GEMP - UFC Quixadá - ICPC Library

Contents

1 Data Structures

1.1 BIT

```cpp
#include <bits/stdc++.h>
using namespace std;
class Bit{
private:
  typedef long long t_bit;
  int nBit;
  int nLog;
  vector<t_bit> bit;
public:
  Bit(int n){
    nBit = n;
    nLog = 20;
    bit.resize(nBit + 1, 0);
  }
  //1-indexed
  t_bit get(int i){
    t_bit s = 0;
    for (; i > 0; i -= (i & -i))
      s += bit[i];
    return s;
  }
  //1-indexed [l, r]
  t_bit get(int l, int r){
    return get(r) - get(l - 1);
  }
  //1-indexed
  void add(int i, t_bit value){
    assert(i > 0);
    for (; i <= nBit; i += (i & -i))
      bit[i] += value;
  }
  t_bit lower_bound(t_bit value){
    t_bit sum = 0;
    int pos = 0;
    for (int i = nLog; i >= 0; i--){
      if ((pos + (1 << i) <= nBit) and (sum + bit[pos + (1 << i)] <
          value)){
        sum += bit[pos + (1 << i)];
        pos += (1 << i);
      }
    }
    return pos + 1;
  }
};
```

1.2 BIT 2D

```cpp
#include <bits/stdc++.h>
using namespace std;
class Bit2d{
private:
  typedef long long t_bit;
  vector<vector<t_bit>> bit;
  int nBit, mBit;
public:
  Bit2d(int n, int m){
    nBit = n;
    mBit = m;
    bit.resize(nBit + 1, vector<t_bit>(mBit + 1, 0));
  }
  //1-indexed
  t_bit get(int i, int j){
    t_bit sum = 0;
    for (int a = i; a > 0; a -= (a & -a))
      for (int b = j; b > 0; b -= (b & -b))
        sum += bit[a][b];
    return sum;
  }
  //1-indexed
  t_bit get(int a1, int b1, int a2, int b2){
    return get(a2, b2) - get(a2, b1 - 1) - get(a1 - 1, b2) + get(a1 -
        1, b1 - 1);
  }
  //1-indexed [i, j]
  void add(int i, int j, t_bit value){
    for (int a = i; a <= nBit; a += (a & -a))
      for (int b = j; b <= mBit; b += (b & -b))
        bit[a][b] += value;
  }
};
```

1.3 BIT In Range

```cpp
#include <bits/stdc++.h>
using namespace std;
class BitRange{
private:
  typedef long long t_bit;
  vector<t_bit> bit1, bit2;
  t_bit get(vector<t_bit> &bit, int i){
    t_bit sum = 0;
    for (; i > 0; i -= (i & -i))
      sum += bit[i];
    return sum;
  }
  void add(vector<t_bit> &bit, int i, t_bit value){
    for (; i < (int)bit.size(); i += (i & -i))
      bit[i] += value;
  }
public:
  BitRange(int n){
    bit1.assign(n + 1, 0);
    bit2.assign(n + 1, 0);
  }
  //1-indexed [i, j]
  void add(int i, int j, t_bit v){
    add(bit1, i, v);
    add(bit1, j + 1, -v);
    add(bit2, i, v * (i - 1));
    add(bit2, j + 1, -v * j);
  }
  //1-indexed
  t_bit get(int i){
    return get(bit1, i) * i - get(bit2, i);
  }
  //1-indexed [i,j]
  t_bit get(int i, int j){
    return get(j) - get(i - 1);
  }
};
```

1.4 Dynamic Median

```cpp
#include <bits/stdc++.h>
using namespace std;
class DinamicMedian{
  typedef int t_median;
private:
  priority_queue<t_median> mn;
  priority_queue<t_median, vector<t_median>, greater<t_median>> mx;
public:
  double median(){
    if (mn.size() > mx.size())
      return mn.top();
    else
      return (mn.top() + mx.top()) / 2.0;
  }
  void push(t_median x){
    if (mn.size() <= mx.size())
      mn.push(x);
    else
      mx.push(x);
    if ((!mx.empty()) and (!mn.empty())){
      while (mn.top() > mx.top()){
        t_median a = mx.top();
        mx.pop();
        t_median b = mn.top();
        mn.pop();
        mx.push(b);
        mn.push(a);
      }
    }
  }
};
```

