1. (20%) Assume there are Q users, each user's transmitted symbol is s and $\mathbb{E}\left\{\mathbf{s}\mathbf{s}^H\right\}=\mathbf{I}$, and $\mathbf{n}$ is independent and identically distributed (i.i.d.) additive Gaussian noise. The received signal chip sampled over the kth symbol is

$$\mathbf{x}(k)=\mathbf{H}\mathbf{A}\mathbf{s}(k)+\mathbf{n}(k),$$

where $\mathbf{H}=[\mathbf{h}_1,\mathbf{h}_2,\ldots,\mathbf{h}_Q]$: channel matrix,

$\mathbf{A}=\mathrm{diag}[\sigma_1,\sigma_2,\ldots,\sigma_Q]$: power control factor,

$\mathbf{s}(k)=[s_1(k),s_2(k),\ldots,s_Q(k)]^T$: transmit symbol.

Based on the Minimum Mean Square Error (MMSE) detector design criterion on p. 29 of Chapter 2, please derive the Wiener solution expressed in $\mathbf{R}_{xx}$ and $\mathbf{r}_{xs}$, which are signal correlation matrix and cross-correlation vector, respectively.

$x(k) = HAs(k)+n(k)$ receive signal

$H=[h_1, h_2, \cdots, h_Q]$ channel matrix

$A = \mathrm{diag}[\sigma_1, \sigma_2, \cdots \sigma_Q]$ power control factor

$s(k) : [s_1(k) \cdots s_Q(k)]^T$ transmit symbol

△ MMSE criterion : jointly minimize MAI and noise

$$d_{MS} = \arg\min_{d_{MS}} E\{\|\sigma_1 s_1(k) - d_{MS}^H x(k)\|^2\}$$

$$= E\{(\sigma_1 s_1(k) - d^H x(k))(\sigma_1 s_1(k) - d^H x(k))^H\}$$

$$= \sigma_1^2 - r_{xs} d - (r_{xs} d)^H + d^H R_{xx} d$$

let $\frac{\partial}{\partial D} E\{\|\sigma_1 s_1(k) - d^H x(k)\|^2\} = 0$

$$\Rightarrow R_{xx} d - r_{xs} = 0 \Rightarrow d_{MS} = R_{xx}^{-1} r_{xs}$$

solution $\Rightarrow d_{MS} = R_{xx}^{-1} r_{xs}$

① $R_{xx} = E\{x(k)x^H(k)\}$

$$= E\{(HAs(k)+n(k))(HAs(k)+n(k))^H\}$$

$$= E\{HAs(k)s^H(k)A^H H^H\} + E\{HAs(k)n^H(k)\} + E\{n(k)s^H(k)A^H H^H\} + \overbrace{E\{n(k)n^H(k)\}}^{\sigma_n^2 I}$$

$HA\,E\{s(k)s^H(k)\}A^H H^H$

$= HA\sigma_s^2 A^H H^H$ $\quad E\{ss^H\}=I$

$= H\|A\|^2 H^H$

↳ $HA\,E\{s(k)n^H(k)\} = 0$ (∵ s 和 n indep)

↳ $E\{n(k)s^H(k)A^H H^H\} = \overset{0}{E\{n(k)\}}\, E\{s^H(k)A^H H^H\} = 0$

$$= H\|A\|^2 H^H + \sigma_n^2 I$$

$$= \sum_{q=1}^{Q} \sigma_q^2 h_q h_q^H + \sigma_n^2 I$$

② $r_{xs} = E\{x(k)\sigma_1 s_1^*(k)\}$, from P18 $\Rightarrow x(k) = \sigma_1 h_1 s_1(k) + \sum_{q=2}^{Q} \sigma_q h_q s_q(k) + n(k)$

代入 $r_{xs} = E\{\sigma_1 h_1 s_1(k)\sigma_1 s_1^*(k)\} + \overset{0}{E\{\sum_{q=2}^{Q}\sigma_q h_q s_q(k)\sigma_1 s_1^*(k)\}} + \overset{0}{E\{n(k)\sigma_1 s_1^*(k)\}}$

$$= \sigma_1^2 h_1 \underbrace{E\{s_1(k)s_1^*(k)\}}_{1}$$

$$= \sigma_1^2 h_1$$

訊號之間彼此獨立

(i.e. $E\{s_i(k)s_j(k)\} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$)

