Simulating Collisional Dark Matter

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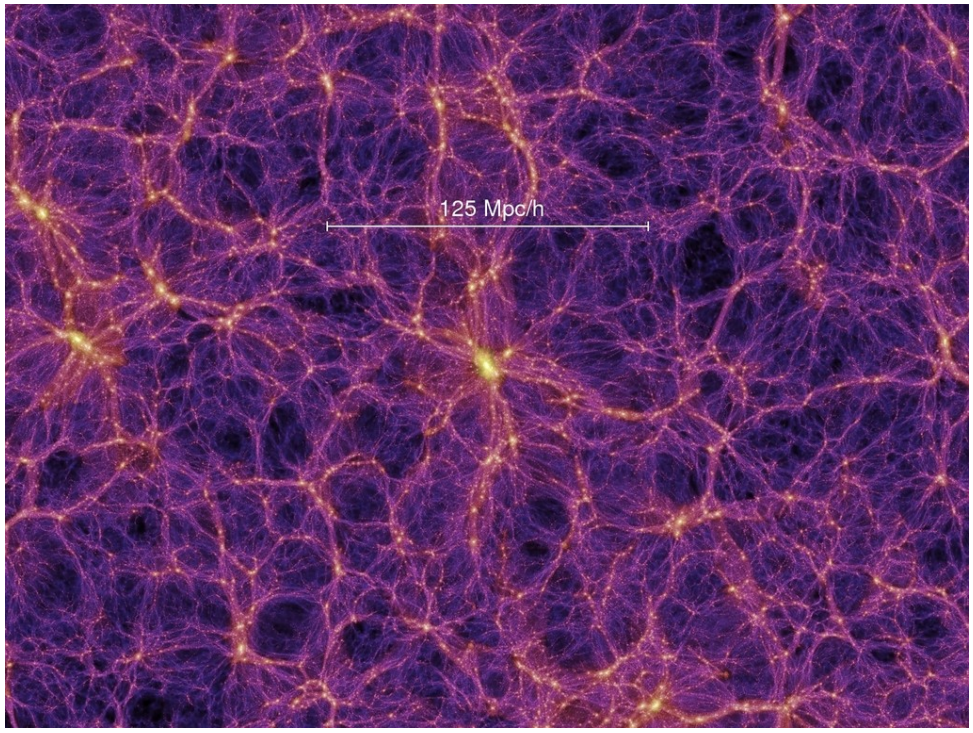
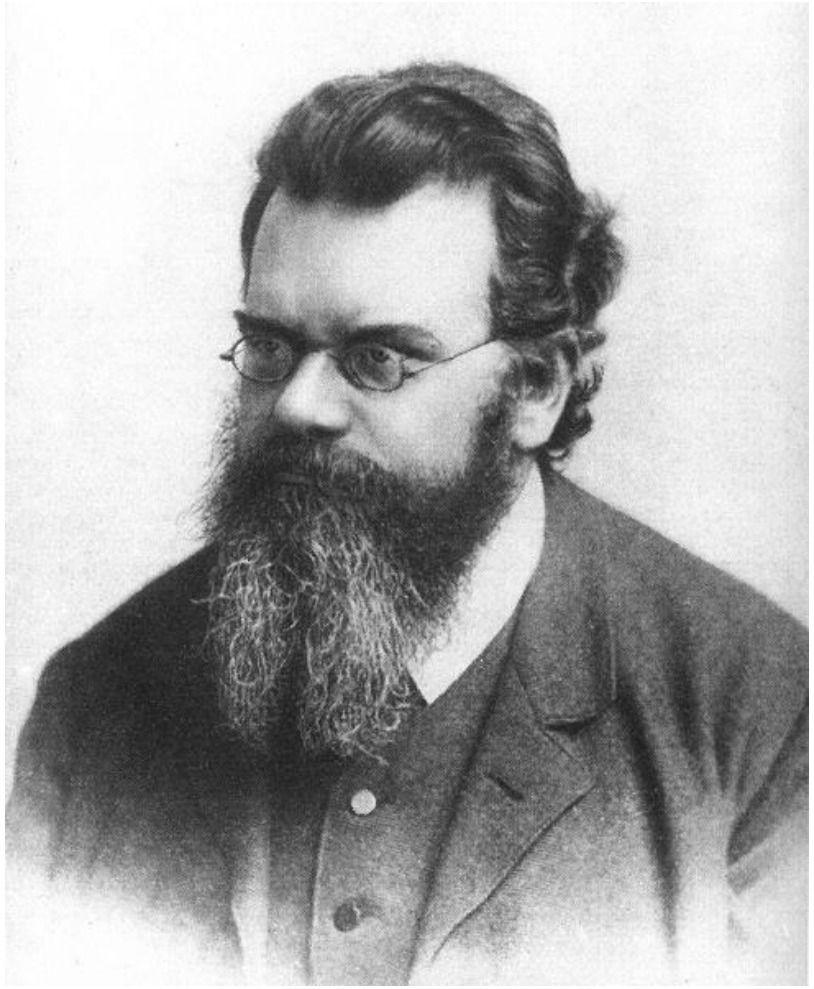
Collisional and Collisionless Dark Matter



Other efforts on simulating Collisional DM

The Boltzmann Equation

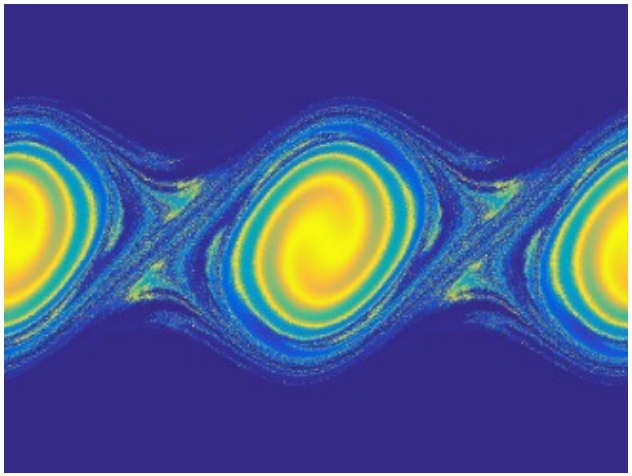


$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \cdot \vec{\nabla}_{\vec{r}} f + F \cdot \vec{\nabla}_{\vec{p}} = C[f]$$

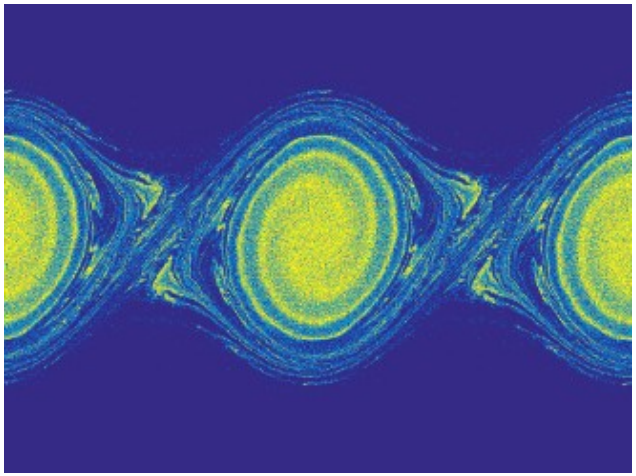
$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \cdot \vec{\nabla}_{\vec{r}} f + F \cdot \vec{\nabla}_{\vec{p}} = 0$$

