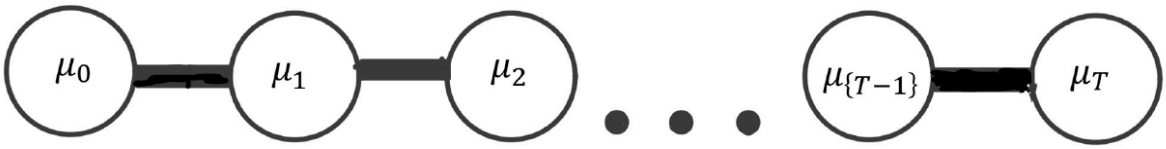
STA465 Homework 2

Ze Qian(Claire) Wu

1002192254

Question 1:

(1)



(2)

$$
\begin{aligned}
\because u_t &= au_{t-1} + \epsilon_t \\
&= exp(-\frac{1}{2}\frac{a^2u_1^2 + \sum_{t=2}^{T-1}(1+a^2)u_t^2 + u_n^2 - 2a\sum_{t=1}^{T-1}u_t u_{t+1}}{\sigma^2}) \\
&= a(au_{t-2} + \epsilon_{t-1}) + \epsilon_t \\
&= a(a(au_{t-3} + \epsilon_{t-2}) + \epsilon_{t-1}) + \epsilon_t \\
&\dots \\
&= \epsilon_t + a\epsilon_{t-1} + a^2\epsilon_{t-2} + \ldots + +a^{t-1}\epsilon_1 + a^i u_0
\end{aligned} \tag{1}
$$

$$
\therefore E(u_t) = 0, \quad t = 1, \ldots, T \tag{2}
$$

Then as

$$
\begin{aligned}
p(\mu_1, \mu_2, \mu_3 \ldots \mu_T) &\propto exp(-\frac{1}{2}\frac{\sum_{t=2}^{T}(u_t - au_{t-1})^2}{\sigma^2}) \\
&= exp(-\frac{1}{2}\frac{a^2u_1^2 + \sum_{t=2}^{T-1}(1+a^2)u_t^2 + u_n^2 - 2a\sum_{t=1}^{T-1}u_t u_{t+1}}{\sigma^2}) \\
&= exp(-\frac{1}{2}\{u_1, u_2, \ldots u_T\}Q^{-1}\{u_1, u_2, \ldots u_T\}^T)
\end{aligned} \tag{3}
$$

The precision matrix will be

$$\left\{\begin{array}{cccccccc} a^2/\sigma^2 & -a/\sigma^2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -a/\sigma^2 & (1+a^2)/\sigma^2 & -a/\sigma^2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -a/\sigma^2 & (1+a^2)/\sigma^2 & -a/\sigma^2 & \cdots & 0 & 0 & 0 \\ 0 & 0 & -a/\sigma^2 & (1+a^2)/\sigma^2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & (1+a^2)/\sigma^2 & -a/\sigma^2 & 0 \\ 0 & 0 & 0 & 0 & \cdots & -a/\sigma^2 & (1+a^2)/\sigma^2 & -a/\sigma^2 \\ 0 & 0 & 0 & 0 & \cdots & 0 & -a/\sigma^2 & 1/\sigma^2 \end{array}\right\} \quad (4)$$