Group 6 0201 作业:

Problem1: 首先做第二阶段(t+1期)优化

$$\mathcal{L}=\frac{C_{t+1}^{1-\eta}-1}{1-\eta}+\chi\frac{[(1-\xi)b_{t+1}]^{1-\eta}-1}{1-\eta}-\lambda[C_{t+1}+b_{t+1}-(1+r_{t+1})S_t]$$

$$\begin{cases}\frac{\partial\mathcal{L}}{\partial C_{t+1}}=C_{t+1}^{-\eta}-\lambda=0\\ \frac{\partial\mathcal{L}}{\partial b_{t+1}}=\chi(1-\xi)^{1-\eta}b_{t+1}^{-\eta}-\lambda=0\end{cases}\Rightarrow 联立解得: \begin{cases}b_{t+1}=\frac{S_t}{\chi^{-\frac{1}{\eta}}(1-\xi)^{\frac{\eta-1}{\eta}}+1}\\ C_{t+1}=\frac{\chi^{-\frac{1}{\eta}}(1-\xi)^{\frac{\eta-1}{\eta}}}{\chi^{-\frac{1}{\eta}}(1-\xi)^{\frac{\eta-1}{\eta}}+1}S_t\end{cases}$$

再做第一阶段优化: 令 $\alpha=\chi^{-\frac{1}{\eta}}(1-\xi)^{\frac{\eta-1}{\eta}}$ $A=w_tl_t+(1-\xi)b_t+g_t$

$$\mathcal{L}(C_t,S_t)=\frac{C_t^{1-\eta}-1}{1-\eta}+\beta\left[\frac{\left(\frac{\alpha S_t}{\alpha+1}\right)^{1-\eta}-1}{1-\eta}+\chi\frac{\left[(1-\xi)\frac{S_t}{\alpha+1}\right]^{1-\eta}-1}{1-\eta}\right]-\lambda(C_t+S_t-A)$$

$$\begin{cases}\frac{\partial\mathcal{L}}{\partial C_t}=C_t^{-\eta}-\lambda=0\\ \frac{\partial\mathcal{L}}{\partial S_t}=\beta\left(\frac{\alpha}{\alpha+1}\right)^{1-\eta}S_t^{-\eta}+\beta\chi\left(\frac{1-\xi}{\alpha+1}\right)^{1-\eta}S_t^{-\eta}-\lambda=0\end{cases}$$

联立解得 $$S_t=\frac{w_tl_t+(1-\xi)b_t+g_t}{\left[\beta\left(\frac{\alpha}{\alpha+1}\right)^{1-\eta}+\beta\chi\left(\frac{1-\xi}{\alpha+1}\right)^{1-\eta}\right]^{-\frac{1}{\eta}}+1}$$

$$C_t=\left[\beta\left(\frac{\alpha}{\alpha+1}\right)^{1-\eta}+\beta\chi\left(\frac{1-\xi}{\alpha+1}\right)^{1-\eta}\right]^{-\frac{1}{\eta}}S_t$$

$$C_{t+1}=\frac{\chi^{-\frac{1}{\eta}}(1-\xi)^{\frac{\eta-1}{\eta}}}{\chi^{-\frac{1}{\eta}}(1-\xi)^{\frac{\eta-1}{\eta}}+1}S_t$$

$$b_{t+1}=\frac{S_t}{\chi^{-\frac{1}{\eta}}(1-\xi)^{\frac{\eta-1}{\eta}}+1}$$

令 $\mu=\left[\beta\left(\frac{\alpha}{\alpha+1}\right)^{1-\eta}+\beta\chi\left(\frac{1-\xi}{\alpha+1}\right)^{1-\eta}\right]^{-\frac{1}{\eta}}$

$S_t=\frac{A}{\mu+1}$ $b_{t+1}=\frac{S_t}{\alpha+1}$

$\therefore S_t=\frac{1}{\mu+1}w_tl_{it}+\frac{1-\xi}{\mu+1}b_t+\frac{g_t}{\mu+1}$