∵ $\sigma_1 \overset{0}{E\{n(k)\}} E\{s_1^*(k)\} = 0$

($n(k)$, $s_1(k)$ indep)

2. (30%) Consider the single-user RAKE, ZF, and MMSE detectors in CDMA systems. The simulation environment is assumed as follows:

- It is assumed that user 1 is the desired signal with power σ_1^2, and the others are the MAI with the same transmit power σ_i^2.
- The input SNR_i and SIR_i are defined as

$$\mathrm{SNR}_i = \sigma_1^2 / \sigma_n^2 \ ; \ \mathrm{SIR}_i = \sigma_1^2 / \sigma_i^2 .$$

- Q active user signals are transmitted via independent Rayleigh fading paths with path delays assumed to be uniform over $[0, 5T_c]$ (i.e., $L = 5$).
- All CDMA signals are BPSK modulated and spread by a random code of length 48 (a random sequence of ± 1).
- Perfect channel estimation is assumed.

Perform the following tasks and **COMMENT on your results**:

(a) Plot SINR_o (output SINR in dB) of the detectors as a function of SNR_i (from -20 to 20 dB with an increment of 2 dB) with $\mathrm{SIR}_i = 0$ dB and $Q = 8$. (Using HW2_1_a_2025.m)

(b) Plot SINR_o (output SINR in dB) of the detectors as a function of Q (from 1 to 10) with $\mathrm{SNR}_i = 0$ dB and $\mathrm{SIR}_i = 0$ dB. (Using HW2_1_b_2025.m)

criteria：

- Single-user system 去detect user1 訊號，其餘user (Q-1) 為Multi-Access interference
- Detector : RAKE, ZF, MMSE
- Path delays L = 5
- BPSK , processing gain(spreading factor) = N = 48
- Perfect channel estimation assumed.

$$h_q = \sum_{l=1}^{L} \alpha_{q,l} c_{q,l} = C_q f_q$$

$$f_q = [\alpha_{q,1}, \alpha_{q,2} \dots \alpha_{q,L}]^T$$

$$C_q = \begin{pmatrix} c_q[0] & 0 & \cdots & 0 \\ \vdots & c_q[0] & \cdots & \vdots \\ c_q[N-1] & \vdots & \cdots & 0 \\ 0 & c_q[N-1] & \cdots & c_q[0] \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_q[N-1] \end{pmatrix}$$

RAKE detector：$d_{RAKER} = h_1$

Zero-forcing detector ：$d_{ZF} = H(H^H H)^{-1} e_1$

MMSE detector ：$d_{MS} = R_{xx}^{-1} r_{xs}$，如第一題推導的

2.(a)Q = 8，$SIR_i = 0dB$畫出$SIR_o(-20dB \sim 20dB)$，去比對$SINR_o$隨SNR_i變化有什麼改變，看不同的detector對$SINR_o$提升效果

參數設置：

```
%% Parameter Setting

SIR = 0;                          % signal-to-CCI ratio
IPW = 10^(-SIR/10);                 % interference power
Q = 8                             % number of active users
L = 5                             % number of paths (fingers)
N = 48                            % random code length
trial = 5000;                       % number of Monte Carlo runs
SNR = -20:2:20;                     % signal to noise ratio
```

這裡有用到Monte Carlo runs，原因是因為無線通訊中的fading. Interference . noise. Spreading code 都是隨機變數，所以只做一次模擬的話，結果較不準確，故做5000次並取平均。

