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CS1800 Final Write Up

I did a problem set based on the board game Scrabble. The first problem I had set for myself was finding the number of draws until a vowel is found, this was pretty easy. I found the expected number of vowels on each draw until the sum of the expected values was greater than 1. The second question was asking about the number of draws until a pair, this question was easy too in that it was more a counting problem, I would have liked to have made this question harder, but the theme did not lend itself to a better alternative. Using the pigeonhole principle here to show that you can have each letter once is very clean. The next question was asking about finding a letter to make a long word, this one I wanted to list out the letters needed so there wasn't any other information needed. This one is a little bit sneaky with needing to notice that some of the letters listed are duplicates and the one you already have must not be counted in the probability. Additionally, the blank tiles should be counted too. The replacement questions are straightforward probability questions. The solutions differ by the denominator only, this makes sense because the number of success has not changed, only the number of failures. The next question that I tried to do was to find the probability of drawing a 7-tile hand with no valid words. The first step I did was find a list of 2-letter words, then I found all 3 letter combinations that did not have any of those words. When I attempted to find combinations of 3-letter combinations to make 6-letter combinations, the runtime became too large to finish in a reasonable time. I tried finding some better algorithm but could not find a better asymptotic behavior. I could not figure out how to deal with the large sample space or how to avoid finding these 7-letter sets. This led me to ditching the question. The hardest question is the one to find the best play. I had expected the best play to be replacing

an A so you have a chance of drawing a Q so you could play MOSQUES, but that was actually one of the worst performances that I found. The ability to replace tiles after playing means that you can get points while still trying to draw the Q. Most of the trouble is making sure that the Q is played as often as possible, and the M is always played. Leaving an S out of the first word lets you get the word SUQ, not a word I personally know, but the Scrabble websites I used claim is a real word. https://word.tips/ and https://www.thewordfinder.com/anagram-solver/ as well as the Hasbro rules https://scrabble.hasbro.com/en-us/rules were where I sourced the legitimacy of words and the rules. Finding the expected number of points was straight forward, using the AEMOSSU letters make a word worth some number of points, then find what letters are left over and how many new tiles to draw, using this find all possible letter combinations of EEELONQ and their frequency. Using each possible letter set find the maximum word, combine this with the frequency of that point total to get the expected value for each strategy. The resulting solution really surprised me, I would have thought playing one of the longer words would be worth more in order to play Q more frequently. Also comparing SOME to OASES, the reason SOME is better is because of being able to guarantee that M is played for 3 is worth more than the extra 1 point from the length and the S is needed for SUQ and can be tacked to most words. I do not think that this question part 6 is well suited for anyone else to do because of how tedious the problem became with checking the best words for so many letter combinations.