Prob1.

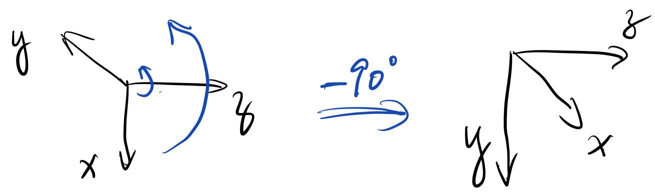
1. Moon's position in Frame Wall-e is ${}^{w}Moon = (6, 0, 4)$ (x y z)
Eve's position in Frame Wall-e is ${}^{w}Eve = (6, 10, 4)$

2. ${}^{m}T_{w}$: Wall-e's pose in the Moon's coordinate frame

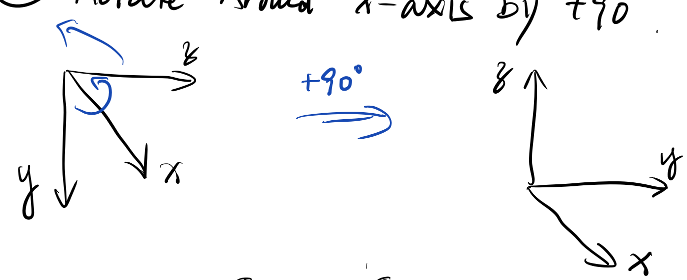
⇒ relative transform from Wall-e's to Moon's frame

⇒ Suppose Wall-e's coordinate is aligned w/ moon's coordinate, then to transform from frame Moon to the real Frame Wall-e takes:

① Rotate Around z-axis by $-90°$



② Rotate Around x-axis by $+90°$



sanity check: what about 先x再y? The same R?

$$R = R_y\left(\tfrac{\pi}{2}\right) R_x\left(-\tfrac{\pi}{2}\right)$$

$$= \begin{pmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} & 0 \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{-\pi}{2} & -\sin\frac{-\pi}{2} \\ 0 & \sin\frac{-\pi}{2} & \cos\frac{-\pi}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

③ Translate "rotated Moon Frame" to the real Wall-e frame, along the real moon-frame

(Wording is disastrous, will fix later).

$$d = {}^{m}\text{Wall-e} - {}^{w}\text{Wall-e}$$

$$= \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix}$$

④ Hence
$$ {}^{m}T_{w} = \begin{pmatrix} 0 & 0 & -1 & 4 \\ -1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Sanity Check:

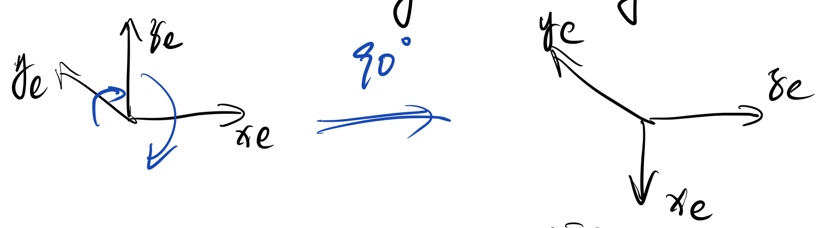
$$ {}^{m}\text{Wall-e} = {}^{m}T_{w} \, {}^{w}\text{Wall-e}$$

$$\begin{pmatrix} 6 \\ 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 6 \\ 0 & 0 & -1 & 4 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \checkmark$$

${}^{e}T_{m}$: Follow the same step as above:

① Rotation:

Rotate around y-axis by 90°



$$R = R_y\left(-\frac{\pi}{2}\right) = \begin{bmatrix} \cos(\frac{\pi}{2}) & 0 & \sin(\frac{\pi}{2}) \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{2} & 0 & \cos(\frac{\pi}{2}) \end{bmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

