1. Solve:

(a)

By using the time vector **t** and the bandpass signal **xat**, I can plot the bandpass signal as shown in the Fig. 1.

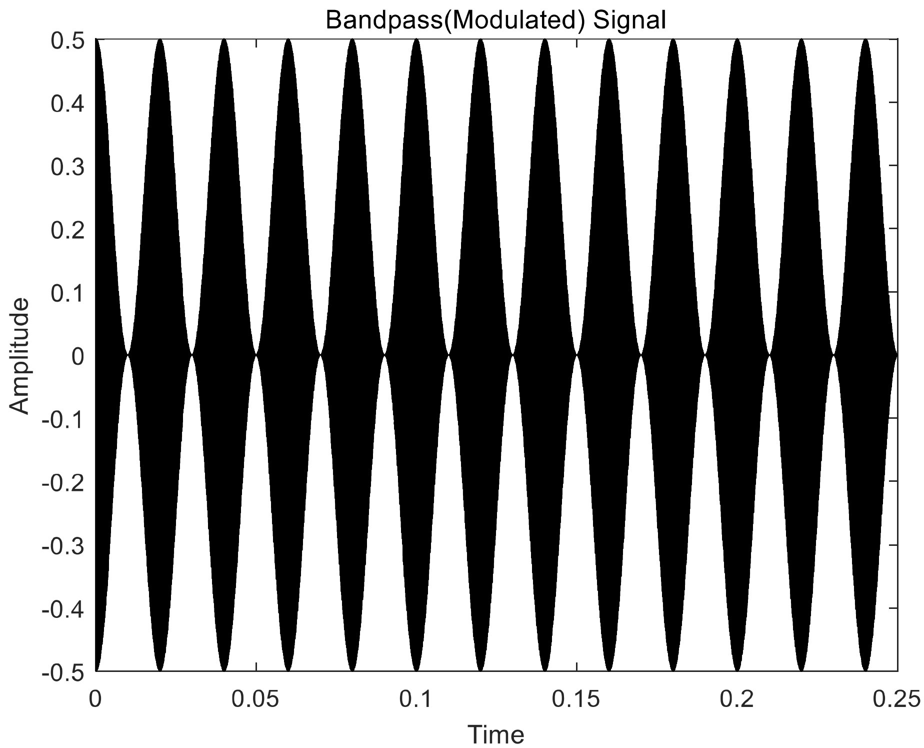


Fig. 1. Bandpass signal xat

The Matlab code is as follows:

```
t;                        % t from hwk3prob1data
xat;                      % xat from hwk3prob1data
figure(1)                 % plot the band pass signal xat
plot(t,xat,'k')
title('Bandpass(Modulated) Signal')
xlabel('Time')
ylabel('Amplitude')
```

(b) From given conditions, we can know:

$F_c = 65kHz$, $B =$ bandwidth $= F_H - F_L = 1950Hz$

The sampling frequency $F_s = \frac{4}{2k+1} F_c$

The maximum k: $k = [\frac{F_c}{B} - \frac{1}{2}] = [32.833] = 32$

So an appropriate sampling frequency $F_s = \frac{4}{32\times2+1} \times 65kHz = 4kHz$,

and an appropriate sampling period $T_s = \frac{1}{F_s} = 0.00025s$

(c)

By using the Matlab code below, I can obtain a sampled sequence **xanT**, which is shown in Fig. 2., from the sequence **xat**.

```
figure(2)                           % sampled sequence xanT
deltaT=1e-6;
Ts=1/4000;                          % obtained from question(b)
tsampstep=Ts/deltaT;
xanT=xat(1:tsampstep:2.5*10^5+1);  % obtain every tsampstep
                                      samples from xat
n=0:1:2.5*10^5/250;
plot(n,xanT,'k')                    % plot the xanT sequence
title('xanT Obtained from Sampling xat')
xlabel('n')
ylabel('Amplitude')
```

