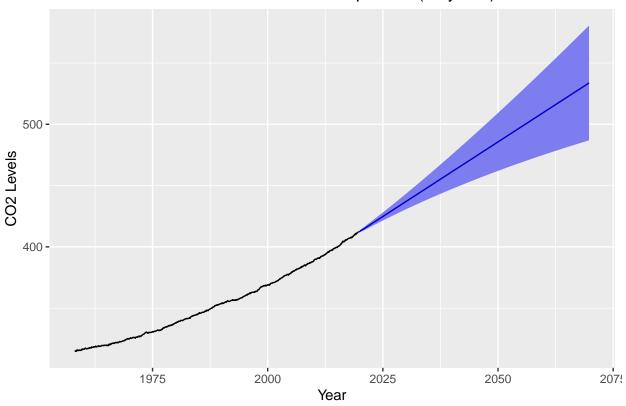
# Analysis

## Predict $CO_2$ :

#### **ARIMA** - $CO_2$ :

```
##
##
    Box-Ljung test
##
## data: c02.ts
## X-squared = 736.72, df = 1, p-value < 0.0000000000000022
```

## ARIMA Forecast CO2 - 95% CI for 600 periods (50 years)

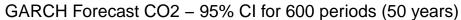


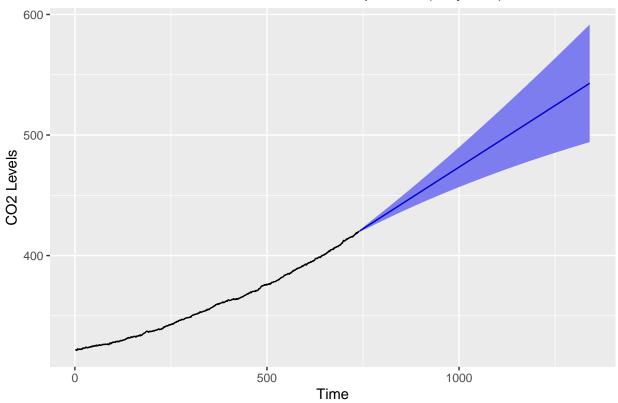
#### GARCH - CO2:

```
##
##
    **** ESTIMATION WITH ANALYTICAL GRADIENT ****
##
##
              INITIAL X(I)
                                   D(I)
##
        Ι
##
              7.133030e+02
                                1.000e+00
##
        1
              5.000000e-02
                                1.000e+00
##
        2
              5.000000e-02
                                1.000e+00
##
```

