1 Normalization in the NEO-ORB Fortran Routine

Integrating orbits the following system of differential equations is solved, where $\rho_0(\boldsymbol{R},t) = \frac{m_\alpha v}{ZeB(\boldsymbol{R},t)}$.

$$\boldsymbol{b}^*(\boldsymbol{R},{v_{||}}^n,t) = \boldsymbol{b}(\boldsymbol{R},t) + {v_{||}}^n \rho_0(\boldsymbol{R},t)\boldsymbol{\nabla}\times\boldsymbol{b}(\boldsymbol{R},t) \tag{1.1}$$

$$\begin{aligned}\frac{d{v_{||}}^n}{dt^n} &= -\frac{\boldsymbol{b}^*(\boldsymbol{R},{v_{||}}^n,t)\cdot\left(\mu^n\boldsymbol{\nabla}B(\boldsymbol{R},t) - \frac{\boldsymbol{E}^*(\boldsymbol{R},v_{||},t)}{m_\alpha v^2 B(\boldsymbol{R},t)}\right)}{\boldsymbol{b}^*(\boldsymbol{R},{v_{||}}^n,t)\cdot\boldsymbol{b}(\boldsymbol{R},t)} \\ \frac{d\boldsymbol{R}}{dt^n} &= \frac{{v_{||}}^n\boldsymbol{b}^*(\boldsymbol{R},{v_{||}}^n,t) + \boldsymbol{b}(\boldsymbol{R},t)\times\left(\mu^n\rho_0(\boldsymbol{R},t))\boldsymbol{\nabla}B(\boldsymbol{R},t) - \frac{\boldsymbol{E}^*(\boldsymbol{R},v_{||},t)}{eZvB(\boldsymbol{R},t)}\right)}{\boldsymbol{b}^*(\boldsymbol{R},{v_{||}}^n,t)\cdot\boldsymbol{b}(\boldsymbol{R},t)}\end{aligned} \tag{1.2}$$

Equations 1.2 are not independent of the magnetic field magnitude B, because B appears in the denominator of ρ_0 in equation 1.1. In the fortran routine test_orbits_vmec.f90 ρ_0 is multiplied by a value $\frac{bmod00}{bmod\_ref} \sim 5.63$. Strictly following the mathematical derivation would demand a factor of 2π.

I compared the results of my matlab integrator with the ones of the fortran routine once using $\frac{bmod00}{bmod\_ref} \sim 5.63$ and once using 2π to normalize ρ_0 for a trapped orbit and a passing orbit. The deviations of s in the trapped case originate from the different differentiation methods.

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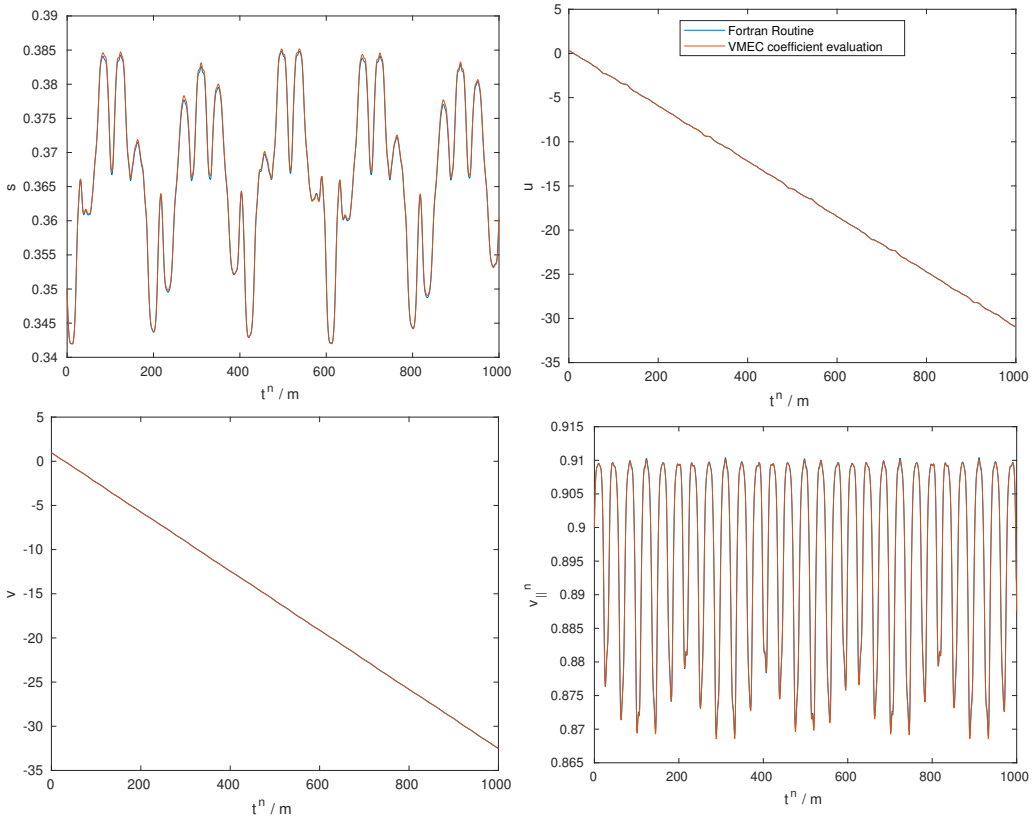