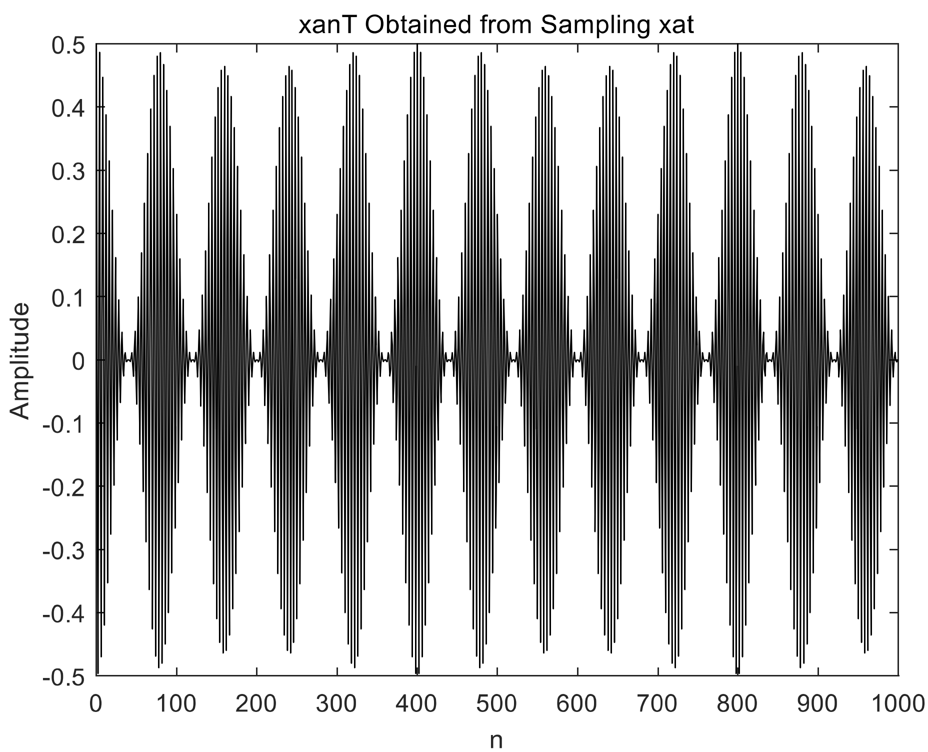
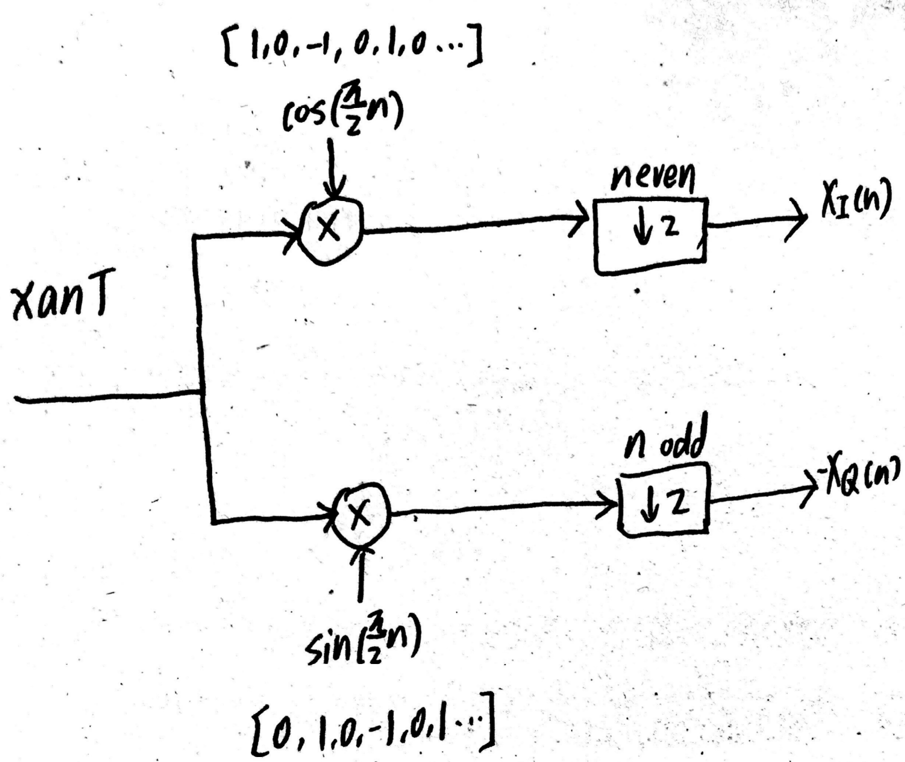


Fig. 2. A sampled sequence xanT

(d)



Because k=32 is even, we scale -1.

