Qubit dynamics under alternating controls: applications in diamond

Clarice D. Aiello's PhD thesis defense, July 25th, 2014

Quantum Engineering Group @ MIT

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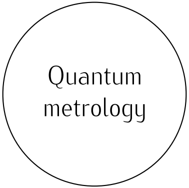
Qubit

Controls

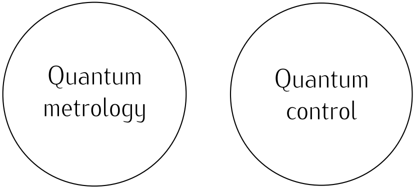
Qubit

Controls

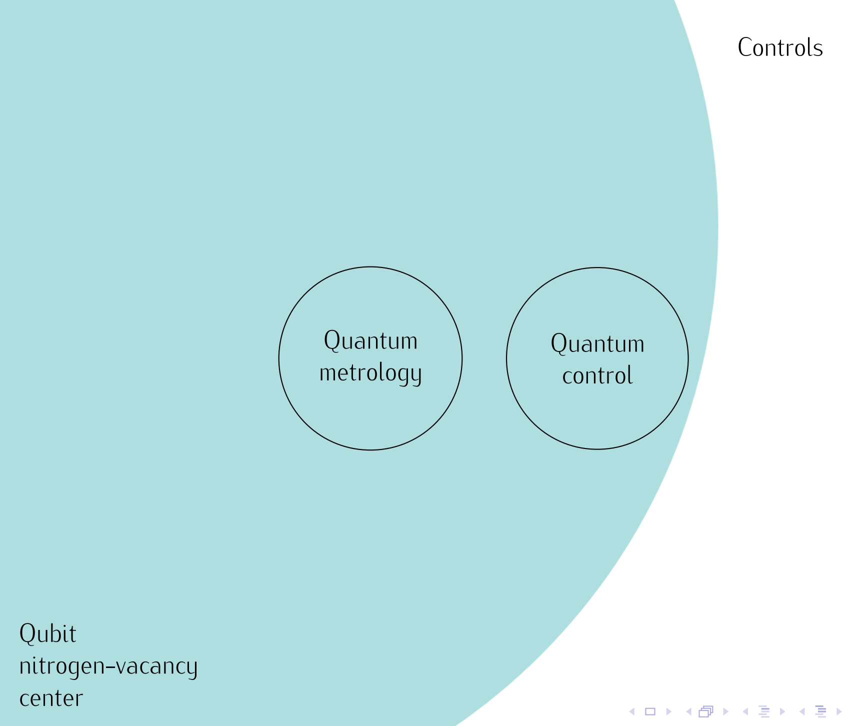


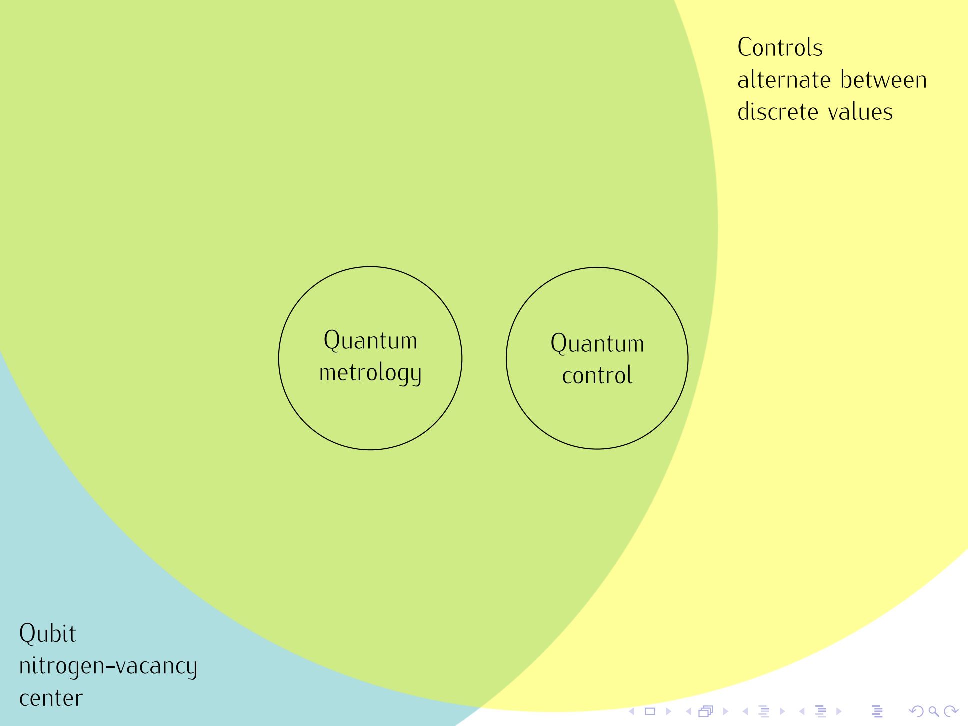
Qubit

Controls



Qubit


Controls
Quantum metrology
Quantum control
Qubit nitrogen-vacancy center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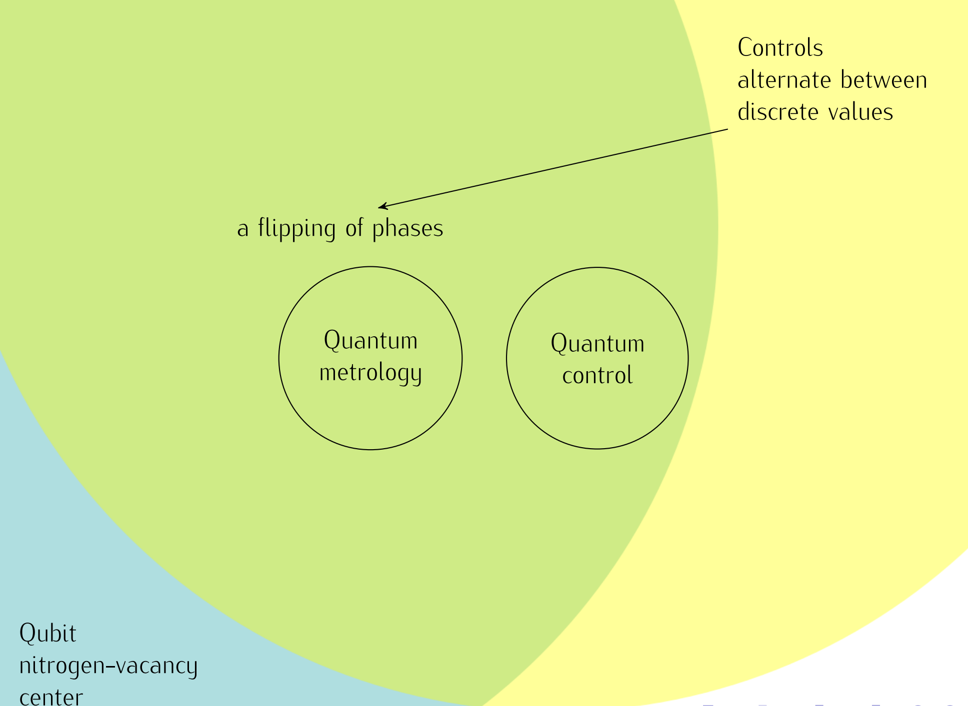
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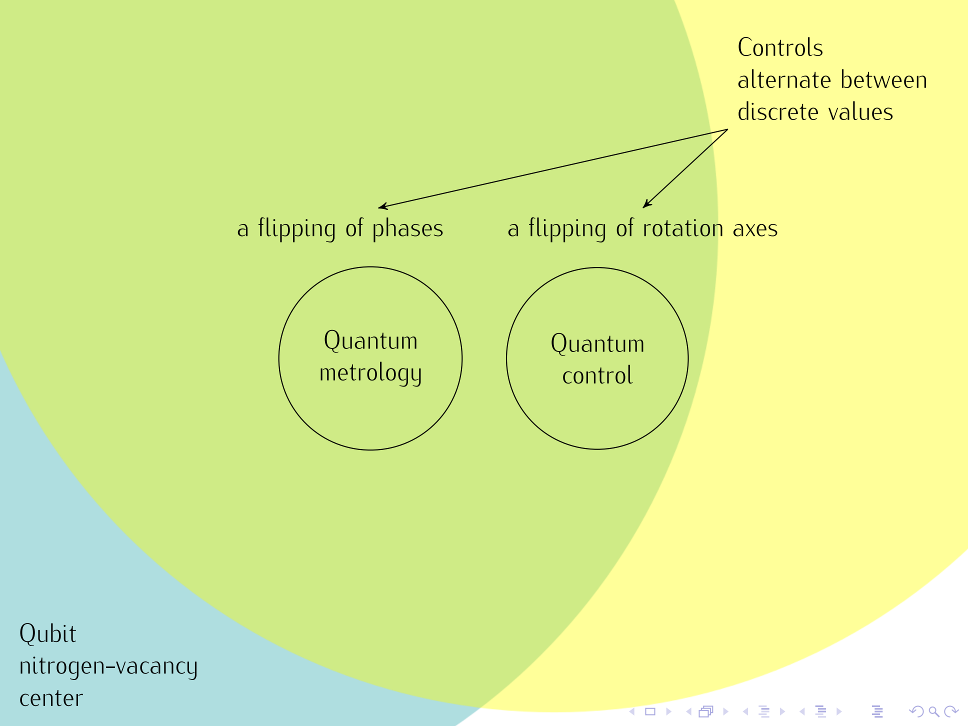
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Qubit
'NV'
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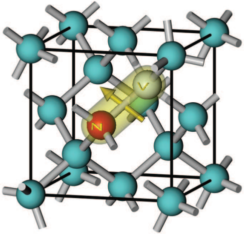
NV center is one of many defects in diamond

500+ naturally occurring defects in diamond!

- vacancy next to substitutional nitrogen
- localized
- also created by irradiation + annealing
- absorbs light and fluoresces



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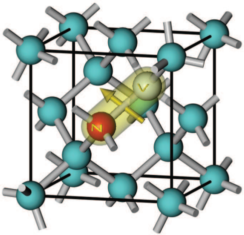

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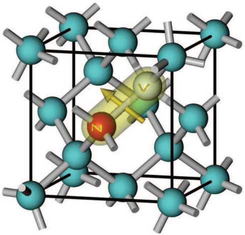

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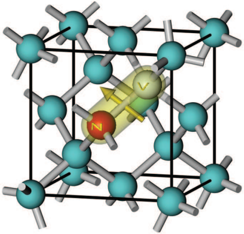

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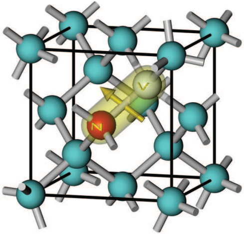

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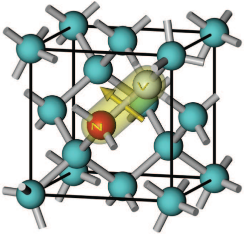

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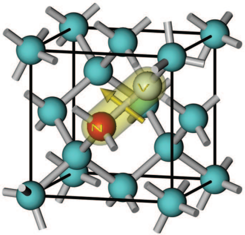

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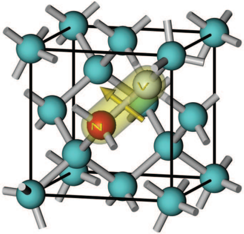
  $\Rightarrow$ state-dependent fluorescence intensity
- very long $T_2 \approx$ ms *at room temperature*



Tracking individual NV centers:

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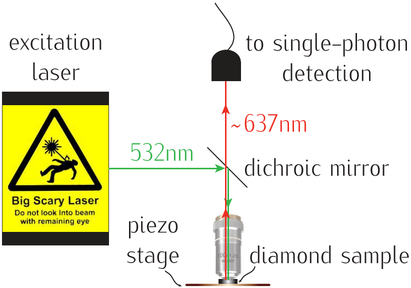
Tracking individual NV centers: confocal imaging



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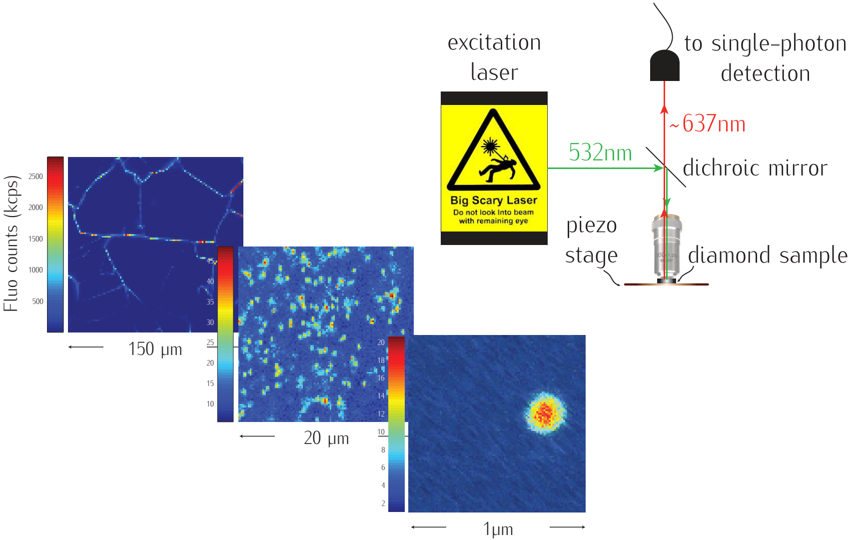