Predict  $CO_2$ :

```
##
                                 RELDF
                                                      RELDX
##
             NF
                      F
                                          PRELDF
                                                              STPPAR
                                                                        D*STEP
                                                                                  NPRELDF
       ΙT
                 9.584e+03
##
        0
              1
                 4.711e+03
                                                   7.0e-04
                                                                       1.0e+00
                                                                                 5.91e+05
##
        1
              2
                             5.08e-01
                                        1.11e+01
                                                             1.1e+05
##
        2
                 4.708e+03
                             5.43e-04
                                        6.41e-04
                                                   3.5e-05
                                                             2.0e+00
                                                                       5.0e-02
                                                                                 3.88e-01
##
        3
                 4.707e+03
                             3.67e-04
                                        3.46e-04
                                                   3.5e-05
                                                             1.3e+00
                                                                       5.0e-02
                                                                                 4.33e-04
##
        4
                 4.707e+03
                             2.76e-05
                                        2.64e-05
                                                    2.4e-06
                                                             6.0e+00
                                                                       4.8e-03
                                                                                 1.20e-04
##
        5
              9
                 4.707e+03
                             5.13e-06
                                        5.13e-06
                                                   4.9e-07
                                                              4.0e+01
                                                                       9.6e - 04
                                                                                 1.02e-04
##
        6
                 4.706e+03
                             9.86e-06
                                        9.86e-06
                                                   9.8e-07
                                                             5.8e+00
                                                                       1.9e-03
                                                                                 9.66e-05
             11
        7
##
             13
                 4.706e+03
                             1.81e-05
                                        1.81e-05
                                                    1.9e-06
                                                              3.3e+00
                                                                       3.8e-03
                                                                                 8.66e-05
##
        8
                 4.706e+03
                             3.37e-06
                                        3.37e-06
                                                    3.9e-07
                                                              4.1e+01
                                                                       7.7e-04
                                                                                 6.82e-05
             15
        9
##
             17
                 4.706e+03
                             6.64e-07
                                        6.64e-07
                                                    7.8e-08
                                                              2.0e+02
                                                                       1.5e-04
                                                                                 6.46e-05
##
       10
                 4.706e+03
                             1.32e-06
                                        1.32e-06
                                                    1.6e-07
                                                              2.5e + 01
                                                                       3.1e-04
                                                                                 6.39e-05
             19
                                        2.62e-07
##
       11
             21
                 4.706e+03
                             2.62e-07
                                                    3.1e-08
                                                              4.8e+02
                                                                       6.1e-05
                                                                                 6.26e-05
                             5.22e-07
                                                                                 6.23e-05
##
       12
             23
                 4.706e+03
                                        5.22e-07
                                                    6.2e-08
                                                              6.1e+01
                                                                       1.2e-04
##
       13
             26
                 4.706e+03
                             1.04e-08
                                        1.04e-08
                                                    1.2e-09
                                                              1.2e+04
                                                                       2.5e-06
                                                                                 6.18e-05
##
       14
             28
                 4.706e+03
                             2.08e-08
                                        2.08e-08
                                                   2.5e-09
                                                              1.5e + 03
                                                                       4.9e-06
                                                                                 6.18e-05
##
       15
                 4.706e+03
                             4.16e-09
                                        4.16e-09
                                                    5.0e-10
                                                              3.0e + 04
                                                                       9.8e-07
                                                                                 6.17e-05
             30
##
                 4.706e+03
                             8.33e-09
                                        8.33e-09
                                                   9.9e-10
                                                             3.7e + 03
                                                                       2.0e-06
                                                                                 6.17e-05
       16
             32
##
       17
             34
                 4.706e+03
                             1.67e-08
                                        1.67e-08
                                                   2.0e-09
                                                              1.9e+03
                                                                       3.9e-06
                                                                                 6.17e-05
##
       18
             36
                 4.706e+03
                             3.33e-09
                                        3.33e-09
                                                   4.0e-10
                                                             3.7e + 04
                                                                       7.9e-07
                                                                                 6.17e-05
##
       19
                 4.706e+03
                             6.66e-10
                                        6.66e-10
                                                              1.9e+05
                                                                       1.6e-07
                                                   8.0e-11
                                                                                 6.17e-05
##
       20
                 4.706e+03
                             1.33e-10
                                        1.33e-10
                                                   1.6e-11
                                                             9.3e+05
                                                                       3.1e-08
                                                                                 6.17e-05
             40
##
       21
             42
                 4.706e+03
                             2.66e-10
                                        2.66e-10
                                                   3.2e-11
                                                             1.2e+05
                                                                       6.3e-08
                                                                                 6.17e-05
##
       22
             44
                 4.706e+03
                             5.33e-11
                                        5.33e-11
                                                   6.4e-12
                                                             2.3e + 06
                                                                       1.3e-08
                                                                                 6.17e-05
##
       23
             46
                 4.706e+03
                             1.07e-11
                                        1.07e-11
                                                    1.3e-12
                                                             1.2e+07
                                                                       2.5e-09
                                                                                 6.17e-05
##
       24
                                        8.53e-11
                                                              3.6e+05
                                                                       2.0e-08
             49
                 4.706e+03
                             8.53e-11
                                                    1.0e-11
                                                                                 6.17e-05
##
       25
             53
                 4.706e+03
                             1.70e-13
                                        1.71e-13
                                                   2.0e-14
                                                             7.2e+08
                                                                       4.0e-11
                                                                                 6.17e-05
##
       26
             55
                 4.706e+03
                             3.38e-14
                                        3.41e-14
                                                    4.1e-15
                                                              3.6e + 09
                                                                       8.1e-12
                                                                                 6.17e-05
##
       27
                             6.80e-14
                                        6.82e-14
                                                              4.5e+08
                                                                       1.6e-11
             57
                 4.706e+03
                                                   8.1e-15
                                                                                 6.17e-05
##
       28
             58
                 4.706e+03 -2.12e+06
                                        1.36e-13
                                                   1.6e-14
                                                             9.0e+08
                                                                       3.2e-11
                                                                                 6.17e-05
##
##
    **** FALSE CONVERGENCE ****
##
##
    FUNCTION
                   4.706351e+03
                                   RELDX
                                                 1.629e-14
##
    FUNC. EVALS
                       58
                                   GRAD. EVALS
                                                      28
##
    PRELDF
                   1.364e-13
                                   NPRELDF
                                                 6.171e-05
##
                FINAL X(I)
                                    D(I)
                                                    G(I)
##
        Ι
##
                                 1.000e+00
##
        1
              7.133030e+02
                                                1.122e-04
##
        2
              1.035450e+00
                                 1.000e+00
                                                1.380e+01
##
        3
              1.536755e-11
                                 1.000e+00
                                                1.436e+01
```



