GARCH Model and Comparison with AR,ARIMA

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Autoregressive Model:

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t-i} + \varepsilon_t$$

Moving-average Model:

$$X_t = \mu + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t$$

ARMA model: AR(p) and MA(q):

$$X_t = c + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \sum_{i=1}^{p} \varphi_i X_{t-i}$$

GARCH Model

Autoregressive conditional heteroskedasticity(ARCH):

$\varepsilon_t = \sigma_t z_t$, z_t is white noise, σ_t is time-dependent standard deviation

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \varphi_i \varepsilon_{t-i}^2$$

Generalized Autoregressive conditional heteroskedasticity(GARCH):

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2$$

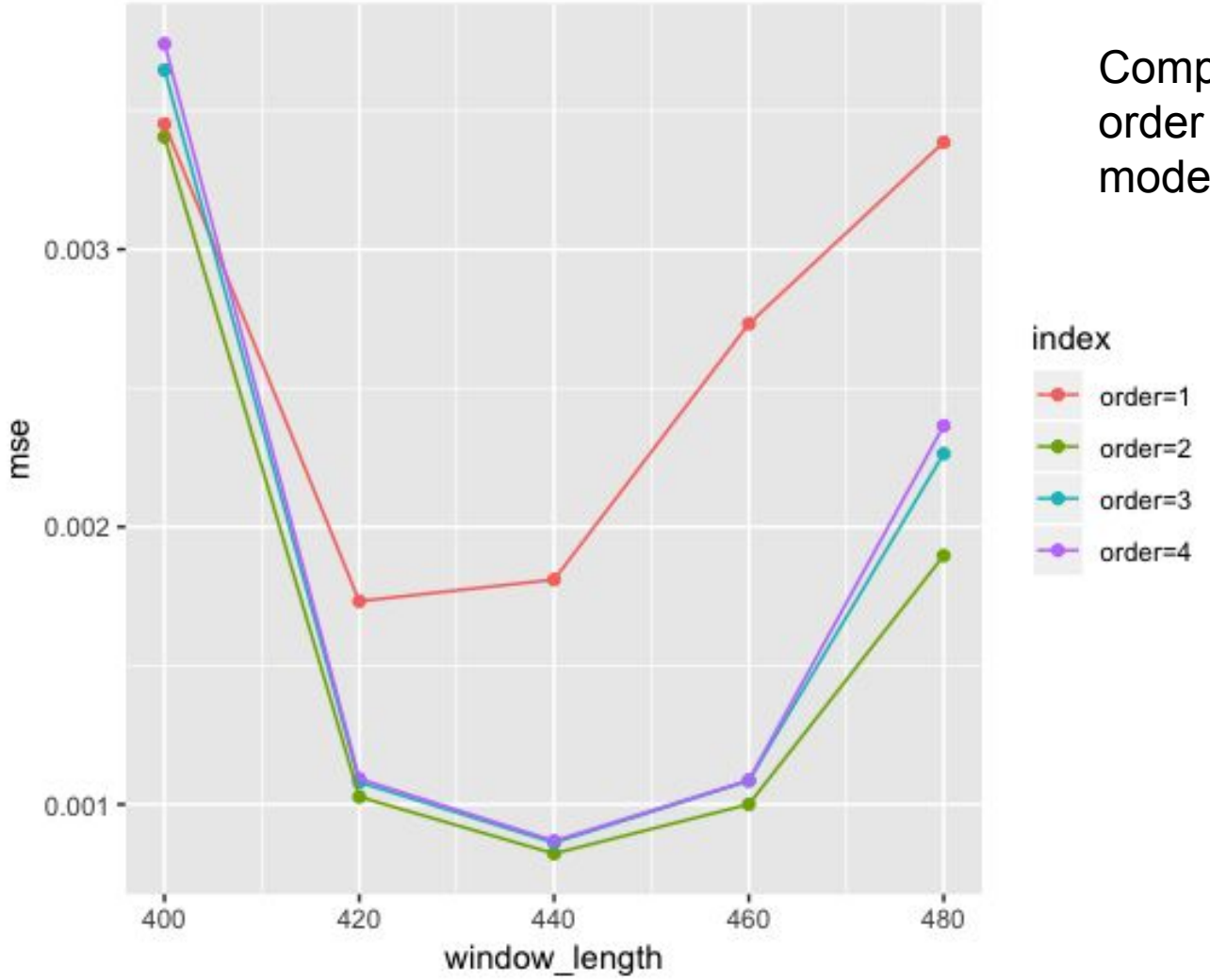
GARCH Model

Package: rugarch

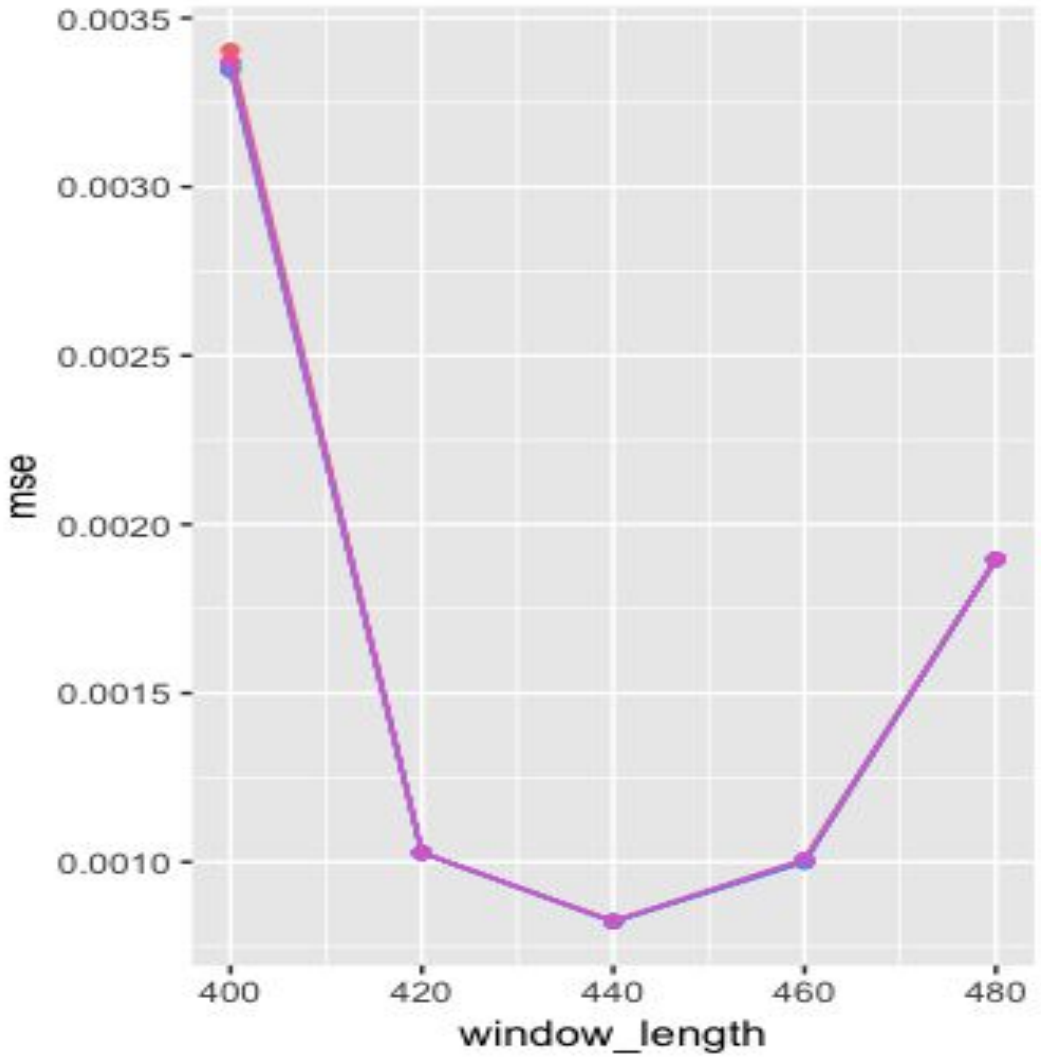
spec <- ugarchspec(mean.model=list(armaOrder=c(2,0)), variance.model = list(model="fGARCH", garchOrder=c(1,1), submodel="GARCH"))

m1 = ugarchfit(data=price[(i-window):(i-1)], spec=spec)