Solving Boltzmann Equation

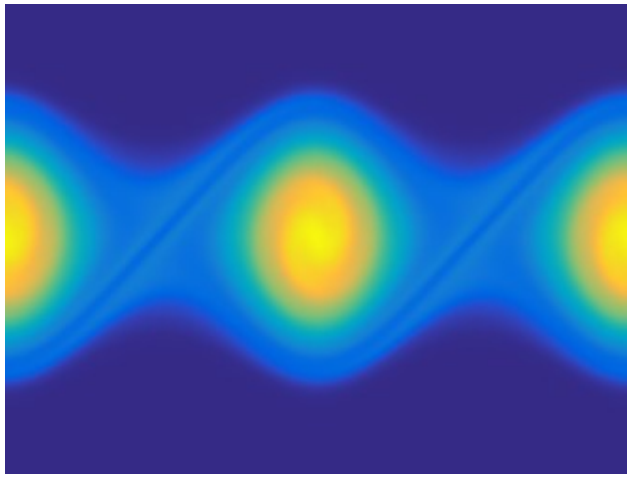
LB



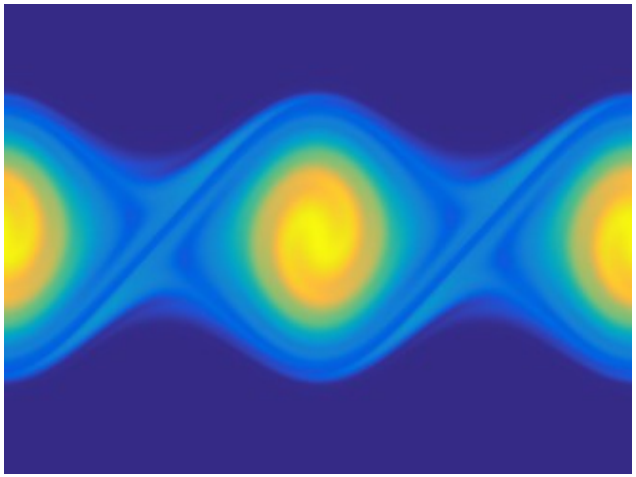
PM



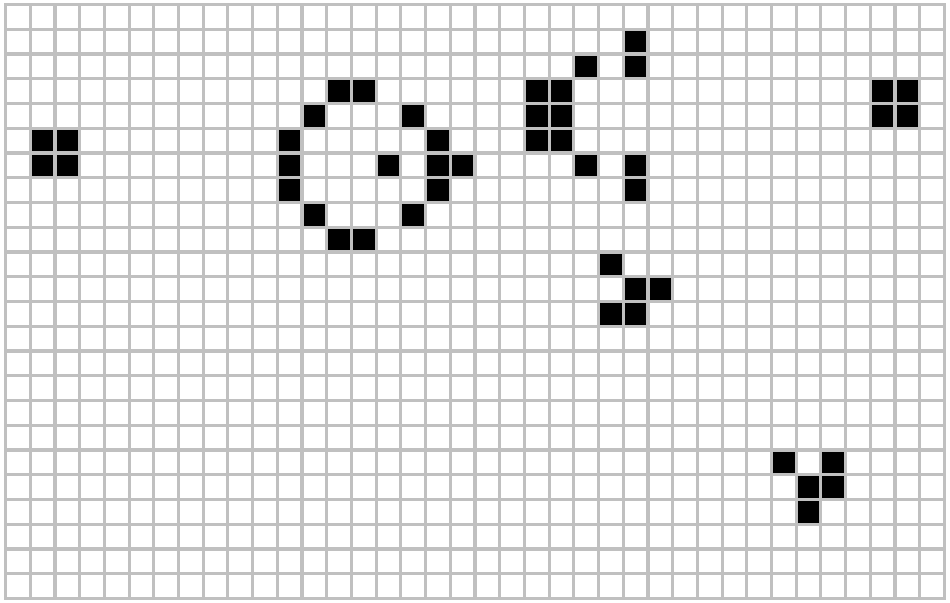
FV

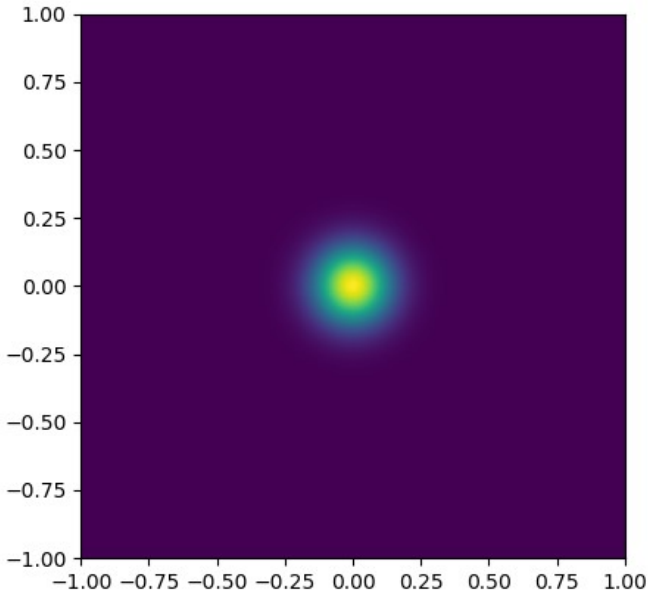


MM

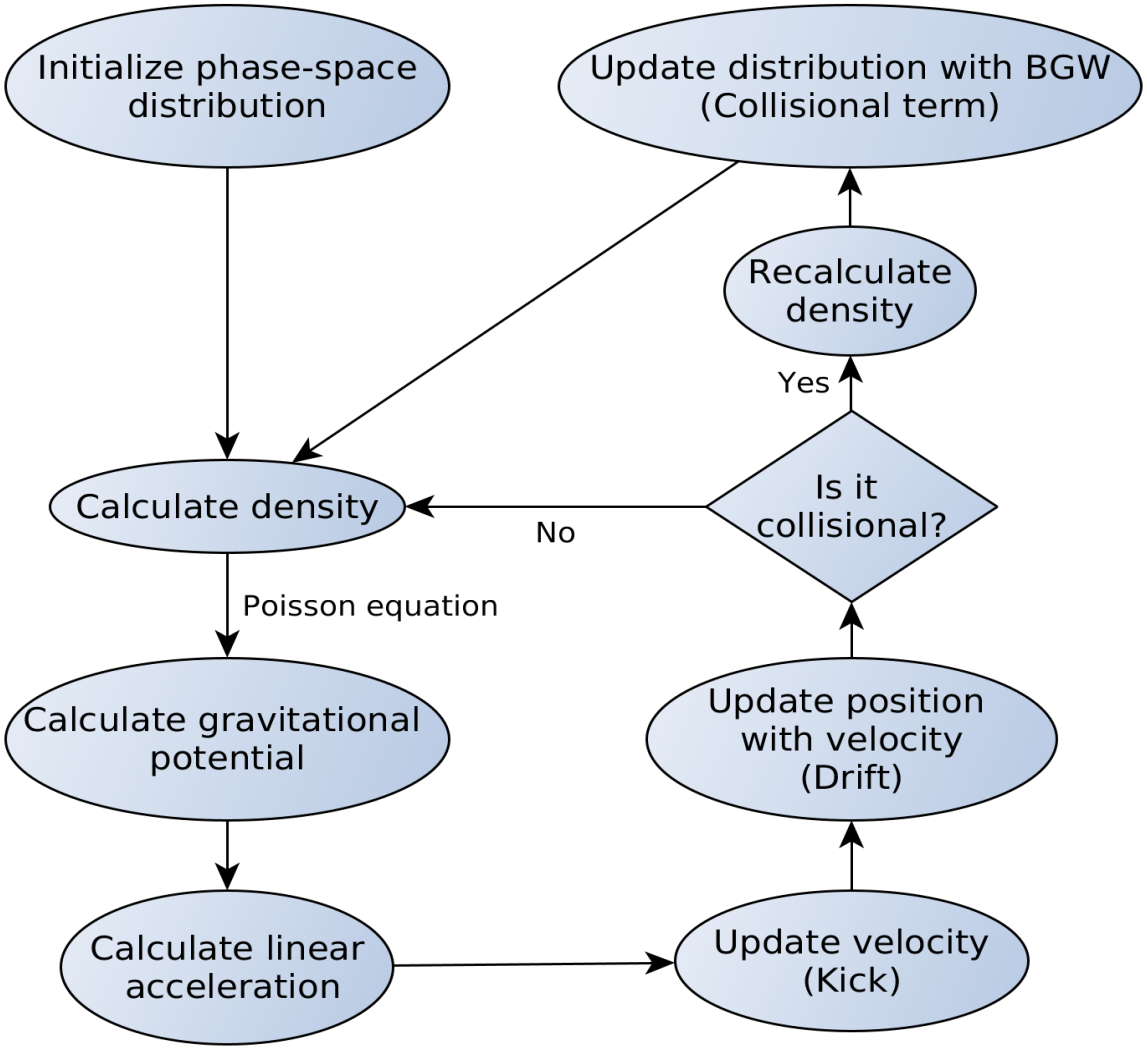


Lattice-Bolztmann and Automatas





Overview of the Algorithm



Some Equations