1.5 Dynamic Wavelet Tree

```cpp
#include <bits/stdc++.h>
using namespace std;
struct SplayTree{
  struct Node{
    int x, y, s;
    Node *p = 0;
    Node *l = 0;
    Node *r = 0;
```

```cpp
  Node(int v){
    x = v;
    y = v;
    s = 1;
  }
  void upd(){
    s = 1;
    y = x;
    if (l){
      y += l->y;
      s += l->s;
    }
    if (r){
      y += r->y;
      s += r->s;
    }
  }
  int left_size(){
    return l ? l->s : 0;
  }
};
Node *root = 0;
void rot(Node *c){
  auto p = c->p;
  auto g = p->p;
  if (g)
    (g->l == p ? g->l : g->r) = c;
  if (p->l == c){
    p->l = c->r;
    c->r = p;
    if (p->l)
      p->l->p = p;
  }
  else{
    p->r = c->l;
    c->l = p;
    if (p->r)
      p->r->p = p;
  }
  p->p = c;
  c->p = g;
  p->upd();
  c->upd();
}
void splay(Node *c){
  while (c->p){
    auto p = c->p;
    auto g = p->p;
    if (g)
      rot((g->l == p) == (p->l == c) ? p : c);
    rot(c);
  }
  c->upd();
  root = c;
}
Node *join(Node *l, Node *r){
  if (not l)
    return r;
  if (not r)
    return l;
  while (l->r)
    l = l->r;
  splay(l);
  r->p = l;
  l->r = r;
  l->upd();
  return l;
}
pair<Node *, Node *> split(Node *p, int idx){
  if (not p)
    return make_pair(nullptr, nullptr);
  if (idx < 0)
    return make_pair(nullptr, p);
  if (idx >= p->s)
    return make_pair(p, nullptr);
  for (int lf = p->left_size(); idx != lf; lf = p->left_size()){
    if (idx < lf)
      p = p->l;
    else
      p = p->r, idx -= lf + 1;
  }
  splay(p);
  Node *l = p;
  Node *r = p->r;
  if (r){
    l->r = r->p = 0;
    l->upd();
  }
  return make_pair(l, r);
}
Node *get(int idx){
  auto p = root;
  for (int lf = p->left_size(); idx != lf; lf = p->left_size()){
    if (idx < lf)
      p = p->l;
    else
      p = p->r, idx -= lf + 1;
  }
  splay(p);
  return p;
}
int insert(int idx, int x){
  Node *l, *r;
  tie(l, r) = split(root, idx - 1);
  int v = l ? l->y : 0;
  root = join(l, join(new Node(x), r));
  return v;
}
void erase(int idx){
  Node *l, *r;
  tie(l, r) = split(root, idx);
  root = join(l->l, r);
  delete l;
}
int rank(int idx){
  Node *l, *r;
  tie(l, r) = split(root, idx);
  int x = (l && l->l ? l->l->y : 0);
  root = join(l, r);
  return x;
}
int operator[](int idx){
```

```
    return rank(idx);
  }
  ~SplayTree(){
    if (!root)
      return;
    vector<Node *> nodes{root};
    while (nodes.size()){
      auto u = nodes.back();
      nodes.pop_back();
      if (u->l)
        nodes.emplace_back(u->l);
      if (u->r)
        nodes.emplace_back(u->r);
      delete u;
    }
  }
};
class WaveletTree{
private:
  int lo, hi;
  WaveletTree *l = 0;
  WaveletTree *r = 0;
  SplayTree b;
public:
  WaveletTree(int min_value, int max_value){
    lo = min_value;
    hi = max_value;
    b.insert(0, 0);
  }
  ~WaveletTree(){
    delete l;
    delete r;
  }
  //0-indexed
  void insert(int idx, int x){
    if (lo >= hi)
      return;
    int mid = (lo + hi - 1) / 2;
    if (x <= mid){
      l = l ?: new WaveletTree(lo, mid);
      l->insert(b.insert(idx, 1), x);
    }else{
      r = r ?: new WaveletTree(mid + 1, hi);
      r->insert(idx - b.insert(idx, 0), x);
    }
  }
  //0-indexed
  void erase(int idx){
    if (lo == hi)
      return;
    auto p = b.get(idx);
    int lf = p->l ? p->l->y : 0;
    int x = p->x;
    b.erase(idx);
    if (x == 1)
      l->erase(lf);
    else
      r->erase(idx - lf);
  }
  //kth smallest element in range [i, j[
  //0-indexed
  int kth(int i, int j, int k){
    if (i >= j)
      return 0;
    if (lo == hi)
      return lo;
    int x = b.rank(i);
    int y = b.rank(j);
    if (k <= y - x)
      return l->kth(x, y, k);
    else
      return r->kth(i - x, j - y, k - (y - x));
  }
  //Amount of numbers in the range [i, j[ Less than or equal to k
  //0-indexed
  int lte(int i, int j, int k){
    if (i >= j or k < lo)
      return 0;
    if (hi <= k)
      return j - i;
    int x = b.rank(i);
    int y = b.rank(j);
    return l->lte(x, y, k) + r->lte(i - x, j - y, k);
  }
  //Amount of numbers in the range [i, j[ equal to k
  //0-indexed
  int count(int i, int j, int k){
    if (i >= j or k < lo or k > hi)
      return 0;
    if (lo == hi)
      return j - i;
    int mid = (lo + hi - 1) / 2;
    int x = b.rank(i);
    int y = b.rank(j);
    if (k <= mid)
      return l->count(x, y, k);
    return r->count(i - x, j - y, k);
  }
  //0-indexed
  int get(int idx){
    return kth(idx, idx + 1, 1);
  }
};
```

