ECE 558 Hmwk 04

Problem 1:

Solve: (1) The characterization of the system is shown as follows:

An input image: $f(x,y)$, size $M \times N$, and its DFT is $F(u,v)$

A filter in frequency domain: $H(u,v)$

where $H(u,v) = e^{-\left[\frac{u^2}{150}+\frac{v^2}{150}\right]} + 1 - e^{-\left[\frac{(u-50)^2}{150}+\frac{(v-50)^2}{150}\right]}$

An output image: $g(x,y) = IDFT\{F(u,v) \cdot H(u,v)\}$

(2) Since $e^{-\left[\frac{u^2}{150}+\frac{v^2}{150}\right]}$ .... (1) is the Gaussian lowpass filter in the frequency domain

and $1 - e^{-\left[\frac{(u-50)^2}{150}+\frac{(v-50)^2}{150}\right]}$ .... (2) is the Gaussian highpass filter in the frequency domain

we can know $H(u,v) = (1) + (2)$ is a band-stop filter in the frequency domain

Scanned with CamScanner

Problem 2:

Solve: (1) From $|G(u,v)|^2 = |F(u,v)|^2 \cdot |H(u,v)|^2 + |N(u,v)|^2$,

due to that $|\hat{F}(u,v)|^2 = |F(u,v)|^2$ and $|\hat{F}(u,v)|^2 = |R(u,v)|^2 \cdot |G(u,v)|^2$

So $|F(u,v)|^2 = |R(u,v)|^2 \cdot \left[|F(u,v)|^2 \cdot |H(u,v)|^2 + |N(u,v)|^2\right]$

Therefore $R(u,v) = \sqrt{\dfrac{|F(u,v)|^2}{|F(u,v)|^2 \cdot |H(u,v)|^2 + |N(u,v)|^2}}$

(2) $\hat{F}(u,v) = R(u,v) \cdot G(u,v)$

$$= \sqrt{\frac{|F(u,v)|^2}{|F(u,v)|^2 \cdot |H(u,v)|^2 + |N(u,v)|^2}} \cdot G(u,v)$$

$$= \sqrt{\frac{1}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}}} \cdot G(u,v)$$

Since the power spectrum $|N(u,v)|^2$'s density is $S_\eta(u,v)$

the power spectrum $|F(u,v)|^2$'s density is $S_f(u,v)$

Therefore $\hat{F}(u,v) = \sqrt{\dfrac{1}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}} \cdot G(u,v)$