$$\rho(x, v, t) = \sum_{V_{min}}^{V_{max}} f(x, v, t)\Delta v$$

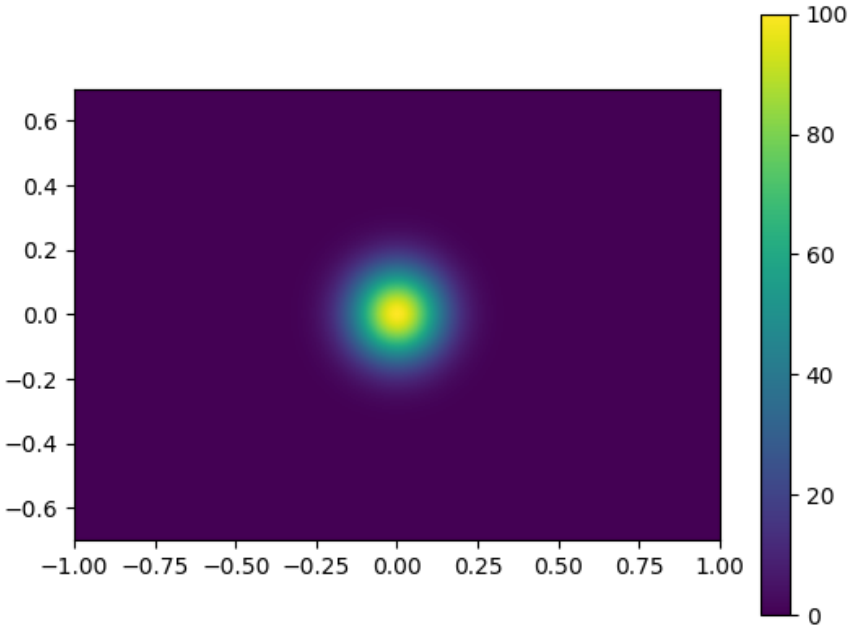
$$\nabla^2\Phi(x) = 4\pi G\rho(x)$$

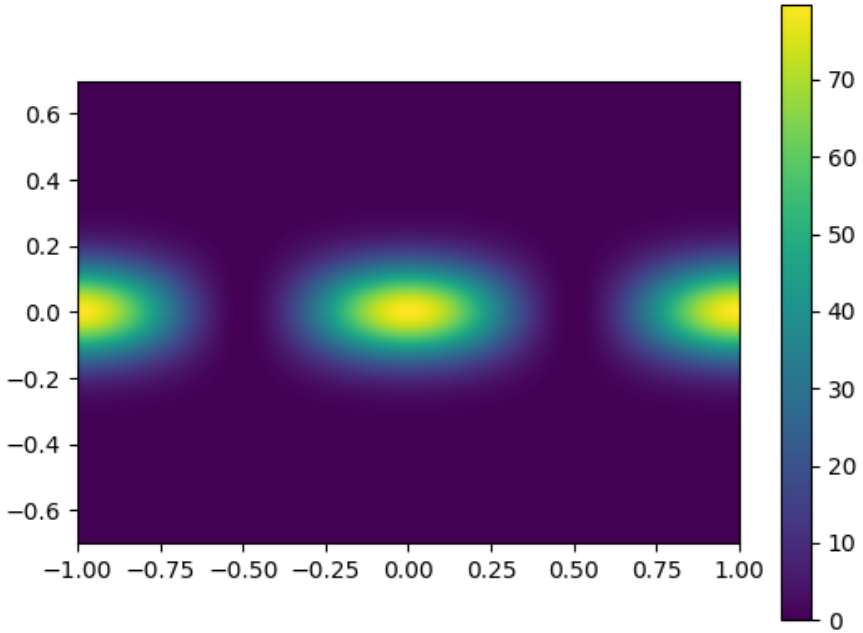
$$a(x) = -\frac{\mathrm{d}\Phi(x)}{\mathrm{d}x}$$

$$v_{n+1} = v_n + \lfloor a_n\ \delta t \rceil$$

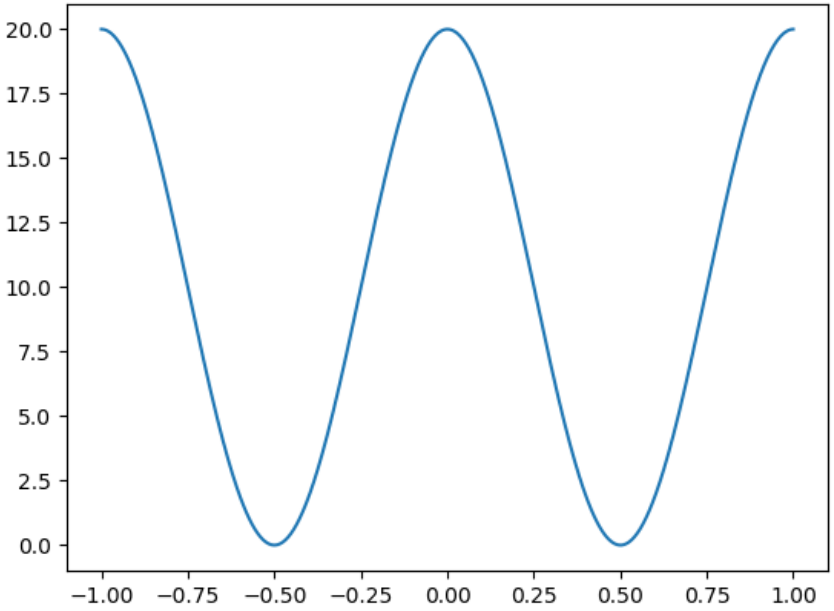
$$x_{n+1} = x_n + \lfloor v_n\ \delta t \rceil$$