結果：

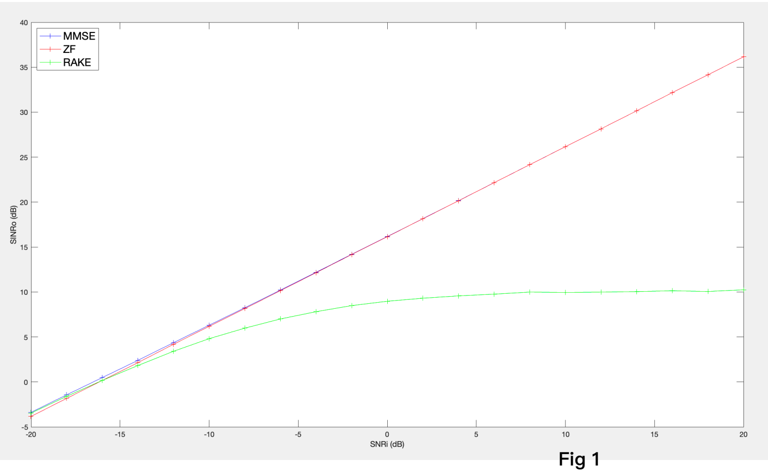

Fig 1

Comment: 從Fig 1可以觀察到

- 在低SNR_i時ZF的表現最差，這是因為ZF用比較暴力（$\times H^{-1}$）的方式去消掉ISI（$As(k) \xrightarrow{channel} HAs(k) \xrightarrow{ZF} H^{-1}HAs(k) = As(k)$），但他並沒有考慮到雜訊的部分，所以當雜訊很大($H \rightarrow singular, H^{-1} \rightarrow \infty, H^{-1}n(k) \rightarrow \infty$)時，他就會被放大，使得整個系統效能下降。而當$SNR_i$很高時，ZF的表現很好，因為他完美消除ISI
- MMSE他權衡noise和ISI，在高低SNR_i表現都很不錯，表現得比ZF更穩定，不過他的計算量會比ZF高很多，所以適合在低SNR_i使用
- RAKE他比較單純在對接收到的訊號進行合併（做match filter$\times H^H$），沒有對MAI進行處理，抗干擾能力較弱（$As(k) \xrightarrow{channel} HAs(k) \xrightarrow{RAKE} H^H HAs(k) = GAs(k)$），$G = H^H H = \begin{pmatrix} h_1^H h_1 & h_1^H h_2 & \cdots & h_1^H h_Q \\ \vdots & \vdots & \ddots & \vdots \\ h_Q^H h_1 & h_Q^H h_2 & \cdots & h_Q^H h_Q \end{pmatrix}$對角線元素$h_i^H h_i$為processing gain，非對角元素為$h_i^H h_j$（不同user的干擾），雖然這裡的IPW是固定值$SIR_i = 0dB$，$SINR_{RAKE} \approx \dfrac{K}{1 + \frac{P_i}{P_s}}$，，當高$SNR_i$時（$P_s$ –> 無限大），表現會逐漸趨於漸近線，但反觀ZF.MMSE可透過干擾消除、抑制去提高$SINR$。但他的計算量會是三者中最低的，所以實施簡單計算量低。
- 通常會先用RAKE最大化SNR，再用ZF.MMSE做二次處理去消掉MAI。

結論：

- ZF適合在高SNR_i時使用，低SNR反而會放大雜訊
- MMSE適合在低SNR_i時使用，會去權衡雜訊和Interference
- RAKE無法有效對抗MAI，通常用於multi–path補償

2.(b)不同的Q(1~10)，固定$SNR_i = 0dB$ 和 $SIR_i = 0dB$ 畫出$SINR_o$，去看當越來越多user加入後，RAKE, ZF, MMSE三個的抗干擾能力為何