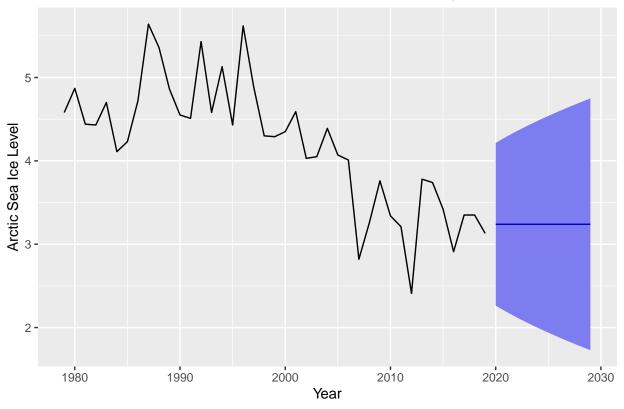


#### Predict Arctic Sea Ice Level:

#### ARIMA - Arctic Sea Ice Level:

```
##
## Box-Ljung test
##
## data: ice.ts
## X-squared = 87.039, df = 5, p-value < 0.00000000000000022</pre>
```





#### GARCH - Arctic Sea Ice Level:

## ## I INITIAL X(I) D(I) ## ## 1 5.367499e-01 1.000e+00 ## 2 5.000000e-02 1.000e+00	
## 1 5.367499e-01 1.000e+00 ## 2 5.000000e-02 1.000e+00	
## 2 5.000000e-02 1.000e+00	
## 3 5.000000e-02 1.000e+00	
##	
## IT NF F RELDF PRELDF RELDX STPPAR D*STEP NPRELD	)F
## 0 1 2.390e+02	
## 1 2 7.768e+01 6.75e-01 1.09e+01 8.8e-01 2.6e+03 1.0e+00 1.41e+0	)4
## 2 4 7.750e+01 2.36e-03 2.65e-03 2.4e-02 1.9e+00 5.0e-02 1.10e-0	)1
## 3 6 7.738e+01 1.50e-03 1.44e-03 2.1e-02 0.0e+00 5.0e-02 2.09e-0	)3
## 4 7 7.731e+01 9.48e-04 8.91e-04 4.0e-02 3.7e-01 1.0e-01 9.20e-0	)4
## 5 10 7.731e+01 1.32e-05 1.22e-05 4.6e-04 6.8e+00 1.0e-03 1.17e-0	)4
## 6 12 7.730e+01 2.53e-05 2.53e-05 9.2e-04 5.5e+00 2.0e-03 3.18e-0	)4
## 7 14 7.730e+01 4.88e-06 4.88e-06 1.9e-04 8.9e+01 4.0e-04 2.96e-0	)4
## 8 17 7.730e+01 3.71e-05 3.71e-05 1.5e-03 3.8e+00 3.2e-03 3.01e-0	)4
## 9 19 7.730e+01 7.01e-06 7.01e-06 3.1e-04 5.6e+01 6.4e-04 2.65e-0	)4
## 10 22 7.730e+01 1.39e-07 1.39e-07 6.1e-06 2.8e+03 1.3e-05 2.77e-0	)4
## 11 24 7.730e+01 2.78e-08 2.78e-08 1.2e-06 1.4e+04 2.6e-06 2.81e-0	