② Translation:

$$d = {}^{e}\text{Moon} - {}^{m}\text{moon} = \begin{pmatrix} -10 \\ 0 \\ 0 \end{pmatrix}$$

③ Pose $${}^{e}T_{m} = \begin{pmatrix} 0 & 0 & 1 & -10 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Sanity Check:

$${}^{e}\text{Moon} = {}^{e}T_{m} \, {}^{m}\text{Moon}$$

$$= \begin{pmatrix} 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -10 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

${}^{e}T_{w}$: ① Rotate around z-axis by $-90°$.

$$R = R_z(-\tfrac{\pi}{2}) = \begin{pmatrix} \cos -\frac{\pi}{2} & -\sin\frac{\pi}{2} & 0 \\ \sin -\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

② Translation $d = {}^{e}\text{Wall-e} - {}^{w}\text{Wall-e} = \begin{pmatrix} -10 \\ 6 \\ -4 \end{pmatrix}$

③ $${}^{e}T_{w} = \begin{pmatrix} 0 & 1 & 0 & -10 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. As mentioned above, ${}^{e}T_{w}$ represents translation from Eve's frame to Wall-e's frame, which is $-90°$ around z_e, which is $R = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Then the $\begin{pmatrix} -10 \\ 6 \\ -4 \end{pmatrix}$ represent translation from Eve's frame's position to Wall-e's frame's position, which is just ^{e}Wall-e.

4. ${}^{e}T_{m} \, {}^{m}T_{w}$

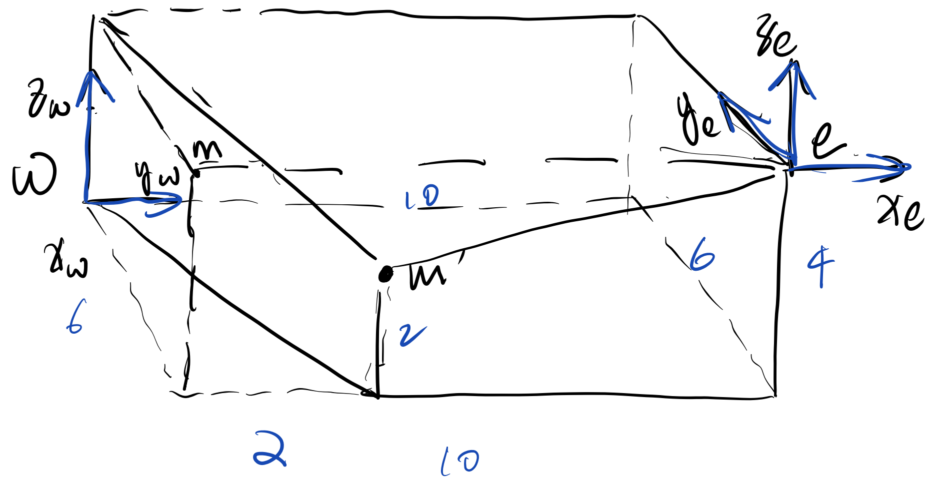
$$= \begin{pmatrix} 0 & 0 & 1 & -10 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 & 4 \\ -1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 & -10 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{pmatrix} = {}^{e}T_{m} \checkmark$$

5. $$({}^{w}T_{e})^{-1} = \begin{pmatrix} 0 & -1 & 0 & 6 \\ 1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 1 & 0 & -10 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= {}^{e}T_{w} \checkmark$$

6. ◯ Does moon's frame change?



${}^{e}T_{w}$ is derived exactly the same as in part 2, hence the result is the same. Changing the moon's frame does not have anything to do w/ Wall-e and Eve's Frame.

Moreover, as we have seen,

$$ {}^{e}T_{w} = {}^{e}T_{m} \, {}^{m}T_{w} = {}^{e}T_{m'} \, {}^{m'}T_{e}. $$

Both m (orginal moon) and m' (New moon) are canceled out. Hence ${}^{e}T_{w}$ doesn't change.