Main Features of an Option Market Making Algo

1) It needs to derive realized/future volatility predictions for the future

It needs to utilize skew/kurtosis as an input this is an optional component of the model

2) It needs determine the theoretical pricing of an option using volatility, skew/kurtosis, and standard features (expiration, strike, etc) under a formula (europeans are often under BSM, but some other models include the Binomial Option Model, I don't recommend this for European options)
3) It needs to determine what type of position it wants to order - short vol, long vol, neutral vol(this would be under a skew trading; vega neutral)
4) It needs to properly design the position so that it can hedge its greeks (delta for equity exposure, vega for volatility exposure etc)
5) Risk management on the position as it continues onwards into expiration (spread positions can switch as the equity moves in different directions so it is fundamental that the algo realizes and hedges accordingly)

* the algo needs to be constantly checking for thresholds due to greek limitations
*There are other features to be implemented when considering theo. price such as arbitrage opportunities

Deriving/Predicting future volatility:

- Implementations/:
  - We can import multiple volatility models and regress the forecasted variance versus the realized variance in a given data set to determine which is the best volatility model to use in a given time period.
    - https://www.math.vu.nl/~sbhulai/papers/thesis-ladokhin.pdf
    - ^ 2.3; First use RMSE/MHSE to compare the realized and forecasted volatility, second use, using the minimized error values then Model Blending - combines different models to make up for their weaknesses (some models over estimate volatility during periods others underestimate)
  - We can have one volatility model that best suits our scenario (short term volatility within a one month period)
    - This strategy is hard to tell since we don't have much information about the underlying to begin with, most likely not the most efficient solution

- Adjustments/deriving historical volatility:
  - When getting volatility from the underlying we have to adjust the volatility based on time remaining on the option where $\sigma^{new} = \sigma^{calculated} * \sqrt{T}$
    - E.g suppose an underlying has a volatility of 30% yearly. If we wanted a week's duration in volatility we would get $30 \ * \sqrt{\frac{1}{52}} \ = 4.16\%$
  - To estimate the volatility from historical data with stock prices
    - $s = \sqrt{\frac{1}{n+1}\sum_{i=1}^{n}(u - \overline{u})^2}$, where n+1 = number of observations,

      u = $ln(\frac{Si}{Si-1})$, S = stock price at each i interval, $\overline{u} = mean\ of\ u$
    - Once s is found, it should be noted that it has to be transformed based off the time duration (refer to 1A), so $\sigma_{\ estimated} = \frac{s}{\sqrt{t}}$
- Deriving volatility through implied:
  - Note that implied volatility is FORWARD looking whereas historical volatility has already occured
  - This must be done through brute force or an estimator you cannot invert the BSM equation; meaning you must guess the volatility and lower the margin of error until you reach the correct volatility

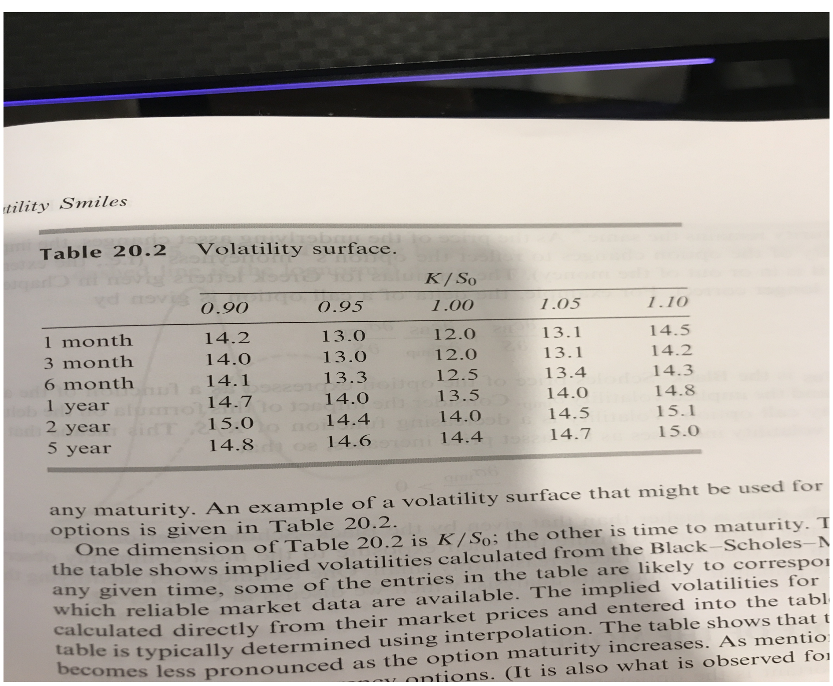
- Types of volatility models:

