ECE 513 homework2

1. solve: We can know $X(z) = \frac{3z^3+5z^2-z}{(z+0.7)(z^2-3z+2.25)}$

So we get $\frac{X(z)}{z} = \frac{3z^2+5z-1}{(z+0.7)(z^2-3z+2.25)}$

let $\frac{X(z)}{z} = \frac{C_1}{z+0.7} + \frac{C_2}{(z-1.5)^2} + \frac{C_3}{z-1.5}$

So $C_1 = \frac{X(z)}{z}(z+0.7)\Big|_{z=-0.7} \approx -0.626$

$C_2 = \frac{X(z)}{z}(z-1.5)^2\Big|_{z=1.5} \approx 6.023$

$C_3 = \frac{d\left[\frac{X(z)}{z}(z-1.5)^2\right]}{dz}\Big|_{z=1.5} = \frac{(6z+5)(z+0.7)-3z^2-5z+1}{(z+0.7)^2}\Big|_{z=1.5}$

≈ 3.626

So we get $\frac{X(z)}{z} = \frac{-0.626}{z+0.7} + \frac{6.023}{(z-1.5)^2} + \frac{3.626}{z-1.5}$

So $X(z) = \underbrace{\frac{-0.626z}{z+0.7}}_{X_1(z)} + \underbrace{\frac{6.023z}{(z-1.5)^2}}_{X_2(z)} + \underbrace{\frac{3.626z}{z-1.5}}_{X_3(z)}$

$0<|z|<0.7$, $X_1(z) = -0.626\left(1-\frac{0.7}{z+0.7}\right)$

$= -0.626\left[1-\frac{1}{1+(-0.7^{-1}z)}\right]$

$= 0.626\left[\sum_{n=0}^{+\infty}(-0.7^{-1}z)^n - 1\right]$

$= 0.626\sum_{n=1}^{+\infty}(-0.7)^{-n}z^n$

$\xRightarrow{m=-n} 0.626\sum_{-\infty}^{m=-1}(-0.7)^m z^{-m}$

So $X_1(n) = 0.626(-0.7)^n u(-n-1)$

$X_3(z) = 3.626\left(1+\frac{1.5}{z-1.5}\right) = -3.626\left(\frac{1}{1-1.5^{-1}z} - 1\right)$

$= -3.626\left[\sum_{n=0}^{+\infty}(1.5^{-1}z)^n - 1\right]$

$= -3.626\sum_{n=1}^{+\infty}(1.5)^{-n}z^n$

$\xRightarrow{m=-n} -3.626\sum_{-\infty}^{m=-1}1.5^m z^{-m}$

So $X_3(n) = -3.626\times(1.5)^n u(-n-1)$

$X_2(z) = \frac{6.023z}{(z-1.5)^2} = \frac{6.023}{1.5}\cdot\frac{1.5z^{-1}}{(1-1.5z^{-1})^2}$

Due to $-na^n u(-n-1) = \frac{az^{-1}}{(1-az^{-1})^2}$

So $X_2(n) = -4.015n(1.5)^n u(-n-1)$.

Therefore, when $0<|z|<0.7$

$X(n) = [0.626(-0.7)^n - 4.015n(1.5)^n - 3.626\cdot(1.5)^n]u(-n-1)$

This signal is unstable

$0.7<|z|<1.5$, $X_1(z) = -0.626\cdot\frac{1}{1-(-0.7)z^{-1}}$

$\therefore X_1(n) = -0.626(-0.7)^n u(n)$

Therefore, when $0.7<|z|<1.5$

$X(n) = -0.626(-0.7)^n u(n) - (4.015n+3.626)1.5^n u(-n-1)$

This signal is stable

$|z|>1.5$

$X_2(z) = \frac{6.023}{1.5}\cdot\frac{1.5z^{-1}}{(1-1.5z^{-1})^2}$

Due to $na^n u(n) = \frac{az^{-1}}{(1-az^{-1})^2}$

So $X_2(n) = 4.015\cdot n\cdot(1.5)^n u(n)$

$X_3(z) = 3.626\cdot\frac{1}{1-1.5z^{-1}}$

So $X_3(n) = 3.626\cdot(1.5)^n u(n)$

Therefore, when $|z|>1.5$

$X(n) = [-0.626\cdot(-0.7)^n + 4.015n\cdot(1.5)^n + 3.626\cdot(1.5)^n]u(n)$

This signal is unstable.

Scanned with CamScanner

2.Solve:

(a)

In order to make the magnitude of a stable discrete time system and the magnitude of an unstable discrete time system have the same frequency response, we could stabilize the poles, which are outside the unit circle in an unstable discrete time system, by replacing them with their reciprocals and then adjusting the magnitude of the discrete time system appropriately.