(e)

By using the Matlab code below, I can plot the discrete sequences **xIn** and **xQn** as is shown in Fig. 3.

```
n=0:1:2.5*10^5/250;
xanTcos=xanT.*cos(0.5*pi*n);
xanTsin=xanT.*sin(0.5*pi*n);
xIn=xanTcos(1:2:2.5*10^5/250+1); % extract values when n is
                                   even
xQn=-xanTsin(2:2:2.5*10^5/250);  % extract values when n is odd
figure(3)
subplot(2,1,1)                    % plot the discrete sequences
                                   xIn
stem(0:length(xIn)-1,xIn)
title('xI(n) Obtained from Sampling xat')
xlabel('n')
ylabel('Amplitude')
subplot(2,1,2)
stem(0:length(xQn)-1,xQn)         % plot the discrete sequences
                                   xQn
title('xQ(n) Obtained from Sampling xat')
xlabel('n')
ylabel('Amplitude')
```

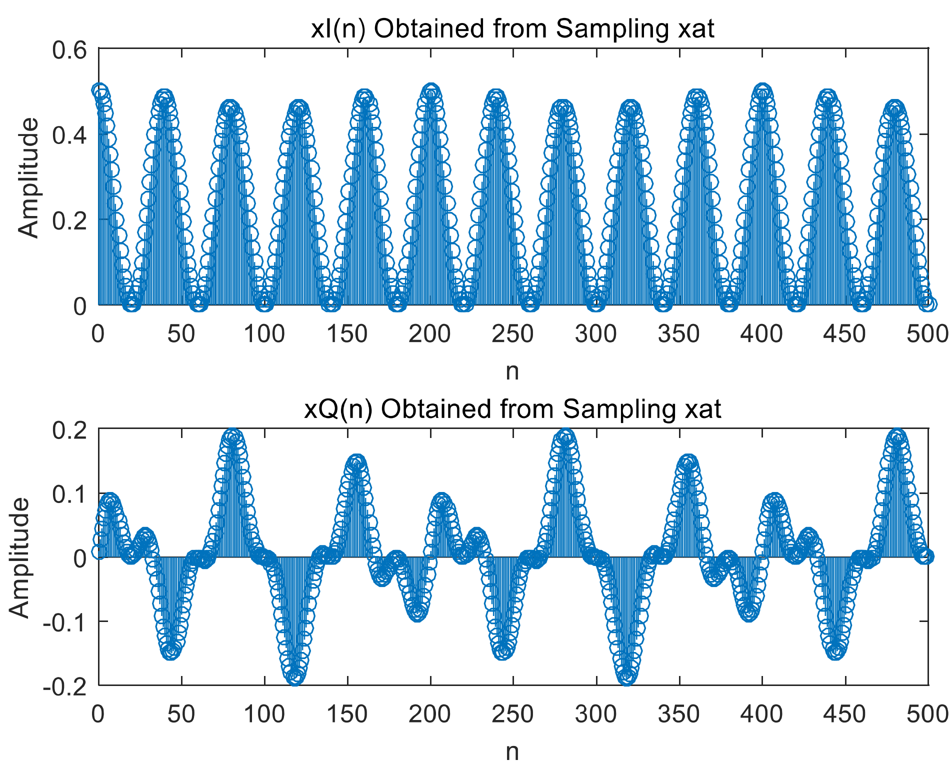


Fig. 3. Discrete sequences **xIn** and **xQn**

(f)

In order to calculate the absolute error between the in-phase component already calculated and xIn_true loaded from hwk3prob1data.mat, and the absolute error between the quad-phase component already calculated and xQn_true loaded from hwk3prob1data.mat, I write the Matlab code below:

```
figure(4)
subplot(2,1,1)
stem(0:length(xIn_true)-1,xIn_true-xIn)  % the absolute error
                                         between the in-phase
                                         component and xIn_true
title('In-phase Component Absolute Error')
xlabel('n')
ylabel('Absolute Error')
subplot(2,1,2)
stem(0:length(xQn_true)-1,xQn_true-xQn)  % the absolute error
                                         between the quad-phase
                                         component and xQn_true
title('Quad-phase Component Absolute Error')
xlabel('n')
ylabel('Absolute Error')
```

(g)

Plot the absolute error as is shown in Fig. 4.