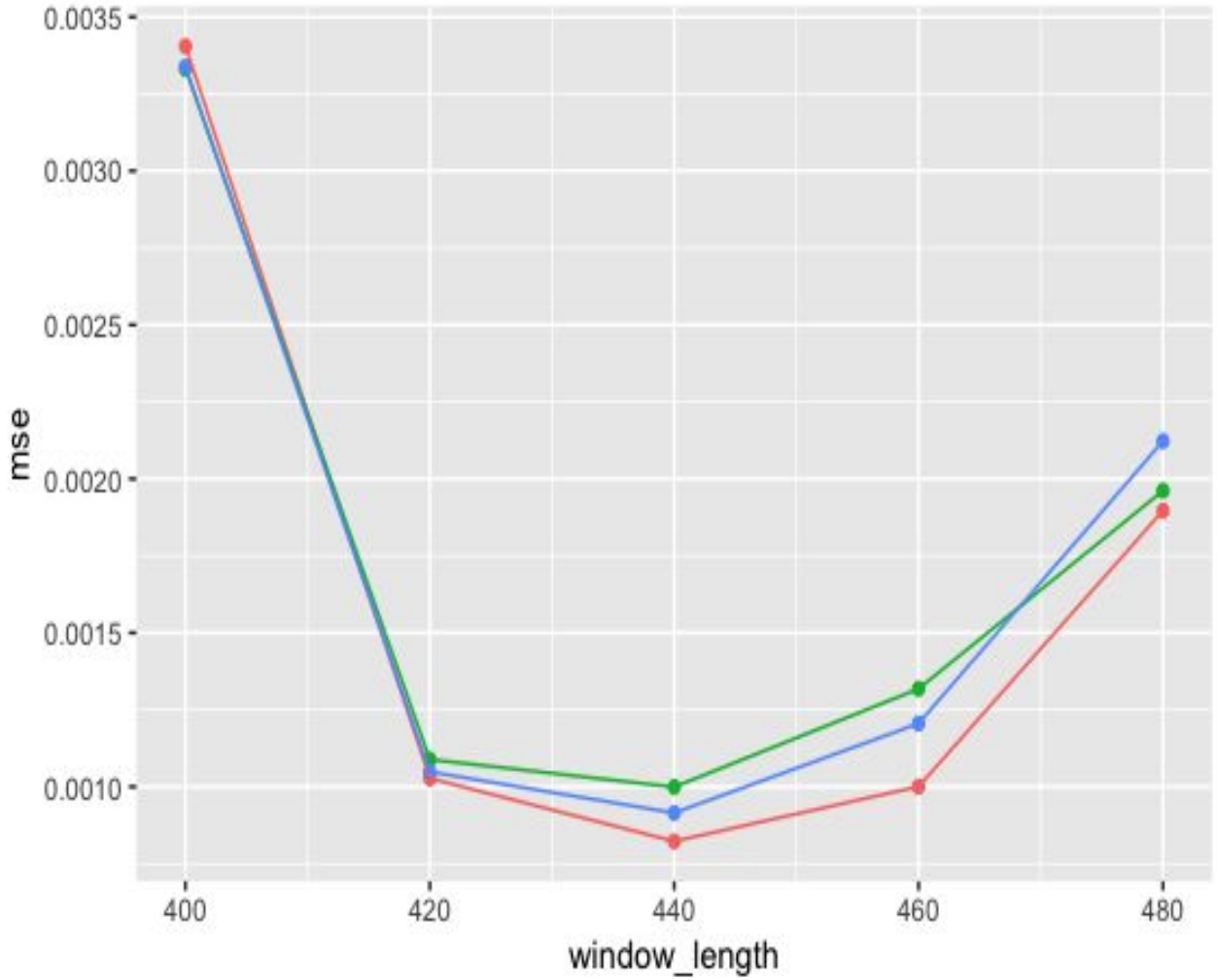
v1 = ugarchforecast(m1, n.ahead=1)