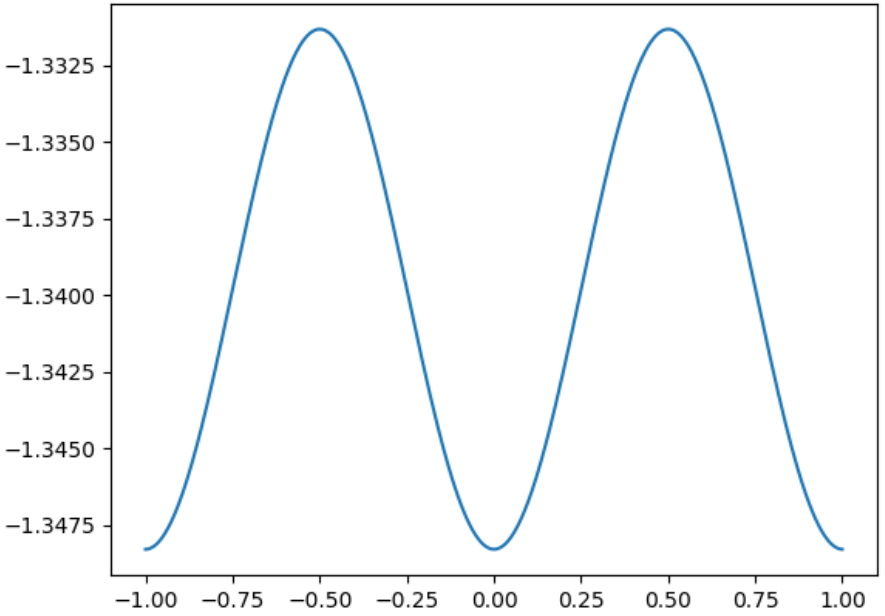
Initial Conditions



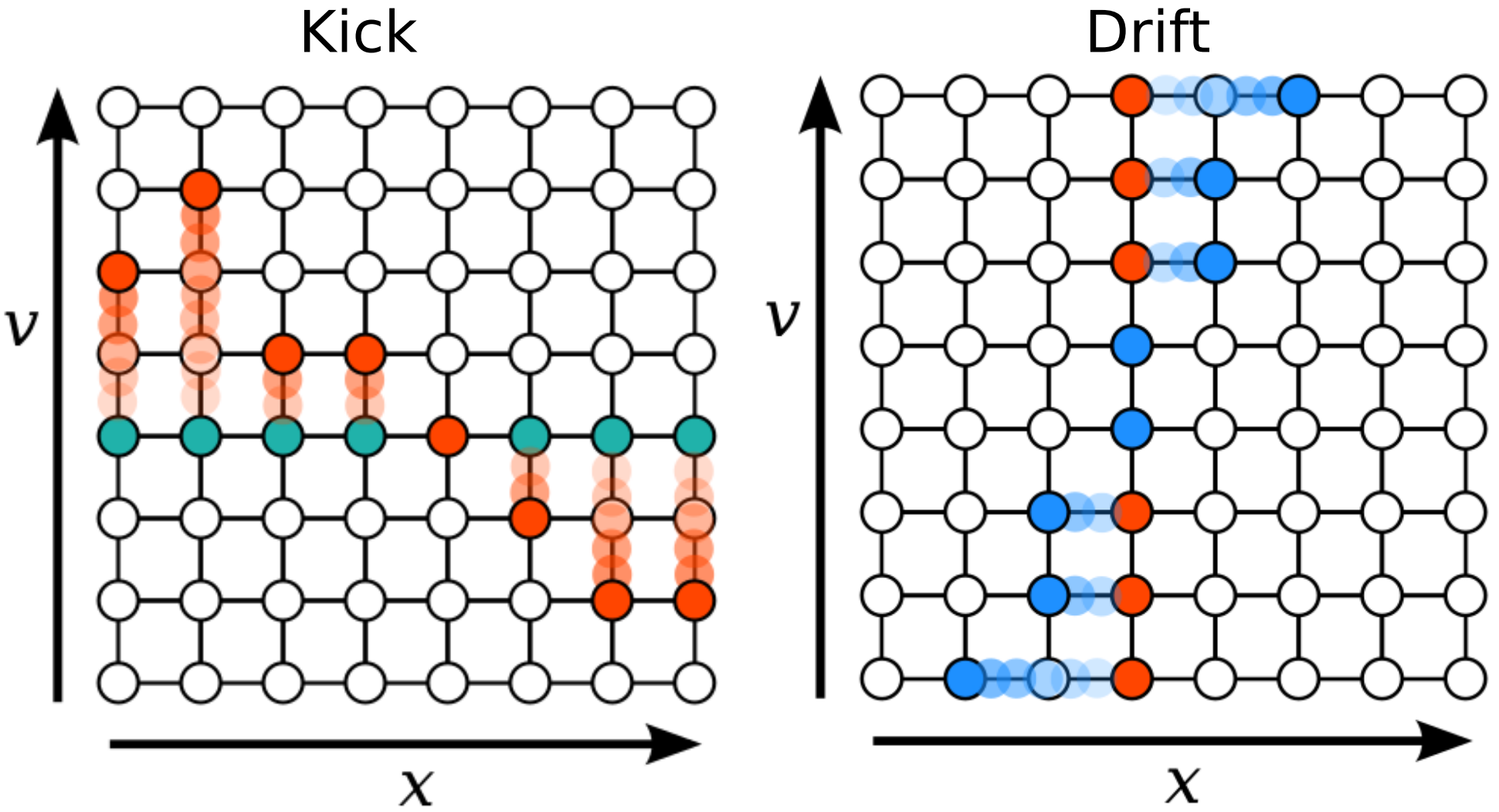


Density and Potential

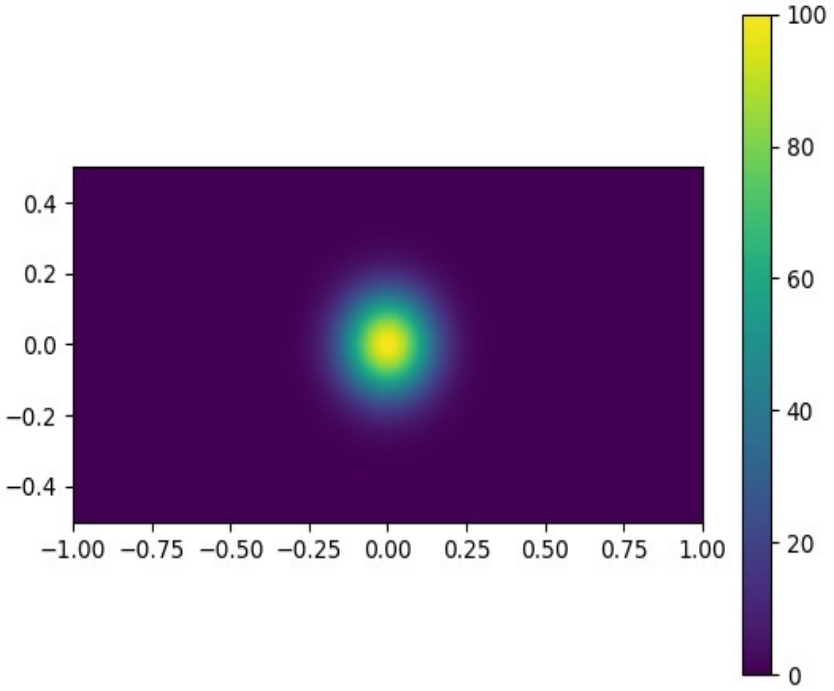


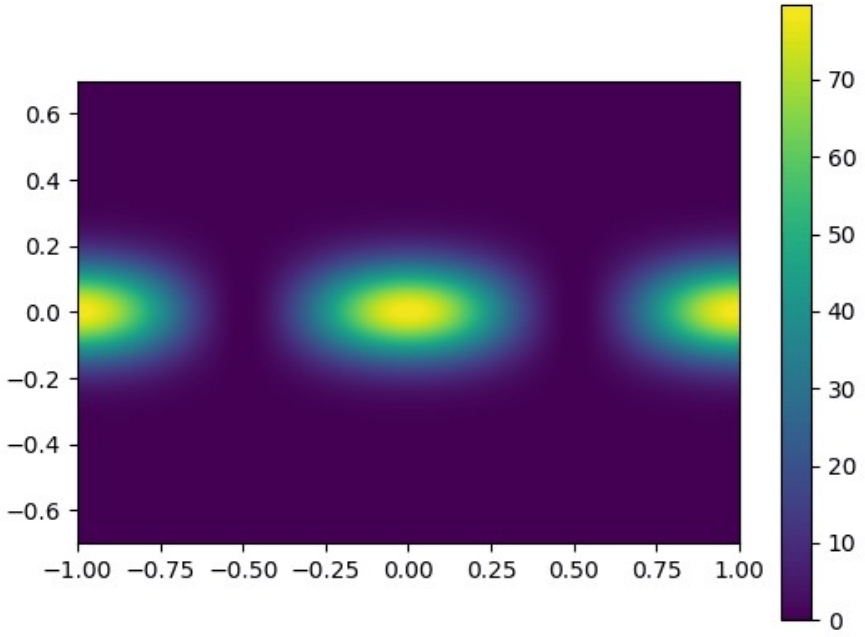


Streaming Step



Collisionless examples





Collisional Term: the BGK Approximation

$$C[f] = -\frac{1}{\tau}(f - f_{eq})$$

$$f(x + v\delta t, v, t + \delta t) - f(x, v, t) =$$

