1.

(a) Describe the assumption of the Jakes model

Jakes model他是一種用來模擬smal-scale fading的統計模型。

他假設訊號在傳播過程中經過多個獨立且隨機分佈的散射路徑，每條路徑產生不同的delay和phase變化，他的輸出會是一個complex random process ，Amplitude會是Rayleigh distribution，phase 是uniform distribution$(0,2\pi)$

假設如下

- 通道經歷Rayleigh fading，適用NLOS, Non-Line-of-Sight。
- 假設接收到的訊號是來自許多不同方向的多條路徑訊號的總和，每條路徑的amplitude和phase隨機變化。
- 所有散射源均勻分佈在一個圓周上，模擬無線訊號在移動環境中的傳播特性。
- 移動設備或環境中的scatter導致doppler shift，其最大f_{max}由相對速度(固定）和載波頻率決定。
- Channel response響應是WSS，即它的統計特性不隨時間變化。
- 假設接收訊號的In-phase 和Quadrature 分量服從零均值、高斯分佈，且相互獨立。
- Jakes 模型的autocorrelation符合Bessel函數的形式，描述了通道增益的時間相關性。

(b) Describe how the simulator generates Rayleigh fading gains with Doppler shift

透過助教提供的HW1_Jakes_2025去產生具有Doppler shift 的Rayleigh fading channel，(1)透過sum-of-sinusoids去模擬Rayleigh fading(2)考慮Doppler shift去模擬移動接收到的訊號變化

Step 1 設定參數

```
N = 4*M + 2;       % no. of paths
alpha = 0;         % a parameter ralated to
n = 1:1:M;
t = 0:Ts:10^3*Ts;    % time interval
beta = n*pi/M;
```

Step 2 去計算每條散射路徑的oscillation frequency

```
fn = f_max*cos(2*pi*n/N); % oscillation frequency
```

模擬結果要確保doppler spectrum形狀接近U-shape

Step 3產生Rayleigh fading channel

```
for i = 1:length(t)
    hi(i) = 2*cos(beta)*cos(2*pi*fn*t(i))'+sqrt(2)*cos(alpha)*cos(2*pi*f_max*t(i));
    hq(i) = 2*sin(beta)*cos(2*pi*fn*t(i))'+sqrt(2)*sin(alpha)*cos(2*pi*f_max*t(i));
end
h = (hi + 1i*hq);  % total channel for all time included real and image part
```

將實部、虛部訊號算出來

$h(t) = \sum_n 2cos(\beta_n)cos(2\pi f_n t) + \sqrt{2}cos(\alpha)cos(2\pi f_{max} t)$，$\beta_n$ α用來控制相位

$h = h_I + jh_q$

Step 4除上power 去normalize他

```
power = sum((abs(h)).^2)/length(h);
hn = h/sqrt(power); % normalized channel
```

(c) Based on $f_{max} = \frac{vf_c}{c}$,try different combinations of parameters and summarize the impact of parameter adjustments on the system.