Figure 1.1: comparison of integration variables: passing case; normalization $2\pi\ \rho_0$

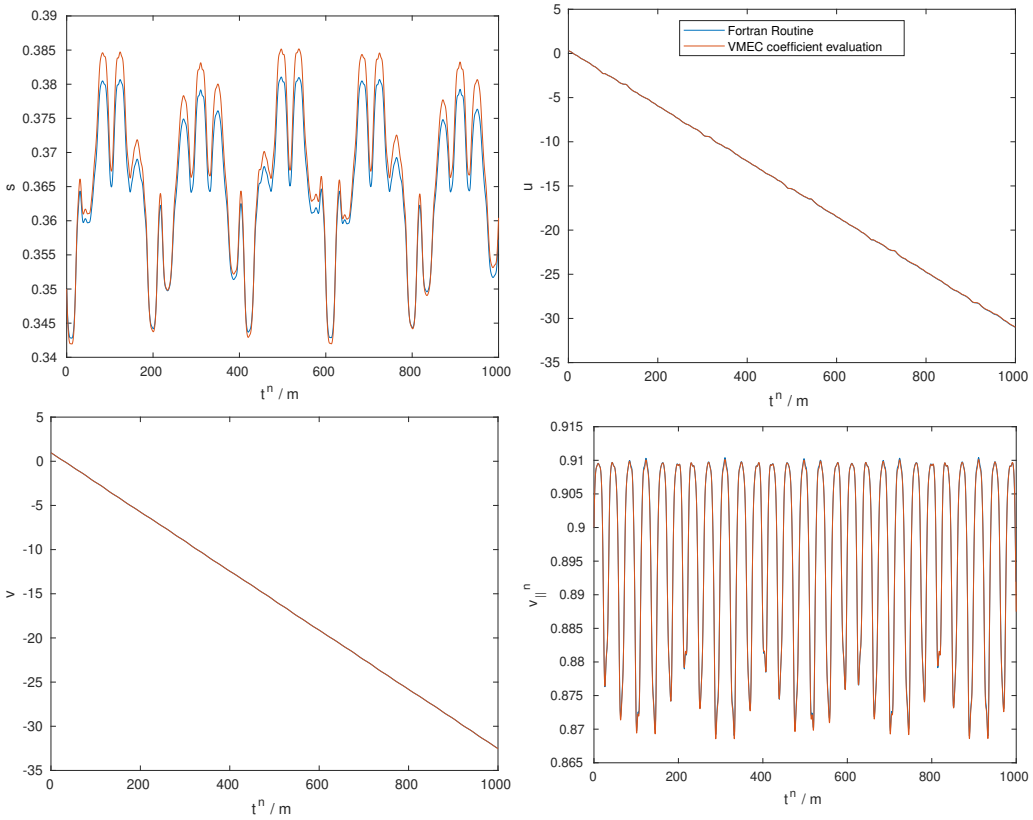

Figure 1.2: comparison of integration variables: passing case; normalization 5.63 ρ_0

