CS437/CS591 – Machine Learning & Soft Computing – Fall 2021

Homework #2
(Due Date: 11/5/21)

The purpose of this homework is to help you learn more about *fuzzy inference systems*. The assignment will have you perform fuzzy inference computations for a simulated clothes dryer that uses fuzzy logic rules for its control system.

1 Membership Functions

A *membership function* is a function over some numeric *domain*, with its *range* being the reals from 0 through 1. This problem will use *triangular* and *trapezoidal* membership functions, which are popular choices due to the ease/cost of computations.

A *triangular* membership function can be defined using three numbers: a, b, c, where $a < b < c$. The membership function is then:

$$\begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x == b \\ \frac{c-x}{c-b} & b < x < c \\ 0 & x \geq c \end{cases}$$

Triangular membership functions may also have:

- a of $-\infty$ and b of *domain minimum*: i.e., negative slope line from minimum to c;
- b of *domain maximum* and c of ∞: i.e., position slope line from a to maximum;

A *trapezoidal* membership function can be defined using four numbers: a, b, c, d, where $a < b < c$. The membership function is then:

$$\begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & b \leq x \leq c \\ \frac{d-x}{d-c} & c < x < d \\ 0 & x \geq d \end{cases}$$

Trapezoidal membership functions may also have:

- a of $-\infty$, b of *domain minimum*: i.e., 1 for $min \leq x \leq c$, then negative slope line from c to d;

- c of *domain maximum*, d of ∞: i.e., positive slope line from a to b, then 1 for $b \leq x \leq max$;

2 Fuzzy Terms for this Problem

2.1 Drum Air Temperature

One of the dryer sensors will be for the temperature of the air inside the drum. Readings can range from 0 to 200 degrees (farenheit).

The following fuzzy terms will be used for the air temperature inside the dryer drum. They will all be represented by *trapezoidal* membership functions as specified:

Cold: $-\infty, 0, 70, 80$

Moderate: $70, 80, 100, 140$

Hot: $120, 140, 150, 160$

Very Hot: $150, 160, 200, \infty$

E.g., *cold* is the trapezoidal function:

$$\begin{cases} 1.0 & 0 \leq t \leq 70 \\ \frac{80-t}{10} & 70 < t < 80 \\ 0 & t \geq 80 \end{cases}$$

2.2 Clothing Weight

The other sensor will be for the weight of the clothing in the drum. Readings can range from 0 to 25 pounds. (While this is not a sensor commonly found on clothes dryers, it is a common sensor with washing machines.)

The following fuzzy terms will be used for the clothing weight. They will all be represented by *trapezoidal* membership functions as specified:

Light: $-\infty, 0, 5, 10$

Average: $5, 10, 15, 20$

Heavy: $15, 20, 25, \infty$

2.3 Heating Coil Power (Input Voltage)

The goal of this simulated clothes dryer controller is to adjust the *voltage* applied to the electric heating unit, to control the amount of heat being applied to dry the clothing.

An electric clothes dryer has a heating coil with a fixed *resistance*, so the heat/power is adjusted by changing the applied voltage. *Ohm's Law* says $I = V/R$ and Power $= I \times V$, so

Power = V^2/R. Electric clothes dryers in the US are designed to receive 240 volt service.

The following fuzzy terms will be used for heating coil votage. They will be represented by *triangular* membership functions as specified:

Low: $70, 90, 100$

Medium Low: $100, 120, 140$

Medium: $150, 170, 190$

Medium High: $180, 200, 220$

High: $220, 240, \infty$

3 Rules

Here is a *Fuzzy Associative Memory* (FAM) table that represents the fuzzy rules that should be used in answering the questions:

	Light	Average	Heavy
Cold:	Medium	Medium High	High
Moderate:	Medium Low	Medium	Medium High
Hot:	Low	Medium	Medium
Very Hot:	Low	Low	Medium Low

For example then, the left-top entry represents the rule:
If Cold temp and Light load Then Medium power/voltage

4 Fuzzy Inference

When questions ask you to compute the voltage the controller wants given a specific air temperature and clothing weight, you are being asked to perform the following fuzzy steps, using *Mamdani* style inference:

1. Fuzzify the inputs
2. Evaluate the fuzzy rules
3. Aggregate the rule outputs
4. Defuzzify the aggregated outputs

In carrying out these steps, use the following approaches unless instructed to use an alternative in the question:

- *clipping* style rule evaluation
- *centroid/COG* style defuzzification

5 Questions to be Answered

Submit your answers to the following questions, via the Homework #2 Dropbox on the course D2L page:

1. Determine the heating coil voltage (exact voltage value) if: air temperature is 75 degrees and clothing weight is 8 pounds. For this problem, show all of your work for the four inference steps.
2. Determine the heating coil voltage (exact voltage value) if: air temperature is 90 degrees and clothing weight is 12 pounds.
3. Determine the heating coil voltage (exact voltage value) if: air temperature is 180 degrees and clothing weight is 5 pounds.
4. Determine the heating coil voltage (exact voltage value) if: air temperature is 155 degrees and clothing weight is 22 pounds.
5. Redo Problem 1, but use *Sugeno* style inference: *zero-order Sugeno fuzzy rule model* and *Sugeno style weighted average aggregation*. To produce the needed single values from the relevant rules, use the *centroid/COG* value of a rule's fuzzy output membership function. Show your work.
6. Did your answer for Problem 5 differ from your answer for Problem 1? Explain why there was or was not a difference.
7. Proponents of fuzzy logic argue that the fuzzy rules (or FAM) approach for building a controller such as this, is superior to conventional approaches for building controllers, since those require "large" tables or "complex" functions. Think about taking the FAM and turning it into a table that took exact temperature and weight values as inputs, and mapped them to an exact voltage. Sketch a couple of rows of such a table, and comment on how large such a table would be (e.g., for a modern microprocessor). How many entries would such a table need given likely precision of clothes dryer sensors and required precision of voltage?
8. If the controller were to use a function to determine heater voltage, that function would be: $f(t, w) \rightarrow v$ (temp and weight inputs, voltage output). By trying a few sets of inputs, do you have any sense of how complex such a function would be?