1.6 Implicit Treap

```
#include <bits/stdc++.h>
using namespace std;
class ImplicitTreap{
private:
  typedef int t_treap;
  const t_treap neutral = 0;
  inline t_treap join(t_treap a, t_treap b, t_treap c){
    return a + b + c;
  }
  struct Node{
    int y, size;
    t_treap v, op_value;
    bool rev;
    Node *l, *r;
    Node(t_treap _v){
```

```
    v = op_value = _v;
    y = rand();
    size = 1;
    l = r = NULL;
    rev = false;
  }
};
Node *root;
int size(Node *t) { return t ? t->size : 0; }
t_treap op_value(Node *t) { return t ? t->op_value : neutral; }
Node *refresh(Node *t){
  if (t == NULL)
    return t;
  t->size = 1 + size(t->l) + size(t->r);
  t->op_value = join(t->v, op_value(t->l), op_value(t->r));
  if (t->l != NULL)
    t->l->rev ^= t->rev;
  if (t->r != NULL)
    t->r->rev ^= t->rev;
  if (t->rev){
    swap(t->l, t->r);
    t->rev = false;
  }
  return t;
}
void split(Node *&t, int k, Node *&a, Node *&b){
  refresh(t);
  Node *aux;
  if (!t){
    a = b = NULL;
  }else if (size(t->l) < k){
    split(t->r, k - size(t->l) - 1, aux, b);
    t->r = aux;
    a = refresh(t);
  }else{
    split(t->l, k, a, aux);
    t->l = aux;
    b = refresh(t);
  }
}
Node *merge(Node *a, Node *b){
  refresh(a);
  refresh(b);
  if (!a || !b)
    return a ? a : b;
  if (a->y < b->y){
    a->r = merge(a->r, b);
    return refresh(a);
  }else{
    b->l = merge(a, b->l);
    return refresh(b);
  }
}
Node *at(Node *t, int n){
  if (!t)
    return t;
  refresh(t);
  if (n < size(t->l))
    return at(t->l, n);
  else if (n == size(t->l))
    return t;
    else
      return at(t->r, n - size(t->l) - 1);
  }
  void del(Node *&t){
    if (!t)
      return;
    if (t->l)
      del(t->l);
    if (t->r)
      del(t->r);
    delete t;
    t = NULL;
  }
public:
  ImplicitTreap() : root(NULL){
    srand(time(NULL));
  }
  ~ImplicitTreap() { clear(); }
  void clear() { del(root); }
  int size() { return size(root); }
  //0-indexed
  bool insert(int n, int v){
    Node *a, *b;
    split(root, n, a, b);
    root = merge(merge(a, new Node(v)), b);
    return true;
  }
  //0-indexed
  bool erase(int n){
    Node *a, *b, *c, *d;
    split(root, n, a, b);
    split(b, 1, c, d);
    root = merge(a, d);
    if (c == NULL)
      return false;
    delete c;
    return true;
  }
  //0-indexed
  t_treap at(int n){
    Node *ans = at(root, n);
    return ans ? ans->v : -1;
  }
  //0-indexed [l, r]
  t_treap query(int l, int r){
    if (l > r)
      swap(l, r);
    Node *a, *b, *c, *d;
    split(root, l, a, d);
    split(d, r - l + 1, b, c);
    t_treap ans = op_value(b);
    root = merge(a, merge(b, c));
    return ans;
  }
  //0-indexed [l, r]
  void reverse(int l, int r){
    if (l > r)
      swap(l, r);
    Node *a, *b, *c, *d;
    split(root, l, a, d);
    split(d, r - l + 1, b, c);
```

```
    if (b != NULL)
      b->rev ^= 1;
    root = merge(a, merge(b, c));
  }
};
```