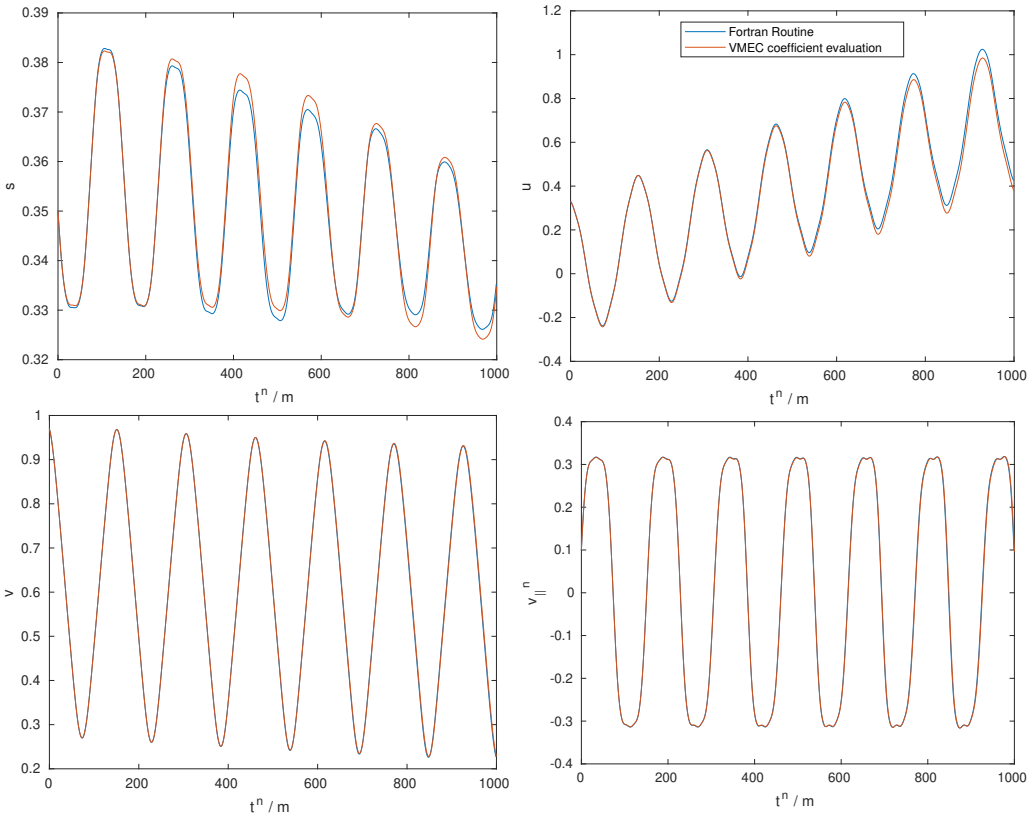

Figure 1.3: comparison of integration variables: trapped; normalization with $2\pi\ \rho_0$

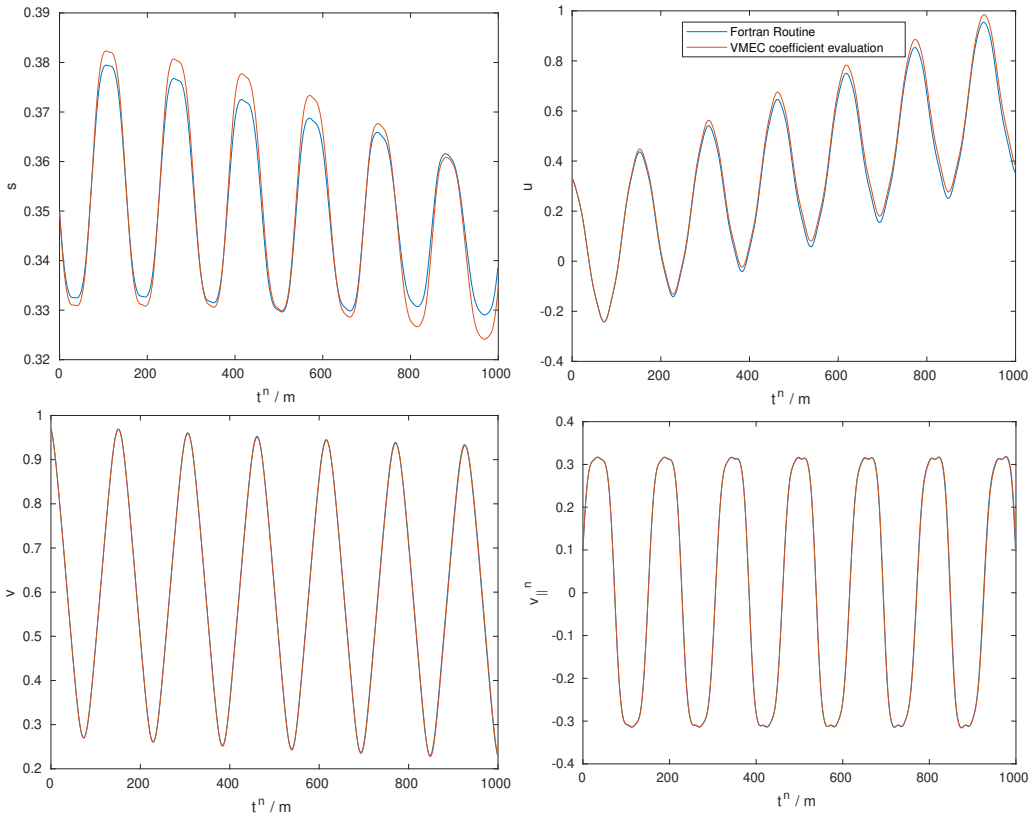

Figure 1.4: comparison of integration variables: trapped case; normalization with 5.63 ρ_0