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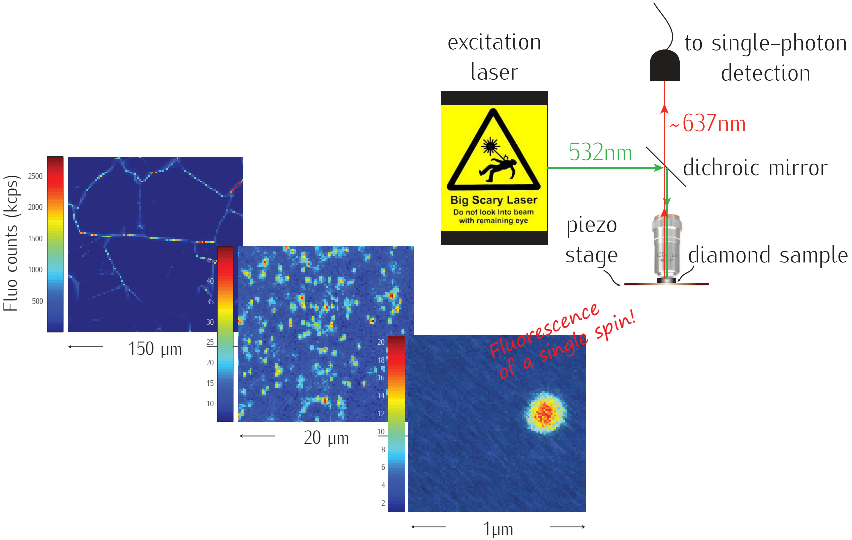
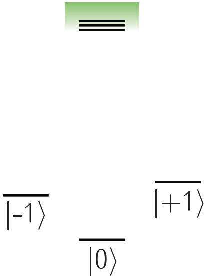


State-dependent fluorescence is linked to energy levels

non-resolvable
excited states

ground
states



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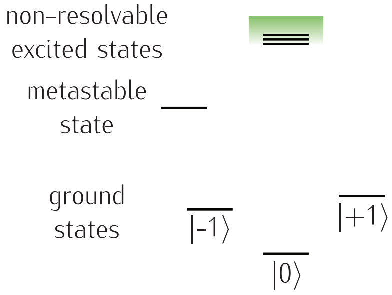

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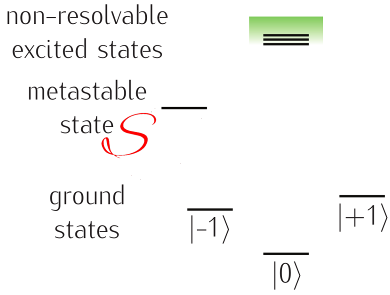

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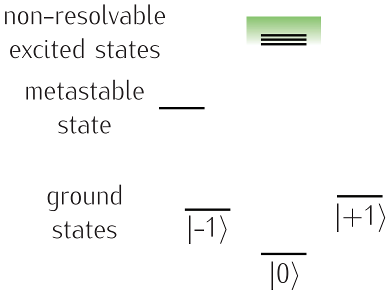

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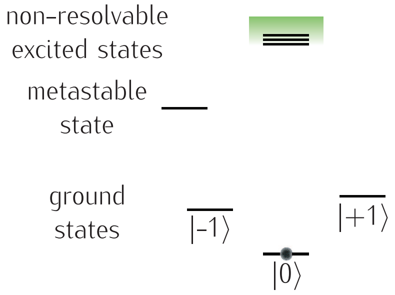

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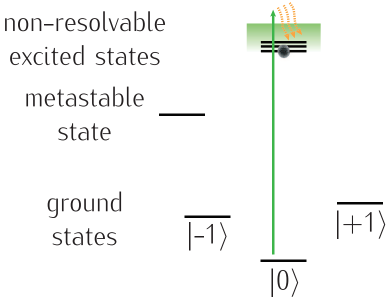

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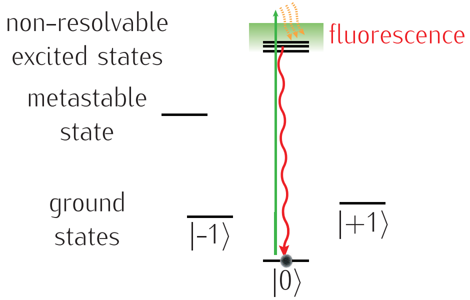

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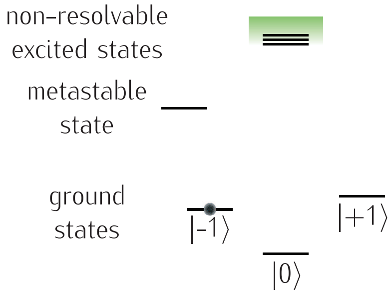

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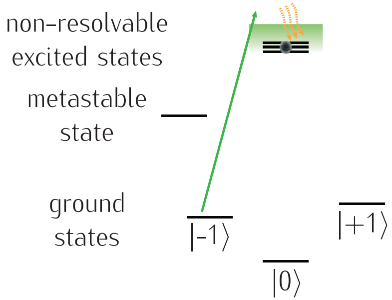

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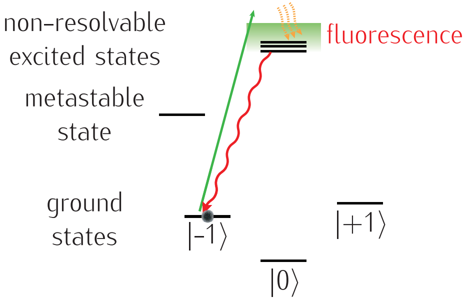

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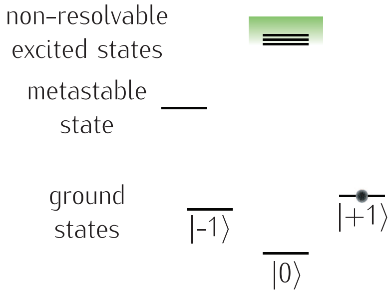

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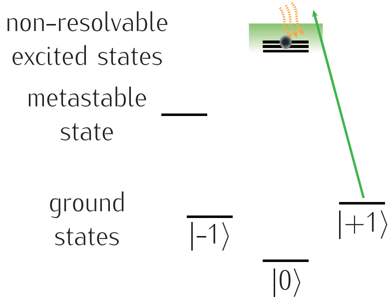

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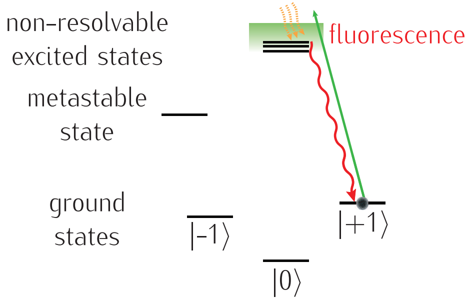

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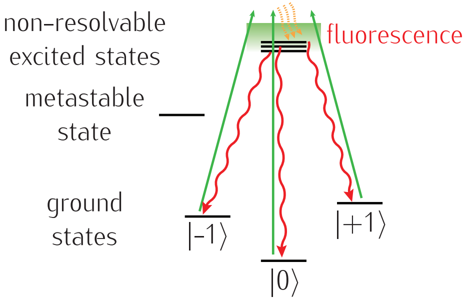

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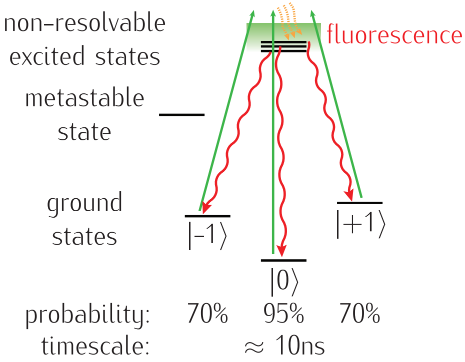

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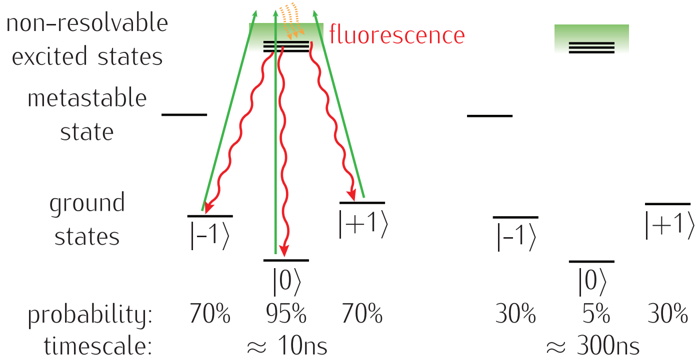

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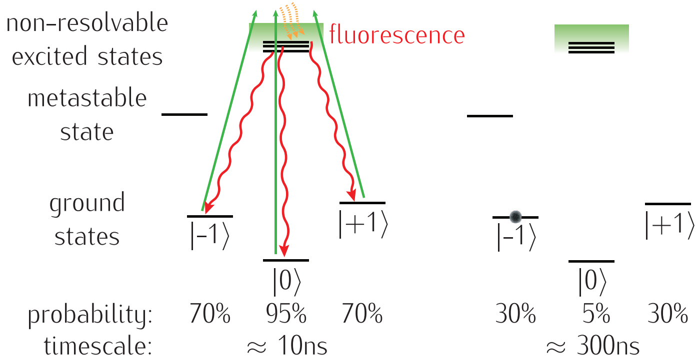

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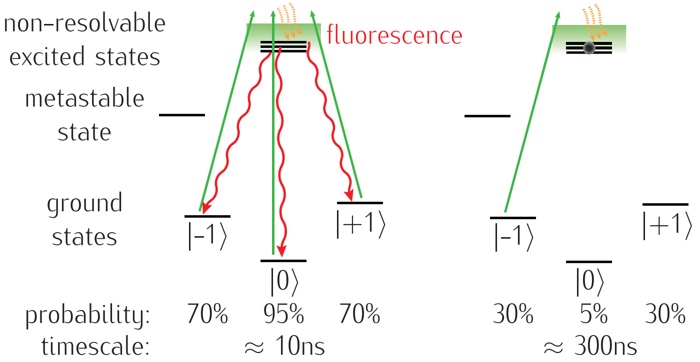

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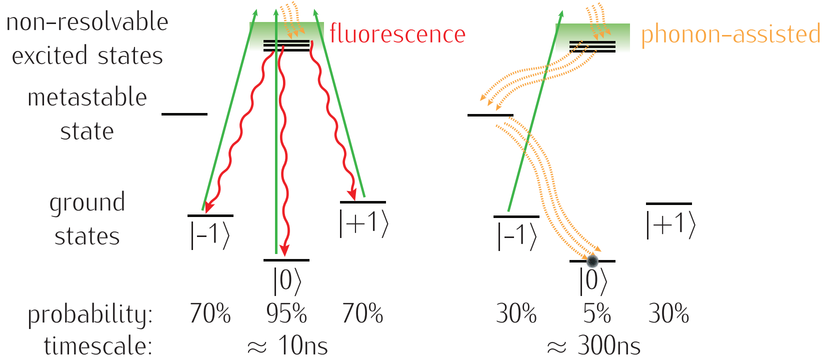

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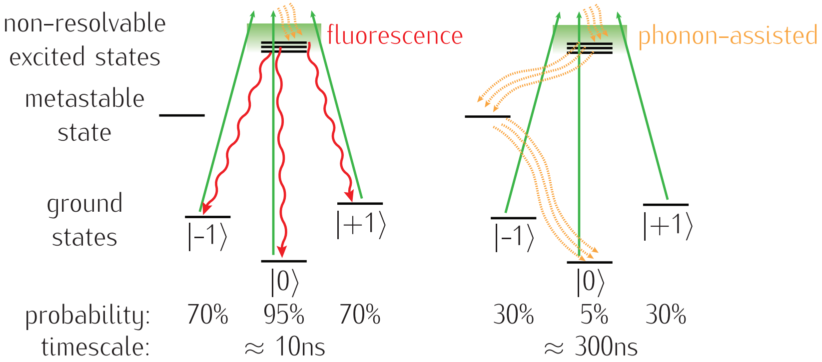

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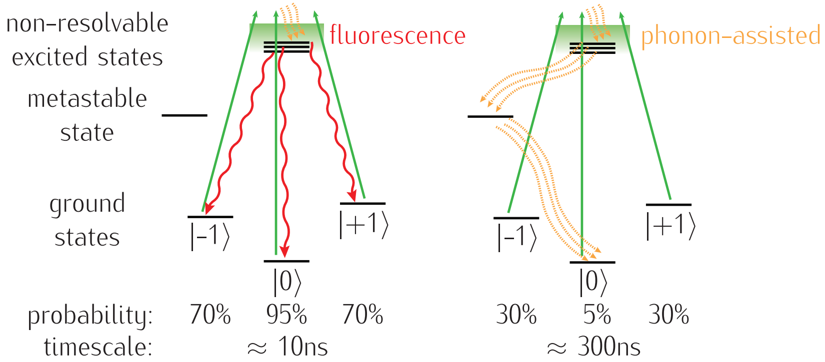

- after some time, regardless of the NV initial state, ends up in $|0\rangle$
  'initialization'

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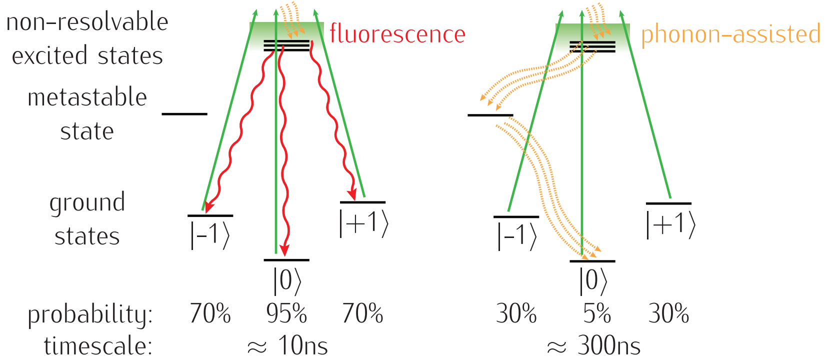
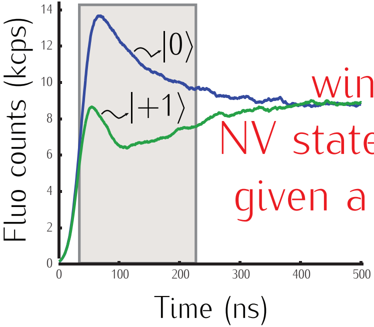