1.7 LiChao Tree

```
#include <bits/stdc++.h>
using namespace std;
const int INF = 0x3f3f3f3f;
class LiChaoTree{
private:
  typedef int t_line;
  struct Line{
    t_line k, b;
    Line() {}
    Line(t_line k, t_line b) : k(k), b(b) {}
  };
  int n_tree, min_x, max_x;
  vector<Line> li_tree;
  t_line f(Line l, int x){
    return l.k * x + l.b;
  }
  void add(Line nw, int v, int l, int r){
    int m = (l + r) / 2;
    bool lef = f(nw, l) > f(li_tree[v], l);
    bool mid = f(nw, m) > f(li_tree[v], m);
    if (mid)
      swap(li_tree[v], nw);
    if (r - l == 1)
      return;
    else if (lef != mid)
      add(nw, 2 * v, l, m);
    else
      add(nw, 2 * v + 1, m, r);
  }
  int get(int x, int v, int l, int r){
    int m = (l + r) / 2;
    if (r - l == 1)
      return f(li_tree[v], x);
    else if (x < m)
      return max(f(li_tree[v], x), get(x, 2 * v, l, m));
    else
      return max(f(li_tree[v], x), get(x, 2 * v + 1, m, r));
  }
public:
  LiChaoTree(int mn_x, int mx_x){
    min_x = mn_x;
    max_x = mx_x;
    n_tree = max_x - min_x + 5;
    li_tree.resize(4 * n_tree, Line(0, -INF));
  }
  void add(t_line k, t_line b){
    add(Line(k, b), 1, min_x, max_x);
  }
  t_line get(int x){
    return get(x, 1, min_x, max_x);
  }
};
```

1.8 Policy Based Tree

```
#include <bits/stdc++.h>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
using namespace std;
typedef tree<int, null_type, less<int>, rb_tree_tag,
  tree_order_statistics_node_update> OrderedSet;
typedef tree<int, int, less<int>, rb_tree_tag,
  tree_order_statistics_node_update> OrderedMap;
//order_of_key (k) : Number of items strictly smaller than k .
//find_by_order(k) : K-th element in a set (counting from zero).
```

1.9 Queue Query

```
#include <bits/stdc++.h>
using namespace std;
class QueueQuery{
private:
  typedef long long t_queue;
  stack<pair<t_queue, t_queue>> s1, s2;
  t_queue cmp(t_queue a, t_queue b){
    return min(a, b);
  }
  void move(){
    if (s2.empty()){
      while (!s1.empty()){
        t_queue element = s1.top().first;
        s1.pop();
        t_queue result = s2.empty() ? element : cmp(element, s2.top().
            second);
        s2.push({element, result});
      }
    }
  }
public:
  void push(t_queue x){
    t_queue result = s1.empty() ? x : cmp(x, s1.top().second);
    s1.push({x, result});
  }
  void pop(){
    move();
    s2.pop();
  }
  t_queue front(){
    move();
    return s2.top().first;
  }
  t_queue query(){
    if (s1.empty() || s2.empty())
      return s1.empty() ? s2.top().second : s1.top().second;
    else
      return cmp(s1.top().second, s2.top().second);
  }
  t_queue size(){
    return s1.size() + s2.size();
  }
```

```
};
```