By using Matlab, we could know that the poles of the unstable system are as follows:

$$p_1 = -0.6979 + j1.3800 \quad p_1^* = -0.6979 - j1.3800$$
$$p_3 = -0.7951 + j0.0708 \quad p_3^* = -0.7951 - j0.0708 \quad p_5 = -0.1078$$

So $H(z)$ can be written in the form

$$H(z) = \frac{B(z)}{(z-p_1)(z-p_1^*)(z-p_3)(z-p_3^*)(z-p_5)}$$

A stable transfer function, with the same magnitude response, can be determined as

$$H_1(z) = \frac{z^{-2}B(z)}{(z^{-1}-p_1)(z^{-1}-p_1^*)(z-p_3)(z-p_3^*)(z-p_5)}$$

$H_1(z)$ can be simplified to obtain

$$H_1(z) = \frac{B(z)}{|p_1|^2 (z-\frac{1}{p_1})(z-\frac{1}{p_1^*})(z-p_3)(z-p_3^*)(z-p_5)}$$

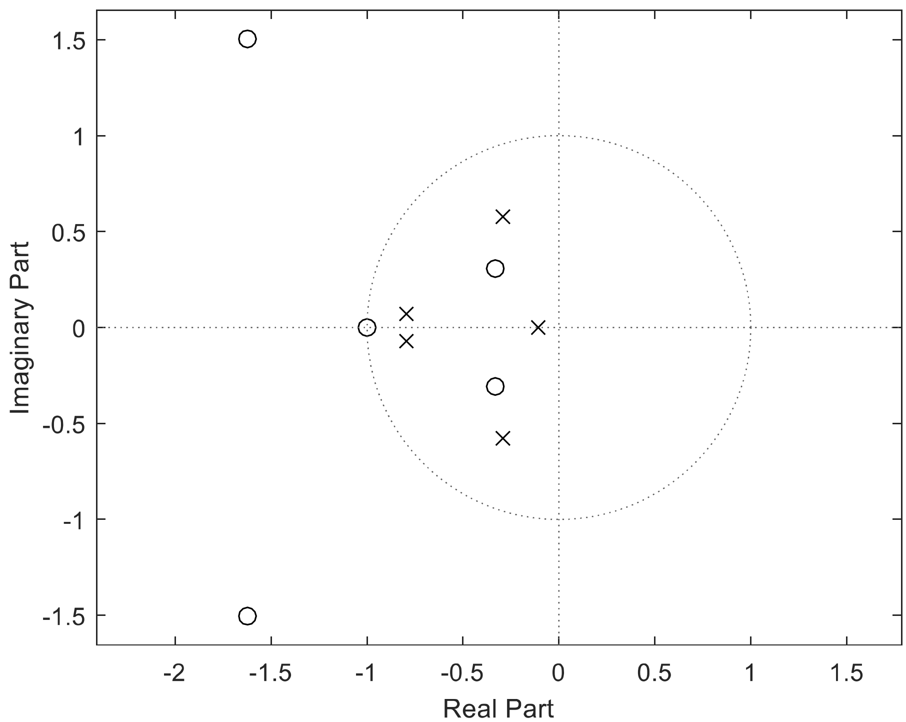
So we can get

$$H_1(z) = \frac{B_1(z)}{A_1(z)}$$
$$B_1(z) = 0.3588 + 1.7617z^{-1} + 4.0056z^{-2} + 4.0056z^{-3} + 1.7617z^{-4} + 0.3588z^{-5}$$
$$A_1(z) = 1.0000 + 2.2817z^{-1} + 2.2179z^{-2} + 1.2507z^{-3} + 0.3782z^{-4} + 0.0287z^{-5}$$

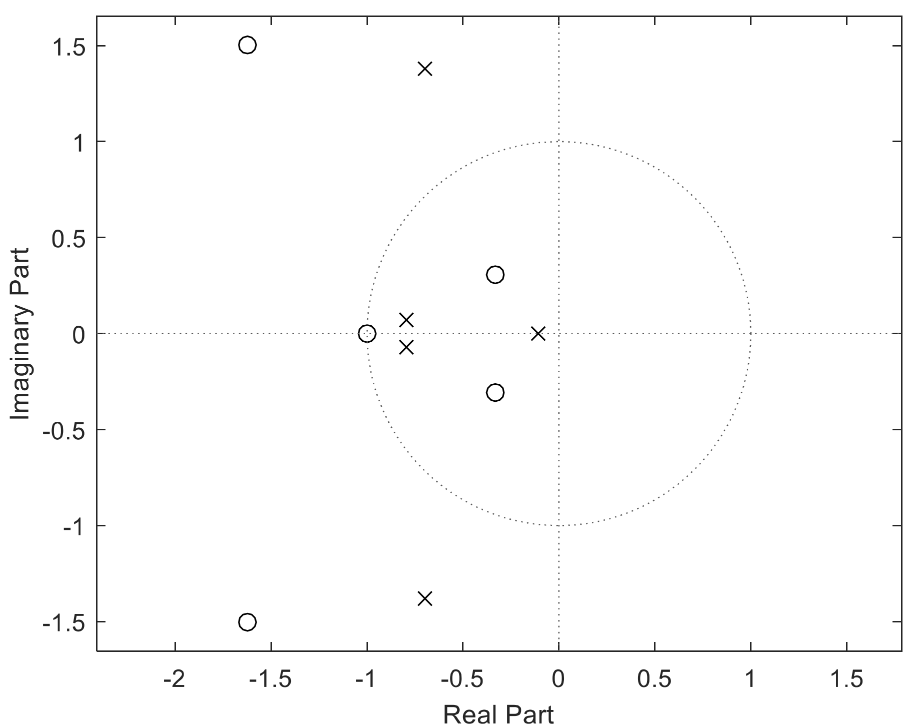
This system is stable and has the same magnitude response as the unstable system.