$$-\frac{\delta t}{\tau}[f(x, v, t) - f_{eq}(x, v, t)]$$

Equilibrium Distribution

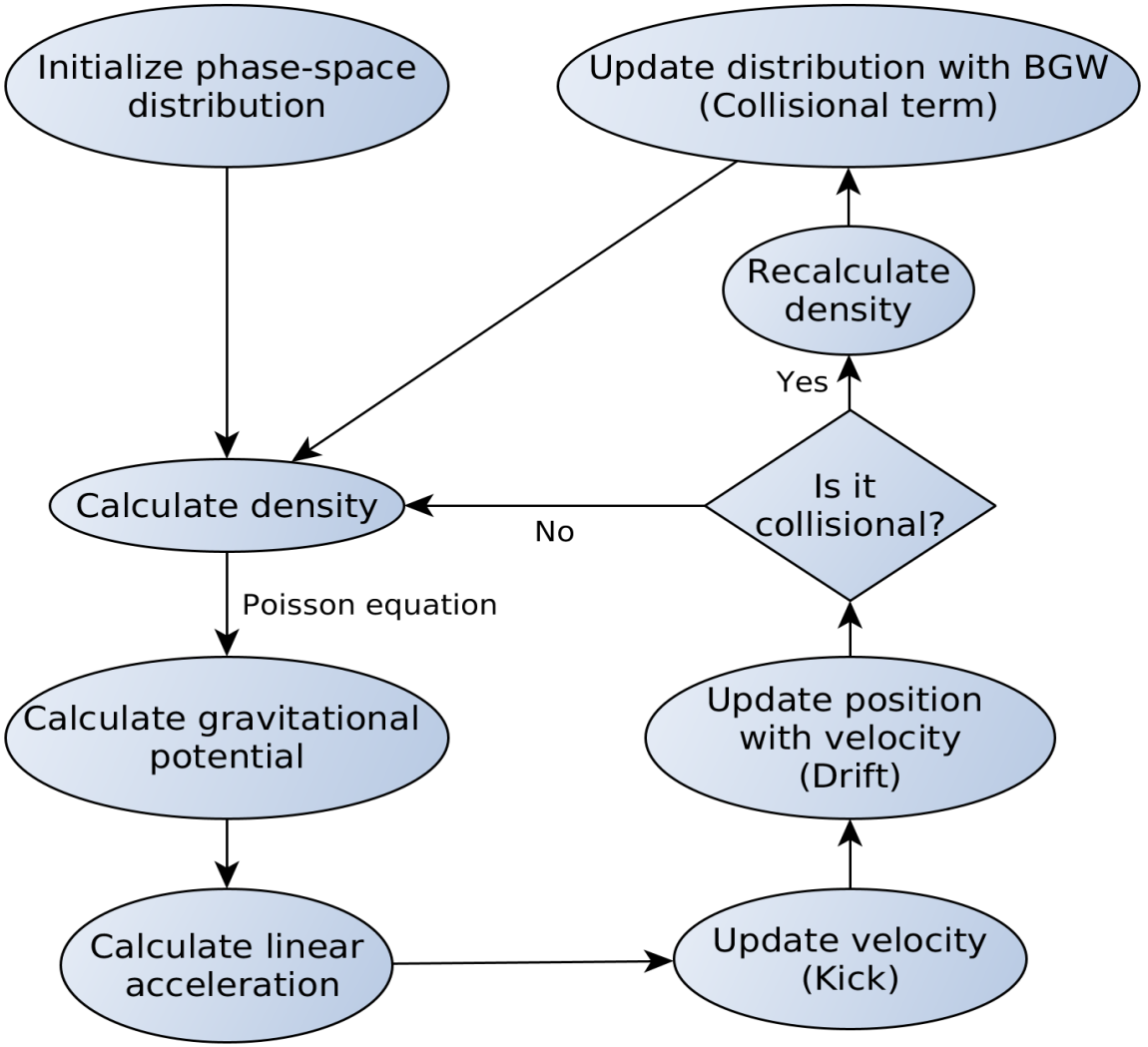
$$\rho(x,t) = \int m f(x,v,t) dv$$

$$\rho(x,t)u(x,t) = \int m v f(x,v,t) dv$$

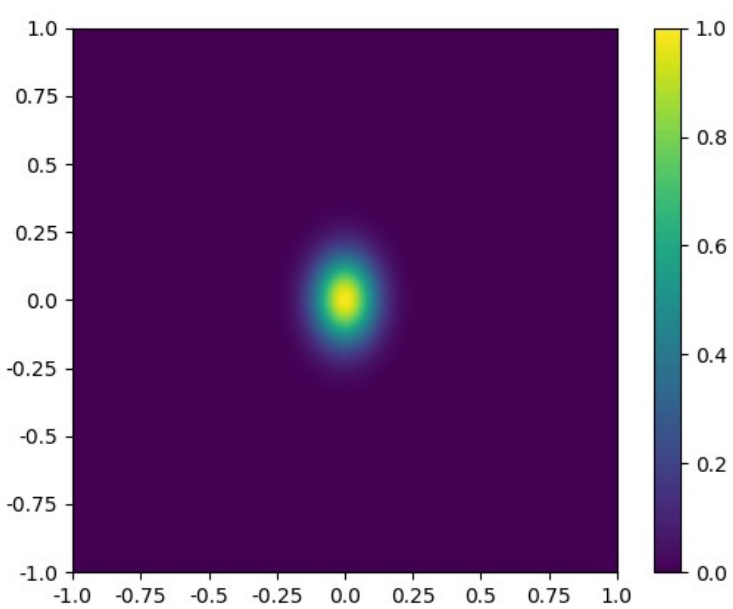
$$\rho(x,t)e(x,t) = \frac{1}{2}\int m(v-u)^2 f(x,v,t) dv$$

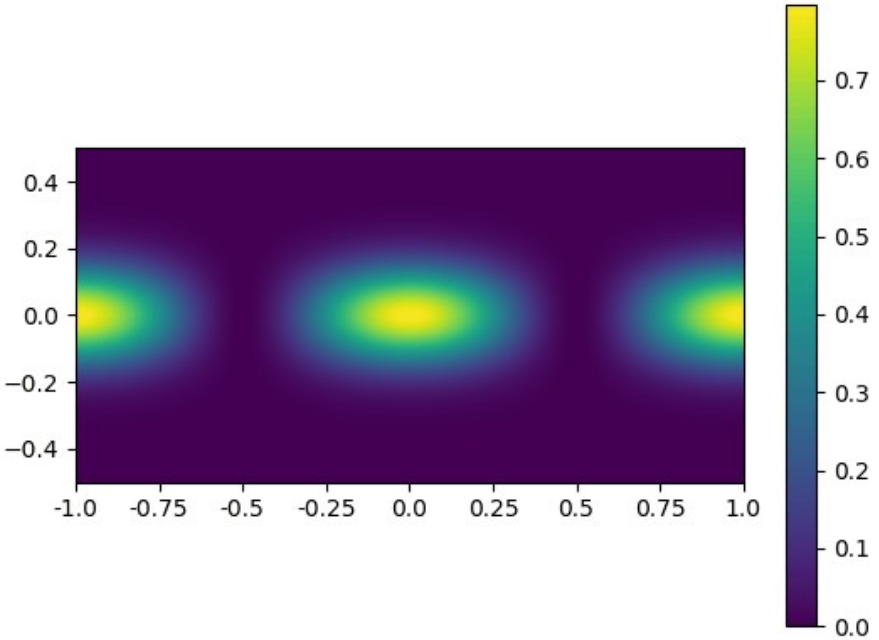
$$f_{eq}(x,v,t) = \frac{\rho(x,t)}{m\sqrt{2\pi e(x,t)}} \exp\left[-\frac{(v-u)^2}{2e(x,t)}\right]$$