- after some time, regardless of the NV initial state, ends up in $|0\rangle$
  'initialization'
- before that time, state is distinguishable by fluorescence intensity
  'read-out'

Single NV state is observed by fluorescence



window during which
NV states can be distinguished
given a fluorescence intensity

compare:
unobservable NMR signal from the tiny magnetic moment of a single spin

The NV center can be controlled with photons

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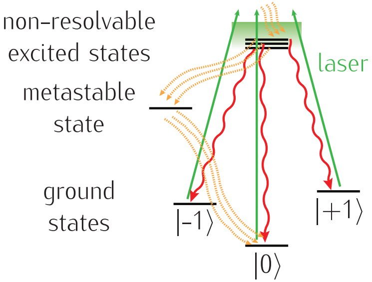
- laser excites NV off-resonantly



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- laser excites NV off-resonantly
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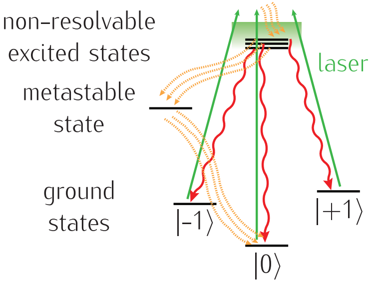

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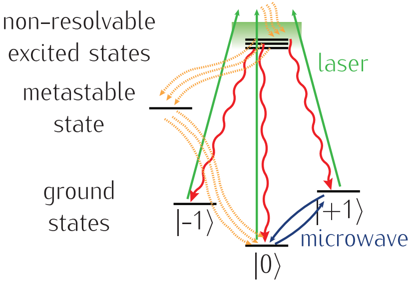

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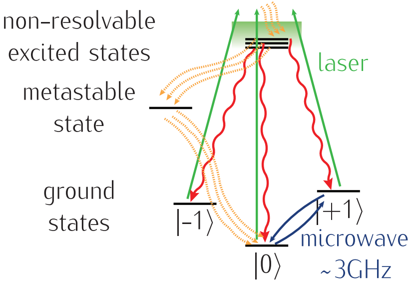

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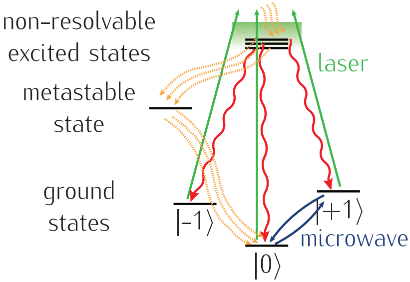

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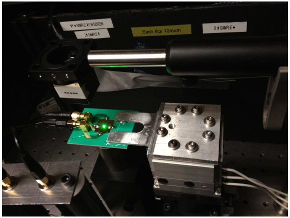

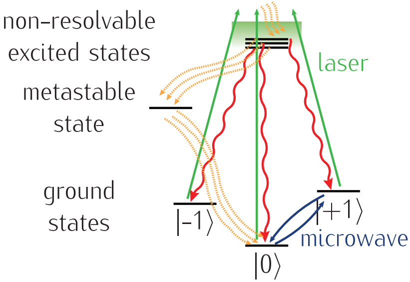

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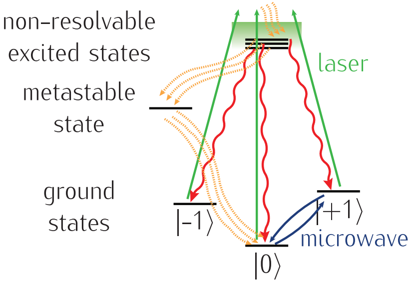

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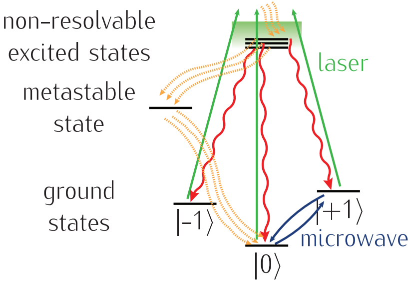
  $\Rightarrow$ coherent control



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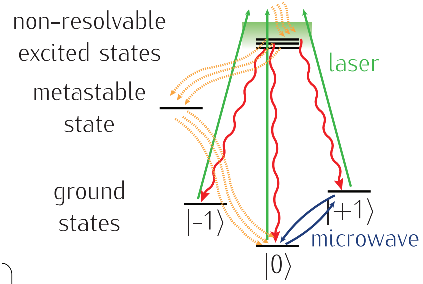

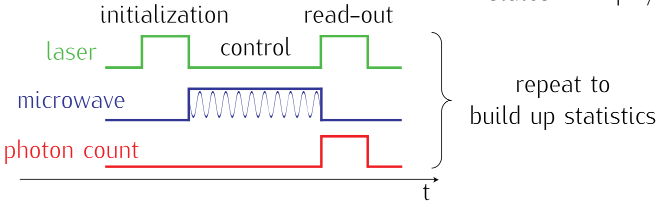

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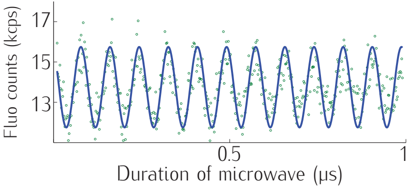

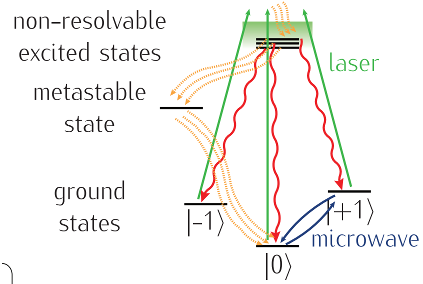

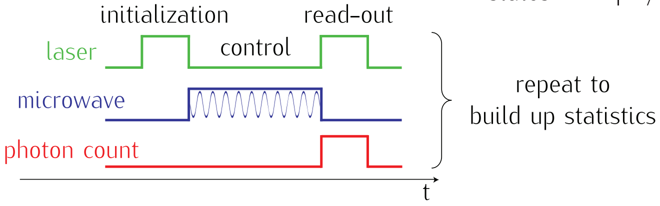

To measure things better is to understand nature better

better frequencies

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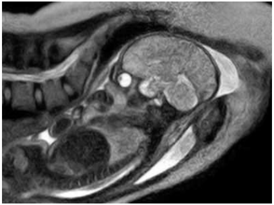

better magnetic fields

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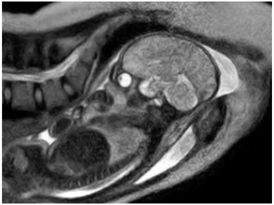

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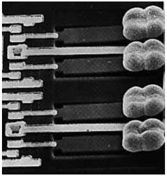

better accelerations

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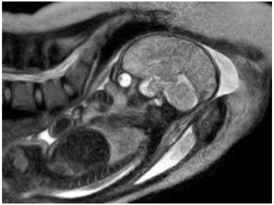

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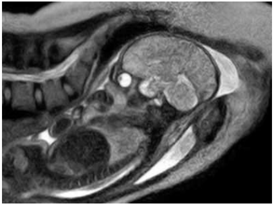

better magnetic fields

better accelerations

what if...

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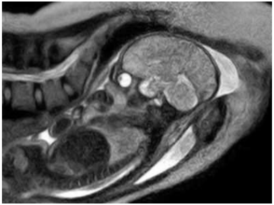

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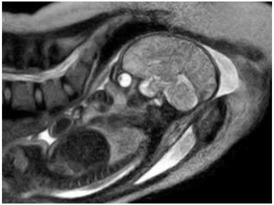
better accelerations

what if...

...the quantity to be measured is way too small?

To measure things better is to understand nature better

better frequencies



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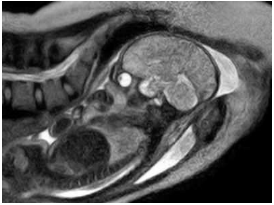

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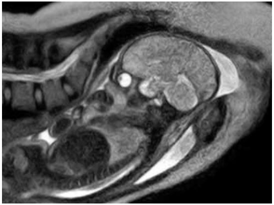

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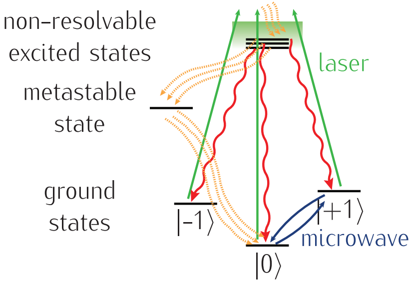
here:

sensing tiny magnetic fields with the NV center in diamond

NV magnetometer: principle of operation



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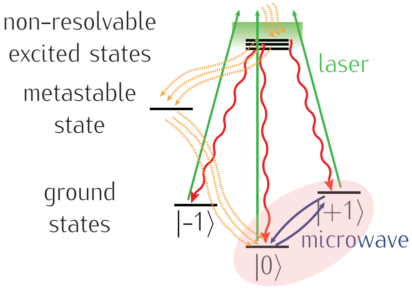

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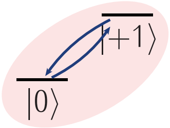

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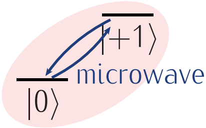

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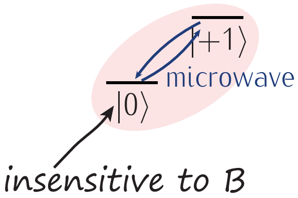

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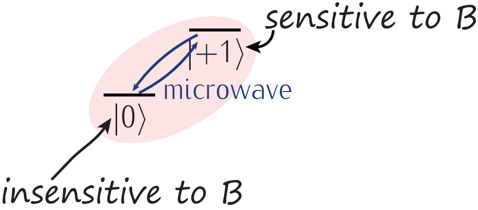

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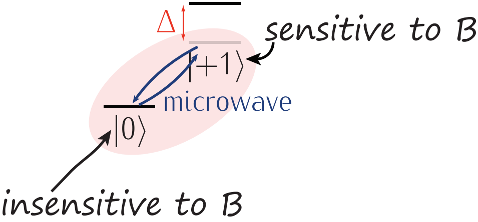

Zeeman shift $\Delta \propto B$
(gyro $\gamma_e \sim 2.8$MHz/G)

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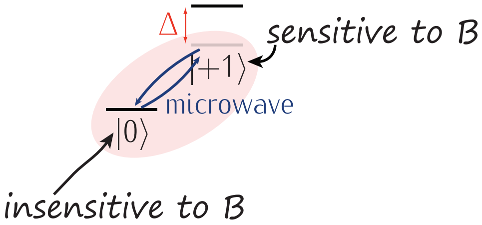


Zeeman shift $\Delta \propto$ B
(gyro $\gamma_e \sim 2.8$MHz/G)

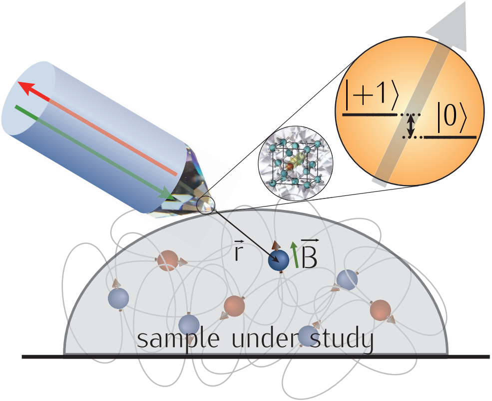
measure B
$\Updownarrow$
measure
detuning from resonance Δ

The tiny NV magnetometer senses tiny magnetic fields

The tiny NV magnetometer senses tiny detunings

The tiny NV magnetometer senses tiny detunings

- nanoscale probe brought to proximity of sample
- senses detunings caused by few or individual spins



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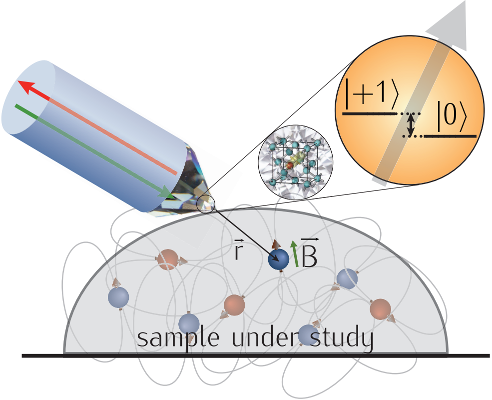
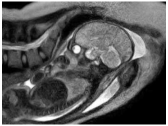
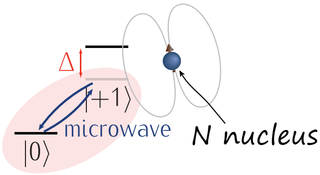
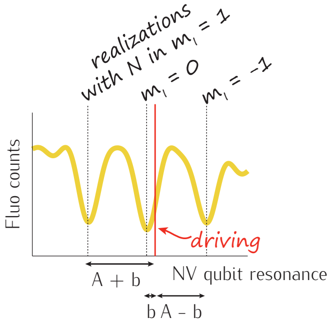




The NV has a built-in testbed magnetic field source





- hyperfine with spin-1 ^{14}N, A $\sim$ 2MHz

- drive center resonance (^{14}N in $m_I = 0$), expect 3 detunings:

$$\color{red}{\Delta} = \{ \text{ b, A} \pm \text{b } \}, \text{ b detuning from presumed resonance}$$

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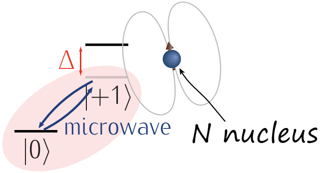

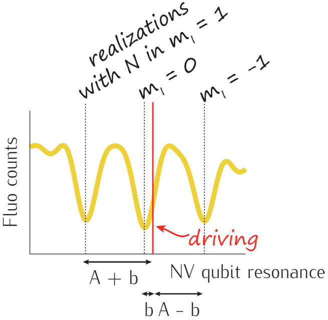

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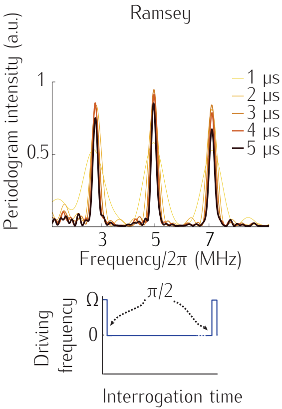
... but what's the best way to measure those detunings?

