1. The general form of MIMO channel capacity is given by

$$C = \log_2[\det(\mathbf{I} + \mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\mathbf{R}_{nn}^{-1})].$$

(a) (5%) Let $\mathbf{H}^H\mathbf{R}_{nn}^{-1}\mathbf{H} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H$, $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1,\dots,\lambda_Q,0,\dots,0)$. Show that the $\mathbf{R}_{xx}$ given on p. 113 of Chap 5 leads to $C = \sum_{i=1}^{Q}\log_2(1+P_i\lambda_i)$.

(b) (5%) Find P_i in terms of λ_i, $1 \le i \le Q$, to maximize C under the total power constraint $\sum_i P_i = P$.

(a) 已知 $H^H R_{nn}^{-1} H = V \Lambda V^H$

λ^{-1} is chosen to satisfy transmit power constraint $\Sigma_i P_i = P$

λ_i's are eigenvalues of $\mathbf{H}^H\mathbf{R}_{nn}{}^{-1}\mathbf{H} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H$

Transmitted signal satisfies $\mathbf{R}_{xx} = \mathbf{V}\mathrm{diag}\{[P_1,\dots,P_Q,0\dots,0]^T\}\mathbf{V}^H$

Precoding power allocation

f x > 0 and 0 otherwise

114

令 $\mathbb{P} = \mathrm{diag}\{[P_1, \cdots P_Q, 0, \cdots 0]^T\}$

$$C = \log_2[\det(I + H R_{xx} H^H R_{nn}^{-1})]$$

By $\det(I+AB) = \det(I+BA)$

$$= \log_2[\det(I + R_{xx} H^H R_{nn}^{-1} H)]$$

$$= \log_2[\det(I + V \mathbb{P} V^H V \Lambda V^H)]$$

$$= \log_2[\det(I + V \mathbb{P} \Lambda V^H)]$$

$\because$ V is Unitary matrix $V^H V = I$

$$= \log_2[\det(I + \mathbb{P}\Lambda)]$$

By $\det(I+AB) = \det(I+BA)$

$$= \log_2\left(\prod_{i=1}^{Q}(1 + P_i\lambda_i)\right)$$

$$= \sum_{i=1}^{Q}\log_2(1 + P_i\lambda_i)$$

(b) By water-filling

$C = \sum_{i=1}^{Q}\log_2(1+P_i\lambda_i)$ subject to $\sum_{i=1}^{Q} P_i = P$

$$\mathcal{L} = \sum_{i=1}^{Q}\log_2(1+P_i\lambda_i) - \mu\left(\sum_{i=1}^{Q}P_i - P\right)$$

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{\lambda_i}{(1+\lambda_i P_i)\ln 2} - \mu = 0$$

$$\Rightarrow P_i = \frac{1}{\mu \ln 2} - \frac{1}{\lambda_i}$$

$$\Rightarrow P_i = \left(\lambda^{-1} - \frac{1}{\lambda_i}\right)^+$$

$(\cdot)^+$ 為負數就設0

λ^{-1} 由滿足 $\Sigma P_i = P$ 解得

2. (10%) Show that minimizing $\left\|(\mathbf{G}_n)_j\right\|^2$ is equivalent to maximizing SNR in V-BLAST with ZF+OSIC.

$\arg\min_{j \notin \{a_1 \cdots a_{n-1}\}} \|(G_n)_j\|^2$

$G = (H^+)^H = W$

$\underline{y} = \underline{H}\,\underline{x} + \underline{n}$, let W to be ZF decoder, $W^H = (H^H H)^{-1} H^H$, $W = H(H^H H)^{-1}$

$W = [\underline{w_1}, \underline{w_2} \cdots \underline{w_{NT}}]$

$W^H = [\underline{w_1}^H, \underline{w_2}^H \cdots \underline{w_{NT}}^H]$

$$z = W^H \underline{y} = \underline{x} + W^H \underline{n} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{NT} \end{bmatrix} + \begin{bmatrix} \underline{w_1}^H \underline{n} \\ \underline{w_2}^H \underline{n} \\ \vdots \\ \underline{w_{NT}}^H \underline{n} \end{bmatrix}$$, $\underline{w_i}$: ith column of W

The SNR of $z_i[n]$ is $SNR_i = \dfrac{E\{x_i[n]\, x_i^H[n]\}}{E\{\underline{w_i}^H \underline{n}[n]\, \underline{n}^H[n]\, w_i\}} = \dfrac{\sigma_x^2}{\sigma_n^2\, w_i^H w_i} = \dfrac{\sigma_x^2}{\sigma_n^2 \|w_i\|^2}$

$$\Rightarrow \arg\max_i SNR_i = \arg\max_i \frac{\sigma_x^2}{\sigma_n^2 \|w_i\|^2} = \arg\min \|w_i\|^2$$

$\therefore$ minimize $\|(G_n)_j\|^2$ is equivalent to maximize SNR in VBLAST with ZF+OSIC

3. (10%) Consider the following space-time code matrix for a MIMO system with 4 Tx antennas and 1 Rx antenna:

$$\mathbf{X} = \begin{pmatrix} x_1 & -x_2^* & -x_3^* & x_4 \\ x_2 & x_1^* & -x_4^* & -x_3 \\ x_3 & -x_4^* & x_1^* & -x_2 \\ x_4 & x_3^* & x_2^* & x_1 \end{pmatrix}$$

Suggest a decoding strategy for this code.

$$X \cdot X^H = \begin{bmatrix} x_1 & -x_2^* & -x_3^* & x_4 \\ x_2 & x_1^* & -x_4^* & -x_3 \\ x_3 & -x_4^* & x_1^* & -x_2 \\ x_4 & x_3^* & x_2^* & x_1 \end{bmatrix} \begin{bmatrix} x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & -x_4 & x_1 & x_2 \\ x_4^* & -x_3^* & -x_2^* & x_1^* \end{bmatrix} \neq (|x_1|^2 + |x_2|^2 + |x_3|^2 + |x_4|^2) I$$

$N=4,\ M=1,\ L_s=4$

$$y = \begin{bmatrix} y_1 \\ y_2^* \\ y_3 \\ y_4^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} = \underline{\underline{H}} \cdot \underline{x} + \underline{n}$$

$$z = H^H y = G s + H^H n$$

$$G = H^H H = \begin{bmatrix} h_1^* & h_2 & h_3 & h_4^* \\ h_2^* & -h_1 & h_4 & -h_3^* \\ h_3^* & h_4 & -h_1 & -h_2^* \\ h_4^* & -h_3 & -h_2 & h_1^* \end{bmatrix} \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \neq \sum_{n=1}^{4} |h_n|^2 I_4$$

match filter cannot be applied to this code

→ maximum likelihood decoder is applied as $\hat{x} = \arg\min_{\hat{x} \in \mathcal{X}} \| \underline{y} - \underline{\underline{H}} \underline{x} \|^2$

$\mathcal{X}$ is the set of all possible symbols