Overview of the Algorithm

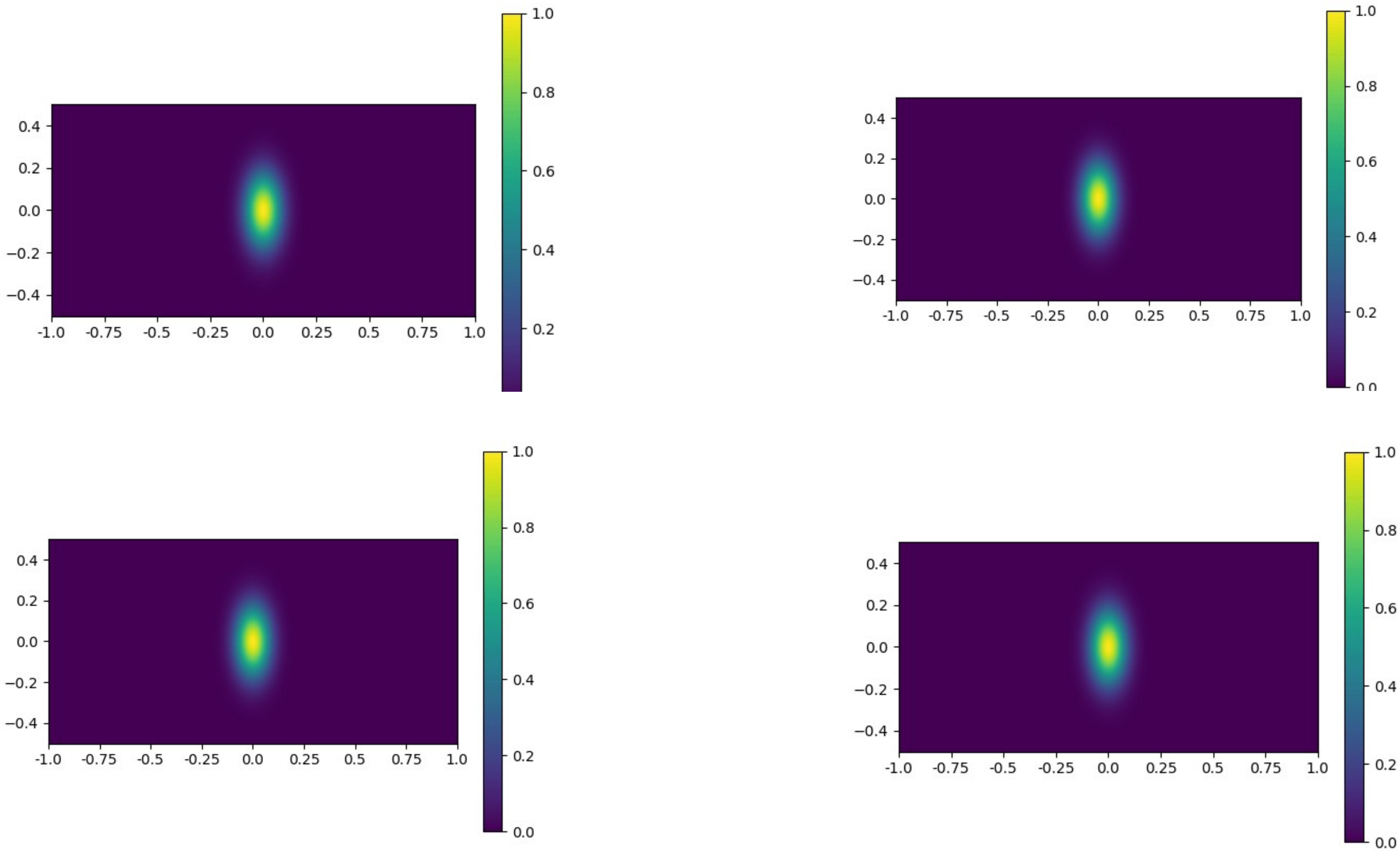


Collisional Examples

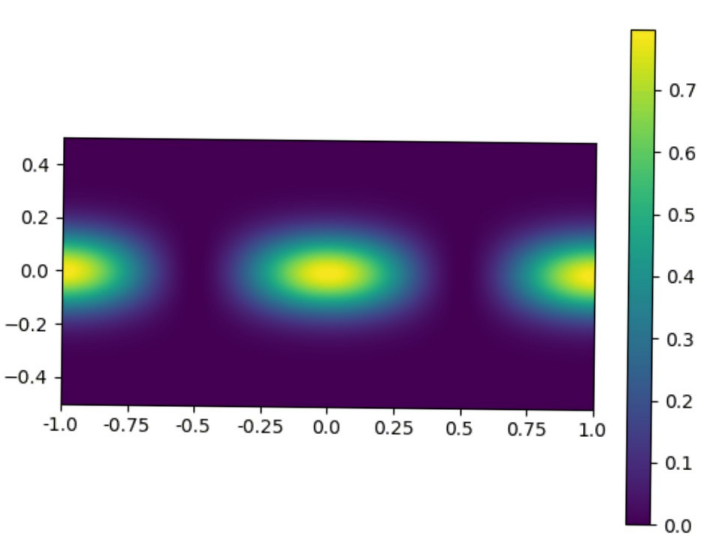


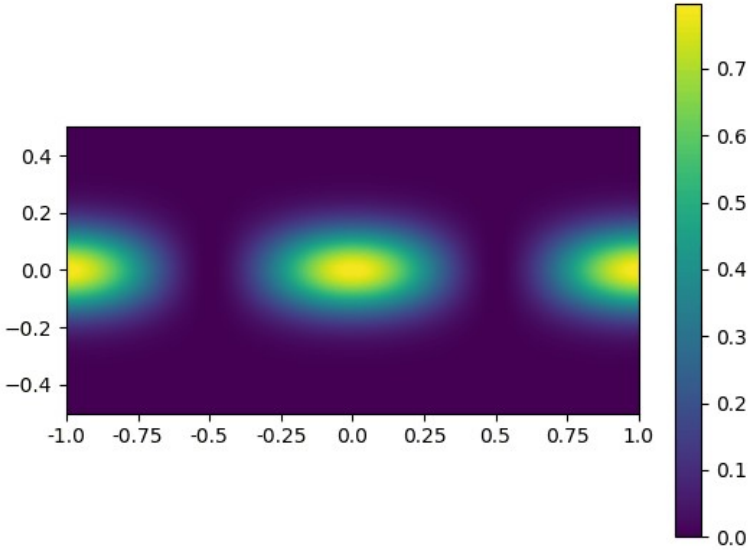


Different Taus