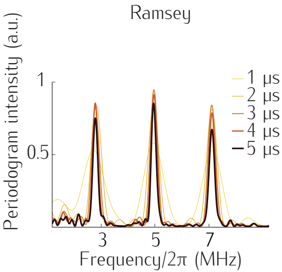
Benchmark answer: Ramsey spectroscopy

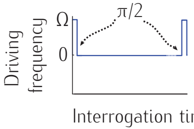


✓ Signal $\sim \cos(t\Delta)$

✗ $T_2^\star$–limited

Alternative answer: Rabi spectroscopy





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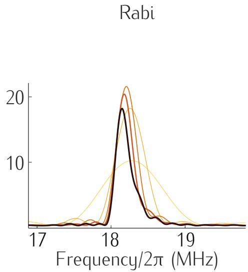
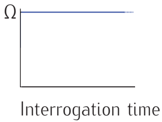


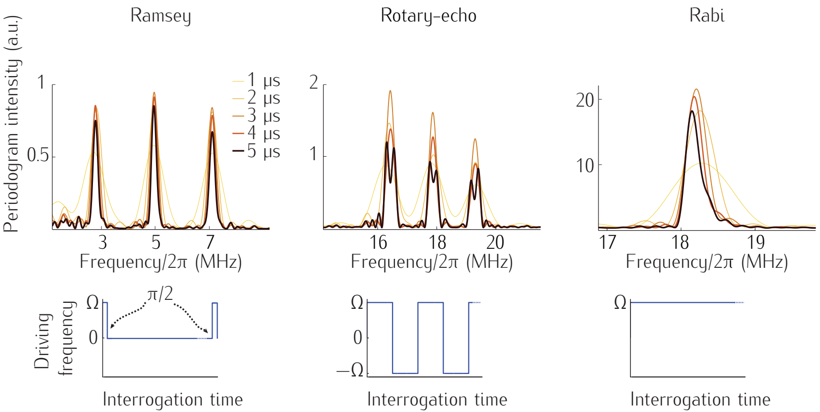


✗ Signal $\sim \cos\left(\frac{t\Delta^2}{\Omega}\right)$

✓ T_1–limited

Fedder et al., Appl. Phys. B **102**, 497 (2011)

Our answer: Rotary-echo spectroscopy



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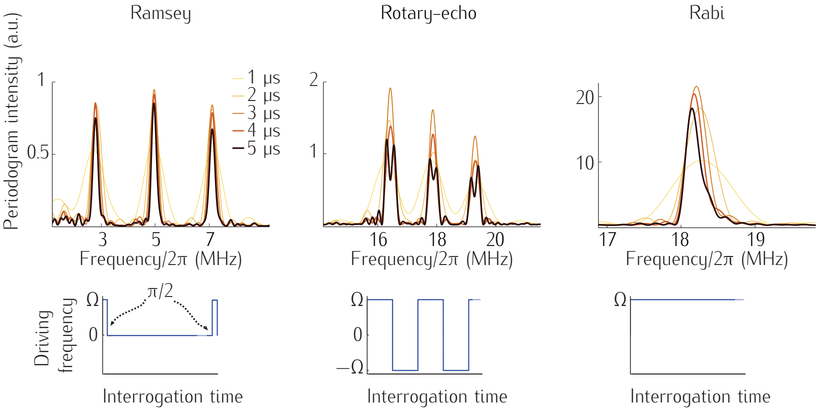
Fedder et al., Appl. Phys. B **102**, 497 (2011)

Our answer: 'RE' spectroscopy



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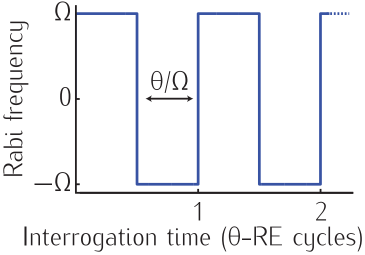
Fedder et al., Appl. Phys. B **102**, 497 (2011)

Dynamics under RE sequence is now better understood

- NMR origins Solomon, Phys. Rev. **110**, 61 (1958)
- corrects driving field imperfections
- does not correct detunings from resonance ✓
- length of pulse $\sim \theta$



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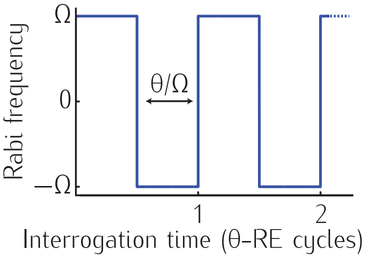

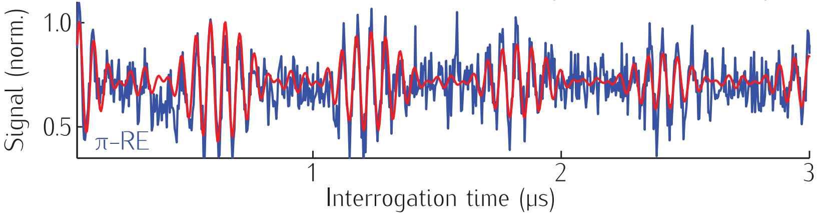

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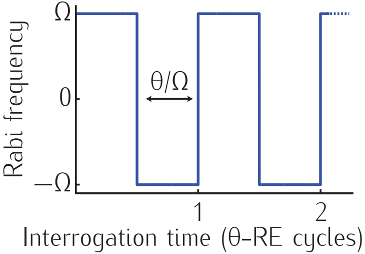

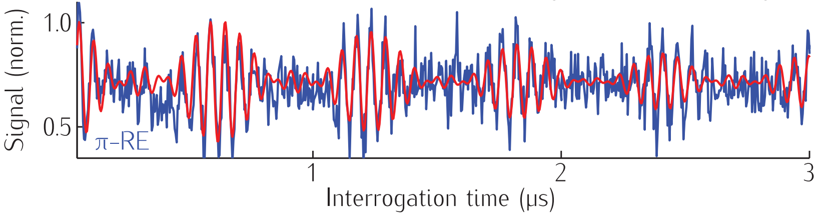

$$\text{Signal} \simeq \frac{1}{2}\left[1+\cos^2\left(\frac{\theta}{2}\right)+\sin^2\left(\frac{\theta}{2}\right)\cos\left(\frac{2t\Delta}{\theta}\sin\left(\frac{\theta}{2}\right)\right)\cos\left(\frac{\pi t\Omega}{\theta}\right)\right]$$

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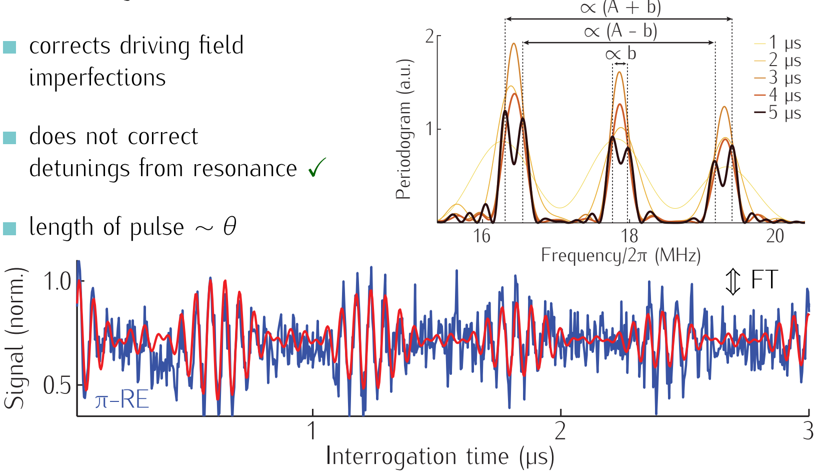

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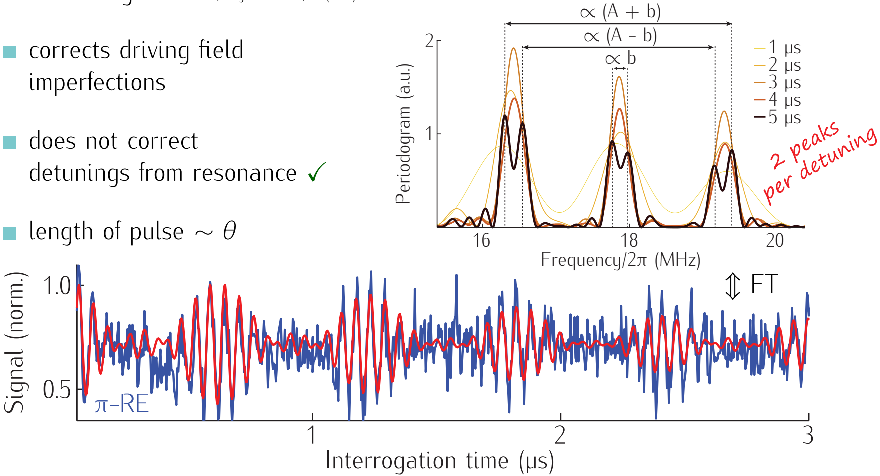

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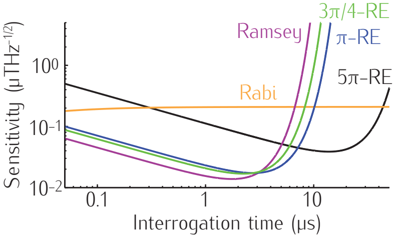
Sensitivities of RE can be comparable to Ramsey's...

$$\eta = \frac{1}{\gamma_e} \lim_{\Delta \to 0} \frac{\sigma_S}{\left|\frac{\partial S}{\partial \Delta}\right|} \sqrt{t},$$

for signal S and interrogation time t



(assuming static bath noise)

$$\eta_{Rabi} \sim \frac{\sqrt{2\Omega}}{\gamma_e} \qquad \eta_{Ram} = \frac{1}{\gamma_e \sqrt{t}} \qquad \eta_{RE} = \frac{1}{\gamma_e \sqrt{t}} \frac{\theta}{1 - \cos\theta}$$

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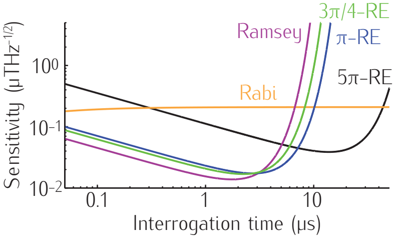

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$\uparrow$ with $\uparrow \theta$

min for $\theta = 3\pi/4$

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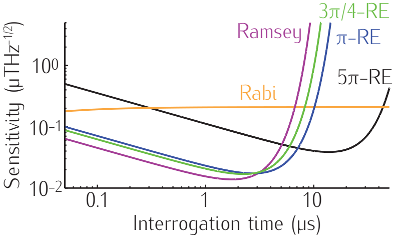

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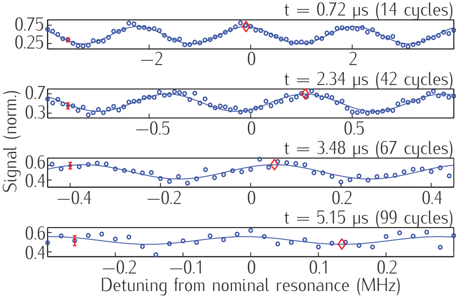

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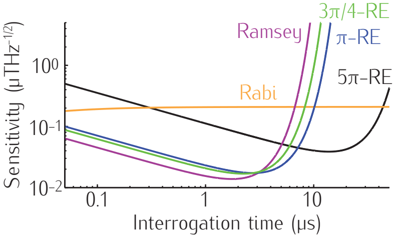

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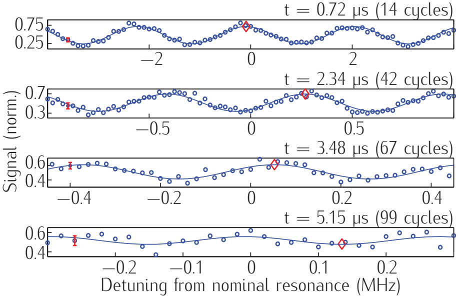

