Wireless Communication Signal Processing—Homework 5

Due on June 6

May 23, 2025

1. The general form of MIMO channel capacity is given by

$$C = \log_2[\det(\mathbf{I} + \mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\mathbf{R}_{nn}^{-1})].$$

   (a) (5%) Let $\mathbf{H}^H\mathbf{R}_{nn}^{-1}\mathbf{H} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H$, $\mathbf{\Lambda} = \text{diag}(\lambda_1,\ldots,\lambda_Q,0,\ldots,0)$. Show that the $\mathbf{R}_{xx}$ given on p. 113 of Chap 5 leads to $C = \sum_{i=1}^{Q}\log_2(1+P_i\lambda_i)$.

   (b) (5%) Find P_i in terms of λ_i, $1 \le i \le Q$, to maximize C under the total power constraint $\sum_i P_i = P$.

2. (10%) Show that minimizing $\left\|(\mathbf{G}_n)_j\right\|^2$ is equivalent to maximizing SNR in V-BLAST with ZF+OSIC.

3. (10%) Consider the following space-time code matrix for a MIMO system with 4 Tx antennas and 1 Rx antenna:

$$\mathbf{X} = \begin{pmatrix} x_1 & -x_2^* & -x_3^* & x_4 \\ x_2 & x_1^* & -x_4^* & -x_3 \\ x_3 & -x_4^* & x_1^* & -x_2 \\ x_4 & x_3^* & x_2^* & x_1 \end{pmatrix}$$

   Suggest a decoding strategy for this code.