clear trends or divergences. This indicates good mixing and suggests the MCMC algorithm has likely converged.

Figure 7b is a Rhat Plot that shows the potential scale reduction factor, which assesses convergence of the MCMC chains. The Rhat statistic should be close to 1, typically below 1.1, to indicate good convergence. In this plot, the Rhat values for all parameters are very close to 1, suggesting the MCMC algorithm has converged, and there is no significant difference between the variances within and across chains.

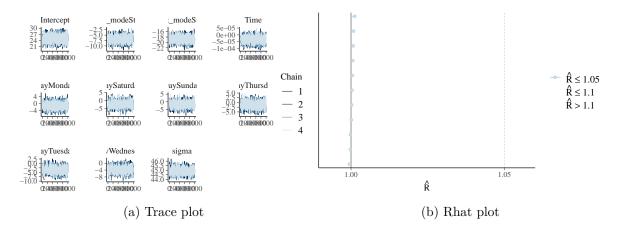


Figure 7: Checking the convergence of the MCMC algorithm

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