1.10 Range Color

```
#include <bits/stdc++.h>
using namespace std;
class RangeColor{
private:
  typedef long long ll;
  struct Node{
    ll l, r;
    int color;
    Node() {}
    Node(ll l, ll r, int color) : l(l), r(r), color(color) {}
  };
  struct cmp{
    bool operator()(Node a, Node b){
      return a.r < b.r;
    }
  };
  std::set<Node, cmp> st;
  vector<ll> ans;
public:
  RangeColor(ll first, ll last, int maxColor){
    ans.resize(maxColor + 1);
    ans[0] = last - first + 1LL;
    st.insert(Node(first, last, 0));
  }
  //set newColor in [a, b]
  void set(ll a, ll b, int newColor){
    auto p = st.upper_bound(Node(0, a - 1LL, -1));
    assert(p != st.end());
    ll l = p->l;
    ll r = p->r;
    int oldColor = p->color;
    ans[oldColor] -= (r - l + 1LL);
    p = st.erase(p);
    if (l < a){
      ans[oldColor] += (a - l);
      st.insert(Node(l, a - 1LL, oldColor));
    }
    if (b < r){
      ans[oldColor] += (r - b);
      st.insert(Node(b + 1LL, r, oldColor));
    }
    while ((p != st.end()) and (p->l <= b)){
      l = p->l;
      r = p->r;
      oldColor = p->color;
      ans[oldColor] -= (r - l + 1LL);
      if (b < r){
        ans[oldColor] += (r - b);
        st.insert(Node(b + 1LL, r, oldColor));
        st.erase(p);
        break;
      }else{
        p = st.erase(p);
      }
    }
    ans[newColor] += (b - a + 1LL);
    st.insert(Node(a, b, newColor));
  }
  ll countColor(int x){
    return ans[x];
  }
};
```

1.11 Segment Tree

```
#include <bits/stdc++.h>
using namespace std;
class SegTree{
private:
  typedef long long Node;
  Node neutral = 0;
  vector<Node> st;
  vector<int> v;
  int n;
  Node join(Node a, Node b){
    return (a + b);
  }
  void build(int node, int i, int j){
    if (i == j){
      st[node] = v[i];
      return;
    }
    int m = (i + j) / 2;
    int l = (node << 1);
    int r = l + 1;
    build(l, i, m);
    build(r, m + 1, j);
    st[node] = join(st[l], st[r]);
  }
  Node query(int node, int i, int j, int a, int b){
    if ((i > b) or (j < a))
      return neutral;
    if ((a <= i) and (j <= b))
      return st[node];
    int m = (i + j) / 2;
    int l = (node << 1);
    int r = l + 1;
    return join(query(l, i, m, a, b), query(r, m + 1, j, a, b));
  }
  void update(int node, int i, int j, int idx, Node value){
    if (i == j){
      st[node] = value;
      return;
    }
    int m = (i + j) / 2;
    int l = (node << 1);
    int r = l + 1;
    if (idx <= m)
      update(l, i, m, idx, value);
    else
      update(r, m + 1, j, idx, value);
    st[node] = join(st[l], st[r]);
  }
public:
  template <class MyIterator>
  SegTree(MyIterator begin, MyIterator end){
```

```
    n = end - begin;
    v = vector<int>(begin, end);
    st.resize(4 * n + 5);
    build(1, 0, n - 1);
  }
  //0-indexed [a, b]
  Node query(int a, int b){
    return query(1, 0, n - 1, a, b);
  }
  //0-indexed
  void update(int idx, int value){
    update(1, 0, n - 1, idx, value);
  }
};
```