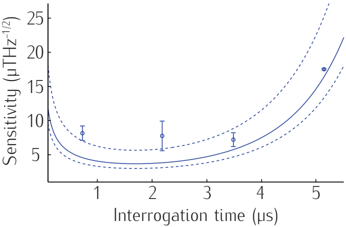

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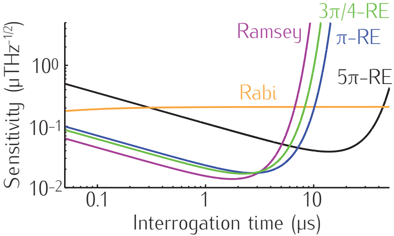

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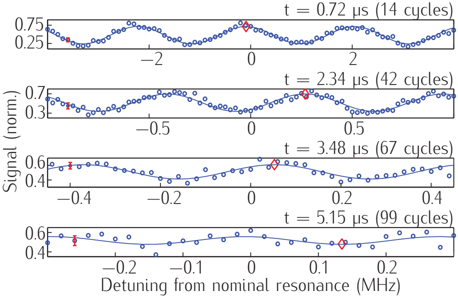

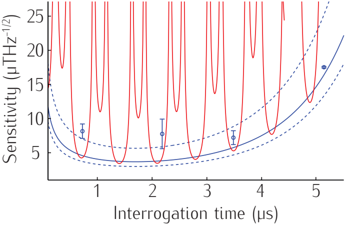

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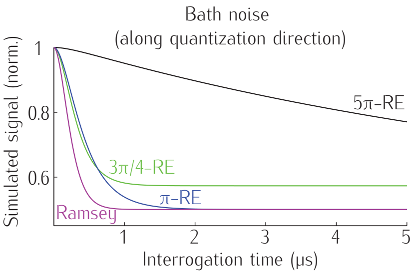

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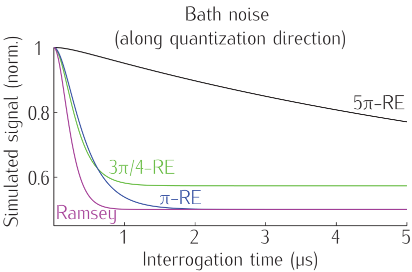


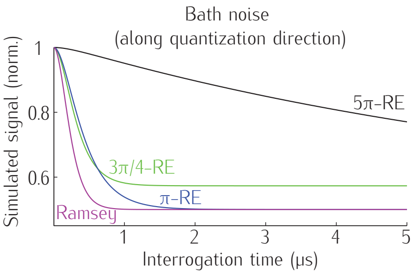
$\Rightarrow$ Trade-off: sensitivity $\times$ coherence times

$\uparrow \theta$

η worsens

$\uparrow$ resilience to bath noise

RE is resilient to different types of noise



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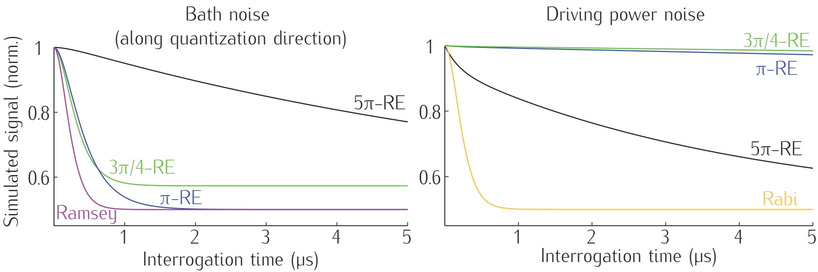

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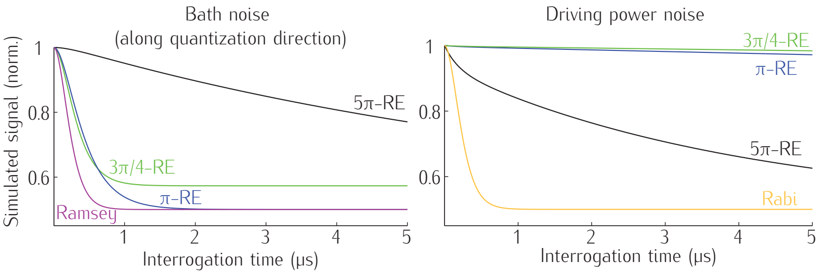

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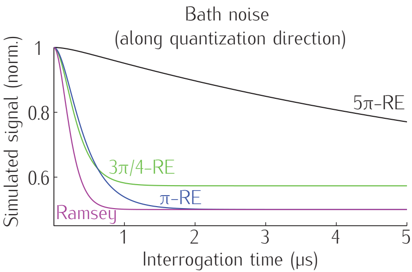

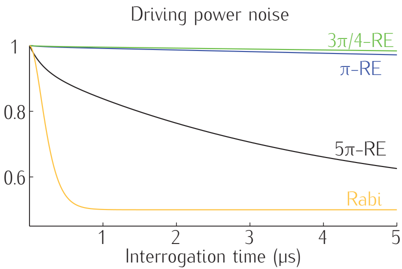

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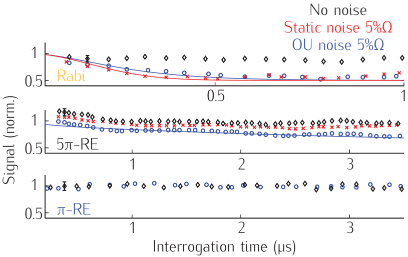

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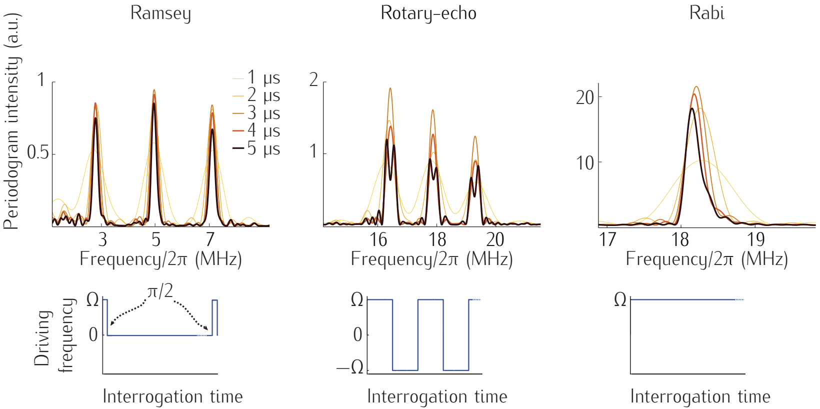
Choice of θ yields flexibility



Ramsey	Rotary-echo	Rabi
✓ Signal $\sim \cos(t\Delta)$	✓ Signal $\sim \cos(\kappa t\Delta)$	✗ Signal $\sim \cos\left(\frac{t\Delta^2}{\Omega}\right)$
✗ $T_2^\star$–limited	✓ intermediate times	✓ T_1–limited

Fedder et al., Appl. Phys. B **102**, 497 (2011)

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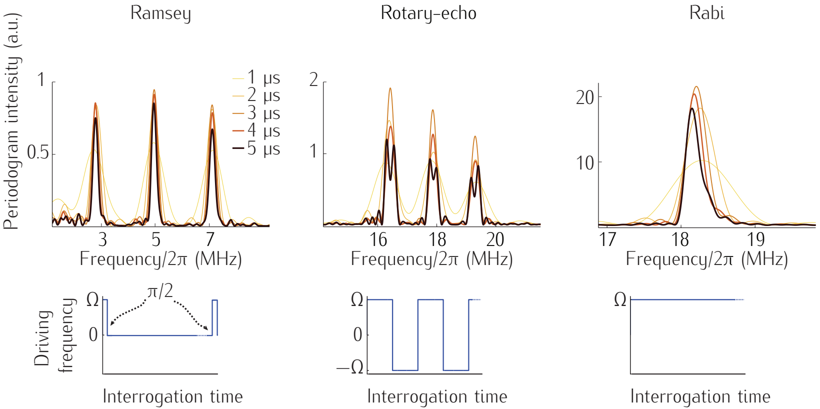

Ramsey	Rotary-echo	Rabi
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✗ $T_2^\star$–limited	✓ intermediate times	✓ T_1–limited
	✓ choice of θ	

Fedder et al., Appl. Phys. B **102**, 497 (2011)

Sensing in variable environments requires flexibility

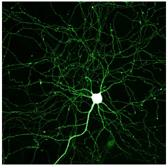
Maybe...

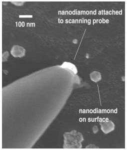
- ...field from samples under study have different lifetimes

  $\Rightarrow$ match RE interrogation time to each particular sample

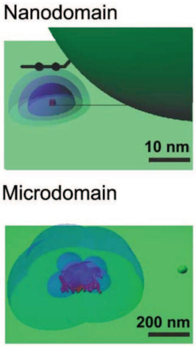
- ...unknown noise sources

  $\Rightarrow$ tune sensor protection by choosing θ accordingly



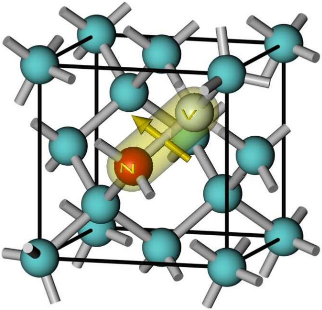


Degen, Appl. Phys. Lett. **92**, 243111 (2008)



Augustine et al., Neuron **40**, 331 (2003)

Placing the NV into the background



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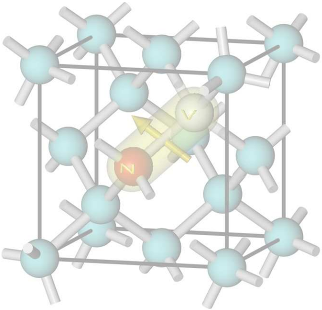

Grandma, here's what I'm up to

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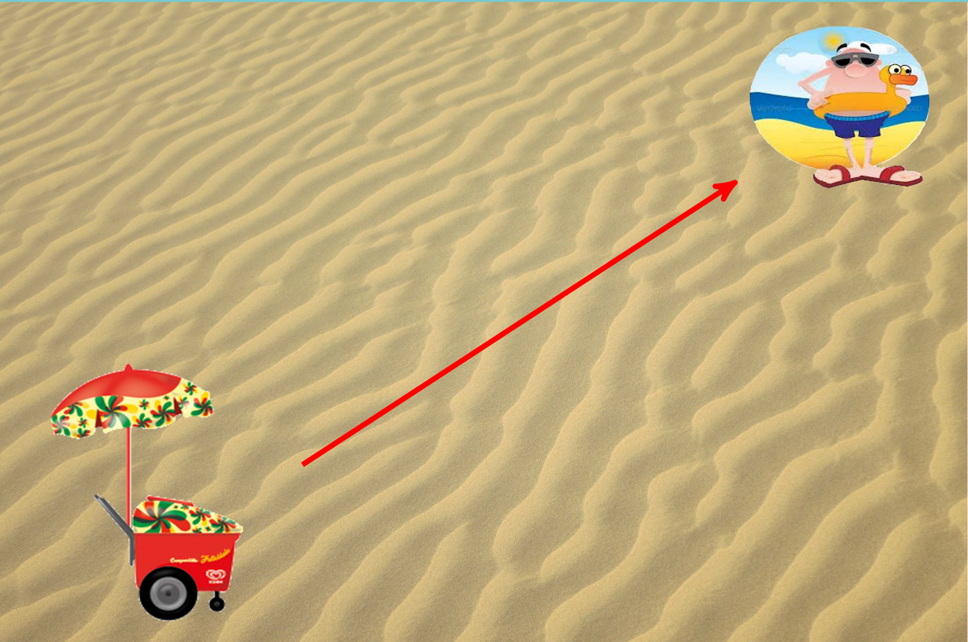
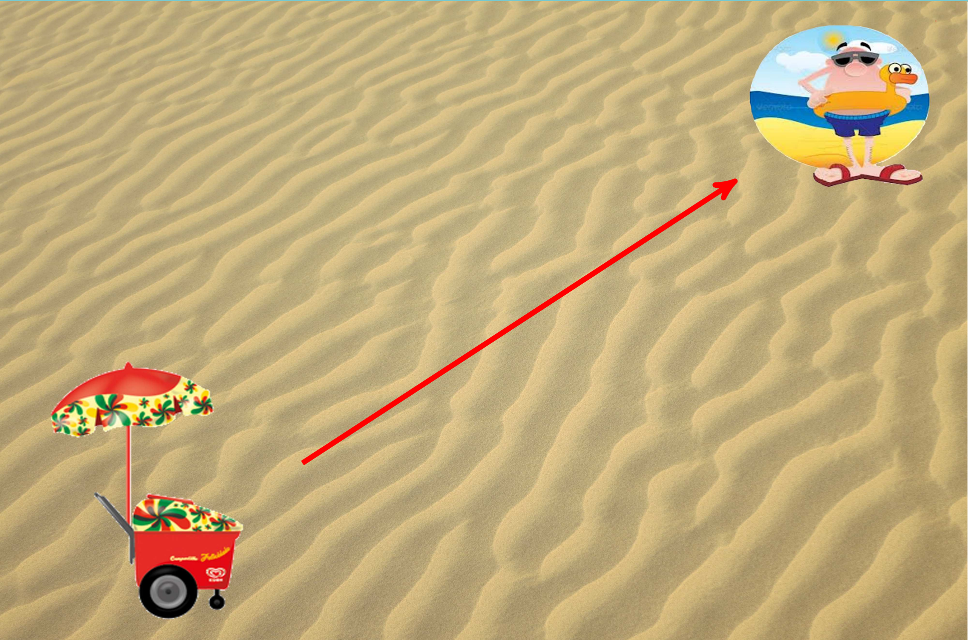


Fast ice-cream delivery... with a broken cart?