comment： 因為c固定，透過調整v和 f_c

- v變大，f_c固定，f_{max}會變大 –>頻譜會散更開
- v固定，f_c變大，f_{max}會變大 –>頻譜會散更開

Rayleigh

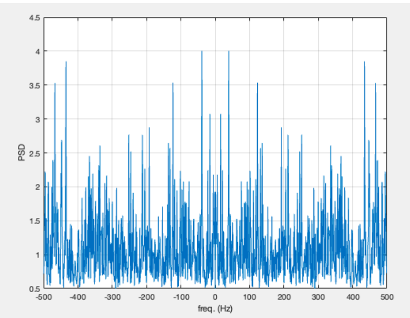

Fig 1

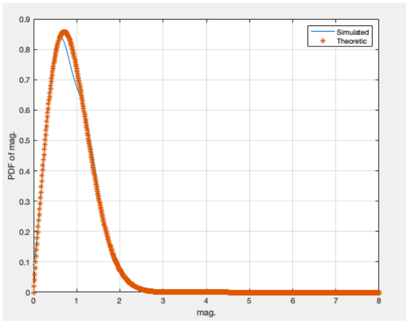

Fig 2

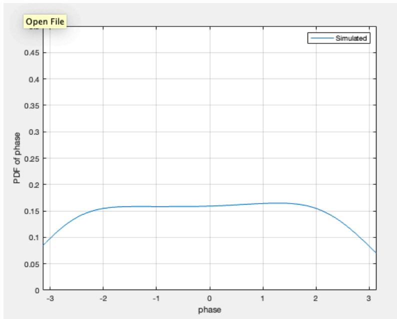

Fig 3

(1)Plot the PSD of the fading gain

根據我們之前推測的，我分別去調整v和f_c的值，去看他PSD的變化

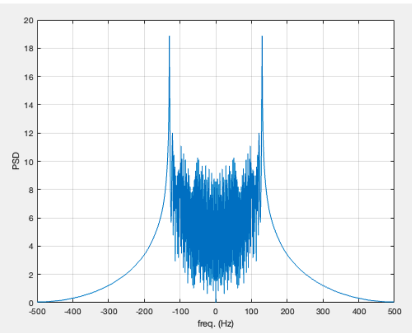

Fig 4
$v = 40km/hr$
$f_{max} = 129HZ$

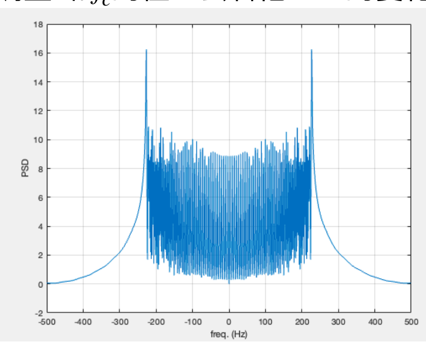

Fig 5
$v = 70km/hr$
$f_{max} = 227HZ$

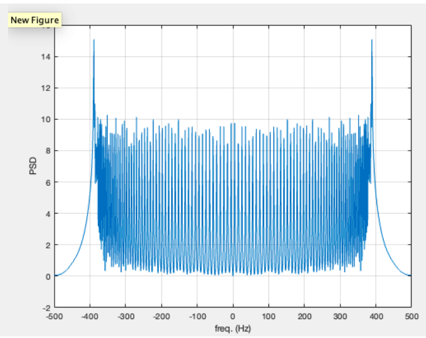

Fig 6
$v = 120km/hr$
$f_{max} = 389HZ$

comment： 從Fig 4~6 可以觀察，因為c固定，透過調整v和 f_c

- v變大，f_c固定 $f_c = 3.5GHZ$，f_{max}會變大 –>頻譜會散更開

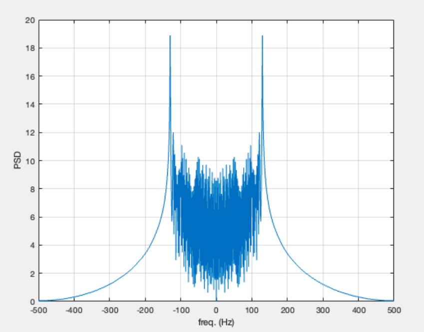

Fig 7
$f_c = 2GHZ$
$f_{max} = 129HZ$

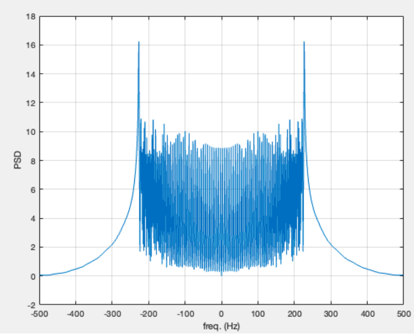

Fig 8
$f_c = 3.5GHZ$
$f_{max} = 227HZ$

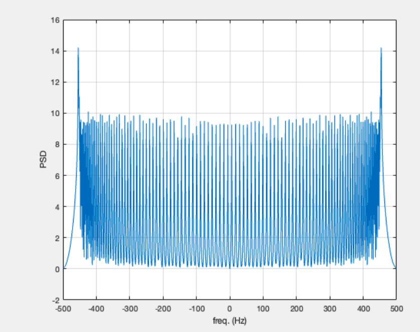

Fig 9
$f_c = 7GHZ$
$f_{max} = 453HZ$

comment： 從Fig 4~6 可以觀察，因為c固定，透過調整v和 f_c

- v固定 $v = 70km/hr$，f_c變大，f_{max}會變大 –>頻譜會散更開
- 並且可以發現 f 的範圍的確在[$f - f_{max}, f + f_{max}$]