(b)

The pole/zero plot of the stable filter is as follows:



The pole/zero plot of the unstable filter is as follows:



(c)

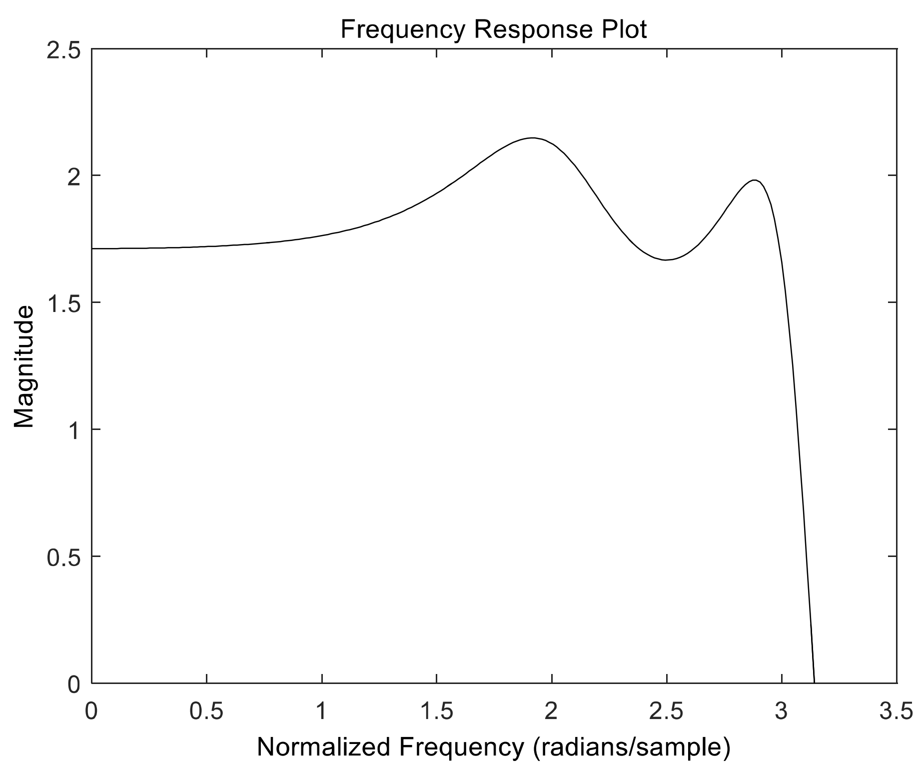
c.1. The original system

c.1.1. The script

```
% Read in the unstable filter
b1=[0.8581 4.2134 9.5802 9.5802 4.2134 0.8581];
a1=[1.0000 3.0937 5.5700 5.2578 2.0294 0.1642];
zplane(b1,a1) % pole/zero plot of the unstable filter
% Compute the frequency response using the Matlab
function freqz
n=0:199;
w=n*pi/199;
h=freqz(b1,a1,w);
hmag=abs(h);
hphase=angle(h);
% Plot the results
figure(2) % Magnitude
plot(w,hmag)
title('Frequency Response Plot')
xlabel('Normalized Frequency (radians/sample)')
ylabel('Magnitude')
print -dps iirfreq.ps
```

c.1.2. The plot

The plot of the magnitude response is as follows:



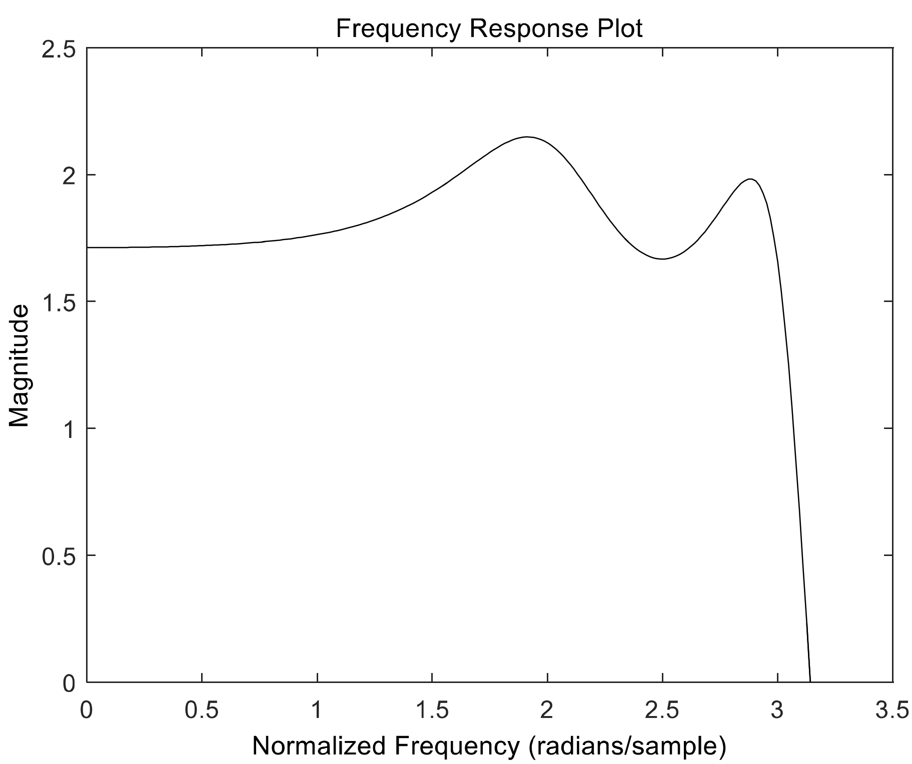
c.2. The stabilized system

c.2.1. The script

```
% Read in the stable filter
b2=[0.3588 1.7617 4.0056 4.0056 1.7617 0.3588];
a2=[1.0000 2.2817 2.2179 1.2507 0.3782 0.0287];
zplane(b2,a2) % pole/zero plot of the stable filter
% Compute the frequency response using the Matlab
function freqz
n=0:199;
w=n*pi/199;
h=freqz(b2,a2,w);
hmag=abs(h);
hphase=angle(h);
% Plot the results
figure(5) % Magnitude
plot(w,hmag)
title('Frequency Response Plot')
xlabel('Normalized Frequency (radians/sample)')
ylabel('Magnitude')
print -dps iirfreq.ps
```

c.2.2. The plot

The plot of the magnitude response is as follows:



ECE 513 homework 2

3. solve:

(a) From give conditions, we can get:

$$H(z) = \frac{G(z-q_1)(z-q_2)(z-q_3)(z-q_4)(z-q_5)(z-q_6)(z-q_7)(z-q_8)(z-q_9)}{(z-P_1)(z-P_2)(z-P_3)(z-P_4)(z-P_5)(z-P_6)(z-P_7)(z-P_8)(z-P_9)}$$

Observing that $q_1 = q_2^*$, $q_3 = q_4^*$, $q_5 = q_6^*$, $q_7 = q_8^*$
$P_1 = P_2^*$, $P_3 = P_4^*$, $P_5 = P_6^*$, $P_7 = P_8^*$

So $H_1(z) = \frac{(z-q_1)(z-q_2)}{(z-P_1)(z-P_2)} = \frac{z^2+1.4172z+1}{z^2+0.861z+0.9973}$ $\quad H_2(z) = \frac{(z-q_3)(z-q_4)}{(z-P_3)(z-P_4)} = \frac{z^2+0.8754z+1}{z^2+0.8366z+0.9837}$

$H_3(z) = \frac{(z-q_5)(z-q_6)}{(z-P_5)(z-P_6)} = \frac{z^2+0.897z+1}{z^2+0.7166z+0.9212}$ $\quad H_4(z) = \frac{(z-q_7)(z-q_8)}{(z-P_7)(z-P_8)} = \frac{z^2+1.0018z+1}{z^2+0.183z+0.6439}$

$H_5(z) = \frac{z-q_9}{z-P_9} = \frac{z+1}{z-0.3854}$ $\quad A = \sqrt[5]{G} \approx 1.2$

Therefore, we can write: $H(z) = 1.2H_1(z) \cdot 1.2H_2(z) \cdot 1.2H_3(z) \cdot 1.2H_4(z) \cdot 1.2H_5(z)$

(b) The corresponding difference equations with the use of A are given by

$$S_1(n) = 1.2X(n) + 1.7006X(n-1) + 1.2X(n-2) - 0.861S_1(n-1) - 0.9973S_1(n-2)$$

$$S_2(n) = 1.2S_1(n) + 1.0505S_1(n-1) + 1.2S_1(n-2) - 0.8366S_2(n-1) - 0.9837S_2(n-2)$$

$$S_3(n) = 1.2S_2(n) + 1.0764S_2(n-1) + 1.2S_2(n-2) - 0.7166S_3(n-1) - 0.9212S_3(n-2)$$

$$S_4(n) = 1.2S_3(n) + 1.2022S_3(n-1) + 1.2S_3(n-2) - 0.183S_4(n-1) - 0.6439S_4(n-2)$$