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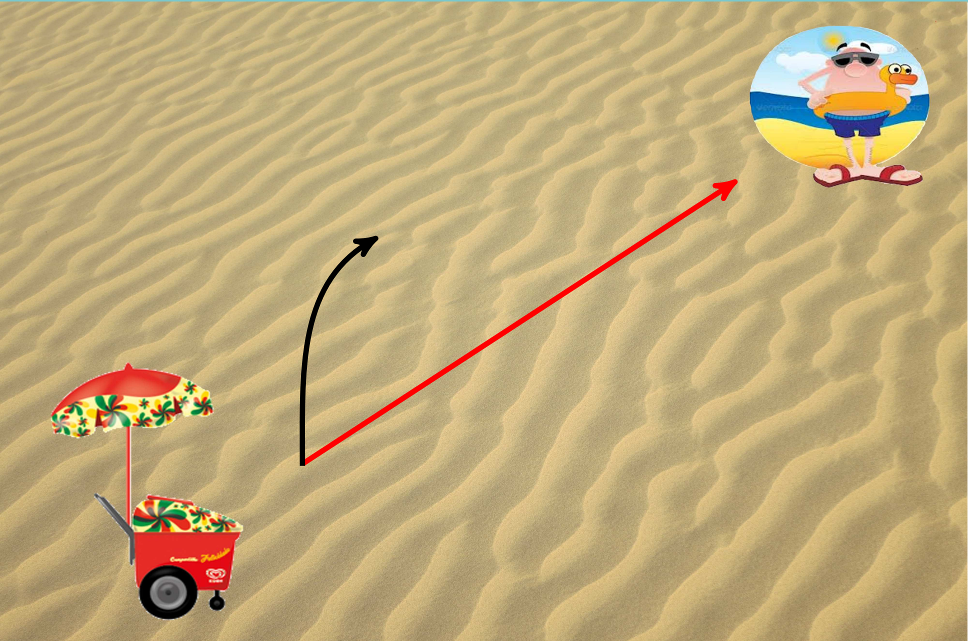

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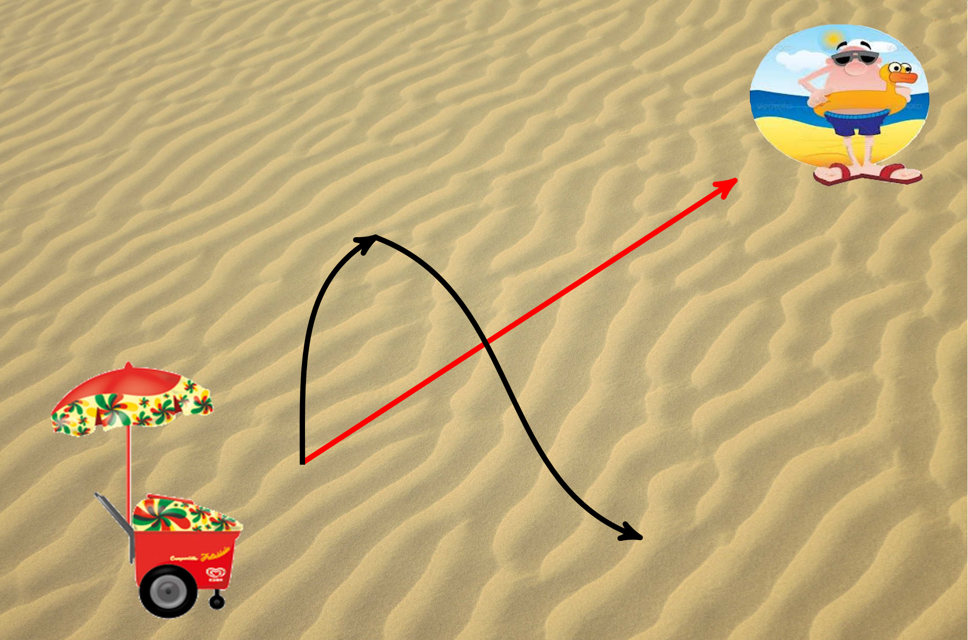

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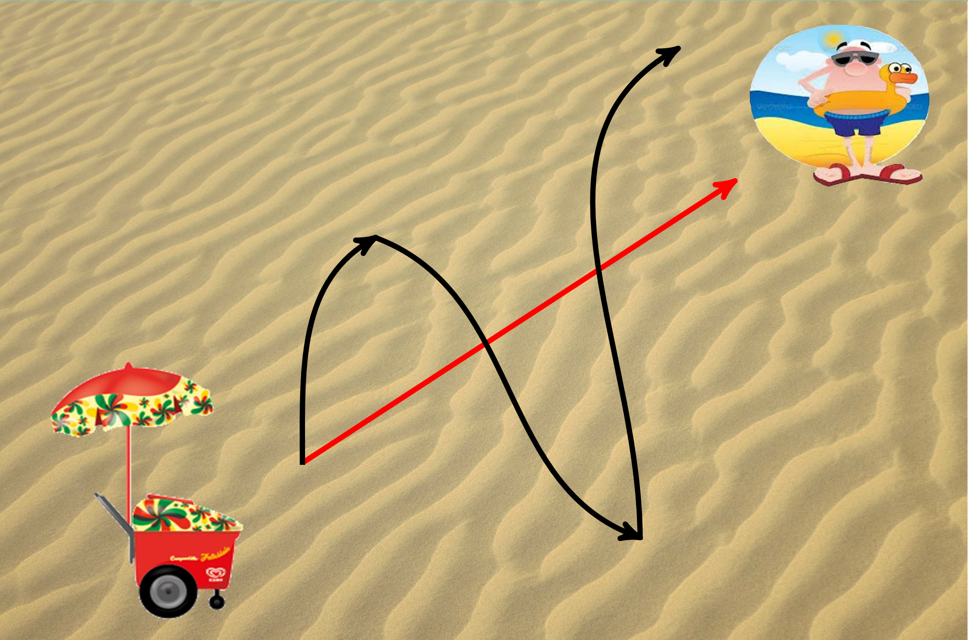

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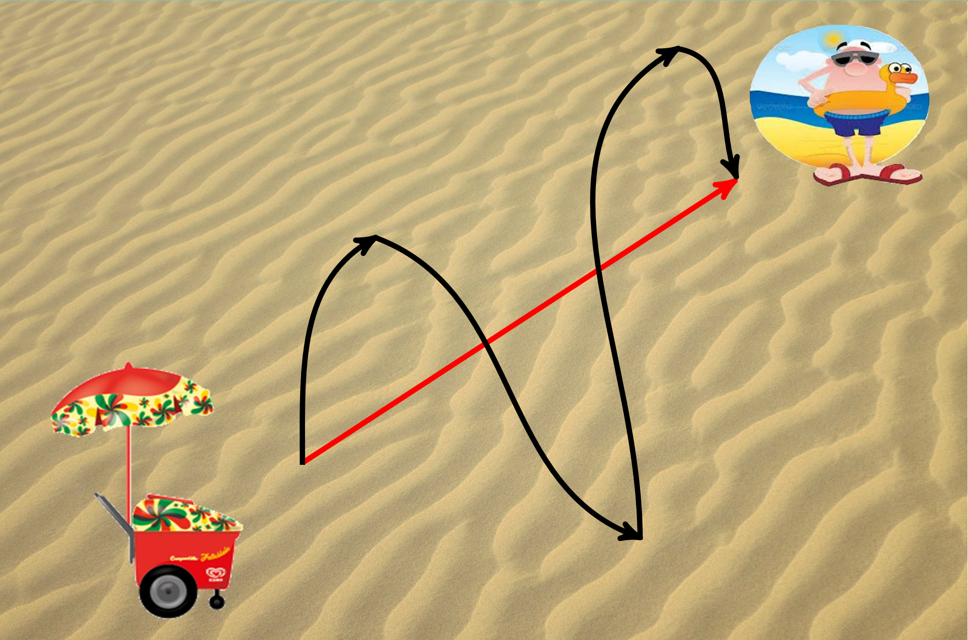
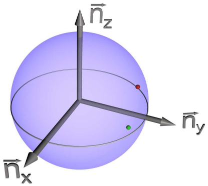


Physics equivalent:

Time-optimal qubit transfer with restricted controls

Time-optimal qubit transfer with restricted controls

- qubit state: point on surface of Bloch sphere



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- qubit state: point on surface of Bloch sphere
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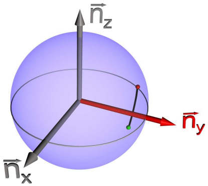

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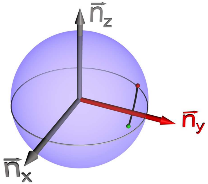

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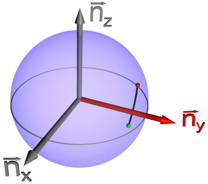

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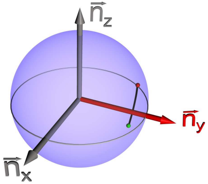

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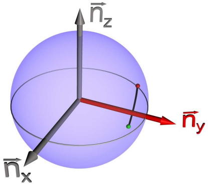

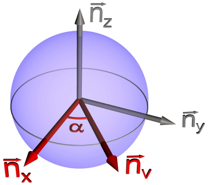

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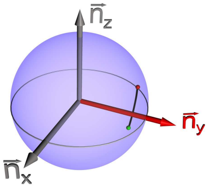

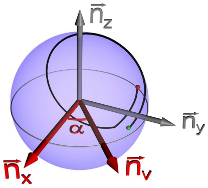

$\vec{n}_v \longrightarrow \vec{n}_x \longrightarrow \vec{n}_v$

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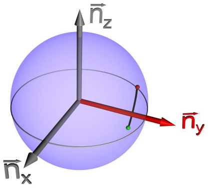

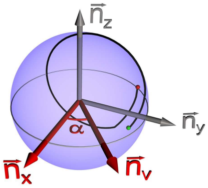

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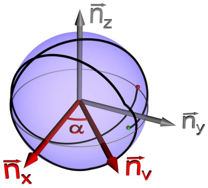
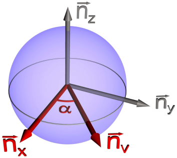


$\vec{n}_x \longrightarrow (\vec{n}_v \longrightarrow \vec{n}_x)^2 \longrightarrow \vec{n}_v$

We went further:

Time-optimal unitary synthesis with restricted controls

- qubit rotations are parametrized by unitaries $U \in SU(2)$
- unitary U: point on a 3-sphere



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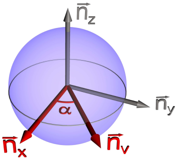

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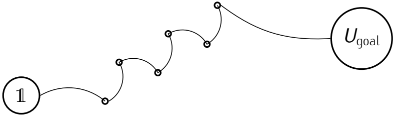

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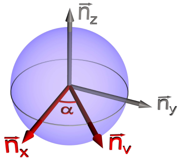

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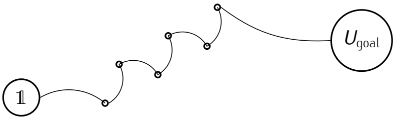

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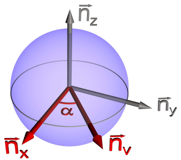

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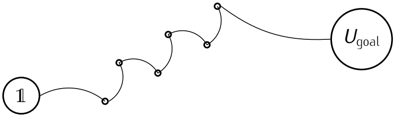

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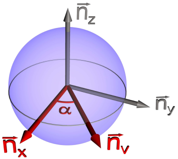

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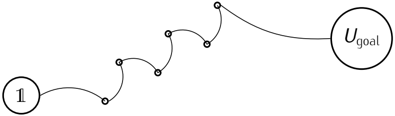

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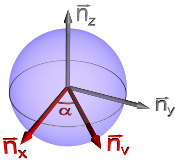

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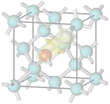

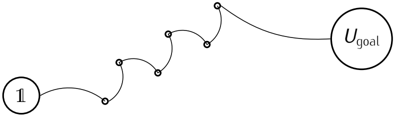

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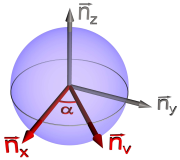

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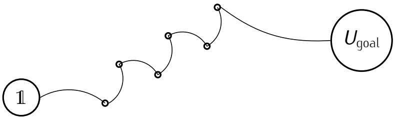

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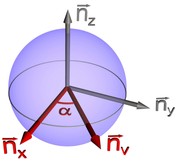

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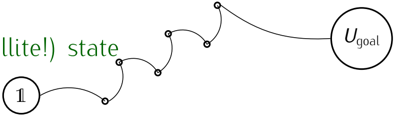
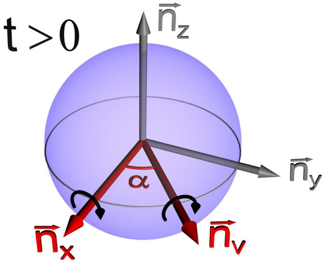
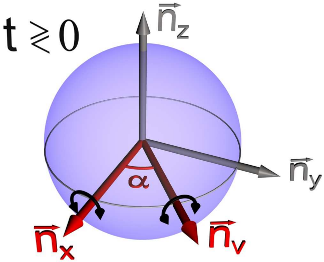
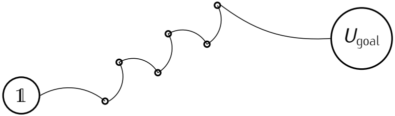
✓ valid solution regardless of qubit (or satellite!) state