1.12 Segment Tree 2D

```
#include <bits/stdc++.h>
using namespace std;
struct SegTree2D{
private:
  int n, m;
  typedef int Node;
  Node neutral = -0x3f3f3f3f;
  vector<vector<Node>> seg;
  Node join(Node a, Node b){
    return max(a, b);
  }
public:
  SegTree2D(int n1, int m1){
    n = n1, m = m1;
    seg.assign(2 * n, vector<Node>(2 * m, 0));
  }
  void update(int x, int y, int val){
    assert(0 <= x && x < n && 0 <= y && y < m);
    x += n, y += m;
    seg[x][y] = val;
    for (int j = y / 2; j > 0; j /= 2)
      seg[x][j] = join(seg[x][2 * j], seg[x][2 * j + 1]);
    for (x /= 2; x > 0; x /= 2){
      seg[x][y] = join(seg[2 * x][y], seg[2 * x + 1][y]);
      for (int j = y / 2; j > 0; j /= 2){
        seg[x][j] = join(seg[x][2 * j], seg[x][2 * j + 1]);
      }
    }
  }
  vector<int> getCover(int l, int r, int N){
    l = std::max(0, l);
    r = std::min(N, r);
    vector<int> ans;
    for (l += N, r += N; l < r; l /= 2, r /= 2){
      if (l & 1)
        ans.push_back(l++);
      if (r & 1)
        ans.push_back(--r);
    }
    return ans;
  }
  Node query(int x1, int y1, int x2, int y2){
    auto c1 = getCover(x1, x2 + 1, n);
    auto c2 = getCover(y1, y2 + 1, m);
    Node ans = neutral;
    for (auto i : c1){
      for (auto j : c2){
        ans = join(ans, seg[i][j]);
      }
    }
    return ans;
  }
};
```

1.13 Segment Tree Iterative

```
#include <bits/stdc++.h>
using namespace std;
class SegTreeIterative{
private:
  typedef long long Node;
  Node neutral = 0;
  vector<Node> st;
  int n;
  inline Node join(Node a, Node b){
    return a + b;
  }
public:
  template <class MyIterator>
  SegTreeIterative(MyIterator begin, MyIterator end){
    int sz = end - begin;
    for (n = 1; n < sz; n <<= 1);
    st.assign(n << 1, neutral);
    for (int i = 0; i < sz; i++, begin++)
      st[i + n] = (*begin);
    for (int i = n + sz - 1; i > 1; i--)
      st[i >> 1] = join(st[i >> 1], st[i]);
  }
  //0-indexed
  void update(int i, Node x){
    st[i += n] = x;
    for (i >>= 1; i; i >>= 1)
      st[i] = join(st[i << 1], st[1 + (i << 1)]);
  }
  //0-indexed [l, r]
  Node query(int l, int r){
    Node ans = neutral;
    for (l += n, r += n + 1; l < r; l >>= 1, r >>= 1){
      if (l & 1)
        ans = join(ans, st[l++]);
      if (r & 1)
        ans = join(ans, st[--r]);
    }
    return ans;
  }
};
```

1.14 Segment Tree Lazy

```
#include <bits/stdc++.h>
using namespace std;
```

```
class SegTreeLazy{
private:
  typedef long long Node;
  vector<Node> st;
  vector<long long> lazy;
  vector<int> v;
  int n;
  Node neutral = 0;
  inline Node join(Node a, Node b){
    return a + b;
  }
  inline void upLazy(int &node, int &i, int &j){
    if (lazy[node] != 0){
      st[node] += lazy[node] * (j - i + 1);
      //tree[node] += lazy[node];
      if (i != j){
        lazy[(node << 1)] += lazy[node];
        lazy[(node << 1) + 1] += lazy[node];
      }
      lazy[node] = 0;
    }
  }
  void build(int node, int i, int j){
    if (i == j){
      st[node] = v[i];
      return;
    }
    int m = (i + j) / 2;
    int l = (node << 1);
    int r = l + 1;
    build(l, i, m);
    build(r, m + 1, j);
    st[node] = join(st[l], st[r]);
  }
  Node query(int node, int i, int j, int a, int b){
    upLazy(node, i, j);
    if ((i > b) or (j < a))
      return neutral;
    if ((a <= i) and (j <= b)){
      return st[node];
    }
    int m = (i + j) / 2;
    int l = (node << 1);
    int r = l + 1;
    return join(query(l, i, m, a, b), query(r, m + 1, j, a, b));
  }
  void update(int node, int i, int j, int a, int b, int value){
    upLazy(node, i, j);
    if ((i > j) or (i > b) or (j < a))
      return;
    if ((a <= i) and (j <= b)){
      lazy[node] = value;
      upLazy(node, i, j);
    }else{
      int m = (i + j) / 2;
      int l = (node << 1);
      int r = l + 1;
      update(l, i, m, a, b, value);
      update(r, m + 1, j, a, b, value);
      st[node] = join(st[l], st[r]);
    }
  }
public:
  template <class MyIterator>
  SegTreeLazy(MyIterator begin, MyIterator end){
    n = end - begin;
    v = vector<int>(begin, end);
    st.resize(4 * n + 5);
    lazy.assign(4 * n + 5, 0);
    build(1, 0, n - 1);
  }
  //0-indexed [a, b]
  Node query(int a, int b){
    return query(1, 0, n - 1, a, b);
  }
  //0-indexed [a, b]
  void update(int a, int b, int value){
    update(1, 0, n - 1, a, b, value);
  }
};
```