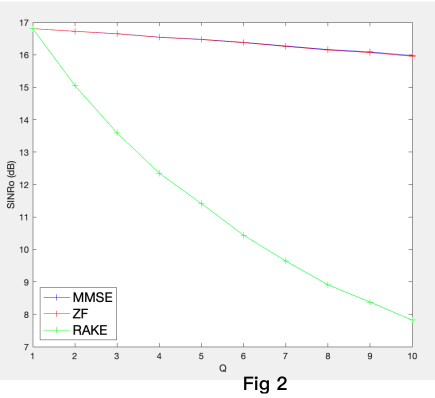

Fig 2

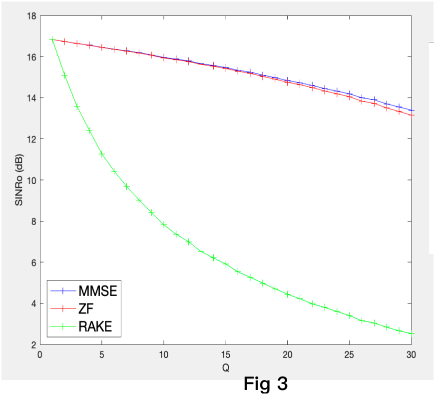

Fig 3

```
%% Parameter Setting

SIR = 0;                                  % signal-to-CCI ratio
IPW = 10^(-SIR/10);                         % interference power
L = 5                                     % number of paths (fingers)
N = 48                                     % random code length
trial = 5000;                               % number of Monte Carlo runs
SNR = 0;
NPW = 10^(-SNR/10);
QQ = 1:10                                  % number of active users

rMS = zeros(trial,length(QQ));
rZF = zeros(trial,length(QQ));
rRAKE = zeros(trial,length(QQ));
```

Fig2 是這次助教要求的數字，Fig 3是我想知道若是user數量再上升會有什麼事發生

Comment: 從Fig 2.3可以觀察到

- RAKE下降的最快，因為他不會主動去消除MAI，他是用在合併multi-path，因此當Q增多，干擾變強，RAKE沒辦法對抗，所以$SINR_o$表現就會急劇下降（對干擾適應差）
- ZF他是完美消除ISI，當Q增加時，ZF會去inverse Q*Q的矩陣，計算量有變大。SINR仍然會下降，但下降的比RAKE緩慢
- MMSE有去權衡干擾和雜訊，去做整體最佳化的選擇，抗干擾能力最好，並且他不像ZF會放大雜訊，表現也較穩定，尤其在Q很大時（看Fig3）

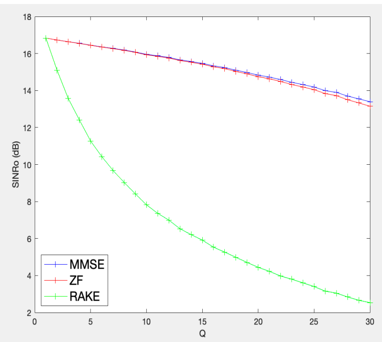

Fig 4
SNR = 0dB

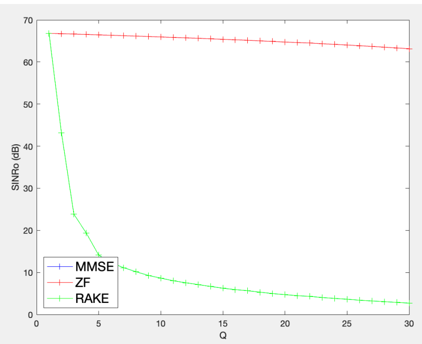

Fig 5
SNR = 50dB

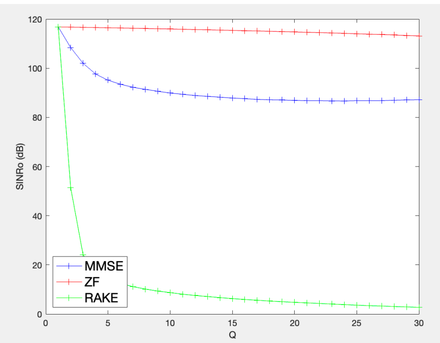

Fig 6
SNR = 100dB

這次助教要我們用的是$SNR = 0dB$但若是我們把SNR調大，如上三圖所示，可以很明顯看到當我們$SNR = 100dB$，因為ZF只要考慮干擾，SNR又很大，雜訊幾乎可忽略，所以表現比MMSE還要好。

總結：

Detector	概念	優點	缺點
RAKE $d_{RAKER} = h_1$	利用multi-path合併，增強訊號	可以用multi-path增強訊號強度，計算量最低	無法消除MAI，抗干擾能力差
ZF $d_{ZF} = H(H^H H)^{-1} e_1$	完全消除干擾，但可能放大雜訊	在干擾較強時能有效去除干擾(SNR很大時可用)	需要矩陣反轉，數值不穩定，雜訊會被放大
MMSE $d_{MS} = R_{xx}^{-1} r_{xs}$	在干擾和雜訊之間做折衷	平衡干擾與雜訊，穩定性較好	計算量較大，需要雜訊資訊