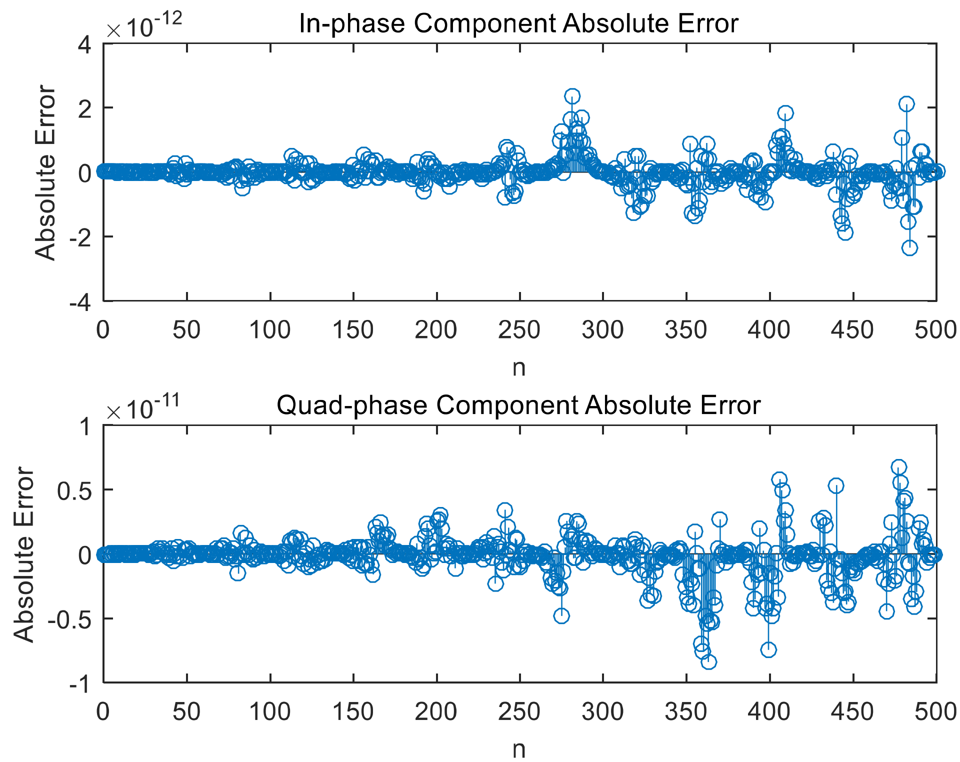


Fig. 4. Absolute Error between Methods

As is clearly demonstrated, the absolute error is definitely small so that people can actually ignore it while conducting research. Obtaining the in-phase and quadrature phase signals directly from the sampled signal can be an efficient method for individuals to study bandpass signals.

2. Solve: by using $x_a(t)=\int_{-\infty}^{+\infty}X_a(F)e^{j2\pi Ft}dF$

let $t=nT=\frac{n}{F_s}$, $x(n)\equiv x_a(nT)=\int_{-\infty}^{+\infty}X_a(F)e^{\frac{j2\pi Fn}{F_s}}dF$

$$=\int_{(m-1)B}^{mB}X_a(F)e^{\frac{j\cdot 2\pi\cdot F\cdot n}{F_s}}dF+\int_{-mB}^{-(m-1)B}X_a(F)e^{\frac{j2\pi Fn}{F_s}}dF$$

by using $x(n)=\int_{-\frac{1}{2}}^{\frac{1}{2}}X(f)e^{j2\pi fn}df$ due to $f=\frac{F}{F_s}\rightarrow F=f\cdot F_s$, to get $dF=F_s\cdot df$

$$\text{So } x(n)=\frac{1}{F_s}\int_{-\frac{1}{2}F_s}^{\frac{1}{2}F_s}X(F)e^{j2\pi\frac{F}{F_s}n}dF=\frac{1}{F_s}\int_{0}^{\frac{1}{2}F_s}X(F)e^{j2\pi\frac{F}{F_s}n}dF+\frac{1}{F_s}\int_{-\frac{1}{2}F_s}^{0}X(F)e^{j2\pi\frac{F}{F_s}n}dF$$

$$=\frac{1}{F_s}\int_{\frac{k-1}{2}F_s}^{\frac{k}{2}F_s}X(F)e^{j2\pi\frac{F}{F_s}n}dF+\frac{1}{F_s}\int_{-\frac{k}{2}F_s}^{-\frac{k-1}{2}F_s}X(F)e^{j2\pi\frac{F}{F_s}n}dF$$

due to $F_s=2B$

$$\text{So we get } x(n)=\frac{1}{F_s}\int_{(k-1)B}^{kB}X(F)e^{j2\pi\frac{F}{F_s}n}dF+\frac{1}{F_s}\int_{-kB}^{-(k-1)B}X(F)e^{j2\pi\frac{F}{F_s}n}dF$$

