1. Solve:

(a)

The Matlab code to use only 2-point FFT to calculate the 8-point FFT of the sequence x(n) with the help of Decimation in Time Approach is shown as follows:

```
xn=[4,-3,2,-1,-5,3,-1,0];
Xk_a=zeros(1,8);
x00=[xn(1),xn(5)];
x01=[xn(3),xn(7)];
x10=[xn(2),xn(6)];
x11=[xn(4),xn(8)];

X00=fft(x00,2);                          % To calculate 2-point
                                         FFT of X00(k)
X01=fft(x01,2);                          % To calculate 2-point
                                         FFT of X01(k)
X10=fft(x10,2);                          % To calculate 2-point
                                         FFT of X10(k)
X11=fft(x11,2);                          % To calculate 2-point
                                         FFT of X11(k)
X00=[X00(1),X00(2),X00(1),X00(2)];       % Utilize the
                                         periodicity to get 4-
                                         point
X01=[X01(1),X01(2),X01(1),X01(2)];       % Utilize the
                                         periodicity to get 4-
                                         point
X10=[X10(1),X10(2),X10(1),X10(2)];       % Utilize the
                                         periodicity to get 4-
                                         point
X11=[X11(1),X11(2),X11(1),X11(2)];       % Utilize the
                                         periodicity to get 4-
                                         point

X0=zeros(1,8);
X1=zeros(1,8);
for k=0:1:3
 X0(k+1)=X00(k+1)+exp(-1i*0.5*pi*k)*X01(k+1);
 X1(k+1)=X10(k+1)+exp(-1i*0.5*pi*k)*X11(k+1);
end
X0=[X0(1),X0(2),X0(3),X0(4),X0(1),X0(2),X0(3),X0(4)]; %Utilize
                                                       the periodicity
                                                       to get 8-point
X1=[X1(1),X1(2),X1(3),X1(4),X1(1),X1(2),X1(3),X1(4)]; %Utilize
                                                       the periodicity
```

```
                                                  to get 8-point
Xk_a=zeros(1,8);
for k=0:1:7
 Xk_a(k+1)=X0(k+1)+exp(-1i*0.25*pi*k)*X1(k+1);
end

w=0:7;
figure(4)          % Plot the magnitude spectrum for Xk_a
plot(w,abs(Xk_a))
title('The magnitude of the resulting spectrum in part(a)')
xlabel('k')
ylabel('Magnitude')
```

The figure of the magnitude of the resulting spectrum for k=0,1,…,7 is shown in Fig. 1:

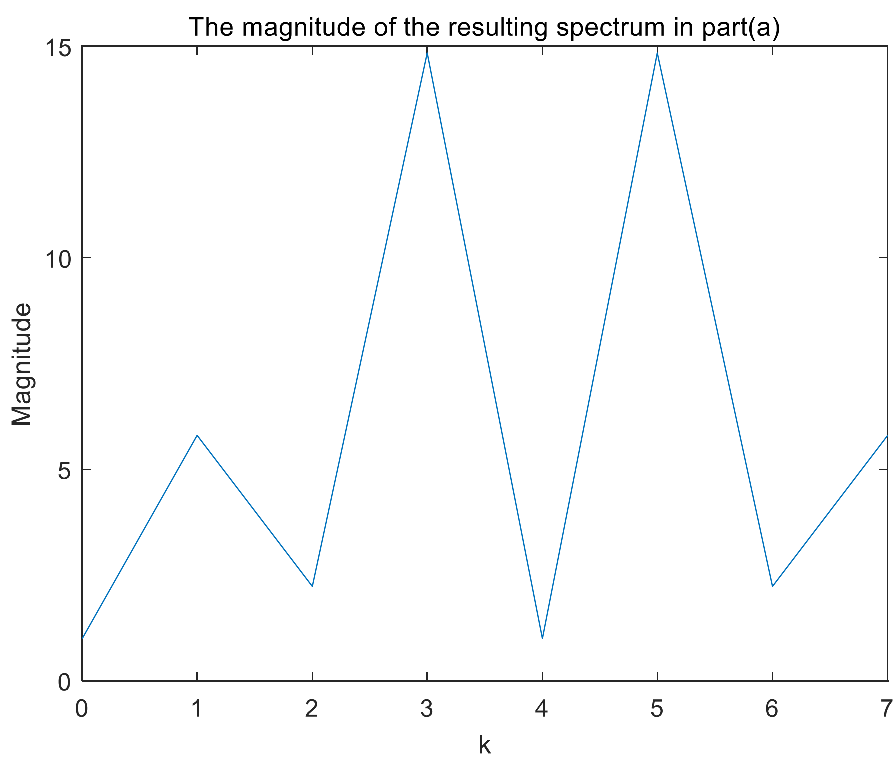


Fig. 1. The magnitude of the resulting spectrum for k=0,1,…,7 in part(a)