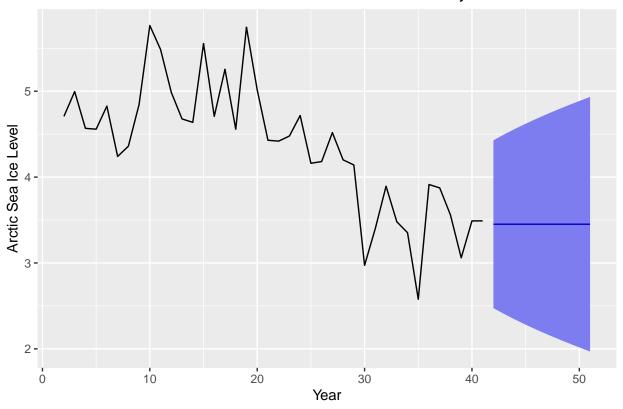
```
##
       12
                7.730e+01
                            5.56e-09
                                       5.56e-09
                                                 2.5e-07
                                                           6.9e+04
                                                                     5.1e-07
                                                                              2.81e-04
            26
##
       13
            28
                7.730e+01
                            1.11e-08
                                       1.11e-08
                                                 4.9e-07
                                                           8.7e+03
                                                                     1.0e-06
                                                                              2.81e-04
                7.730e+01
##
       14
            31
                            2.22e-10
                                       2.22e-10
                                                 9.8e-09
                                                           1.7e+06
                                                                     2.0e-08
                                                                              2.81e-04
##
                                       4.44e-11
                                                 2.0e-09
                                                                              2.81e-04
       15
            33
                7.730e+01
                            4.44e-11
                                                           8.7e + 06
                                                                     4.1e-09
##
       16
            35
                7.730e+01
                            8.89e-11
                                       8.89e-11
                                                 3.9e-09
                                                           1.1e+06
                                                                     8.2e-09
                                                                              2.81e-04
                                                 7.9e-11
                                                                     1.6e-10
##
       17
                7.730e+01
                            1.78e-12
                                       1.78e-12
                                                           2.2e+08
                                                                              2.81e-04
            38
##
       18
            40
                7.730e+01
                            3.56e-12
                                       3.56e-12
                                                 1.6e-10
                                                           2.7e + 07
                                                                     3.3e-10
                                                                              2.81e-04
##
       19
            42
                7.730e+01
                            7.12e-13
                                       7.11e-13
                                                 3.1e-11
                                                           5.4e + 08
                                                                     6.6e-11
                                                                              2.81e-04
##
       20
            44
                7.730e+01
                            1.42e-12
                                       1.42e-12
                                                 6.3e-11
                                                           6.8e+07
                                                                     1.3e-10
                                                                              2.81e-04
##
       21
            46
                7.730e+01
                            2.85e-13
                                       2.84e-13
                                                 1.3e-11
                                                           1.4e+09
                                                                     2.6e-11
                                                                              2.81e-04
##
       22
            48
                7.730e+01
                            5.66e-14
                                       5.69e-14
                                                 2.5e-12
                                                           6.8e+09
                                                                     5.2e-12
                                                                              2.81e-04
       23
##
            50
                7.730e+01
                            1.14e-13
                                       1.14e-13
                                                 5.0e-12
                                                           8.5e + 08
                                                                     1.0e-11
                                                                              2.81e-04
##
       24
            52
                7.730e+01
                            2.24e-14
                                       2.28e-14
                                                 1.0e-12
                                                           1.7e+10
                                                                     2.1e-12
                                                                              2.81e-04
##
       25
                7.730e+01
                            4.52e-14
                                       4.55e-14
                                                 2.0e-12
                                                           2.1e+09
                                                                     4.2e-12
                                                                              2.81e-04
                            9.10e-14
##
       26
            56
                7.730e+01
                                       9.10e-14
                                                  4.0e-12
                                                           1.1e+09
                                                                     8.4e-12
                                                                              2.81e-04
##
       27
            58
                7.730e+01
                            1.80e-14
                                       1.82e-14
                                                 8.0e-13
                                                           2.1e+10
                                                                     1.7e-12
                                                                              2.81e-04
##
       28
            60
                7.730e+01
                            3.86e-15
                                       3.64e-15
                                                  1.6e-13
                                                                     3.4e-13
                                                                              2.81e-04
                                                           1.1e+11
##
       29
                7.730e+01
                            9.19e-16
                                       7.28e-16
                                                  3.2e-14
                                                           5.3e + 11
                                                                     6.7e-14
                                                                              2.81e-04
            62
                7.730e+01
##
       30
                            1.65e-15
                                       1.46e-15
                                                 6.4e-14
                                                           6.6e+10
                                                                     1.3e-13
                                                                              2.81e-04
            64
##
       31
                7.730e+01
                            0.00e+00
                                       2.91e-16
                                                 1.3e-14
                                                           1.3e+12
                                                                     2.7e-14
                                                                              2.81e-04
##
```