$$X_a(F)=\frac{1}{F_s}X(F)$$

$$x_a(t)=\int_{-\infty}^{+\infty}X_a(F)e^{j2\pi Ft}dF=\int_{-\infty}^{+\infty}\frac{X(F)}{F_s}e^{j2\pi Ft}dF$$

$$\text{Due to } X(F)=\sum_{n=-\infty}^{+\infty}x(n)e^{-j2\pi nf}=\sum_{n=-\infty}^{+\infty}x(n)e^{-j2\pi\frac{nF}{F_s}}$$

$$\text{So } x_a(t)=\frac{1}{F_s}\sum_{n=-\infty}^{+\infty}x(n)\int_{-\infty}^{+\infty}e^{-j2\pi n\frac{F}{F_s}}e^{j2\pi Ft}dF$$

$$x_a(t)=\frac{1}{F_s}\sum_{n=-\infty}^{+\infty}x(n)\int_{mB}^{(m+1)B}e^{-j2\pi n\frac{F}{F_s}}\cdot e^{j2\pi Ft}dF+\frac{1}{F_s}\sum_{n=-\infty}^{+\infty}x(n)\int_{-(m+1)B}^{-mB}e^{-j2\pi n\frac{F}{F_s}}\cdot e^{j2\pi Ft}$$

$$\frac{1}{F_s}\int_{mB}^{(m+1)B}e^{j2\pi F(t-\frac{n}{F_s})}dF+\frac{1}{F_s}\int_{-(m+1)B}^{-mB}e^{j2\pi F(t-\frac{n}{F_s})}dF$$

$$=\frac{1}{j2\pi F_s(t-\frac{n}{F_s})}e^{j2\pi F(t-\frac{n}{F_s})}\Big|_{mB}^{(m+1)B}+\frac{1}{j2\pi F_s(t-\frac{n}{F_s})}e^{j2\pi F(t-\frac{n}{F_s})}\Big|_{-(m+1)B}^{-mB}$$

$$=\frac{1}{j2\pi F_s(t-\frac{n}{F_s})}\Big\{\cos[2\pi(t-\tfrac{n}{F_s})(m+1)B]+j\sin[2\pi(t-\tfrac{n}{F_s})(m+1)B]-\cos[2\pi(t-\tfrac{n}{F_s})mB]$$

$$-j\sin[2\pi(t-\tfrac{n}{F_s})mB]+\cos[2\pi(t-\tfrac{n}{F_s})mB]-j\sin[2\pi(t-\tfrac{n}{F_s})mB]$$

$$-\cos[2\pi(t-\tfrac{n}{F_s})(m+1)B]+j\sin[2\pi(t-\tfrac{n}{F_s})(m+1)B]$$

$$=\frac{1}{\pi F_s(t-nT)}\Big\{\sin[2\pi(t-nT)(m+1)B]-\sin[2\pi(t-nT)mB]\Big\}$$

$$= \frac{1}{\pi F_s(t-nT)}\left\{\sin[2\pi(t-nT)(F_c+\tfrac{1}{2}B)] - \sin[2\pi(t-nT)(F_c-\tfrac{1}{2}B)]\right\}$$

$$= \frac{1}{\pi F_s(t-nT)}\left\{\sin[2\pi(t-nT)F_c]\cos[\pi(t-nT)B] + \cos[2\pi(t-nT)F_c]\sin[\pi(t-nT)B]\right.$$
$$\left. - \sin[2\pi(t-nT)F_c]\cos[\pi(t-nT)B] + \cos[2\pi(t-nT)F_c]\sin[\pi(t-nT)B]\right.$$

$$= \frac{1}{\pi F_s(t-nT)} 2\cos[2\pi(t-nT)F_c]\sin[\pi(t-nT)B]$$

$F_s = 2B$, - we get $\dfrac{\sin[\pi(t-nT)B]}{\pi B(t-nT)} \cos[2\pi F_c(t-nT)]$

Therefore $x_a(t) = \sum_{n=-\infty}^{\infty} x(n) \dfrac{\sin[\pi(t-nT)B]}{\pi B(t-nT)} \cos[2\pi F_c(t-nT)]$

3. Solve:

(a) From Figure 3.