1.15 Sparse Table

```
#include <bits/stdc++.h>
using namespace std;
class SparseTable{
private:
  typedef int t_st;
  vector<vector<t_st>> st;
  vector<int> log2;
  t_st neutral = 0x3f3f3f3f;
  int nLog;
  t_st join(t_st a, t_st b){
    return min(a, b);
  }
public:
  template <class MyIterator>
  SparseTable(MyIterator begin, MyIterator end){
    int n = end - begin;
    nLog = 20;
    log2.resize(n + 1);
    log2[1] = 0;
    for (int i = 2; i <= n; i++)
      log2[i] = log2[i / 2] + 1;
    st.resize(n, vector<t_st>(nLog, neutral));
    for (int i = 0; i < n; i++, begin++)
      st[i][0] = (*begin);
    for (int j = 1; j < nLog; j++)
      for (int i = 0; (i + (1 << (j - 1))) < n; i++)
        st[i][j] = join(st[i][j - 1], st[i + (1 << (j - 1))][j - 1]);
  }
  //0-indexed [a, b]
  t_st query(int a, int b){
    int d = b - a + 1;
    t_st ans = neutral;
    for (int j = nLog - 1; j >= 0; j--){
      if (d & (1 << j)){
        ans = join(ans, st[a][j]);
        a = a + (1 << (j));
      }
    }
```

```cpp
    return ans;
  }
  //0-indexed [a, b]
  t_st queryRMQ(int a, int b){
    int j = log2[b - a + 1];
    return join(st[a][j], st[b - (1 << j) + 1][j]);
  }
};
```

1.16 SQRT Decomposition

```cpp
#include <bits/stdc++.h>
using namespace std;
struct SqrtDecomposition{
  typedef long long t_sqrt;
  int sqrtLen;
  vector<t_sqrt> block;
  vector<t_sqrt> v;
  template <class MyIterator>
  SqrtDecomposition(MyIterator begin, MyIterator end){
    int n = end - begin;
    sqrtLen = (int)sqrt(n + .0) + 1;
    v.resize(n);
    block.resize(sqrtLen + 5);
    for (int i = 0; i < n; i++, begin++){
      v[i] = (*begin);
      block[i / sqrtLen] += v[i];
    }
  }
  //0-indexed
  void update(int idx, t_sqrt new_value){
    t_sqrt d = new_value - v[idx];
    v[idx] += d;
    block[idx / sqrtLen] += d;
  }
  //0-indexed [l, r]
  t_sqrt query(int l, int r){
    t_sqrt sum = 0;
    int c_l = l / sqrtLen, c_r = r / sqrtLen;
    if (c_l == c_r){
      for (int i = l; i <= r; i++)
        sum += v[i];
    }else{
      for (int i = l, end = (c_l + 1) * sqrtLen - 1; i <= end; i++)
        sum += v[i];
      for (int i = c_l + 1; i <= c_r - 1; i++)
        sum += block[i];
      for (int i = c_r * sqrtLen; i <= r; i++)
        sum += v[i];
    }
    return sum;
  }
};
```

