Homework-Inequality

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1 Estate taxes and wealth inequality

Our model is an overlapping generations heterogeneous agents economy. Each agent lives for two periods: the young period and the old period. Each old agent gives birth to one child. Each family consists of one parent and one child. There are a continuum of measure 1 families and a government in the economy. The population of the economy keeps constant.

Young agents work and earn labor earnings. Old agents do not have labor income. They consume their savings and leave bequests to their children. Each young agent supplies 1 unit of labor inelastically. But young agents have idiosyncratic labor efficiency risk l_t. We assume

Assumption 1: $\{l_t\}$ is stationary and ergodic.

Assumption 2: $l_t > 0$ has a finite mean. Without loss of generality,

$$E(l_t) = 1.$$

Agents have "joy-of-giving" bequest motives. c_t^y is the consumption in the young period of an agent born at period t, and c_{t+1}^o is his consumption in the old period. He leaves the bequest b_{t+1} to his child at the end of the second priod of his life. s_t denotes his savings. The wage rate per efficiency unit is w_t. g_t is the lump-sum transfer from the government, and $\zeta \in [0, 1)$ is the estate tax rate. The interest rate is r_{t+1}.

1.1 The individual's problem

In period t, an young agent receives bequests b_t, and pays the estate tax ζb_t to the government. He draws his labor efficiency l_t, and receives the government transfer g_t. Then the agent makes consumption and savings decisions. Thus the agent's problem is a deterministic optimization problem

$$\max_{c_t^y, s_t, c_{t+1}^o, b_{t+1}} \frac{(c_t^y)^{1-\eta} - 1}{1-\eta} + \beta \left[\frac{(c_{t+1}^o)^{1-\eta} - 1}{1-\eta} + \chi \frac{\left[(1-\zeta)\, b_{t+1}\right]^{1-\eta} - 1}{1-\eta} \right] \tag{1}$$

$$\begin{aligned} s.t. \quad c_t^y + s_t &= w_t l_t + (1-\zeta)\, b_t + g_t \\ c_{t+1}^o + b_{t+1} &= (1 + r_{t+1}) s_t \end{aligned}$$

where $\eta \geq 1$ is the coefficient of relative risk aversion. $\beta \in (0,1)$ is the time discount factor. χ represents the bequest motive.

Problem 1. Solve the agent's policy functions.

Problem 2. Derive the relationship between s_t and s_{t-1}.

1.2 The firm's problem

There is an aggregate production firm in the economy. The firm has a Cobb-Douglas production function $Y_t = AK_t^\alpha L_t^{1-\alpha}$, where A is the technology level, K_t is the capital, and L_t is the labor. The firm chooses K_t and L_t to maximize its profits

$$\max_{K_t, L_t} \{AK_t^\alpha L_t^{1-\alpha} - w_t L_t - (r_t + \delta)K_t\}$$

where δ is the depreciation rate of capital.

Problem 3. Write down the first order conditions of the firm's problem.

1.3 The government

The government collects the estate tax revenue and gives a lump-sum transfer to the young generation. Each young agent receives the same subsidy g_t. The government has a balanced budget in every period

$$g_t = \zeta \int b_t di \tag{2}$$

where $\int di$ means the aggregation of young agents.

1.4 The general equilibrium

The aggregate population of young agents who are the workers in the economy is 1, and $E(l_t) = 1$. Thus the labor market clearing condition is

$$L_t = \int l_t di = 1 \tag{3}$$

where $\int di$ means the aggregation of young population. The capital market clearing condition is

$$K_{t+1} = \int s_t di \tag{4}$$

where $\int di$ means the aggregation of young agents.

Problem 4. Derive the relationship between K_{t+1} and K_t.

1.5 The wealth distribution

We investigate the stationary distribution of individual wealth accumulation process in the steady-state aggregate economy. Following Bossmann et al. (2007) we use a_{t+1} to denote the individual wealth (before interest) in period

$t+1$. Thus $a_{t+1} = s_t$. The agent's wealth accumulation equation in the steady-state aggregate economy, is

$$a_{t+1} = c_3 l_t + c_4 a_t + c_5. \tag{5}$$

Problem 5. Find the expressions of c_3, c_4, and c_5.

1.6 Estate taxes and wealth inequality

When investigating the impacts of estate taxes on wealth distribution, we concentrate on the logarithmic utility as in Bossmann et al. (2007).

Assumption 3: Utility functions are logarithmic.

The estate tax does not affect aggregate capital. Thus it does not influence the interest rate and the wage rate.

We also assume that labor efficiency is correlated across generations

Assumption 4: The labor efficiency process follows

$$l_{t+1} = \bar{l} + v\left(l_t - \bar{l}\right) + \varepsilon_{t+1}$$

with $\bar{l} = 1$, $E\varepsilon_{t+1} = 0$, and $Var(\varepsilon_{t+1}) = \sigma^2$.

We use the coefficient of variation as the inequality measure. For a random variable X with a finite variance

$$CV(X) = \frac{\sqrt{Var(X)}}{E(X)}.$$

Problem 6. Show that estate taxes reduce wealth inequality in the long run.