(b)

Calculate the 8-point DFT of the sequence x(n) using the MATLAB routine **fft**. Plot the magnitude of the resulting spectrum for k = 0,1,...,7. The Matlab code is shown as follows:

```
Xk_b=fft(xn,8);
w=0:7;
figure(5)          % Plot the magnitude spectrum for Xk_b
plot(w,abs(Xk_b))
title('The magnitude of the resulting spectrum in part(b)')
```

```
xlabel('k')
ylabel('Magnitude')
```

The figure of the magnitude of the resulting spectrum for k=0,1,…,7 is shown in Fig. 2:

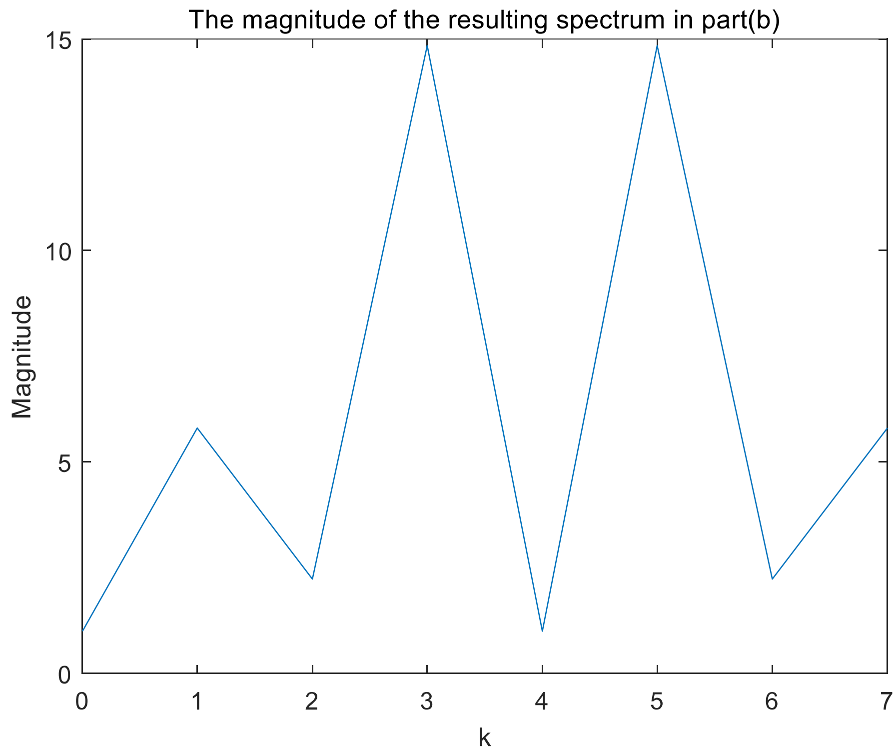


Fig. 1. The magnitude of the resulting spectrum for k=0,1,…,7 in part(b)

(c)

Looking at Fig. 1 and Fig. 2, they almost look the same. To check the conclusion more carefully, I plot the figure of the difference between two figures. The Matlab code is shown as follows:

```
figure(6)          % Plot the difference between Xk_a and Xk_b
plot(w,abs(Xk_a)-abs(Xk_b))
title('The difference betweent results in part(a) and
part(b)')
xlabel('k')
ylabel('Difference')
```

The figure of the difference between two figures is shown in Fig. 3:

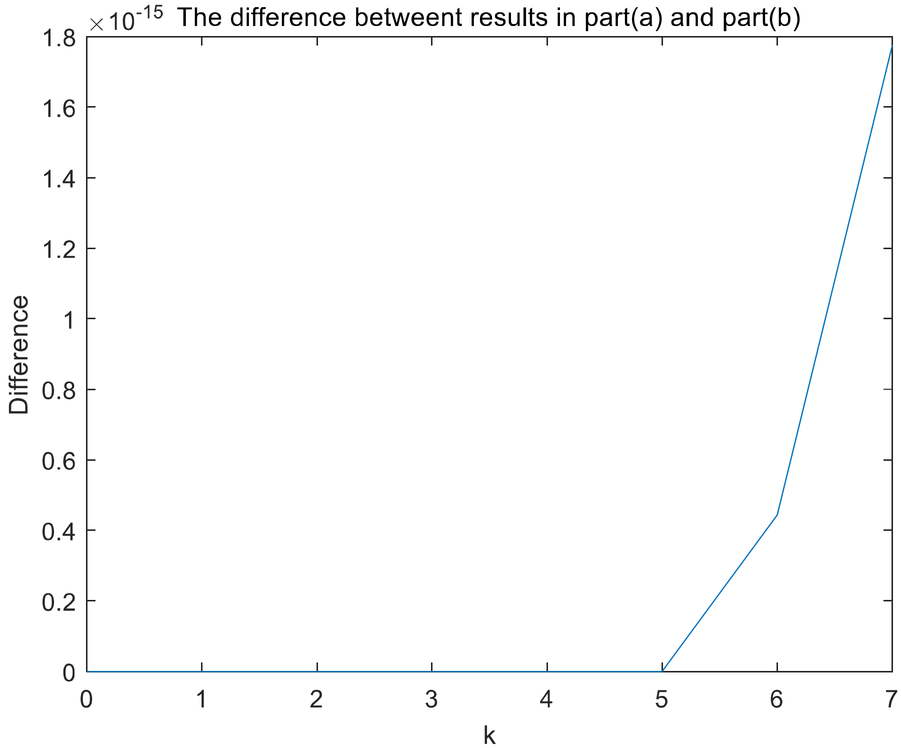


Fig. 3. The difference between Fig.1 and Fig.2

Conclusion: The error is tremendously small, so the result in part(a) is the same as the one in part(b).

2. Solve:

(a) $H(z)=\sum_{k=0}^{14} b(k)z^{-k}$

$$H(z)=b(0)+b(4)z^{-4}+b(8)z^{-8}+b(12)z^{-12}$$
$$+z^{-1}[b(1)+b(5)z^{-4}+b(9)z^{-8}+b(13)z^{-12}]$$
$$+z^{-2}[b(2)+b(6)z^{-4}+b(10)z^{-8}+b(14)z^{-12}]$$
$$+z^{-3}[b(3)+b(7)z^{-4}+b(11)z^{-8}]$$

$$H(z)=E_1(z^4)+z^{-1}E_2(z^4)+z^{-2}E_3(z^4)+z^{-3}E_4(z^4)$$

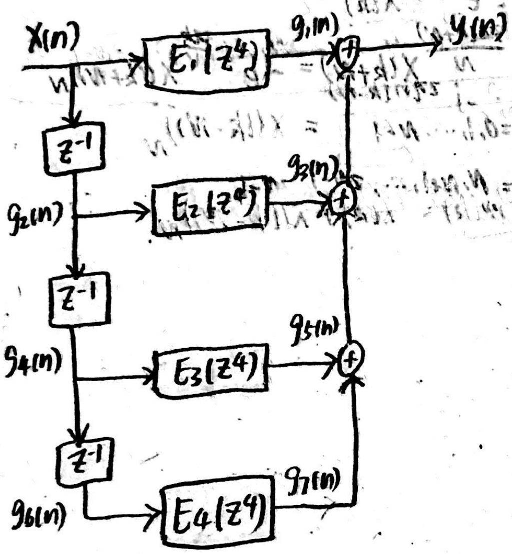
$$E_1(z)=0.0168+0.0287z^{-1}+0.2453z^{-2}-0.0409z^{-3}$$

$$E_2(z)=0.0264-0.1205z^{-1}-0.1205z^{-2}+0.0264z^{-3}$$

$$E_3(z)=-0.0409+0.2453z^{-1}+0.0287z^{-2}+0.0168z^{-3}$$

$$E_4(z)=0.0334+0.6694z^{-1}+0.0334z^{-2}$$

(b)



The corresponding difference equations are:

$$g_1(n)=0.0168x(n)+0.0287x(n-4)+0.2453x(n-8)-0.0409x(n-12)$$

$$g_2(n)=x(n-1)$$

$$g_3(n)=0.0264g_2(n)-0.1205g_2(n-4)-0.1205g_2(n-8)+0.0264g_2(n-12)$$

$$g_4(n)=g_2(n-1)$$

$$g_5(n)=-0.0409g_4(n)+0.2453g_4(n-4)+0.0287g_4(n-8)+0.0168g_4(n-12)$$

$$g_6(n)=g_4(n-1)$$

$$g_7(n)=0.0334g_6(n)+0.6694g_6(n-4)+0.0334g_6(n-8)$$

So $y(n)=g_1(n)+g_3(n)+g_5(n)+g_7(n)$.

