Simulating Collisional Dark Matter

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Contents

Chapter 1

Objetives

1.1 General Objective

To simulate the phase space of a collisional dark matter fluid using a Lattice-Boltzmann Method

1.2 Specific Objectives

- To implement a Lattice-Boltzmann simulation using a 4-dimensional phase space and a varying collisional term.
- To implement a Lattice-Boltzmann simulation using a 6-dimensional phase space and a varying collisional term.
- To study the dynamical behavior of a dark matter fluid using different equilibrium distributions in the collisional term.
- To compare the phase space of a collisional dark matter fluid with its collisionless version.

Chapter 2

Introduction

2.1 Dark Matter, or the Missing Mass Problem

2.2 Types of Dark Matter

2.3 The Boltzmann Equation

2.4 Mesoscopic Modeling

Traditionally, fluid dynamics have been modeled using the Navier-Stokes equation, or some modification of it.

2.5 Lattice Automata and Lattice Boltzmann

2.6 BGK Approximation

Chapter 3

The Lattice Boltzmann Algorithm

As previously asserted, the heart of the Lattice-Boltzmann Algorithm lies on its discretization of the phase space[1] [2]. To discretize the phase space, one must first choose the region to simulate. In this work, we name the extremal values in the k axis of the phase space K_{min} and K_{max}. Then, one has to fix either the size of the grid or the size of the lattice. We named the size of the grid in the k axis N_k (i.e. N_x or N_{vz}). The size of the lattice in the k axis (dk) and the extremal values are related by:

$$dk = \frac{K_{max} - K_{min}}{N_k}$$

In this work we are going to consider the density-velocity phase space. Now that we have properly defined the phase space grid, we can proceed to the initialization. For simplicity, we choose gaussian initial conditions given by:

$$f(\mathbf{r}, \mathbf{v}, 0) = A \exp\left\{-\frac{\mathbf{r}^2}{\sigma_r^2} - \frac{\mathbf{v}^2}{\sigma_v^2}\right\} \tag{3.1}$$

Where $f(\mathbf{r}, \mathbf{v}, 0)$ is initialization of the phase space, A is an indirect measure of the total mass in the system, $\mathbf{r}$ is the vector $= (x, y, z)$, $\mathbf{v}$ is the vector $= (vx, vy, vz)$, and σ_r and σ_v are a measure of the width of the gaussian profile in the given axis. After initialization, the system evolves by classical mechanics and the modelling of the collisional step, which can be seen more easily in figure 3.1

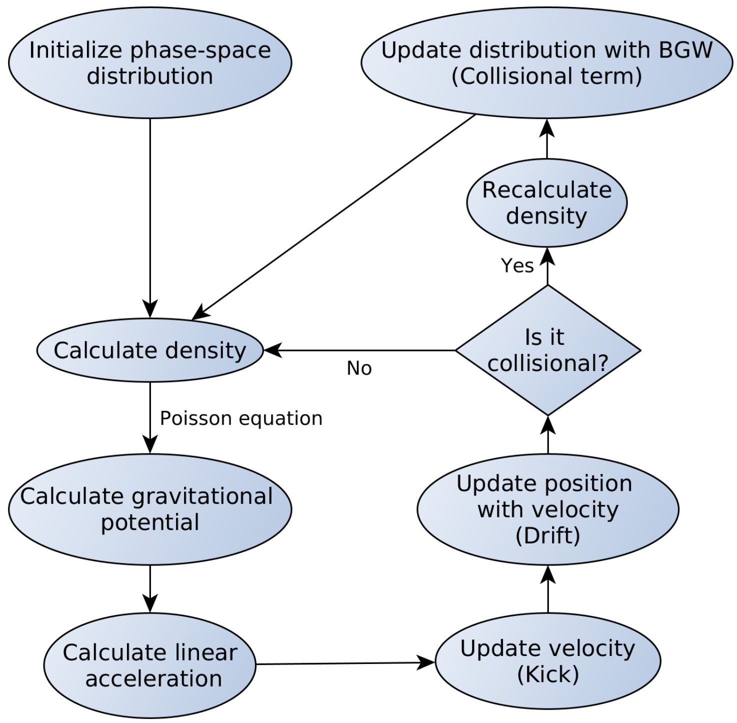

Figure 3.1: Flowchat of the algorithm.

Given the density-velocity phase space, the spatial density of matter is given by the integral:

$$\rho(\mathbf{r}, t) = \int_{-\infty}^{\infty} f(\mathbf{r}, \mathbf{v}, t)\,\mathrm{d}\mathbf{v}.$$

Which, when evaluating in the lattice becomes:

$$\rho(\mathbf{r}, t) = \sum_{\mathbf{V}_{min}}^{\mathbf{V}_{max}} f(\mathbf{r}, \mathbf{v}, t)\,\mathrm{d}\mathbf{v} \tag{3.2}$$

and during initialization:

$$\rho(\mathbf{r}, 0) = \sum_{\mathbf{V}_{min}}^{\mathbf{V}_{max}} f(\mathbf{r}, \mathbf{v}, 0)\,\mathrm{d}\mathbf{v} \tag{3.3}$$

Once we have calculated the density, we can use the Poisson equation to calculate the potential due the gravitational force:

$$\nabla^2 \Phi(\mathbf{r}, t) = 4\pi G \rho(\mathbf{r}, t) \tag{3.4}$$

3.1 The Poisson Equation

3.2 Kick and Drift

asd

3.3 The Collisional Step

asd

3.4 Units and Initial Conditions

Chapter 4

Results

4.1 No Collisional

4.2 $\tau = 10$

4.3 $\tau = 100$

4.4 $\tau = 250$

4.5 $\tau = 500$

4.6 $\tau = 1000$

4.7 Different Equlibrium Distributions

Chapter 5

Conclusions

5.1 A Numerically Stable Simulation

Bibliography

[1] Sebastián Franco Ulloa. Simulaciones de un fluido débilmente auto-interactuante con métodos de lattice-boltzmann, 5 2017.

[2] Philip Mocz and Sauro Succi. Integer lattice dynamics for Vlasov–Poisson. *Mon. Not. Roy. Astron. Soc.*, 465(3):3154–3162, 2017.