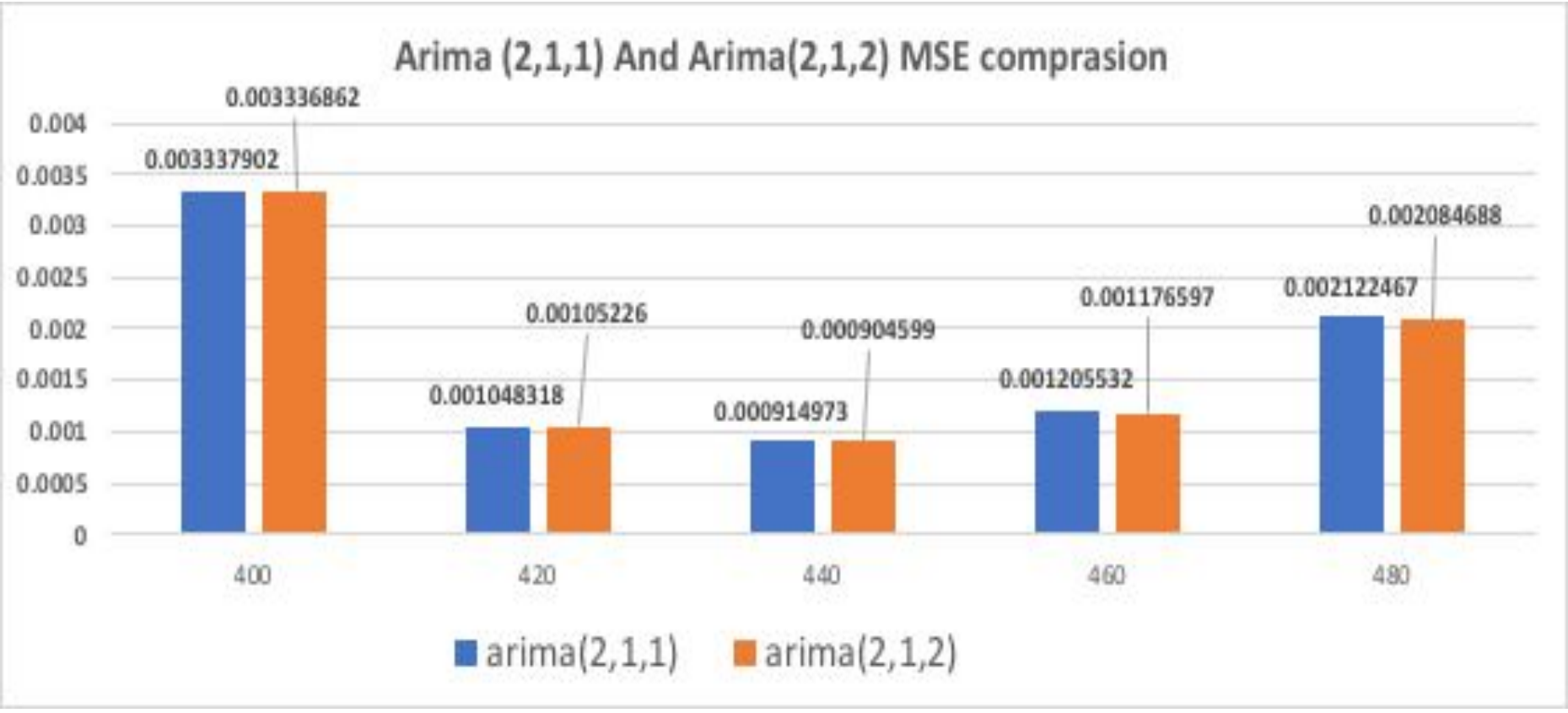
Compare AR order in the mean model.



Compare GARCH order p and q. Nearly no difference.



GARCH(1,1)
AR(2)
ARIMA(2,1,1)
Comparison


Arima (2,1,1) And Arima(2,1,2) MSE comprasion
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
0.003337902
0.003336862
0.001048318
0.00105226
0.000914973
0.000904599
0.001205532
0.001176597
0.002122467
0.002084688
400
420
440
460
480
arima(2,1,1)
arima(2,1,2)

Summary

1. GARCH performs better than ARIMA and AR.
2. ARIMA and GARCH both need large window length to get results. (at least 400)
3. AR order has the largest effect, but fixing AR order, ARIMA and GARCH do improve the result. But it does not show much difference when changing parameters when fixed AR part.
4. So we can pick ARIMA(2,1,1) or GARCH(1,1), good enough basic models.