- GARCH* - garch models are typically better suited for short term volatility; uses conditional volatility (volatility based on prior volatility)
- EGARCH*-
- HAR- https://cran.r-project.org/web/packages/HARModel/HARModel.pdf the har model utilizes realize variances so its better for long term volatility predictions usually utilizes (OLS) to reduce errors
  - https://poseidon01.ssrn.com/delivery.php?ID=920074004105118066118115006064120091062080050007069031088067004076078102105079082096024043056009062004037021066000117089071031011012038076037113009002088068065117030030022009068100027085120021071099019098122003099091100125075103117026099098127000027114&EXT=pdf&INDEX=TRUE paper suggests to use (WLS) weighted least squares over (OLS) and Logarithmic range (LR) over Realized Variance
- HARQ -
- SABR Model* - Adheres the pricing of an option according to the volatility smile https://uu.diva-portal.org/smash/get/diva2:430537/FULLTEXT01.pdf
- https://www.mathworks.com/help/fininst/calibrating-the-sabr-model.html
- https://mfe.baruch.cuny.edu/wp-content/uploads/2015/06/VolWork2-Andrew.pdf

*- stochastic volatility models

- **Volatility is typically adjusted for strike prices which lie outside of the ATM range**
  - This is done using a volatility surface which usually maps $\frac{K}{S_0}$vs Time,

    K is strike price , $S_0$is the stock price
  - For example if you were trying to find the volatility with k/s ratio of 1.05 and an expiration of 9 months you would use a volatility that is interpolated between 9 months and 10 months - 13.7%



tility Smiles

Table 20.2 Volatility surface.

	K/S_0				
	0.90	*0.95*	*1.00*	*1.05*	*1.10*
1 month	14.2	13.0	12.0	13.1	14.5
3 month	14.0	13.0	12.0	13.1	14.2
6 month	14.1	13.3	12.5	13.4	14.3
1 year	14.7	14.0	13.5	14.0	14.8
2 year	15.0	14.4	14.0	14.5	15.1
5 year	14.8	14.6	14.4	14.7	15.0

any maturity. An example of a volatility surface that might be used for options is given in Table 20.2.

One dimension of Table 20.2 is K/S_0; the other is time to maturity. T the table shows implied volatilities calculated from the Black–Scholes–M any given time, some of the entries in the table are likely to correspo which reliable market data are available. The implied volatilities for calculated directly from their market prices and entered into the tabl table is typically determined using interpolation. The table shows that t becomes less pronounced as the option maturity increases. As mentio options. (It is also what is observed fo

- We cannot use a volatility surface for this case due to the fact that all the options expire in the same expiration date
- Instead typically a skew can be used instead
    - Lookup skewadjusted delta - adjusts the delta value of different strike prices dependent on the skew

Simplified Steps for Forecasting volatility

1) Determines volatility using various volatility models
2) Minimize the error values using RMSE/MHSE (essentially compare the estimated vs realized volatility"
3) Blends the estimates volatility values once minimized using a Model Blending
4) Adjust the estimated future volatility based on the skew/surface
    a) The reason we create a volatility surface or a skew is because the B.S.M assumes constant volatility across strike prices; by creating a volatility surface we adjust the values of volatility for different strike prices
5) Use new volatility under theoretical price
    a) Monte Carlo and BSM, and some machine learning models can be used here.