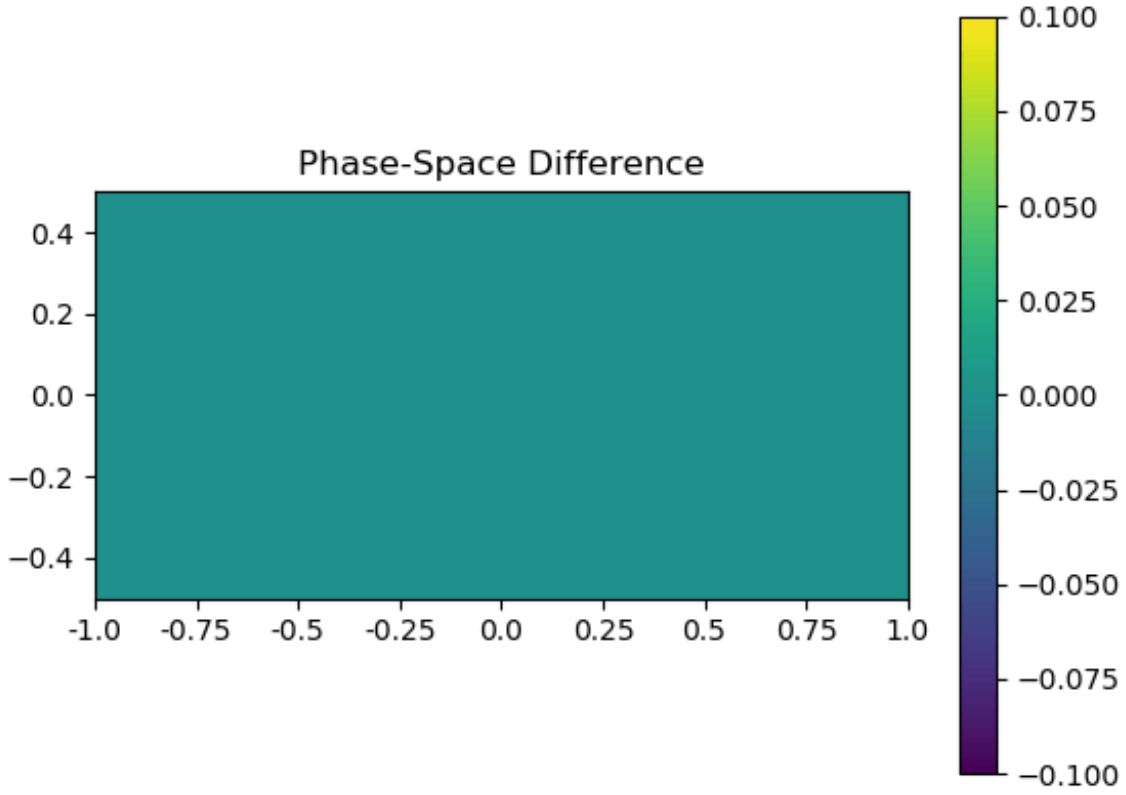
Jeans instability



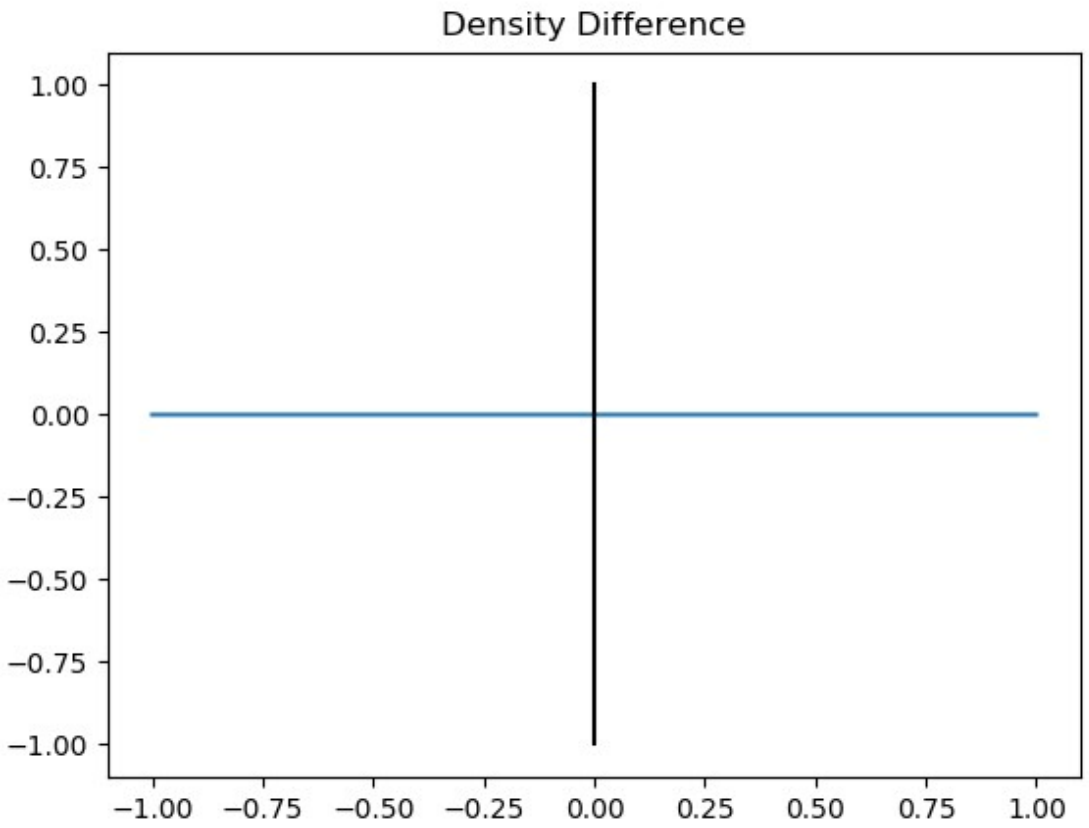


$$f(x,v,0) = \frac{\bar{\rho}}{\sqrt{2\pi\sigma^2}} \exp(-\frac{v^2}{2\sigma^2})(1+A\cos(kx))$$