3. Solve: $X(k)=\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k n}{N}} \quad k=0,1,\cdots,N-1$

(a) due to 2N-point sequence $y(n)=\begin{cases} x(\frac{n+1}{2}) &, n \text{ odd} \\ 0 &, n \text{ even}\end{cases}$

so $y(n)=\{0, x(1), 0, x(2), 0, x(3), \cdots, 0, x(N)\}$ 2N points

$x(N)=x(0)$

$$Y(k)=\sum_{n=0}^{2N-1} y(n) e^{-j\frac{2\pi k n}{2N}} \quad k=0,1,\cdots,2N-1$$

$$=\sum_{n=0}^{N-1} y(2n) e^{-j\frac{2\pi k\cdot 2n}{2N}} + \sum_{n=0}^{N-1} y(2n+1) e^{-j\frac{2\pi k\cdot(2n+1)}{2N}}$$

$$=\sum_{n=0}^{N-1} y(2n+1) e^{-j\frac{2\pi k(2n+1)}{2N}} \quad k=0,1,\cdots,2N-1 \cdots (1)$$

when $k\in[0,N-1]$, (1) is equal to $Y_1(k)=\sum_{n=0}^{N-1} y(2n+1) e^{-j\frac{2\pi k(2n+1)}{N}} \quad k=0,1,\cdots,N-1$

$$=\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k n}{N}} = X(k)$$

when $k\in\{N,2N-1]$, (1) is equal to $Y_2(k)=\sum_{n=0}^{N-1} y(2n+1) e^{-j\frac{2\pi k(2n+1)}{N}}, k=N,N+1,\cdots,2N-1$

$$=\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi n(k-N)}{N}} = X((k-N))_N$$

So $Y(k)=Y_1(k)+Y_2(k)=X(k)+X((k-N))_N$.

(b) $x_3(n)=\frac{1}{N}\sum_{k=0}^{N-1} X_3(k) e^{j\frac{2\pi k n}{N}}, \quad n=0,1,\cdots,N-1$

$$=\frac{1}{N}\sum_{k=0}^{N-1}\frac{1}{N}\sum_{l=0}^{N-1} X_1(l) X_2[((k-l))_N] e^{j\frac{2\pi k n}{N}}$$

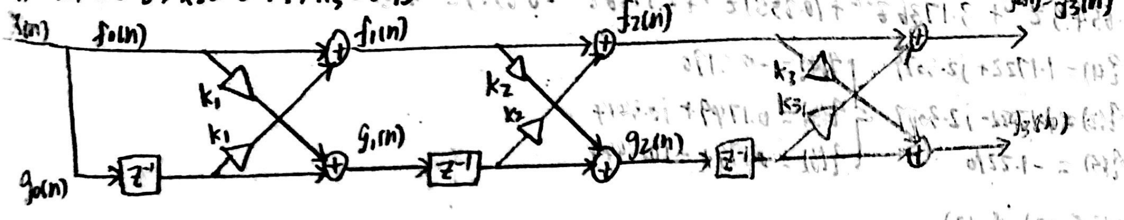
$$=\frac{1}{N}\sum_{k=0}^{N-1} X_1(l) e^{j\frac{2\pi n l}{N}} \frac{1}{N}\sum_{l=0}^{N-1} X_2[((k-l))_N] e^{j\frac{2\pi n(k-l)}{N}}$$

$$=\frac{1}{N}\sum_{k=0}^{N-1} X_1(l) e^{j\frac{2\pi n l}{N}} \cdot x_2(n)$$

$$=x_1(n)\cdot x_2(n)$$

4. Solve:

(a) $K_1 = 0.8,\ K_2 = -0.47,\ K_3 = 0.75$



(b) $f_0(n) = g_0(n) = x(n)$

$f_1(n) = f_0(n) + k_1 g_0(n-1) = x(n) + k_1 x(n-1)$

$g_1(n) = k_1 f_0(n) + g_0(n-1) = k_1 x(n) + x(n-1)$

$f_2(n) = f_1(n) + k_2 g_1(n-1) = x(n) + k_1 x(n-1) + k_2[k_1 x(n-1) + x(n-2)] = x(n) + (k_1 + k_1 k_2) x(n-1) + k_2 x(n-2)$

$g_2(n) = k_2 f_1(n) + g_1(n-1) = k_2[x(n) + k_1 x(n-1)] + k_1 x(n-1) + x(n-2) = k_2 x(n) + (k_1 + k_1 k_2) x(n-1) + x(n-2)$

