- **Problem 1: policy function**

$$\mathcal{L} = \frac{(c_t^y)^{1-\eta}-1}{1-\eta} + \beta \left[\frac{{c_{t+1}^o}^{1-\eta}-1}{1-\eta} + \chi \frac{[(1-\xi)b_{t+1}]^{1-\eta}-1}{1-\eta} \right]$$

$$+\lambda_t (w_t l_t + (1-\xi)b_t + g_t - c_t^y - s_t) + \mu_t [(1+r_{t+1}) s_t - c_{t+1}^o - b_{t+1}]$$

$$\frac{\partial \mathcal{L}}{\partial c_t^y} = 0 \Rightarrow c_t^y = (\lambda_t)^{-\frac{1}{\eta}}$$

$$\frac{\partial \mathcal{L}}{\partial c_{t+1}^o} = 0 \Rightarrow c_{t+1}^o = \left(\frac{\mu_t}{\beta}\right)^{-\frac{1}{\eta}}$$

$$\frac{\partial \mathcal{L}}{\partial b_{t+1}} = 0 \Rightarrow b_{t+1} = \left(\frac{\mu_t}{\beta \chi}\right)^{-\frac{1}{\eta}}$$

$$\frac{\partial \mathcal{L}}{\partial s_t} = 0 \Rightarrow 1 + r_{t+1} = \frac{\lambda_t}{\mu_t}$$

· Combined with Budget Constraints, the Policy Functions are as followings:

① $$c_t^y = \frac{(w_t l_t + (1-\xi) b_t + g_t)(1+r_{t+1})}{(1+r_{t+1}) + (1+\chi^{\frac{1}{\eta}})[\beta(1+r_{t+1})]^{\frac{1}{\eta}}}$$

② $$s_t = \frac{(w_t l_t + (1-\xi) b_t + g_t)(1+\chi^{\frac{1}{\eta}})}{(1+r_{t+1})[\beta(1+r_{t+1})]^{-\frac{1}{\eta}} + (1+\chi^{\frac{1}{\eta}})}$$

③ $$c_{t+1}^o = \frac{s_t(1+r_{t+1})}{1+\chi^{\frac{1}{\eta}}}$$ ④ $$b_{t+1} = \frac{s_t(1+r_{t+1})}{1+\chi^{-\frac{1}{\eta}}}$$

- **Problem 2:**

$$c_t^y + s_t = w_t l_t + (1-\xi) b_t + g_t$$

$$c_{t+1}^o + b_{t+1} = (1+r_{t+1}) s_t \Rightarrow c_t^y + b_t = (1+r_t) s_{t-1}$$

$$> s_t + (1+r_t) s_{t-1} = w_t l_t + (2-\xi) b_t + g_t$$

$$s_t = w_t l_t + g_t + (1+r_t) \frac{\chi^{\frac{1}{\eta}}(1-\xi)-1}{\chi^{\frac{1}{\eta}}+1} \cdot s_{t-1}$$

- **Problem 3:**

$$\max_{k_t, L_t} \{ A k_t^{\alpha} L_t^{1-\alpha} - w_t l_t - (r_t + \delta) k_t \}.$$

① $A \alpha k_t^{\alpha-1} L_t^{1-\alpha} = r_t + \delta.$ $\xrightarrow{k_t = \frac{K_t}{L_t}}$ ① $A \alpha k_t^{\alpha-1} = r_t + \delta.$

② $A(1-\alpha) k_t^{\alpha} L_t^{-\alpha} = w_t$ ② $A(1-\alpha) k_t^{\alpha} = w_t$

- **Problem 4:**

Based on the result of Problem 2:

$$\int s_t di = \int w_t l_t di + \int g_t di + \int (1+r_t) \frac{\chi^{\frac{1}{\eta}}(1-\xi)-1}{\chi^{\frac{1}{\eta}}+1} \cdot s_{t-1} di.$$

Combined with results from Problem 3

Therefore,

$$k_{t+1} = A\left(1 - \frac{2\alpha}{\chi^{\frac{1}{\eta}}+1}\right) k_t^{\alpha} + (1-\alpha) \frac{\chi^{\frac{1}{\eta}}-1}{\chi^{\frac{1}{\eta}}+1} k_t \quad C_1 = A\left(1 - \frac{2\alpha}{\chi^{\frac{1}{\eta}}+1}\right) \quad C_2 = (1-\alpha)\frac{\chi^{\frac{1}{\eta}}-1}{\chi^{\frac{1}{\eta}}+1}$$

- Problem 5

Based on the results from Problem 2. and the steady state calculation.

$$\bar{k} = \left(\frac{c_1}{1-c_2}\right)^{\frac{1}{1-\alpha}} \quad \bar{r} = A\alpha\bar{k}^{\alpha-1} - \delta \quad \bar{w} = A(1-\alpha)\bar{k}^{\alpha}$$

Therefore : $a_{t+1} = c_3 l_t + c_4 a_t + c_5$.

$$c_3 = \bar{w} \quad c_4 = (1+\bar{r})\frac{\chi^{\frac{1}{\sigma}}(1-\xi)-1}{\chi^{\frac{1}{\sigma}}+1} \quad c_5 = \xi(1+\bar{r})\frac{\chi^{\frac{1}{\sigma}}(1-\xi)-1}{\chi^{\frac{1}{\sigma}}+1}\bar{k}$$

- Problem 6.

In long run. $E(a_\infty) = \bar{k}$

$$Var(a_\infty) = c_3^2 \frac{1}{(1-c_4^2)(1-v^2)} \cdot \frac{1+c_4 v}{1-c_4 v}\sigma^2$$

$$Var'(a_\infty) = \frac{c_3^2\sigma^2}{1-v^2}\frac{v(1-c_4^2)(1-v^2)-(1+c_4 v)(-v-2c_4-3vc_4^2)}{\left[(1-c_4^2)(1-c_4 v)\right]^2} > 0.$$

$\therefore$ $c_4 \uparrow$ $Var(a_\infty) \uparrow$

$$c_4^{nt} = (1+\bar{r})\frac{\chi^{\frac{1}{\sigma}}-1}{\chi^{\frac{1}{\sigma}}+1}$$

$$c_4^{wt} = (1+\bar{r})\frac{\chi^{\frac{1}{\sigma}}(1-\xi)-1}{\chi^{\frac{1}{\sigma}}+1}$$

$c_4^{wt} < c_4^{nt}$

$\therefore Var(a_\infty^{wt}) < Var(a_\infty^{nt})$

$\therefore CV(X^{wt}) < CV(X^{nt})$

taxes reduce inequality in long run