*Possibly include Put-Call parity trades using reversals and conversions since there are no commissions to be paid

Most of this is unnecessary to do… the essential idea of an option market making model is to derive the theoretical value of the option and to sell/buy based on that.

- *Gamma Scalping*
- *Arbitrage - Put Call Parity*
- *Skew- Trading*

Full Steps for Forecasting Volatility

1) Derive σ to be used
    a) $\sigma_{historical}$ through underlying:
        i) Refer to Adjustments in **Deriving/Predicting Future Volatility**
    b) $\sigma_{implied}$ through options data:
        i) Brute force the volatility value using the BSM since there is no way to inverse BSM
        ii) The options that will be used for this calculation are the ATM options
        iii) We can use the bid-ask spread or sold options
2) Use σ inputs in specific models to find σ_{future}
    a) HAR model
        i) Uses $\sigma_{historical}$ to predict σ_{future}
    b) GARCH model
        i) Uses $\sigma_{implied}$ to predict σ_{future}
3) Adjust σusing RMSE or MHSE to adjust model in Step 2
4) Average the σ values to get $\sigma_{future-average}$
    a) Model Blending
        i) We might want to use the bid-ask spread also as a parameter but that should already be included when finding $\sigma_{implied}$ through options data (refer to 1.2)

Applications of forecasted Volatility

5) Input the $\sigma_{future-average}$ into a theoretical pricing model to find Cvalues
    a) Blackscholes model and Monte Carlo
    b) Binomial model (Monte Carlo)
    c) Machine Learning
6) P values can be calculated from the Call Values - unsure if this step is necessary I believe there is a shorter way to derive P values in step 4
    a) Call-Put parity describes the relationship between the Calls and Puts to derive Put price

Risk Management

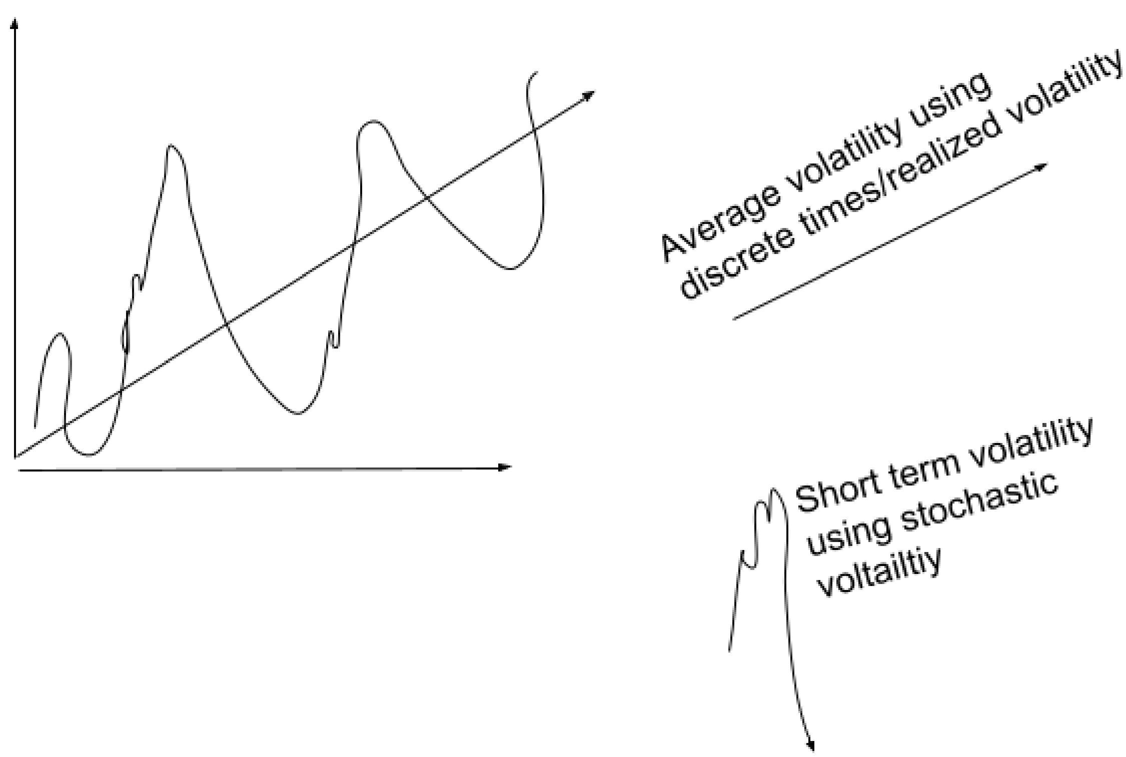
- Implementation:
    - After we decide whether we long/short an option; we need to understand the position we put onto the market. E.G: If we are bullish on the realized volatility and think that the current implied volatility is cheap, we would want to be long on an option. Therefore we would need to be hedged to the equity movements.
    - After the model understands what type of hedge needs to be put on, it needs to understand the frequency on how to hedge.
- An optional implementation:
    - Different strategies can be categorized under different ‘portfolio’, and be all hedged similarly
        - For example all long volatility positions would be placed under the same ‘portfolio’ and the algo will hedge the entire portfolio against delta
- Most likely our main greek hedge will be delta exposure; only if we are trading things like skew would we need vega exposure hedging
- Finally, the algo needs to keep in mind the greek thresholds so that we do not run into a situation where we have too much of one and lost points.