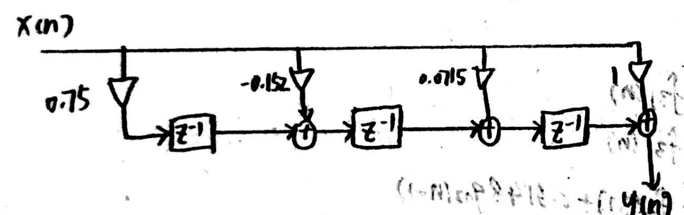
$y(n) = f_3(n) = f_2(n) + k_3 g_2(n-1)$

$= x(n) + (k_1 + k_1 k_2) x(n-1) + k_2 x(n-2) + k_2 k_3 x(n-1) + (k_1 k_3 + k_1 k_2 k_3) x(n-2) + k_3 x(n-3)$

$= x(n) + (k_1 + k_1 k_2 + k_2 k_3) x(n-1) + (k_2 + k_1 k_3 + k_1 k_2 k_3) x(n-2) + k_3 x(n-3)$

So the impulse response $h(n) = \delta(n) + 0.0715\,\delta(n-1) + 0.1528\,\delta(n-2) + 0.75\,\delta(n-3)$

(c) The equivalent direct-form structure:



5. Solve: $H_6(z) = \dfrac{H(z)}{0.1325}$

(1)

$$= 1-0.6543z^{-1}+3.1736z^{-2}+10.7581z^{-3}+3.1736z^{-4}-0.6543z^{-5}+z^{-6}$$

The roots of $H_6(z)$: $\begin{cases} q(1)=1.1722+j2.3079 \\ q(2)=1.1722-j2.3079 \\ q(3)=-1.2210 \end{cases}$ $\begin{cases} q(4)=-0.8190 \\ q(5)=0.1749+j0.3444 \\ q(6)=0.1749-j0.3444 \end{cases}$

$$H(z)=0.1325\,S_1(z)\cdot S_2(z)$$

$$S_1(z)=[z-q(1)][z-q(2)][z-q(3)] = 1-1.1234z^{-1}+3.8379z^{-2}+8.1813z^{-3}$$

$$S_2(z)=[z-q(4)][z-q(5)][z-q(6)] = 1+0.4692z^{-1}-0.1373z^{-2}+0.1222z^{-3}$$

Step 1: $k_{31}=8.1813$

$$B_{31}(z)=z^{-3}S_1(z^{-1})=8.1813+3.8379z^{-1}-1.1234z^{-2}+z^{-3}$$

$$A_{31}(z)=S_1(z)\rightarrow A_{21}(z)=\frac{A_{31}(z)-k_{31}\cdot B_{31}(z)}{1-k_{31}^2}=1+0.4933z^{-1}-0.1976z^{-2}$$

$k_{21}=-0.1976$, $k_{11}=\dfrac{0.4933}{1-0.1976}\approx 0.6148$

Step 2: $k_{32}=0.1222$

$$B_{32}(z)=z^{-3}S_2(z^{-1})=0.1222-0.1373z^{-1}+0.4692z^{-2}+z^{-3}$$

$$A_{32}(z)=S_2(z)\rightarrow A_{22}(z)=\frac{A_{32}(z)-k_{32}B_{32}(z)}{1-k_{32}^2}=1+0.4933z^{-1}-0.1976z^{-2}$$

$k_{22}=-0.1976$ $\quad k_{12}=\dfrac{0.4933}{1-0.1976}\approx 0.6148$

The difference equations:

(2) $f_{01}(n)=x(n)$

$g_{01}(n)=x(n)$

$f_{11}(n)=f_{01}(n)+0.6148g_{01}(n-1)$

$g_{11}(n)=0.6148f_{01}(n)+g_{01}(n-1)$

$f_{21}(n)=f_{11}(n)-0.1976g_{11}(n-1)$

$g_{21}(n)=-0.1976f_{11}(n)+g_{11}(n-1)$

$f_{31}(n)=f_{21}(n)+8.1813g_{21}(n-1)$

$g_{31}(n)=8.1813f_{21}(n)+g_{21}(n-1)$

$f_{02}(n)=f_{31}(n)$

$g_{02}(n)=f_{31}(n)$

$f_{12}(n)=f_{02}(n)+0.6148g_{02}(n-1)$

$g_{12}(n)=0.6148f_{02}(n)+g_{02}(n-1)$

$f_{22}(n)=f_{12}(n)-0.1976g_{12}(n-1)$

$g_{22}(n)=-0.1976f_{12}(n)+g_{12}(n-1)$

$f_{32}(n)=0.1325[f_{22}(n)+0.1222g_{22}(n-1)]$

$g_{32}(n)=0.1222f_{22}(n)+g_{22}(n-1)$