$$y(n) = S_5(n) = 1.2S_4(n) + 1.2S_4(n-1) + 0.3854y(n-1)$$

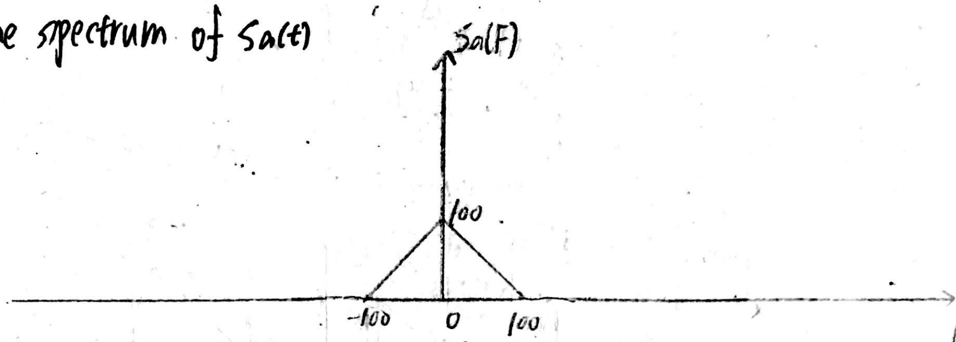
ECE 513 homework 2

4. solve: $S_a(t) = X_a^2(t)$

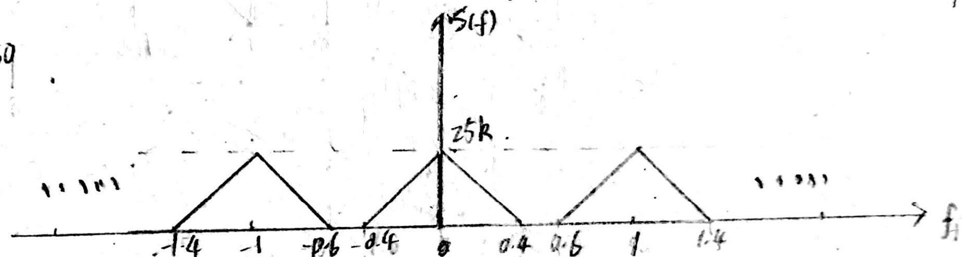
So $S_a(F) = X_a(F) * X_a(F)$

Therefore the spectrum of $S_a(t)$

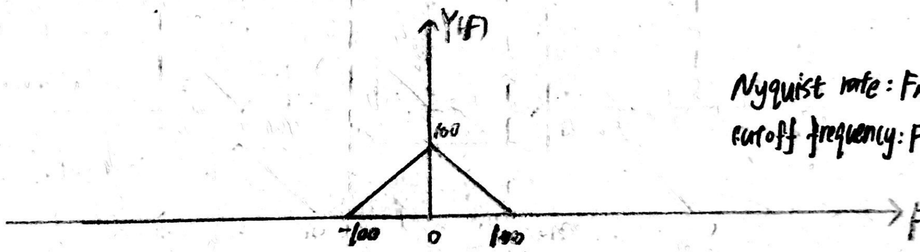
(a)



$F_s = 250$



$F_s = 250$

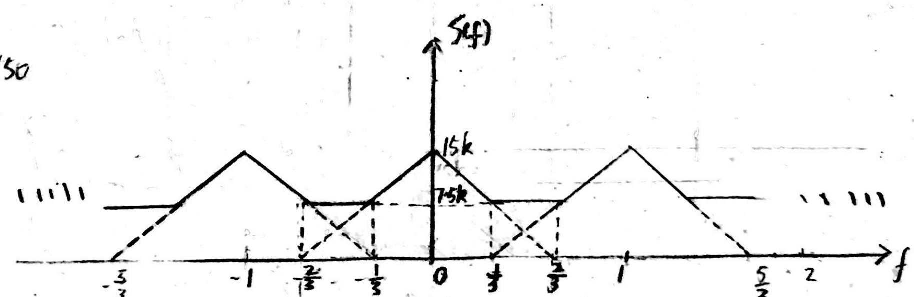


Nyquist rate: $F_N = 200$

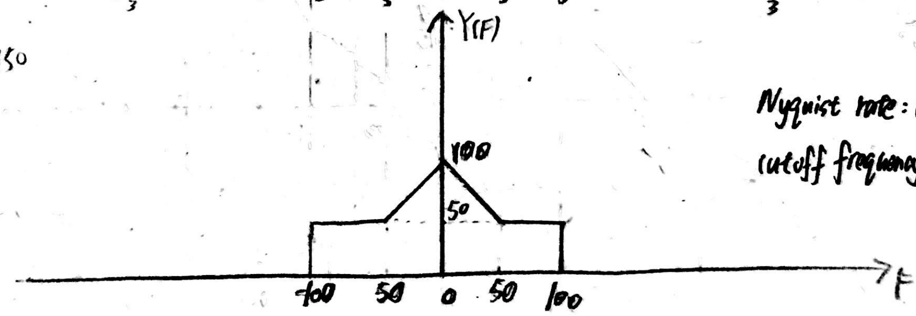
cutoff frequency: $F_c = 125$.

(b)

$F_s = 150$



$F_s = 150$



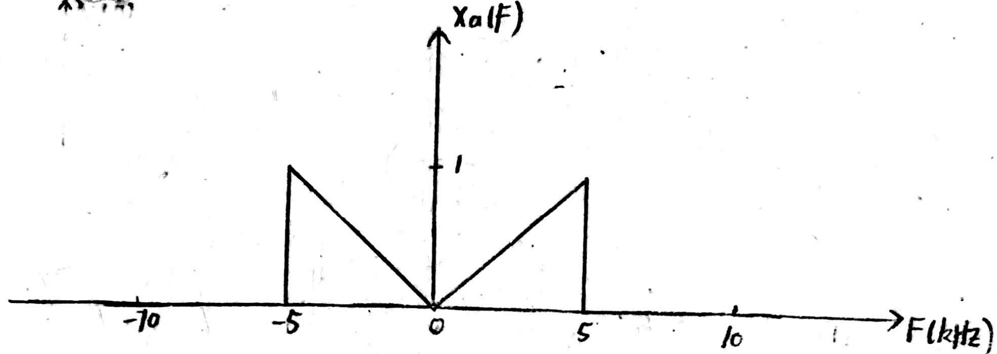
Nyquist rate: $F_N = 200$

cutoff frequency: $F_c = 75$

ECE 513 homework 2

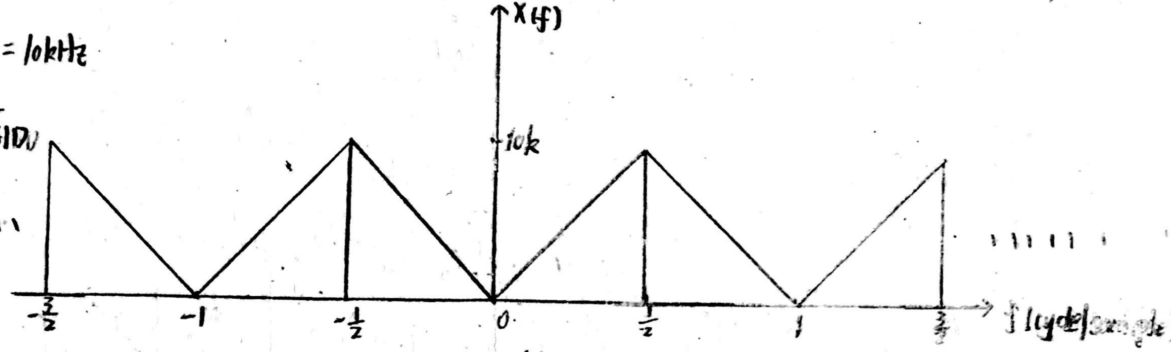
5. solve:

(a)

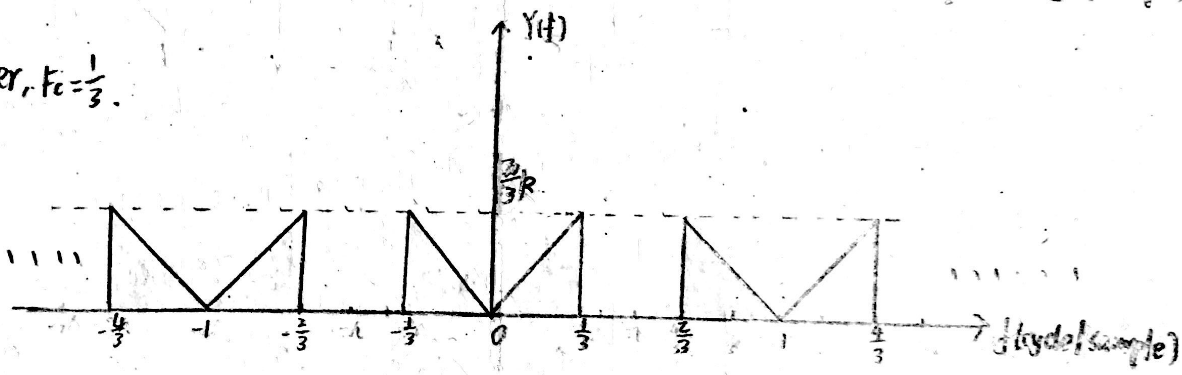


$F_{s,A/D} = 10kHz$

$f = \frac{F}{F_{s,A/D}}$

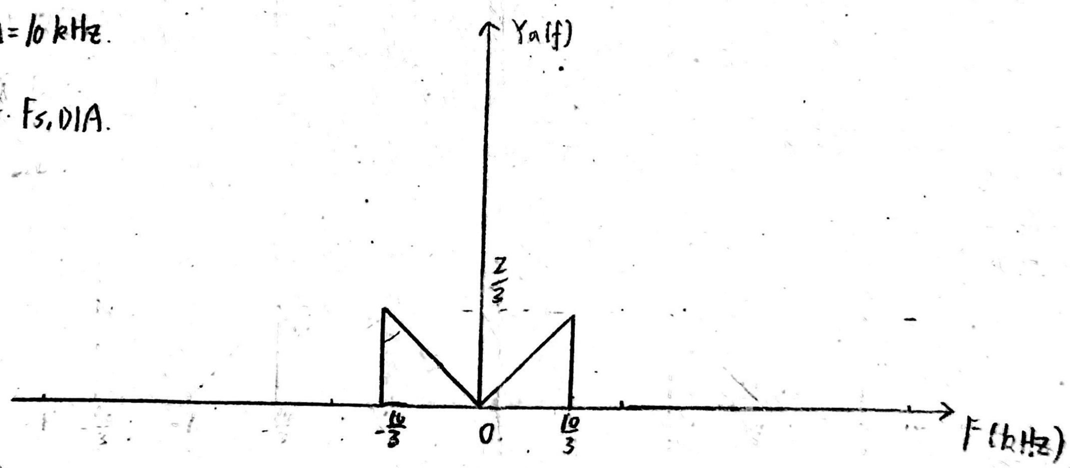


low-pass filter, $F_c = \frac{1}{3}$.

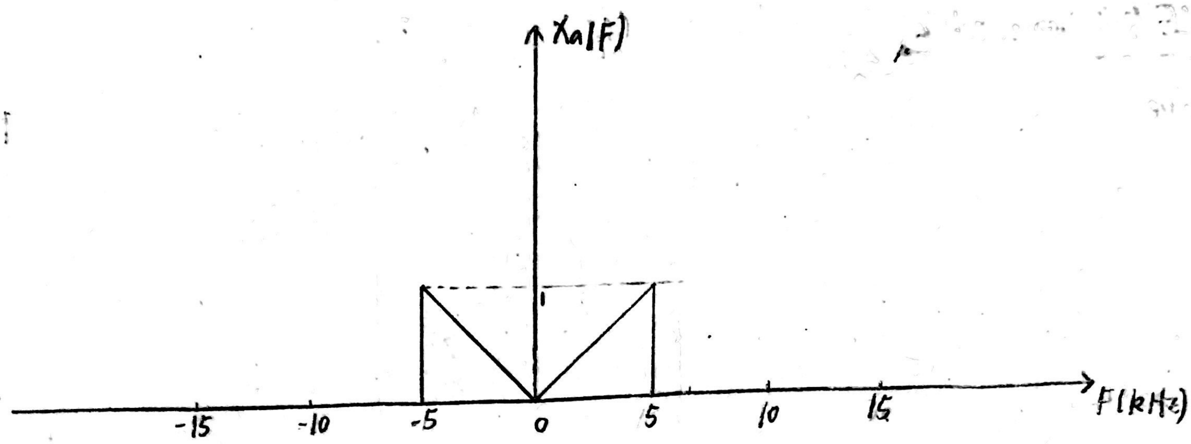


$F_{s,D/A} = 10$ kHz.

$F = f \cdot F_{s,D/A}$.