Algorithm’s end processes

1. Algorithm determines types of volatility model to use ->
2. The algorithm predicts the future volatility ->
3. Algorithm determines fair value using a pricing model ->
4. Determines whether or not to short or long a specific greek ->
5. Opens position ->
6. Manages greek values under pre-established thresholds (e.g, if vegas cannot exceed 100, sell contracts to reduce vega exposure) ->
7. Near end of the case, determine method of closing positions (e.g, just because we are delta hedged at the moment the competition ends, our established positions can change in a future month’s notice which is the expiration date of the options)

Notes 3/20/2021

- We decided that we should Monte Carlo Simulation for a long term prediction of what the theoretical value of the option should be
  - This is because the short-term volatility fluctuates around an average of the volatility. In addition, and most importantly, Monte Carlo is unnecessary for short term predictions
- Because we can use Monte Carlo for long term predictions
  - We might want to use the HARQ model for volatility input for the Montecarlo due to the fact that it uses realized volatility in the past and volatility typically fluctuates around a moving average
    - For example in the short term volatility might spike extremely, but in the long term the volatility should remain consistent with the average volatility level (that can be generated more reliably with the HARQ model; I assume HARQ model is also a discrete model and doesn't use stochastics)



- Using the Monte carlo along with something like the HARQ/HAR model we can decide what the option's predicted volatility should be in the month ahead (where we can't delta hedge), and vega/delta hedge our volatility accordingly
  - We might not gamma hedge because in the intermediate time the option could slowly patternized downwards or upwards; gamma is a measure of curvature of delta hence its short term movement that's of concern
  - Use backwardation to the future volatility -
    - Monte carlo gives you the average price value of the option in the future -> use BSM to find the volatility and other inputs such as future delta, future gamma, etc
  - ^ interesting note if we vega hedge our options then we essentially get rid of potential profit that we can make from the spread of theo vs realized option value, however it prevents us from having an extreme loss where the underlying shifts a large value away from its last known value.