$\frac{B}{2} = 25 \rightarrow B = 50$

So $F_{s,min} = 2B = \boxed{100}$

(b) $g_a(t) = \frac{\sin(\pi B t)}{\pi B t} \cos(2\pi F_c t) \quad F_c = 175$

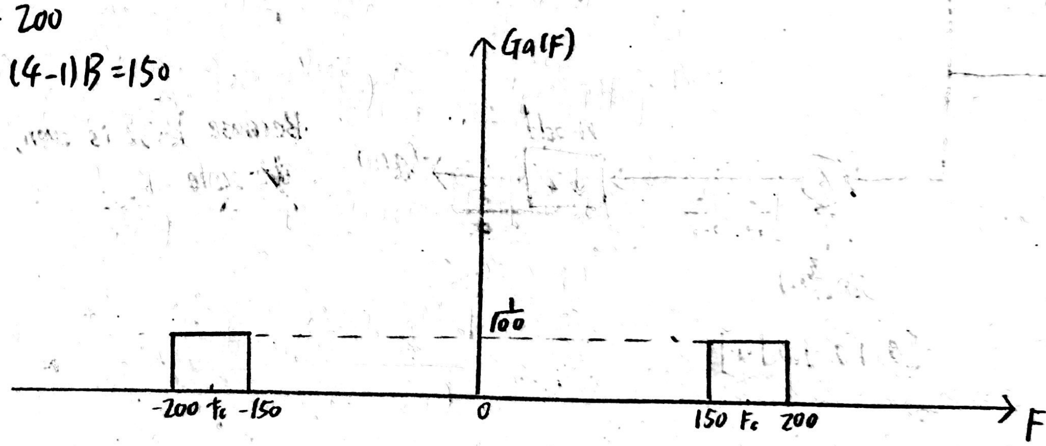
$= \frac{\sin(50\pi t)}{50\pi t} \cos(350\pi t)$.

(c) Due to $F_c = mB - \frac{B}{2}$

so $m = \frac{F_c + \frac{B}{2}}{B} = 4$

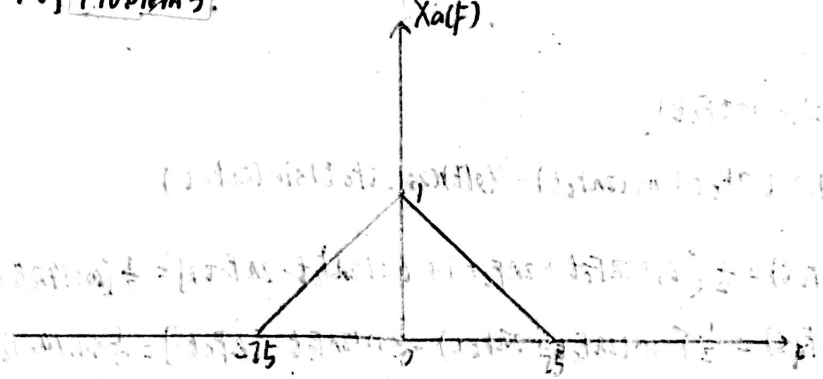
$F_H = 4B = 200$, so we can get the ideal reconstruction function's spectrum as follows:

$F_L = (4-1)B = 150$



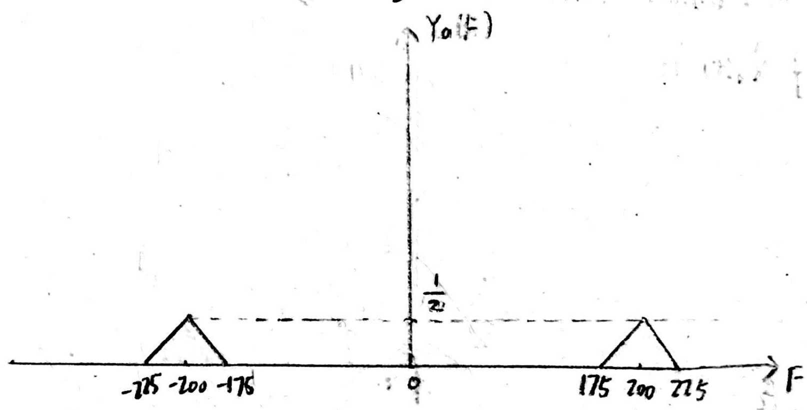
4. Solve: From Figure 1 of Problem 3:

(a)



$y_a(t) = x_a(t)\cos(2\pi\cdot 200t)$

$F_c = 200,\ \frac{B}{2} = 25 \rightarrow B = 50\text{Hz}$



(b)

$F_s = \frac{4}{2k+1} F_c$

$k = \left[\frac{F_c}{B} - 0.5\right] = [3.5] = 3$

So $F_{s,min} = \frac{4}{7} \cdot F_c = \frac{4}{7} \cdot 200\text{ Hz} \approx 114.29\text{Hz}$, which can obtain the in-phase and quadrature-phase components directly from the sampled bandpass signal of $y_a(t)$.