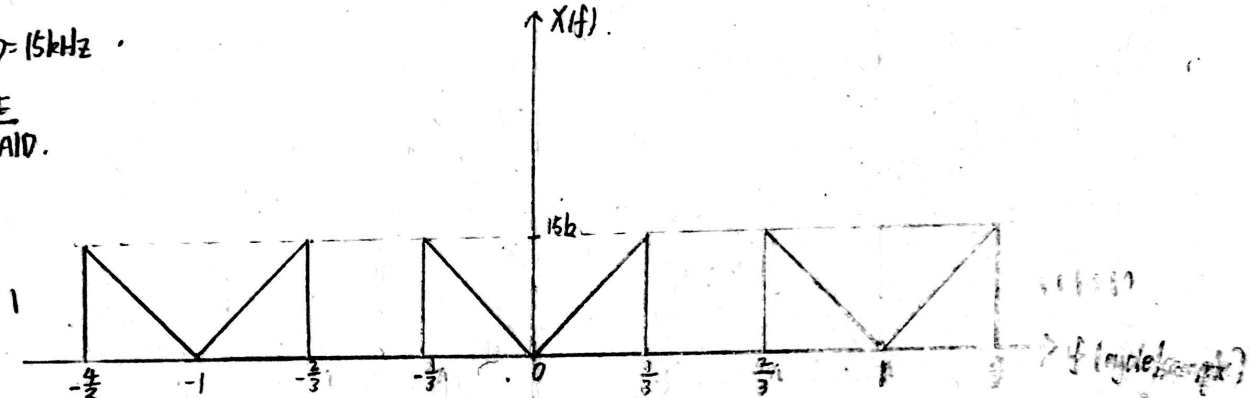


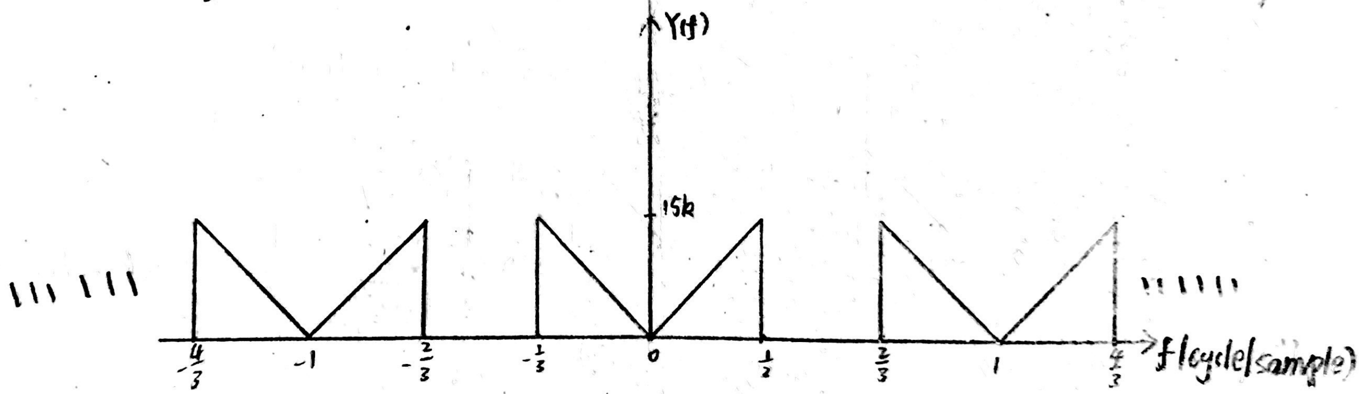
(b)