#### ## \*\*\*\*\* FALSE CONVERGENCE \*\*\*\*\*

##

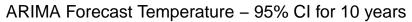
$\pi\pi$				
##	FUNCTION	7.729938e+01	RELDX	1.292e-14
##	FUNC. EVAL	.S 66	GRAD. EVALS	31
##	PRELDF	2.913e-16	NPRELDF	2.808e-04
##				
##	I	FINAL X(I)	D(I)	G(I)
##				
##	1 7	.016953e-01	1.000e+00	-1.738e-02
##	2 1	.022788e+00	1.000e+00	8.226e-01
##	3 1	.478449e-14	1.000e+00	1.629e-01

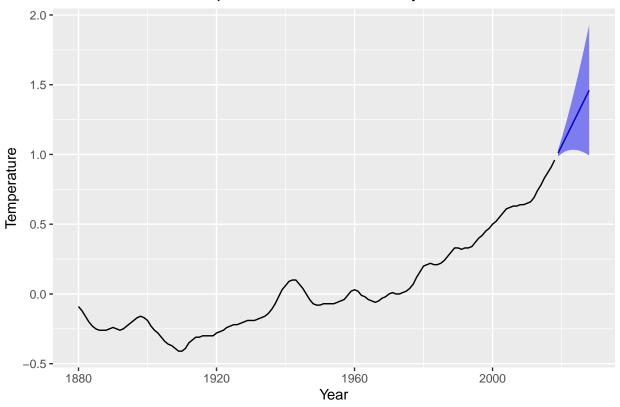
# GARCH Forecast Arctic Sea Ice Level - 95% CI for 10 years



#### **ARIMA** - Temperature

```
##
## Box-Ljung test
##
## data: temp.ts
## X-squared = 588.18, df = 5, p-value < 0.0000000000000000022</pre>
```