3. (50%) Consider the SIC and the PIC detector with the Rake receiver in CDMA systems, respectively. The simulation environment is assumed as follows:

- $Q = 3$ active user signals are transmitted via independent Rayleigh fading paths with path delays assumed to be uniform over $[0, 5T_c]$ (i.e., $L = 5$).
- All CDMA signals are BPSK modulated and spread by a random code of length 48 (a random sequence of ± 1).
- Perfect channel estimation is assumed.
- The input SNR_i is defined as

$$\text{SNR}_i = \sigma_i^2 / \sigma_n^2$$

- The relative power levels of the three users are fixed and given by $\text{SNR}_1 = \text{X dB}$, $\text{SNR}_2 = (\text{X}-6)$ dB, and $\text{SNR}_3 = (\text{X}-12)$ dB.
- The decision is performed according to

$$\text{Dec}(x) = \begin{cases} 1, & \text{if Real}(x) \geq 0 \\ -1, & \text{if Real}(x) < 0 \end{cases}$$

Perform the following tasks and **COMMENT on your results**:

(a) Perform SIC detection on the three users in the order of descending power levels, i.e., 1, 2, 3. Plot the average bit error rate of the three users as a function of input SNR_1 (from -10 to 20 dB with an increment of 5 dB). (Using HW2_2_SIC_2025.m)

(b) Perform SIC detection on the three users in the order of ascending power levels, i.e., 3, 2, 1. Plot the average bit error rate of the three users as a function of input SNR_1 (from -10 to 20 dB with an increment of 5 dB). (Using HW2_2_SIC_2025.m)

(c) Perform PIC detection on the three users. Plot the average bit error rate of the three users as a function of input SNR_1 (from -10 to 20 dB with an increment of 5 dB). (Using HW2_2_PIC_2025.m)

(d) Compare the results in (a)-(c). Which detection scheme is the best for such a scenario? Why?

WCSP HW2

criteria：

- 在multi-users去detect 各個user訊號，並比較用SIC.PIC detector他們的錯誤率
- Path delays L = 5
- BPSK , processing gain(spreading factor) = N = 48
- Perfect channel estimation assumed.
- $SNR_1 = XdB, SNR_2 = X-6dB, SNR_3 = X-12dB$

3(a)(b)