5. Solve:

(a) From Figure.2,

we can get $\hat{X}_a(t) = X_a(t)\cos(2\pi F_c t)$

$$= X_I(t)\cos(2\pi F_c t)\cos(2\pi F_c t) - X_Q(t)\cos(2\pi F_c t)\sin(2\pi F_c t)$$

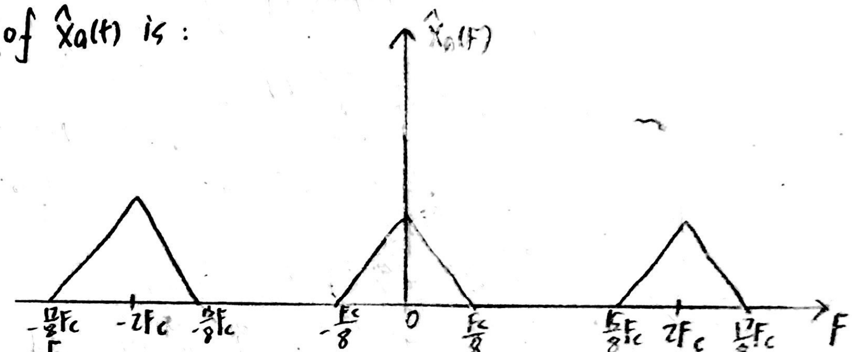
$$\cos(2\pi F_c t)\cos(2\pi F_c t) = \frac{1}{2}[\cos(2\pi F_c t + 2\pi F_c t) + \cos(2\pi F_c t - 2\pi F_c t)] = \frac{1}{2}[\cos(4\pi F_c t) + 1]$$

$$\cos(2\pi F_c t)\sin(2\pi F_c t) = \frac{1}{2}[\sin(2\pi F_c t + 2\pi F_c t) - \sin(2\pi F_c t - 2\pi F_c t)] = \frac{1}{2}\sin(4\pi F_c t)$$

So $\hat{X}_a(t) = X_I(t)\frac{1}{2}[\cos(4\pi F_c t) + 1] - X_Q(t)\cdot\frac{1}{2}\sin(4\pi F_c t)$

$$= \frac{1}{2}X_I(t) + \frac{1}{2}[X_I(t)\cos(4\pi F_c t) - X_Q(t)\sin(4\pi F_c t)]$$

The spectrum of $\hat{X}_a(t)$ is:



$\frac{B}{2} \le \frac{F_c}{8} \Rightarrow B \le \frac{F_c}{4}$, in order to make $y(t) = X_I(t)$

we can make $A=2$ and F_{cutoff} is $\frac{1}{2}(\frac{F_c}{8} + \frac{15}{8}F_c) = F_c$.

(b) $\hat{X}_a(t) = X_I(t)\cos(2\pi F_c t)\sin(2\pi F_c t) - X_Q(t)\sin(2\pi F_c t)\sin(2\pi F_c t)$

$$\cos(2\pi F_c t)\sin(2\pi F_c t) = \frac{1}{2}[\sin(2\pi F_c t + 2\pi F_c t) - \sin(2\pi F_c t - 2\pi F_c t)] = \frac{1}{2}\sin(4\pi F_c t)$$

$$\sin(2\pi F_c t)\sin(2\pi F_c t) = -\frac{1}{2}[\cos(2\pi F_c t + 2\pi F_c t) - \cos(2\pi F_c t - 2\pi F_c t)] = -\frac{1}{2}[\cos(4\pi F_c t) - 1]$$

So $\hat{X}_a(t) = \underbrace{\frac{1}{2}X_I(t)\sin(4\pi F_c t)}_{(1)} + \underbrace{\frac{1}{2}X_Q(t)\cos(4\pi F_c t)}_{(2)} - \underbrace{\frac{1}{2}X_Q(t)}_{(3)}$

The terms (1) and (2) can be filtered,

so now $y(t) = -X_Q(t)$