Assignment 2
Numerical Optimization / Optimization for CS WS2022

Lea Bogensperger, lea.bogensperger@icg.tugraz.at
Nives Križanec, nives.krizanec@student.tugraz.at
Jakob Ederer, ederer@student.tugraz.at

November 15, 2022

Deadline: Dec 6th, 2022 at 23:59

Submission: Upload your report and your implementation to the TeachCenter. Please use the provided framework-file for your implementation. Make sure that the total size of your submission does not exceed 50MB. Include **all** of the following files in your submission:

- `report.pdf`: This file includes your notes for the given task. Keep in mind that we must be able to follow your calculation process. Thus, it is not sufficient to only present the final results. You are allowed to submit hand written notes, however a compiled LaTeX document is preferred. In the first case, please ensure that your notes are well readable.

- `main.py`: This file includes your python code to solve the given task.

1 Simplex Method (25 P.)

The simplex method is a method to solve a constrained linear program (LP) assuming that feasible solutions lie on corners of the constraint polytope. They can be optimized by moving along feasible directions of the edges of the polytope.

Assume a farmer has 1000 acres of land available. There are 4 different types of crops that he wants to consider for planting, corn, wheat, rice, and soybean. Each corn type has its own characteristics regarding preparational costs of one acre, time requirements for work and profit on the market. Corn for example has 100 cost units (CU) required for preparation of the acre while requiring 7 man-days of work, but finally it will yield a profit of 35 CU. The respective information on all four crop types can be found in the table below.

crop type	acre preparation costs [CU]	man-work required [days]	profit [CU]
corn	100	7	35
wheat	120	10	40
rice	70	8	20
soybean	80	8	30

The farmer must consider that he has a maximum preparation budget of 100000 CU. Since he has several workers employed, he has a total maximum of 8000 man-days of work that he can use for labour. Further, take into account that unused preparation budget can *not* be used for additional profit, i.e. there is not direct coupling between them.

How should the farmer cultivate his land considering that he wants to maximize his profit?

With Pen & Paper:

1. Formulate the objective function.

2. Set up the constraints and cast the entire problem as a linear program.

3. Convert the linear program to standard form. Ensure that all requirements are met for the LP to be in standard form.

4. Show how to compute a basic feasible solution without guessing. What criterion exists to ensure that the basic solution is feasible?

In Python:

5. Implement the simplex method as shown on lecture slide 34 to solve the LP.

   - Ensure that all relevant steps are implemented.
   - Check the different options on how to choose an entering index for the new basis. Choose one of the options for your implementation and state the motivation behind your choice.
   - Make sure to check for criteria when to terminate the algorithm.

6. Select a basic solution and verify that it is feasible. Solve the LP using your implemented simplex method.

7. Document all steps in your report regarding current (per-iteration) basic/nonbasic indices, basic solutions and objective cost.

   What is the (final) maximum profit the farmer can obtain and how must he plant his acre?

8. What are the results for your slack variables? Give an explanation and discuss the implicications if these are non-zero.

9. Use an LP solver provided by `scipy`[1] to check your solutions. Discuss your findings with respect to your computed solution and the solution obtained by the LP solver.

---

[1] `https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.linprog.html`