(2)Evaluate the RMS frequency Doppler spread(f_{RMS})

comment：

- 根據公式$f_{RMS} = \sqrt{\frac{\int (f - \bar{f})^2 * S_D(f)df}{\int S_D(f)df}}$ ，$\bar{f}$ 為average frequency
- 看Fig 4–9可以知道，PSD散得越開的話，Doppler spread 就越大，f_{RMS} 越大，故v或f_{max}上升會使f_{RMS} 變大

f_{RMS}

	$f_c = 2GHZ$	$f_c = 3.5GHZ$	$f_c = 7GHZ$
$v = 40km/hr$	52.38	91.66	183.32
$v = 70km/hr$	91.66	160.4	320.81
$v = 120km/hr$	157.13	274.99	549.9

(3)Plot the PDF of the magnitude and the phase of the fading gain.

magnitude

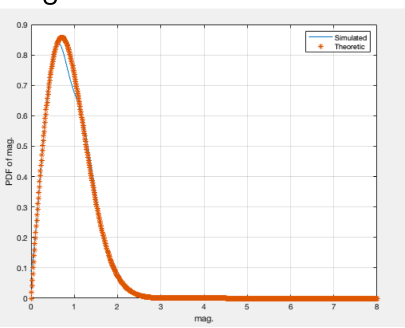

Fig 10 PDF of the magnitude fading coefficient for Rayleigh channel

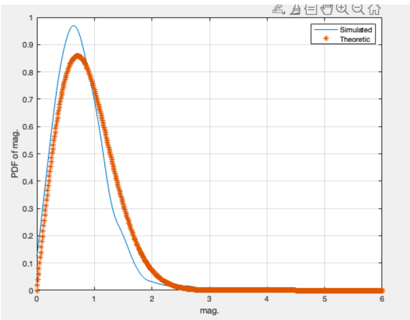

Fig 11 PDF of the magnitude
$v = 60km/hr$ $f_c = 2GHZ$

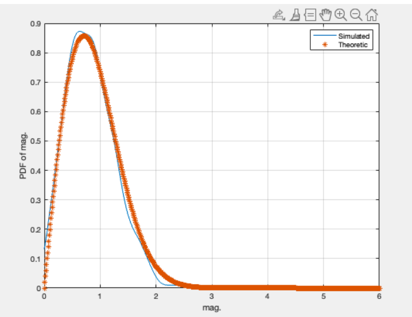

Fig 12 PDF of the magnitude
$v = 70km/hr$ $f_c = 3.5GHZ$

comment：

f_c v皆上升，f_{max}也上升，Doppler spread 會增加，這時候通道中的Doppler effect會更嚴重，fading就會變得更加隨機，magnitude分布更接近Rayleigh distribution（理論值）

phase

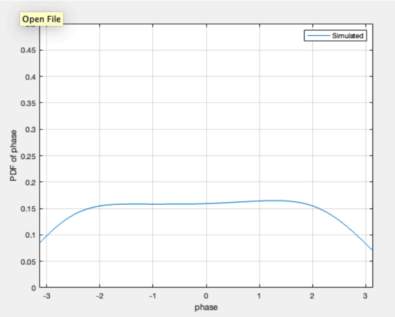

Fig 13 PDF of the phase fading coefficient for Rayleigh channel

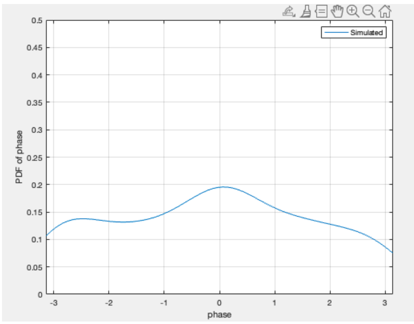

Fig 14 PDF of the magnitude $v = 60km/hr$ $f_c = 2GHZ$

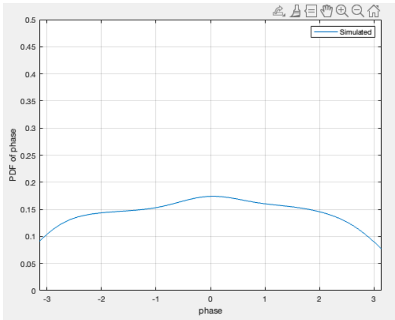

Fig 15 PDF of the magnitude $v = 70km/hr$ $f_c = 3.5GHZ$

comment：

f_c v皆上升，f_{max}也上升，Doppler spread 會增加，這時候通道中的Doppler effect會更嚴重，fading就會變得更加隨機，接收到的phase會更隨機，phase分布更接近Uniform distribution