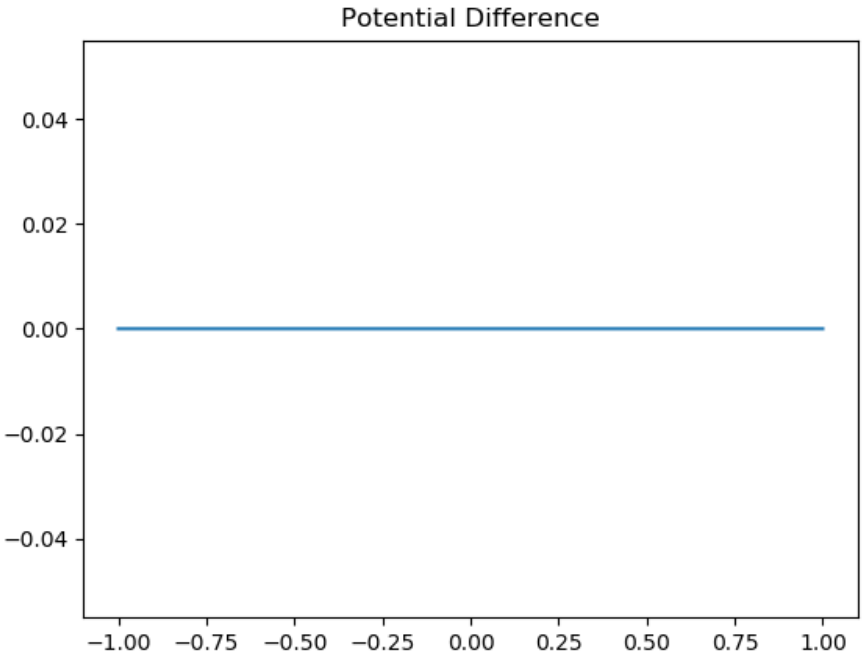
Results on phase-space

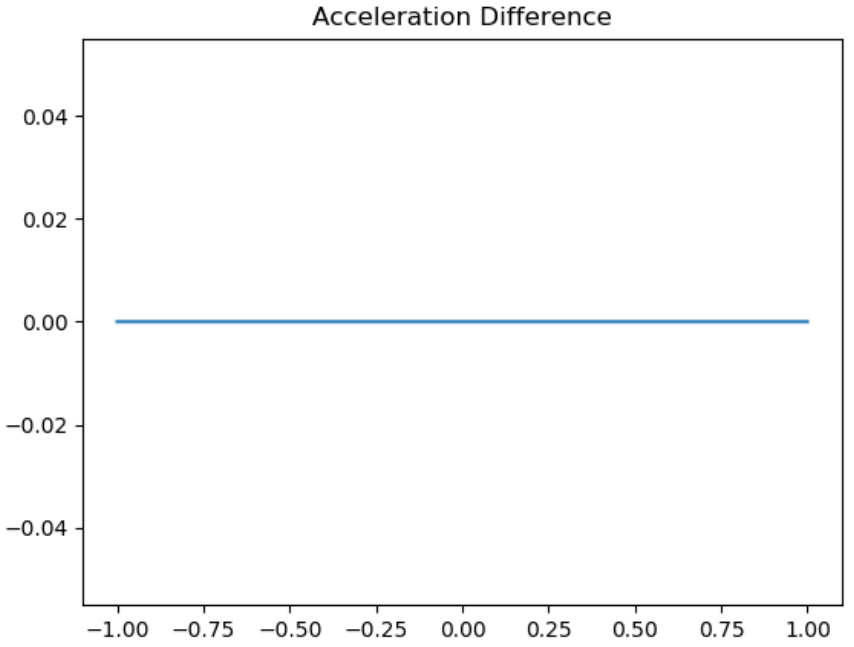


Results on Density



Results on potential and acceleration

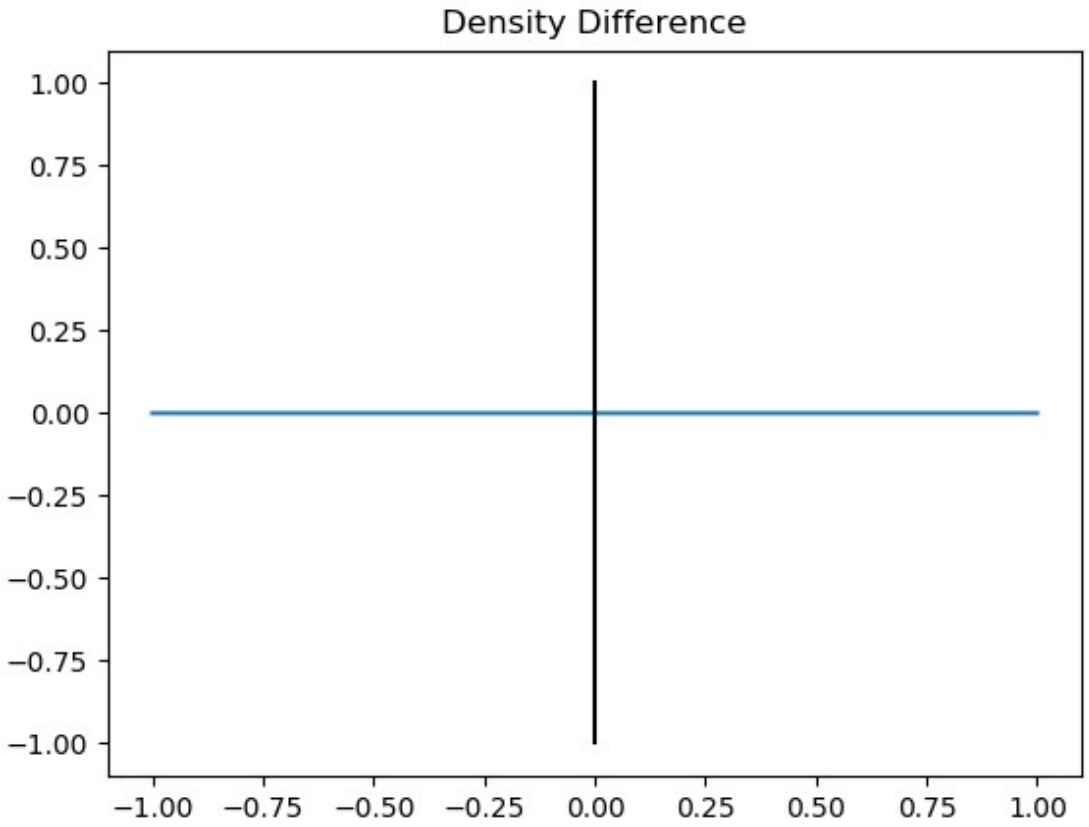




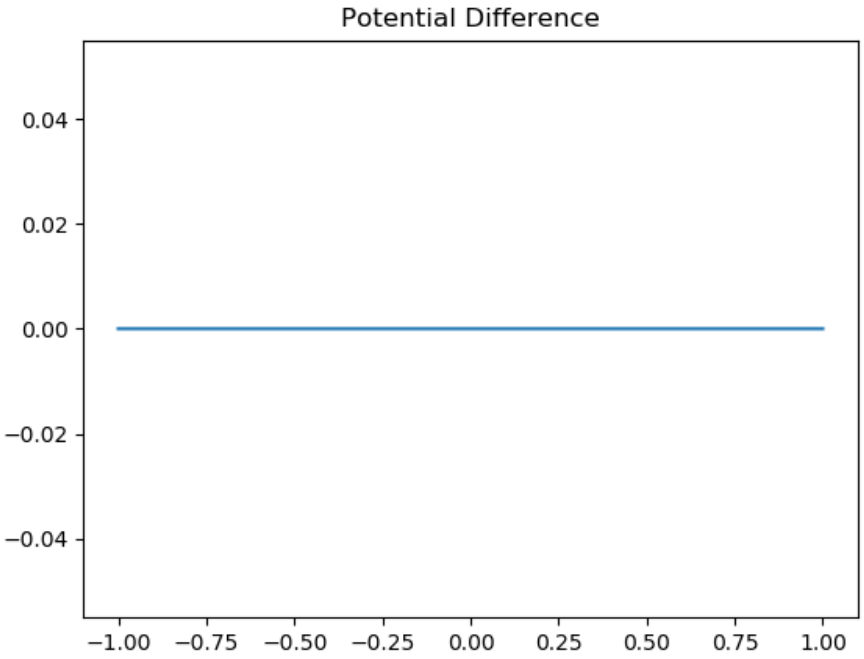
Gaussian distribution

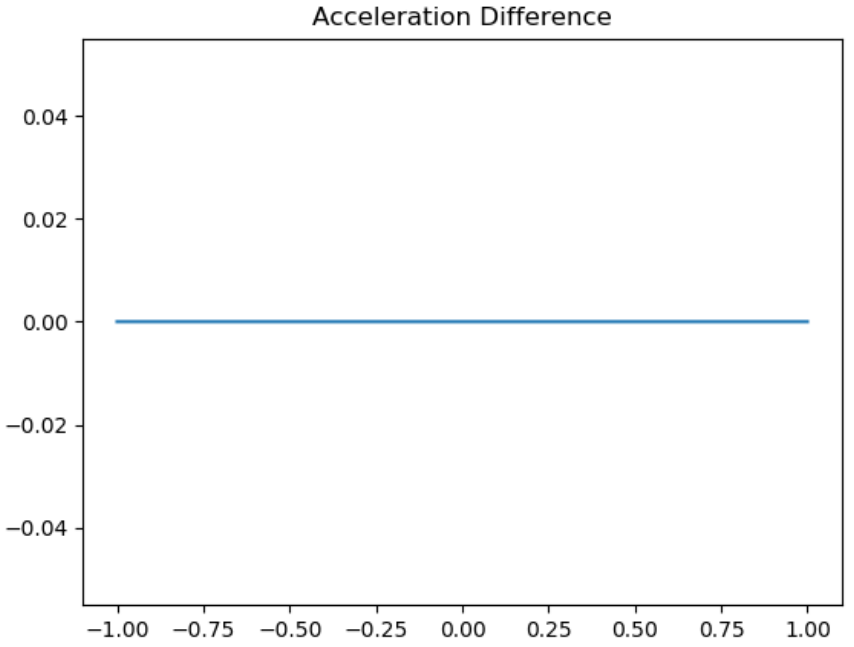
Results on phase-space

Results on Density



Results on potential and acceleration





2D and 3D implementation