$F_{s,A/D} = 15kHz$

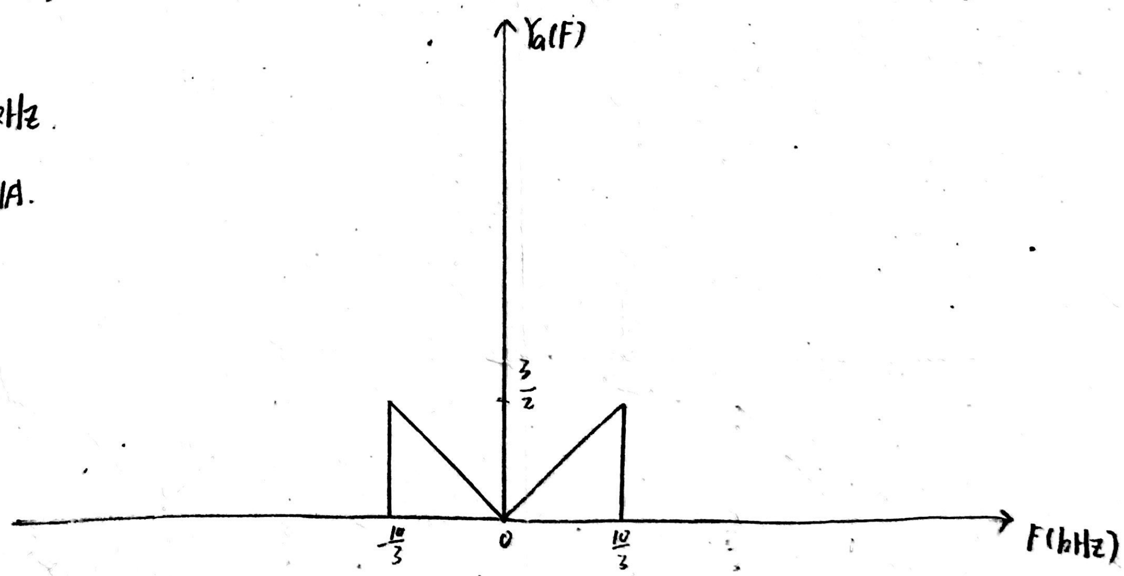
$f = \frac{F}{F_{s,A/D}}$



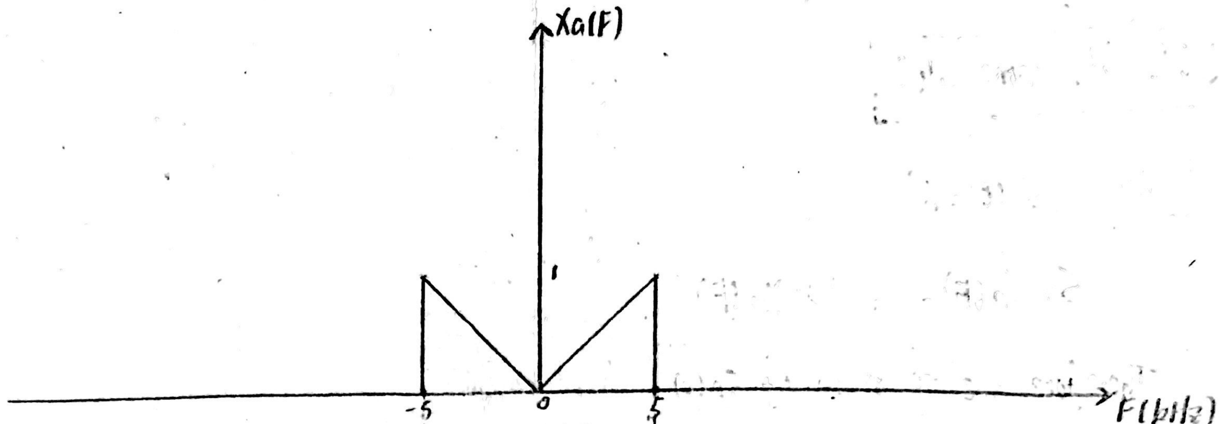


$F_{s\,D/A} = 10kHz$

$F = f \cdot F_{s\,D/A}$

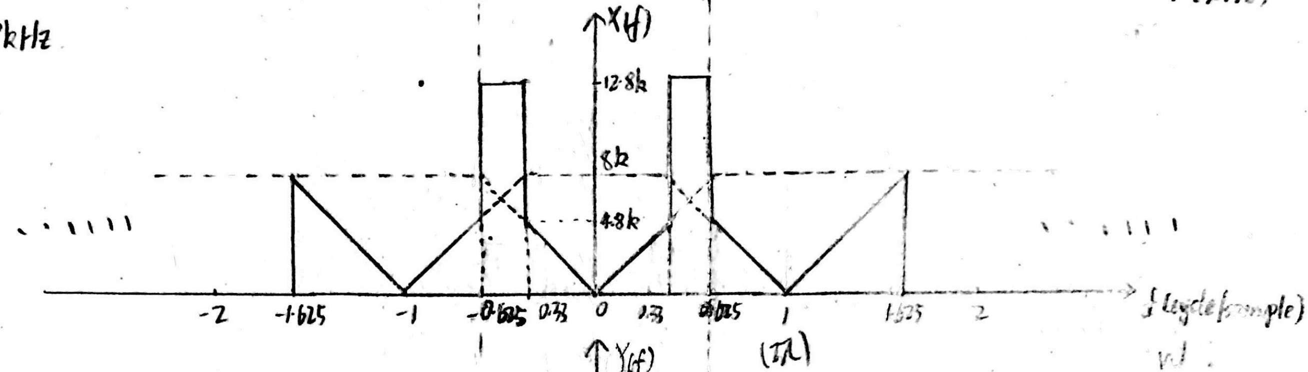


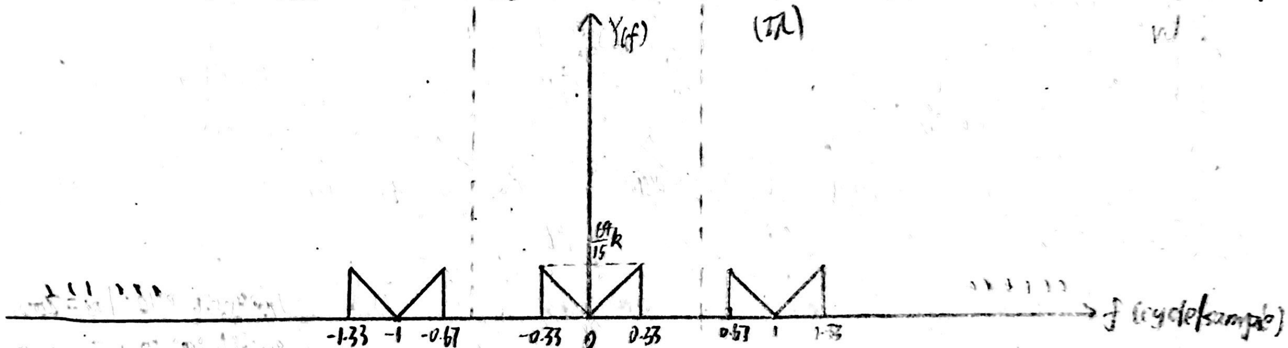
(c)



$F_{s,A/D} = 8kHz$

$f = \frac{F}{F_{s,A/D}}$





$F_{s,D/A} = 16kHz$

$F = f \cdot F_{s,A/D}$