SIC_a是從訊號最強到最弱的開始detect，detect完後重建該訊號從總訊號扣除，繼續detect下一個

SIC_b是從訊號最弱到最強的開始detect，detect完後重建該訊號從總訊號扣除，繼續detect下一個

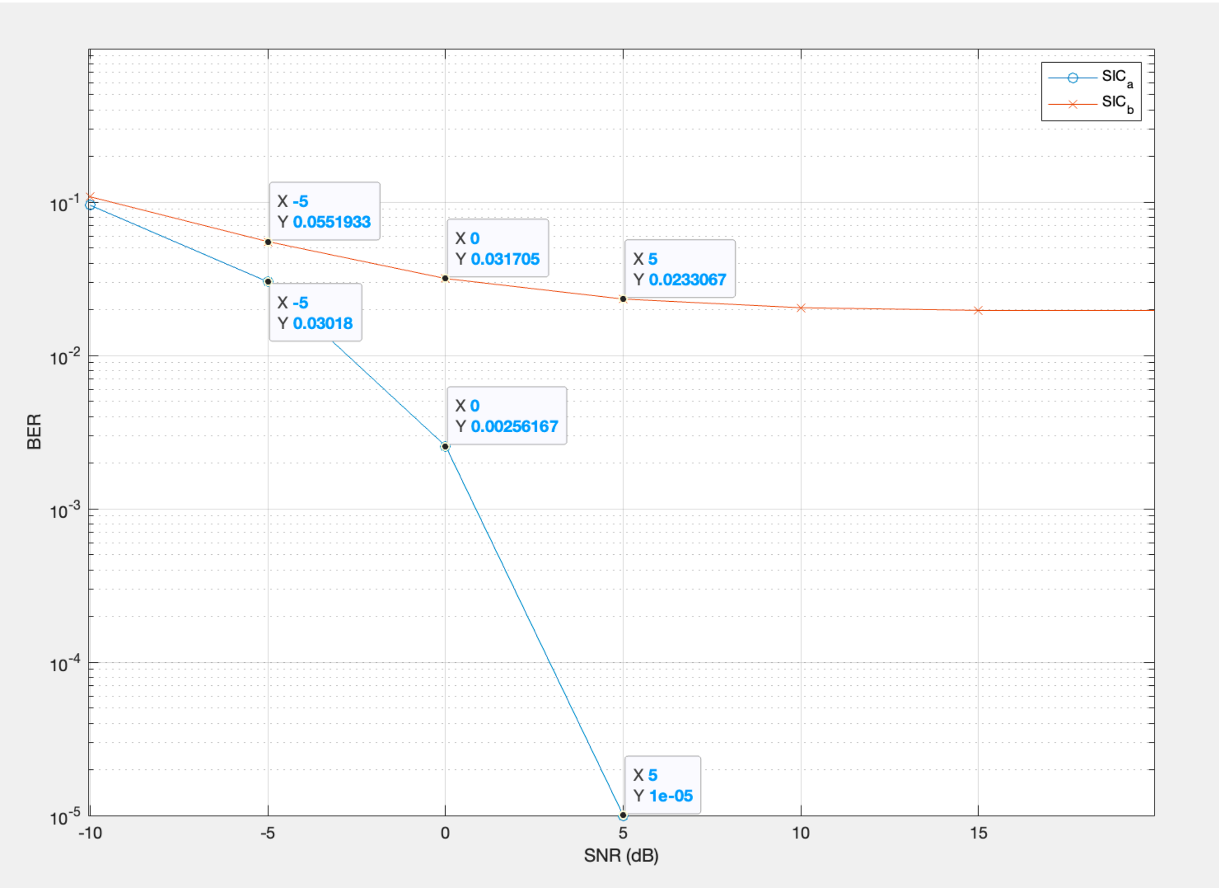

Fig 4

Comment：

可以看到Fig 4在SNR相同的情況下，SIC_a的BER會比SIC_b還低，這是因為從比較強的訊號開始判斷，比較容易判斷正確，所以可以比較有效從總訊號扣除（剩下的干擾也就較小，因為剩下的s中，要判斷的是訊號最強那個）。但若是用SIC_b的方法，弱的比較難解出來，因為還有比自己強的干擾在，所以detect不容易成功。

結論：SIC_a比SIC_b好用

3.(c)