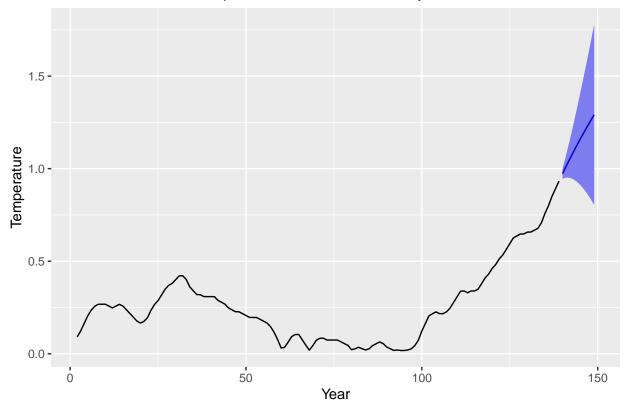


## GARCH - Temperature

## ## ##	****	ESTIMATION WITH	H ANALYTICAL	. GRADIENT	****			
## ## ##	I	INITIAL X(	I) D(	[1)				
##	1	9.807248e-	1.000	e+00				
##	2	5.000000e-	1.000	e+00				
##	3	5.000000e-	1.000	e+00				
##								
##	IT	NF F	RELDF	PRELDF	RELDX	STPPAR	D*STEP	NPRELDF
##	0	1 -9.001e+0	=					
##	1	3 -9.412e+0		1.61e-01	4.3e-01	1.5e+03	1.0e-01	1.23e+02
##	2	4 -1.117e+0	2 1.57e-01	3.17e-01	2.9e-01	2.0e+00	1.0e-01	2.50e+02
##	3	6 -1.295e+0	2 1.37e-01	1.08e-01	1.8e-01	2.0e+00	1.0e-01	2.21e+02
##	4	8 -1.331e+0	2 2.68e-02	2.77e-02	2.6e-02	2.0e+00	2.0e-02	1.60e+04
##	5	10 -1.405e+0	2 5.32e-02	4.76e-02	5.8e-02	2.0e+00	4.0e-02	3.70e+03
##	6	13 -1.405e+0	2 2.27e-05	2.31e-03	2.2e-03	2.4e+00	2.2e-03	7.21e+04
##	7	14 -1.409e+0	2 2.71e-03	4.28e-03	1.5e-03	2.1e+00	1.1e-03	5.77e+04
##	8	19 -1.495e+0	2 5.74e-02	7.14e-02	7.7e-02	2.0e+00	6.1e-02	5.53e+04
##	9	20 -1.550e+0	2 3.55e-02	1.03e-01	6.6e-02	2.0e+00	6.1e-02	3.85e+03
##	10	22 -1.620e+0	2 4.34e-02	4.00e-02	5.9e-02	2.0e+00	6.1e-02	5.41e+02
##	11	26 -1.628e+0	2 4.52e-03	4.64e-03	3.8e-04	7.5e+00	4.1e-04	2.11e+03

```
##
       12
            28 -1.630e+02
                            1.12e-03
                                       1.20e-03
                                                 1.3e-04
                                                           9.5e+00
                                                                     1.4e-04
##
       13
                                       1.14e-03
                                                 2.6e-04
            29 -1.630e+02
                           5.22e-04
                                                           2.3e+00
                                                                     2.8e-04
                                                                              7.02e+02
##
            30 -1.631e+02
                            2.89e-04
                                       4.26e-04
                                                 2.3e-04
                                                           2.0e+00
                                                                     2.8e-04
                                                                              6.59e+02
                            4.83e-02
##
       15
            36 -1.714e+02
                                       9.03e-02
                                                 1.6e-01
                                                           2.0e+00
                                                                              6.34e+02
                                                                     2.1e-01
##
       16
            45 -1.721e+02
                            4.43e-03
                                       2.05e-02
                                                 2.2e-04
                                                           4.5e+00
                                                                     3.4e-04
                                                                              5.21e-01
                            3.93e-03
                                                 1.4e-01
                                                           1.6e+00
##
       17
            51 -1.728e+02
                                       1.01e-02
                                                                     2.1e-01
                                                                              5.73e-02
       18
                                       8.84e-03
                                                 9.3e-02
##
            53 -1.740e+02
                            7.11e-03
                                                           1.0e+00
                                                                     2.1e-01
                                                                              4.37e-02
##
       19
            55 -1.743e+02
                            1.75e-03
                                       2.07e-03
                                                 3.7e-02
                                                           9.2e-01
                                                                     8.6e-02
                                                                              3.51e-03
##
       20
            56 -1.744e+02
                            2.59e-04
                                       2.86e-04
                                                 2.5e-02
                                                           0.0e+00
                                                                     5.7e-02
                                                                              2.86e-04
##
       21
            57 -1.744e+02
                            8.26e-05
                                       7.48e-05
                                                 2.8e-03
                                                           0.0e+00
                                                                     5.9e-03
                                                                              7.48e-05
##
       22
            58 -1.744e+02
                            4.91e-06
                                       4.12e-06
                                                 2.4e-03
                                                           1.6e-01
                                                                     5.9e-03
                                                                              4.16e-06
       23
            59 -1.744e+02
                            1.03e-06
                                                  1.2e-03
##
                                       8.77e-07
                                                           0.0e + 00
                                                                     3.1e-03
                                                                              8.77e-07
##
       24
            60 -1.744e+02
                            1.09e-07
                                       9.27e-08
                                                 3.0e-04
                                                           0.0e + 00
                                                                     6.1e-04
                                                                              9.27e-08
       25
                            8.36e-09
##
            61 -1.744e+02
                                       7.93e-09
                                                 9.3e-05
                                                           0.0e + 00
                                                                     1.9e-04
                                                                              7.93e-09
##
       26
            62 -1.744e+02
                            1.46e-10
                                      9.95e-11
                                                 1.3e-05
                                                           0.0e+00
                                                                     2.9e-05
                                                                              9.95e-11
##
    **** RELATIVE FUNCTION CONVERGENCE ****
##
##
                 -1.744045e+02
                                                1.281e-05
##
    FUNCTION
                                 RELDX
##
    FUNC. EVALS
                      62
                                  GRAD. EVALS
                                                    27
##
    PRELDF
                  9.948e-11
                                 NPRELDF
                                               9.948e-11
##
##
               FINAL X(I)
                                   D(I)
                                                  G(I)
        Ι
##
                               1.000e+00
##
        1
             2.888764e-04
                                              6.755e-02
##
        2
             1.008626e+00
                               1.000e+00
                                              2.168e-05
##
        3
             4.370590e-02
                               1.000e+00
                                              1.087e-04
```

### GARCH Forecast Temperature – 95% CI for 10 years



Alex Clark - Assignment #3