Considered cases: rotation angle t in different domains







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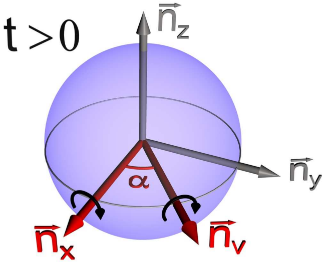
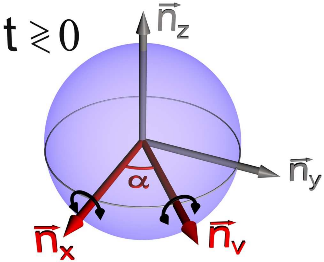
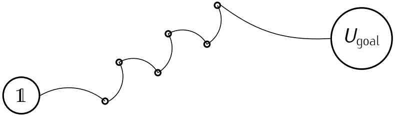




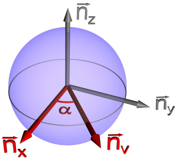
$$\Rightarrow \forall t \in \,]0, 2\pi[$$

$$\Rightarrow \forall t \in \,]-\pi, \pi]$$

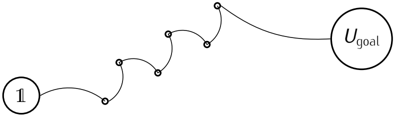


Our novel approach to time-optimal control

$$U_{\text{goal}} = \mathrm{X}(t_n) \cdot \ \ldots \ \cdot \mathrm{V}(t_2) \cdot \mathrm{X}(t_1) \cdot \mathbb{1} \ , \ \text{for min} \ \sum_{i=1}^{n} |t_i|$$



- strategy: if U_{goal} is generated in mininum time,
  find necessary conditions
  on the control sequence parameters $\{t_1, t_2 \ \ldots \ , t_n\}$,
  and length n (note: n finite, or $n \to \infty$)
- tools: algebraic properties of concatenations of control rotations
  - I. time-optimal sequences have like subsequences
  - II. unitaries have alternative algebraic decompositions
- accessible to the experimentalist
- as opposed to geometric control



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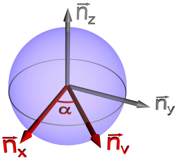

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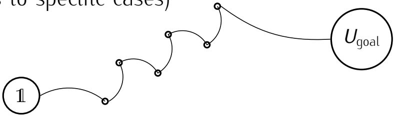
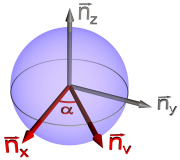
- as opposed to geometric control
  - abstract (challenging to find solutions to specific cases)

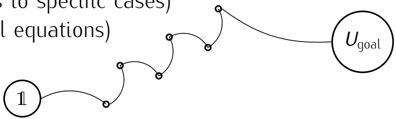


Our novel approach to time-optimal control

$$U_{\text{goal}} = \mathrm{X}(t_n) \cdot \ \ldots \ \cdot \mathrm{V}(t_2) \cdot \mathrm{X}(t_1) \cdot \mathbb{1} \ , \ \text{for min} \ \sum_{i=1}^{n} |t_i|$$



- strategy: if U_{goal} is generated in mininum time,
  find necessary conditions
  on the control sequence parameters $\{t_1, t_2 \ \ldots \ , t_n\}$,
  and length n (note: n finite, or $n \rightarrow \infty$)
- tools: algebraic properties of concatenations of control rotations
  - I. time-optimal sequences have like subsequences
  - II. unitaries have alternative algebraic decompositions
- accessible to the experimentalist
- as opposed to geometric control
  - abstract (challenging to find solutions to specific cases)
  - hard (convolved systems of differential equations)



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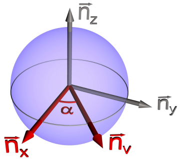

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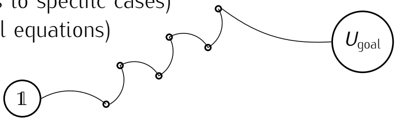
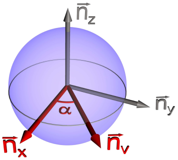
  - typically state-to-state

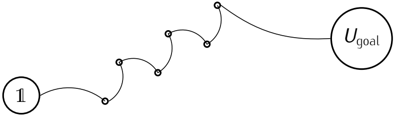


I. Time-optimal sequences have like subsequences



a 4-subsequence of a time-optimal sequence is time-optimal:

$$U_{\text{goal}} = \ \dots\ U^\star\ \dots$$



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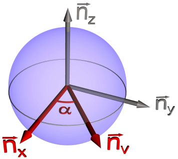

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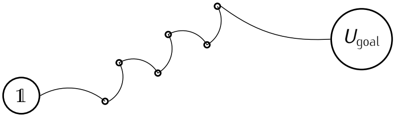
$$U^\star = \mathsf{X}(t_4) \cdot \mathsf{V}(t_3) \cdot \mathsf{X}(t_2) \cdot \mathsf{V}(t_1)$$



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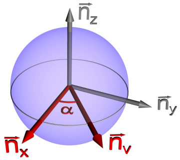

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perturbing times:

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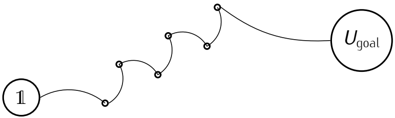

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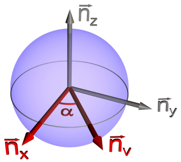

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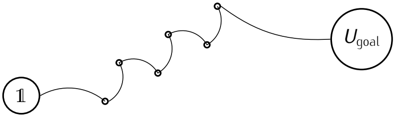
and imposing:

$$\begin{aligned} dU^\star &\approx 0 \\ d\mathcal{T} \equiv \epsilon_4 + \epsilon_3 + \epsilon_2 + \epsilon_1 &= 0 \\ d^2\mathcal{T} &> 0 \end{aligned}$$



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$$U^{\star} = \mathrm{X}(t_4) \cdot \mathrm{V}(t_3) \cdot \mathrm{X}(t_2) \cdot \mathrm{V}(t_1)$$

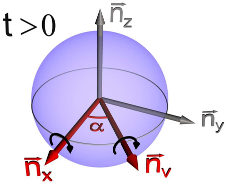
$d\mathcal{T} = 0$

$\downarrow$

$t_2 = t_3 \equiv t_m$

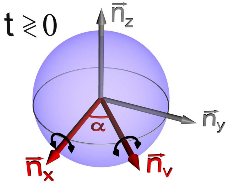
from n to only 3 params!

$d^2\mathcal{T} > 0$



finite n: $t_m > \pi$

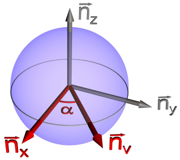
$n \to \infty$: $t_m \to 0$



finite n: t_m bounds signs $\{++--\}$, $\{+--+\}$

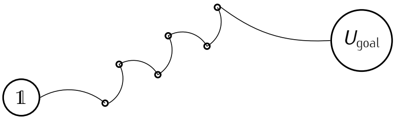
$n \to \infty$: $t_m \to 0$ signs $\{++++\}$, $\{+-+-\}$

II. Unitaries have alternative algebraic decompositions



up to a global phase,

$$U^{\star} = \mathsf{X}(t)$$



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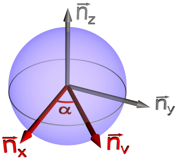
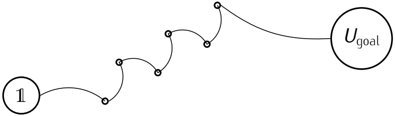


up to a global phase,

$$
\begin{aligned}
U^{\star} &= \mathsf{X}(t) \\
&= \mathsf{V}(t^{*}) \cdot \mathsf{X}(2\pi - t) \cdot \mathsf{V}(t^{*})
\end{aligned}
$$



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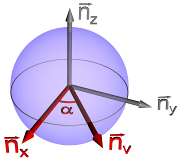
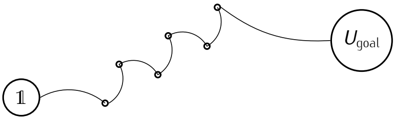


up to a global phase,

$$\begin{aligned} U^\star &= \mathsf{X}(t) \\ &= \mathsf{V}(t^*) \cdot \mathsf{X}(2\pi - t) \cdot \mathsf{V}(t^*) \end{aligned}$$

$$t^* = -2\,\mathrm{arccot}\left(\cos(\alpha)\ \tan\left(\frac{t}{2}\right)\right)$$



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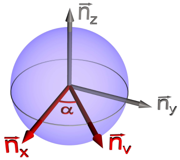

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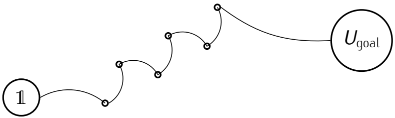
then, compare:

$$|t| \overset{?}{>} |2\pi - t| + 2|t^*|$$



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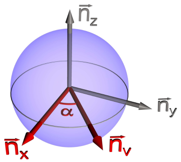

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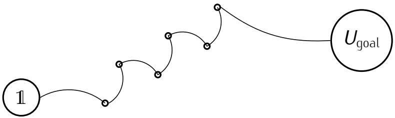
finally, discard non-time-optimal intervals for t



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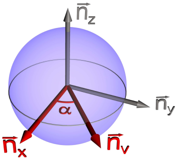

up to a global phase, $\forall |\delta| < |t|$,

$$
\begin{aligned}
U^\star &= \mathrm{X}(\delta) \cdot \mathrm{V}(t) \cdot \mathrm{X}(\delta) \\
&= \mathrm{V}(\tau) \cdot \mathrm{X}(\mu) \cdot \mathrm{V}(\tau)
\end{aligned}
$$

$$
\begin{aligned}
\tau &= \frac{t}{2} + \delta \; cos(\alpha) \left(1 - \cos\left(\frac{t}{2}\right)\right) + \; \mathcal{O}\left(\delta^2\right) \\
\mu &= 2 \; \delta \cos\left(\frac{t}{2}\right) + \; \mathcal{O}\left(\delta^2\right)
\end{aligned}
$$

then, compare:

$$
2|\delta| + |t| \overset{?}{>} 2|\tau| + |\mu|
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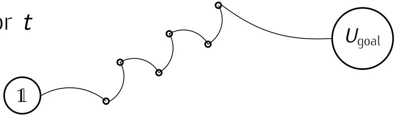

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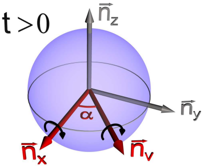
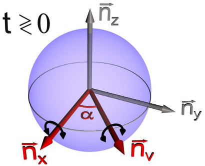
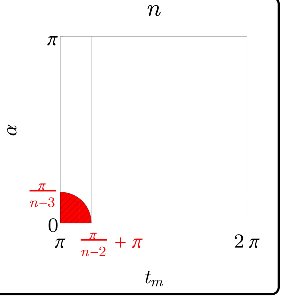




$n = 1, 2, 3, 4, 5$ or ∞



$$n \leq \left\lfloor \frac{\pi}{\alpha} \right\rfloor + 3$$

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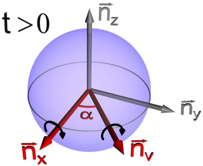
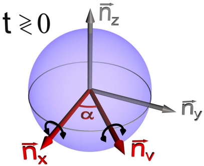
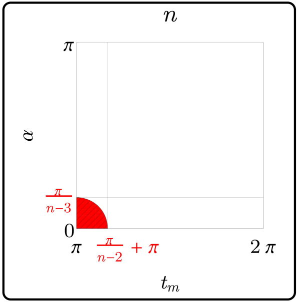


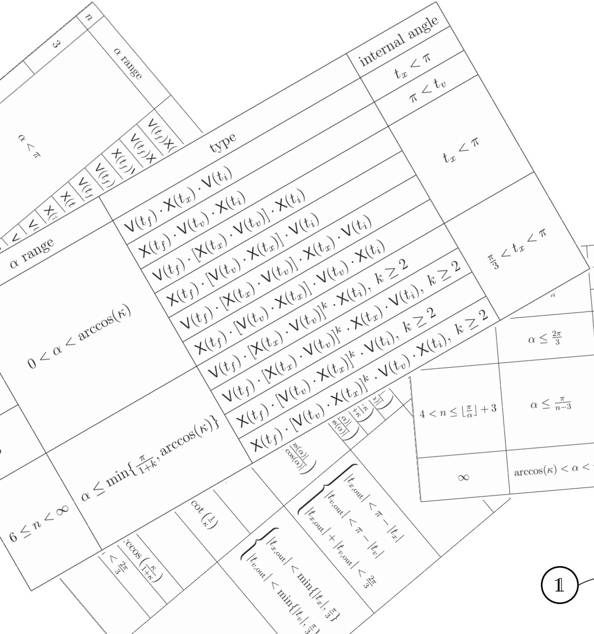


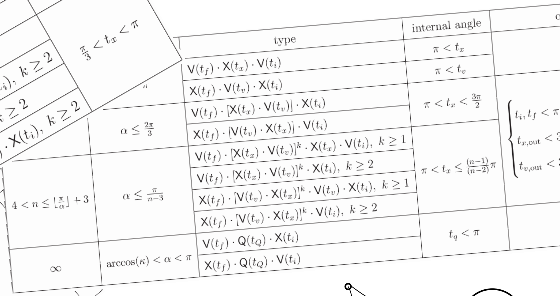


$$n = 1, 2, 3, 4, 5 \text{ or } \infty$$

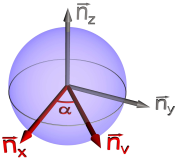
$$n \leq \left\lfloor \frac{\pi}{\alpha} \right\rfloor + 3$$

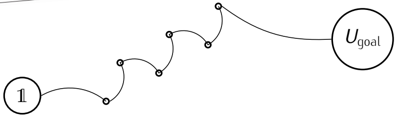
(In reality, time-optimal sequences more constrained)



