Sets

Nodes N

Arcs A - pairs of nodes $(i,j) \in N$

A_S - arcs w/ switches

Data

l_i power load (kW)

λ_i failure rate

n_i no. of customers

$pred_j$ predecessor of node i, $j \in N / \{0\}$

$path_{ij}$ set of nodes betw. i & j

V_i downstream nodes of i

$\tilde{l}_i$ downstream power load of node i

$$\tilde{l}_i = \sum_{j \in V_i} l_j$$

u_i expected duration of interrupts at node i in a year

ENS energy not supplied

$$ENS = \sum (\tilde{l}_i + \tilde{l}_j) f_{ij} + \sum \tilde{l}_i \theta_i$$

ENS energy not supplied

$$ENS = \sum_{i \in N} l_i u_i \text{, equivalently, } ENS = \sum_{(i,j) \in A} (\tilde{l}_i + \tilde{l}_j) f_{ij} + \sum_{i \in N} \tilde{l}_i \theta_i$$

t_i expected restoration time at node i (time to restore)

θ_i duration of interrupts node i due to local faults

$$\theta_i = \lambda_i t_i$$

$\tilde{\theta}_i$ downstream interruption, time node i expected to be interrupted due to downstream faults in a year

$$\tilde{\theta}_i = \theta_i + \sum_{(i,j) \in A \setminus A_s} \tilde{\theta}_j$$

$$u_0 = \tilde{\theta}_0$$

$$u_j - u_i = \begin{cases} 0 & (i,j) \in A \setminus A_s \\ \tilde{\theta}_j & (i,j) \in A_s \end{cases}$$

f_{ij} expected duration of interruptions at node j as a result of faults from node j in a year (?flow)

$$f_{ij} = \begin{cases} 0 & (i,j) \in A_s \\ \tilde{\theta}_j & (i,j) \in A \setminus A_s \end{cases}$$

equivalently,

$$f_{ij} = \theta_i + \sum \tilde{\theta}_k \text{ , } (i,j) \in A \setminus A_s$$

equation 9:

$$f_{ij} = \theta_j + \sum_{(j,k) \in A \setminus A_s} \tilde{\theta}_k \quad , (i,j) \in A \setminus A_s$$

equivalently,

$$f_{ij} = \begin{cases} 0 & (i,j) \in A_s \\ \theta_j + \sum_{(j,k) \in A} f_{jk} & (i,j) \in A \setminus A_s \end{cases}$$

F_j the interruption slack from the j flow

$$f_{ij} + F_j = \theta_j + \sum_{(j,k) \in A} f_{j,k}$$

$$u_0 = F_0$$
$$u_j - u_i = F_j \, , \; (i,j) \in A$$
$$u_j = \sum_{i \in \text{path}_{0,j}} F_i \quad , \; j \in N$$

E_{lb} lower bound of ENS

$$E_{lb} = \sum_{i \in N} \tilde{l}_i \theta_i$$

E_{ub} upper bound of ENS

$$E_{ub} = \tilde{l}_0 \sum_{i \in N} \theta_i$$

N max switches
M_i large constant for node i

Variables

x_{ij} (binary) switch on arc $(i,j) \in A$

f_{ij} flow on arc $(i,j) \in A$

Objective

$$\min \left(\sum_{(i,j) \in A} (\tilde{\ell}_i + \bar{\ell}_j) f_{ij} \right) \tau \quad \varepsilon_{lb}$$

Constraints

$$\sum_{(i,j) \in A} x_{ij} \leq N$$

node balance

$$F_j + \delta_{ij} = \theta_j + \sum_{(j,k) \in A} f_{jk} \qquad (i,j) \in A$$

slack coupling w/ switch alloc.

$$F_j \leq M_j x_{ij} \qquad (i,j) \in A$$