使用PIC去同時估計所有訊號

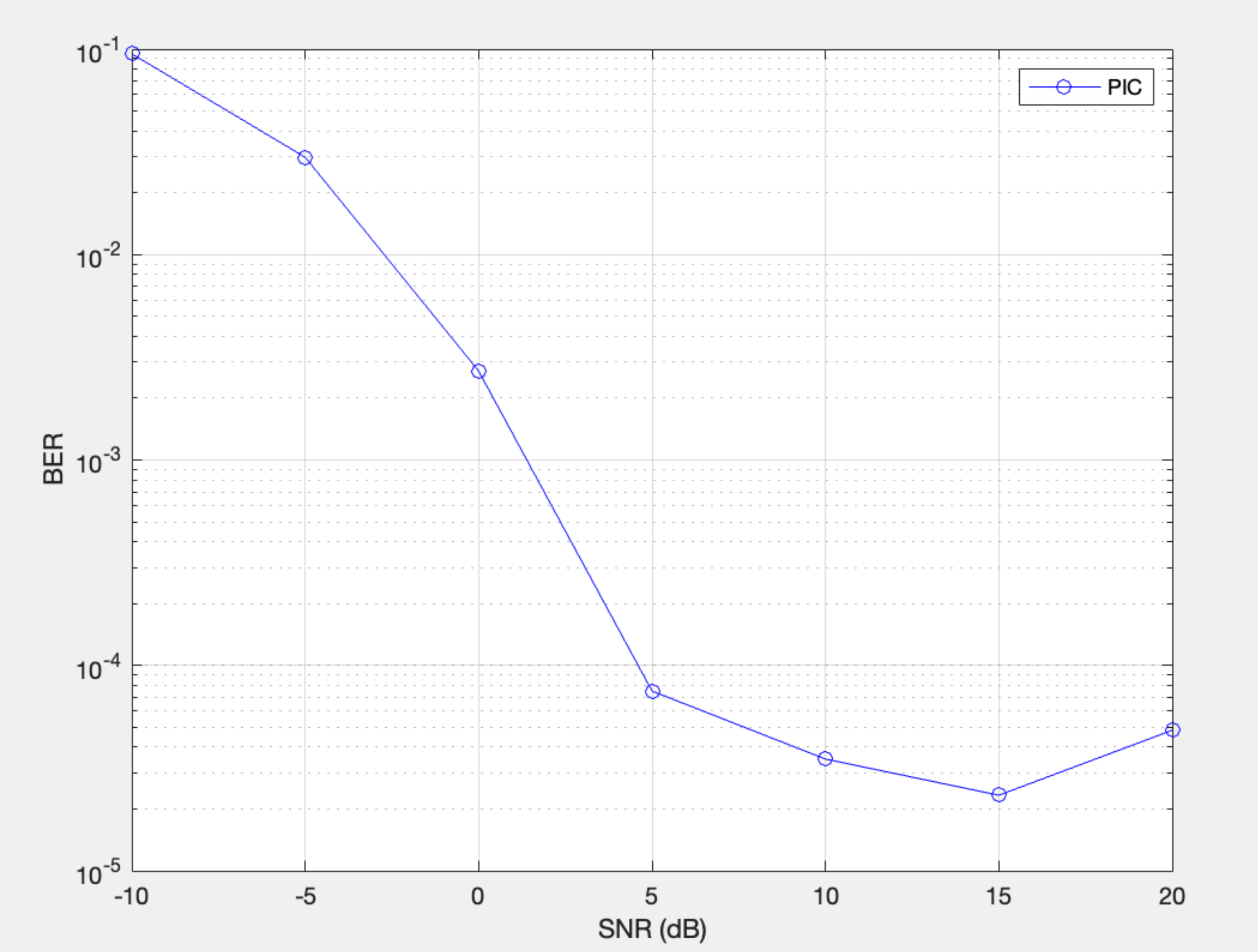

Fig 5

Comment：

- 在低SNR時，因為PIC會同時估計並消除所有user的干擾，這種初始detect錯誤，所以去影響到抵銷的結果，使得產生error progation的效應，所以BER較高。
- 在高SNR時，BER表現都很好10^{-4}以下，所以在高SNR下，PIC干擾消除能力比較好。不過可以看到在SNR15–20反而BER提高了，原因可能是因為PIC會用前一輪的detect 結果去抵銷干擾，若是存在誤差，會影響到最終BER
- 相較於SIC，SIC可以先處理一個訊號，降低干擾後再detect下一個，而PIC同時處理所有訊號，所以更容易受到初始錯誤的影響。

3.(d)

考慮SIC_a , SIC_b , PIC ，我認為SIC_a會是最好的detect，原因如下：

1.在高或低SNR表現最好：因為PIC在低SNR表現較差，SIC_a能先檢測最強的訊號，並從總訊號中移除，降低後續檢測的干擾，因此 BER 在低 SNR 下仍能維持較好的表現。

2.SIC_a vs SIC_b：

- SIC_a透過先檢測強訊號，確保後續檢測時的干擾更少，因此 BER 下降較快。
- SIC_b在低 SNR 時表現最差，因為最弱訊號的檢測誤差較大，影響後續訊號的干擾消除，使得 BER 很高。

3.SIC_a vs PIC

- PIC雖然能夠同時處理所有訊號(delay較小) ，但它在低 SNR 時受到error propagation影響，導致 BER 較高。

結論

Detector	BER	Complexity	Delay	Power control
SIC_a（先detect較強訊號）	最佳	中間（要逐個detect和消除訊號）	長	Reliable detection for weak users.
SIC_b（先detect較弱訊號）	最差	中間（和SIC_a相同）	長	要求高（因為power不平均detect BER很大）
PIC	中間	最高（要同時檢測所有訊號）	最短	要求高（大家power最好差不多）