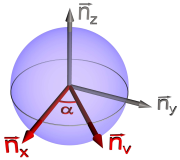


n	α range	type	internal angle	
		$V(t_f)\cdot X(t_x)\cdot V(t_i)$	$\pi < t_x$	
		$X(t_f)\cdot V(t_v)\cdot X(t_i)$	$\pi < t_v$	
	$\alpha \le \frac{2\pi}{3}$	$V(t_f)\cdot[X(t_x)\cdot V(t_v)]\cdot X(t_i)$	$\pi < t_x < \frac{3\pi}{2}$	$\begin{cases} t_i, t_f < \pi \\ t_{x,\text{out}} < \\ t_{v,\text{out}} < \end{cases}$
		$X(t_f)\cdot[V(t_v)\cdot X(t_x)]\cdot V(t_i)$		
$4 < n \le \lfloor\frac{\pi}{\alpha}\rfloor + 3$	$\alpha \le \frac{\pi}{n-3}$	$V(t_f)\cdot[X(t_x)\cdot V(t_v)]^k\cdot X(t_x)\cdot V(t_i),\ k\ge 1$	$\pi < t_x \le \frac{(n-1)}{(n-2)}\pi$	
		$V(t_f)\cdot[X(t_x)\cdot V(t_v)]^k\cdot X(t_i),\ k\ge 2$		
		$X(t_f)\cdot[V(t_v)\cdot X(t_x)]^k\cdot V(t_v)\cdot X(t_i),\ k\ge 1$		
		$X(t_f)\cdot[V(t_v)\cdot X(t_x)]^k\cdot V(t_i),\ k\ge 2$		
∞	$\arccos(\kappa) < \alpha < \pi$	$V(t_f)\cdot Q(t_Q)\cdot X(t_i)$	$t_q < \pi$	
		$X(t_f)\cdot Q(t_Q)\cdot V(t_i)$		



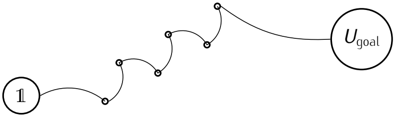


Boost for numerics of time-optimal unitary synthesis



- novel approach to find time-optimal sequences using alternating controls
- mapping of necessary algebraic properties of time-optimal sequences
- parameter search vastly reduced
- optimization problem $\longrightarrow$ algebraic problem

- results applicable to any system with such restricted controls



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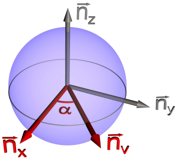

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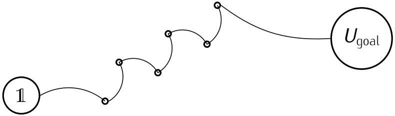
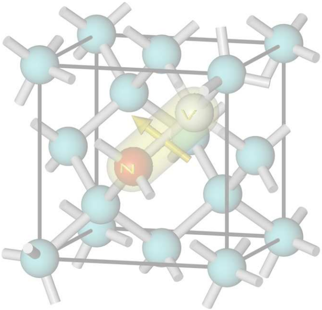


Bringing the NV back from the background



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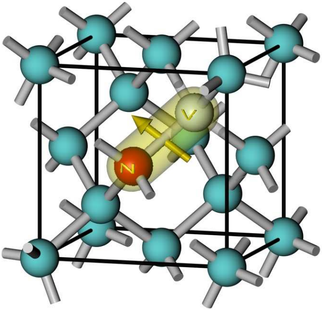

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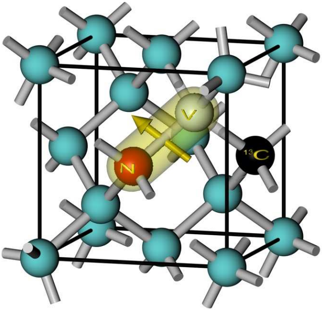

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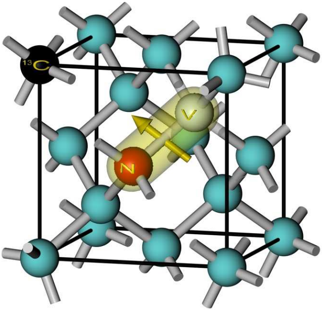

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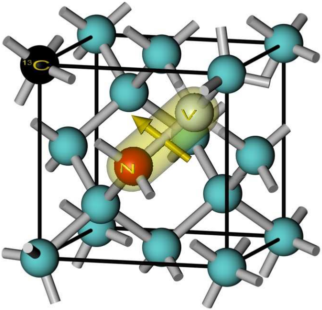

NV couples to (naturally occurring) ^{13}C via hyperfine interaction

Hyperfine mimics discussed control set for ^{13}C qubit

hyperfine interaction:

$$|0\rangle_{\mathrm{NV}} \rightarrow \mathcal{H}_{\mathrm{HF}} = 0$$
$$|\pm 1\rangle_{\mathrm{NV}} \rightarrow \mathcal{H}_{\mathrm{HF}} \sim \mathcal{A} \cdot S_x I_x + \mathcal{B} \cdot S_x I_y$$

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$\vec{n}_x$ quantization axis $\parallel \vec{B}_0$
S, I spin operators for NV and ^{13}C
$\mathcal{A}, \mathcal{B}$ iso and anisotropic parts of hyperfine

Hyperfine mimics discussed control set for ^{13}C qubit

hyperfine interaction:

$$|0\rangle_{\mathrm{NV}} \to \ ^{13}\mathrm{C}\text{ qubit precesses around } \vec{n}_x$$
$$|\pm 1\rangle_{\mathrm{NV}} \to \ ^{13}\mathrm{C}\text{ qubit precesses around axis between } \vec{n}_x, \vec{n}_y : \vec{n}_v$$

$\vec{n}_x$ quantization axis $\parallel \vec{B}_0$
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Hyperfine mimics discussed control set for ^{13}C qubit

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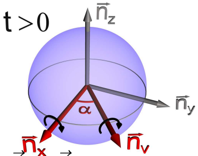

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$\vec{n}_x$ quantization axis $\parallel \vec{B}_0$
S, I spin operators for NV and ^{13}C
$\mathcal{A}, \mathcal{B}$ iso and anisotropic parts of hyperfine

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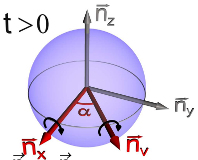

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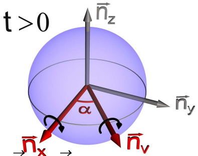

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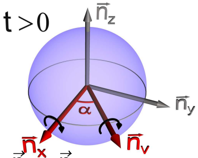

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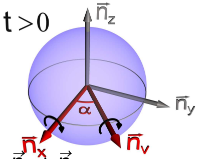

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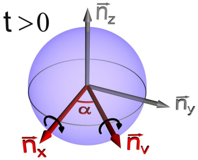
- formalism to design fast unitaries for the ^{13}C is ready!
- caveat: only works if hyperfine anisotropic, since $\alpha = \arctan\left(\frac{\mathcal{B}}{\omega_0 \pm \mathcal{A}}\right)$

NV spin switch also changes the ^{13}C Larmor frequency



Larmor frequency if $|0\rangle_{\mathrm{NV}}$: $\omega_0 \equiv \gamma_n B_0$, $\gamma_n \sim 1\mathrm{kHz/G}$

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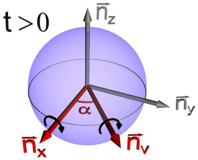

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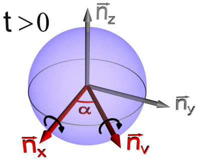

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(my presentation of the control problem assumed $\kappa = 1$;
complete theoretical framework *does* take $\kappa \neq 1$ into account)

Actuator can be faster, cleaner than direct driving of ^{13}C

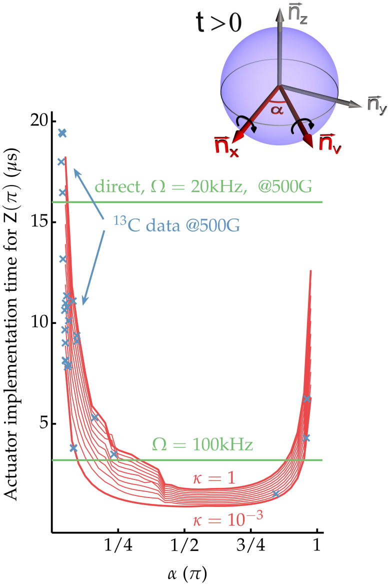
direct driving of ^{13}C qubit:

- (noisy) radio-frequency photons
- Rabi frequency $\Omega \lesssim 100$kHz

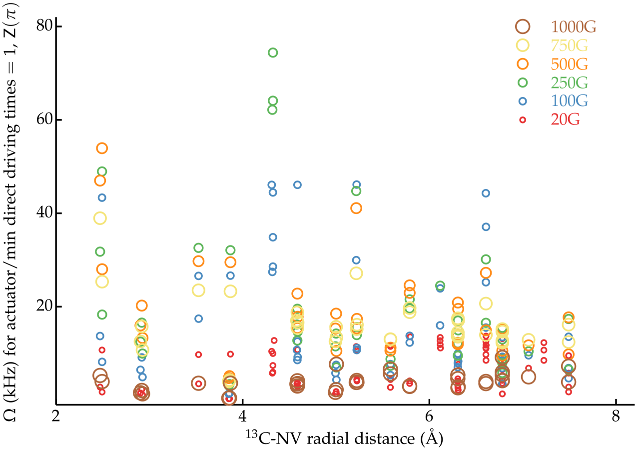
numerical search of time-optimal actuator solutions:

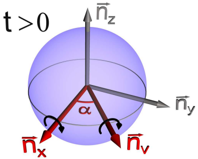
- $U_{\text{goal}} = Z(\pi)$
- large range of $\{\alpha, \kappa\}$

- real $\{\alpha(B_0), \kappa(B_0)\}$: pick $B_0 = 500$G
- comparison with ^{13}C hyperfine data yielded by density functional theory



Actuator beneficial over large ranges of B_0 and distance





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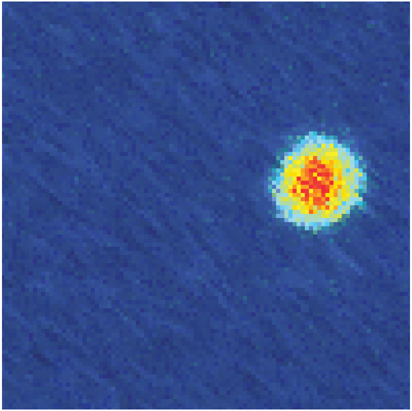
- (analysis made possible via novel approach to time-optimal control)

The future: quantum control and metrology in diamond



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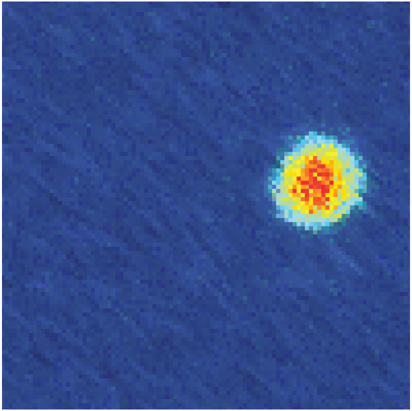

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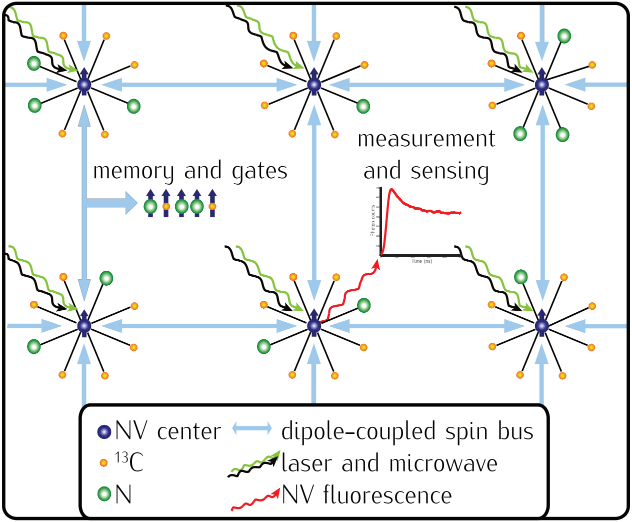

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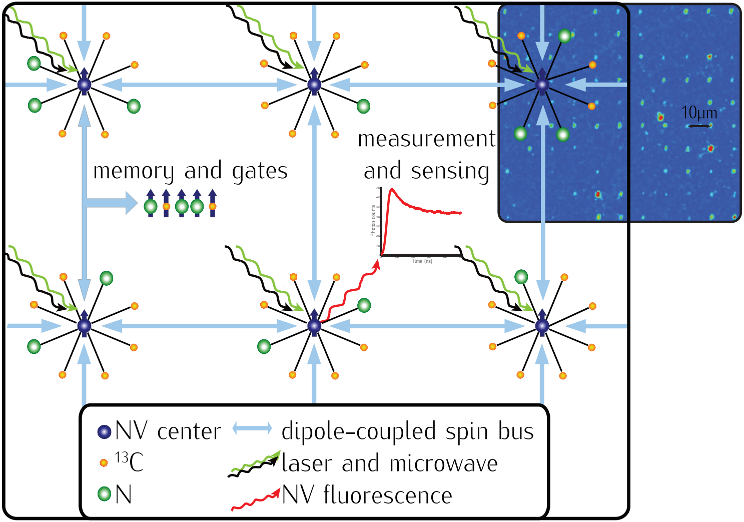
J. Meijer *et al.*, Appl. Phys. A **83**, 321 (2006)



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