$g_t=\frac{1}{n}\xi\sum\frac{S_{t-1}}{\alpha+1}=\frac{\xi}{\alpha+1}k_t$

$\therefore S_t=\frac{1}{\mu+1}w_tl_{it}+\frac{1-\xi}{\mu+1}b_t+\frac{\xi}{(\alpha+1)(\mu+1)}k_t$

$=\frac{1}{\mu+1}w_tl_{it}+\frac{1-\xi}{\mu+1}\frac{S_{t-1}}{\alpha+1}+\frac{\xi}{(\alpha+1)(\mu+1)}k_t$

Problem 3:

$$\begin{cases}A\upsilon k_t^{\upsilon-1}=r_t+\delta\\ A(1-\upsilon)k_t^{\upsilon}=w_t\end{cases}$$

Problem 4: $K_{t+1} = \frac{k_{t+1}}{L_{t+1}} = \frac{\int s_t\, di}{\int l_{t+1}\, di} = \int s_t\, di = K_{t+1}$

同理 $k_t = K_t$

$\therefore k_{t+1} = \dfrac{\frac{1}{u+1}\left(\sum w_t l_{it} + \sum \frac{s_{t-1}}{\alpha+1}\right)/n}{\sum l_{it+1}/n}$

$= \frac{1}{u+1}\left(w_t \frac{\sum l_{it}/n}{\sum l_{it+1}/n} + \frac{1}{\alpha+1} \frac{\sum s_{t-1}/n}{\sum l_{it+1}/n}\right)$

$= \frac{1}{u+1}\left(w_t \frac{\sum l_{it}/n}{\sum l_{it+1}/n} + \frac{1}{\alpha+1} \frac{k_t \sum l_{it}/n}{\sum l_{it+1}/n}\right)$

由大数定律

$k_{t+1} = \frac{1}{u+1} w_t + \frac{1}{(u+1)(\alpha+1)} k_t$

Problem 5:

求 steady state: $k_{t+1} = \frac{1}{u+1} A(1-\nu) k_t^{\nu} + \frac{1}{(u+1)(\alpha+1)} k_t$

$\bar{k} = \sqrt[\nu-1]{\frac{u\alpha+u+\alpha}{A(\alpha+1)(1-\nu)}}$

$\bar{w} = A(1-\nu)\bar{k}^{\nu} = A(1-\nu)\left[\frac{u\alpha+u+\alpha}{A(\alpha+1)(1-\nu)}\right]^{\frac{\nu}{\nu-1}}$

$\bar{r} = A\nu \frac{u\alpha+u+\alpha}{A(\alpha+1)(1-\nu)} - \delta$

$\therefore a_{it+1} = \frac{1}{u+1}\bar{w}\, l_{it} + \frac{1-\tau}{u+1} \frac{a_{it}}{\alpha+1} + \frac{\tau}{(\alpha+1)(u+1)}\bar{k}$

Problem 6:

$a_{it+1} = c_3 l_{it} + c_4 a_{it} + a_5$

$t\to\infty$ 时 $a_{i\infty} = \frac{c_5}{1-c_4} + c_3 \sum_{s=1}^{+\infty} c_4^{s-1} l_{is}$

$E(a_{i\infty}) = \frac{c_3+c_5}{1-c_4} = \bar{k}$

$l_{it+1} = \bar{l} + v(l_{it} - \bar{l}) + \varepsilon_{it+1}$

$Var(a_{i\infty}) = c_3^2 \frac{1}{(1-c_4^2)(1-v^2)} \frac{1+c_4 v}{1-c_4 v}\sigma^2$

$E(a_{i\infty}) = \bar{k}$

有税时 $c_4' = \frac{c_4}{1-\tau}$

$\therefore c_4' < c_4 \quad \therefore Var' < Var$

CV有税 < CV无税 说明税收 decrease inequality.