Notes 3/26

- Make sure that we understand how we want to compute market derived volatility; apparently we aren't given the options data and we only have the underlying price

Notes 4/2

- Need to store past volatility levels for historical volatility/realized
- Starting volatility is the most recently sold options price
  - Options price -> inverse to find volatility through BSM
- Interesting option pricing model - using machine learning MLP https://www.weareworldquant.com/en/thought-leadership/beyond-black-scholes-a-new-option-for-options-pricing/
- the volatilities that are calculated using the underlying stored data needs to be adjusted for the duration of the remaining options time
  - Refer to 1a
- We cannot use SABR model due to limitations of one expiration date
- Put Call Parity trading -
  - When put call parity is violated and the synthetic position is overpriced/underpriced in comparison to the actual underlying you can open a conversion or reverse to lock in profits
  - http://www.optiontradingpedia.com/conversion_reversal_arbitrage.htm#:~:text=Very%20simply%2C%20a%20Conversion%20is,a%20synthetic%20long%20stock%20position.

Put-Call Parity

The equation expressing put-call parity is:

$$C + PV(x) = P + S$$

where:

- C = price of the European call option
- PV(x) = the present value of the strike price (x), discounted from the value on the expiration date at the risk-free rate
- P = price of the European put
- S = spot price or the current market value of the underlying asset

What Is Conversion & Reversal?

Synthetic positioning allows an open options trading position to be synthetically closed without selling the position itself. For example, a synthetic long stock can be synthetically closed by shorting a corresponding amount of the actual stock. Synthetically closed positions are no longer be subject to directional risk and serves to hedge against short term price swings.

Very simply, a Conversion is when stock is being bought in order to synthetically close out a synthetic short stock position and a Reversal is when stock is being shorted in order to synthetically close out a synthetic long stock position. However, Conversion & Reversal do not necessarily apply only to synthetic long and short stocks.

In fact, whenever a synthetically closed position involves a long stock, it is known as a Conversion and whenever it involves a short stock, it is known as a Reversal. For example, a synthetic call option consisting of a long put and a long stock can be synthetically closed by shorting a call option. This is referred to as a Conversion as the whole position involves a long stock. A synthetic put option consisting of a long call and short stock can be synthetically closed by shorting a put option. This is referred to as a Reversal as the position involves a short stock.

Synthetic Position	Components	Closing Instrument	Classification
Synthetic Long Stock	Long Call + Short Put	Short Stock	**Reversal**
Synthetic Short Stock	Short Call + Long Put	Long Stock	**Conversion**
Synthetic Long Call	Long Stock + Long Put	Short Call	**Conversion**
Synthetic Short Call	Short Stock + Short Put	Long Call	**Reversal**
Synthetic Long Put	Short Stock + Long Call	Short Put	**Reversal**
Synthetic Short Put	Short Call + Long Stock	Short Put	**Conversion**

When a position is synthetically closed using Conversion & Reversal, it is subjected only to theta risk, which is time decay on the extrinsic value of the long options involved in the position.

Notes 4/3

- https://github.com/RomanMichaelPaolucci/Algorithmic_Delta_Hedging/blob/master/options/euro_option_analysis.py
-
- ^ delta hedging code
- Note that when delta hedging option positions; we must utilize the correct type of hedge by purchasing the underlying and separating hedging by different strike prices.
- These underlying positions must be offloaded in the case where our delta's reach too high according to the thresholds.

- These contract positions must be offloaded in the case where our gamma, theta, vega, are too high according to the thresholds.
- These contracts/underlyings can be offloaded by taking a position through the other instruments:
- For example if we have 2 long Calls with a delta of .50, we have a total delta of (.50 * 2 * 100). If we delta hedge with 100 underlings (1 * 100), although we reduced our delta exposure, we increased our gamma exposure through option contracts. Now assume we want to go long 1 more Call with .50 deltas, if we were to reach a gamma threshold at that point, we could instead open a short position on the Put and delta hedge with the underlying. Here we opened the same type of delta position but reduced our gamma exposure (since we are shorting the option and not going long on it). This comes at risks including unlimitless loss on the short position where your total loss would equal (premium - (diff between short price and stock price))