1.17 SQRT Tree

```cpp
#include <bits/stdc++.h>
using namespace std;
class SqrtTree{
private:
  typedef long long t_sqrt;
  t_sqrt op(const t_sqrt &a, const t_sqrt &b){
    return a | b;
  }
  inline int log2Up(int n){
    int res = 0;
    while ((1 << res) < n)
      res++;
    return res;
  }
  int n, lg, indexSz;
  vector<t_sqrt> v;
  vector<int> clz, layers, onLayer;
  vector<vector<t_sqrt>> pref, suf, between;
  inline void buildBlock(int layer, int l, int r){
    pref[layer][l] = v[l];
    for (int i = l + 1; i < r; i++)
      pref[layer][i] = op(pref[layer][i - 1], v[i]);
    suf[layer][r - 1] = v[r - 1];
    for (int i = r - 2; i >= l; i--)
      suf[layer][i] = op(v[i], suf[layer][i + 1]);
  }
  inline void buildBetween(int layer, int lBound, int rBound, int
      betweenOffs){
    int bSzLog = (layers[layer] + 1) >> 1;
    int bCntLog = layers[layer] >> 1;
    int bSz = 1 << bSzLog;
    int bCnt = (rBound - lBound + bSz - 1) >> bSzLog;
    for (int i = 0; i < bCnt; i++){
      t_sqrt ans;
      for (int j = i; j < bCnt; j++){
        t_sqrt add = suf[layer][lBound + (j << bSzLog)];
        ans = (i == j) ? add : op(ans, add);
        between[layer - 1][betweenOffs + lBound + (i << bCntLog) + j]
            = ans;
      }
    }
  }
  inline void buildBetweenZero(){
    int bSzLog = (lg + 1) >> 1;
    for (int i = 0; i < indexSz; i++){
      v[n + i] = suf[0][i << bSzLog];
    }
    build(1, n, n + indexSz, (1 << lg) - n);
  }
  inline void updateBetweenZero(int bid){
    int bSzLog = (lg + 1) >> 1;
    v[n + bid] = suf[0][bid << bSzLog];
    update(1, n, n + indexSz, (1 << lg) - n, n + bid);
  }
  void build(int layer, int lBound, int rBound, int betweenOffs){
    if (layer >= (int)layers.size())
      return;
    int bSz = 1 << ((layers[layer] + 1) >> 1);
    for (int l = lBound; l < rBound; l += bSz){
      int r = min(l + bSz, rBound);
      buildBlock(layer, l, r);
      build(layer + 1, l, r, betweenOffs);
    }
```

```cpp
    if (layer == 0)
      buildBetweenZero();
    else
      buildBetween(layer, lBound, rBound, betweenOffs);
  }
  void update(int layer, int lBound, int rBound, int betweenOffs, int
      x){
    if (layer >= (int)layers.size())
      return;
    int bSzLog = (layers[layer] + 1) >> 1;
    int bSz = 1 << bSzLog;
    int blockIdx = (x - lBound) >> bSzLog;
    int l = lBound + (blockIdx << bSzLog);
    int r = min(l + bSz, rBound);
    buildBlock(layer, l, r);
    if (layer == 0)
      updateBetweenZero(blockIdx);
    else
      buildBetween(layer, lBound, rBound, betweenOffs);
    update(layer + 1, l, r, betweenOffs, x);
  }
  inline t_sqrt query(int l, int r, int betweenOffs, int base){
    if (l == r)
      return v[l];
    if (l + 1 == r)
      return op(v[l], v[r]);
    int layer = onLayer[clz[(l - base) ^ (r - base)]];
    int bSzLog = (layers[layer] + 1) >> 1;
    int bCntLog = layers[layer] >> 1;
    int lBound = (((l - base) >> layers[layer]) << layers[layer]) +
        base;
    int lBlock = ((l - lBound) >> bSzLog) + 1;
    int rBlock = ((r - lBound) >> bSzLog) - 1;
    t_sqrt ans = suf[layer][l];
    if (lBlock <= rBlock){
      t_sqrt add;
      if (layer == 0)
        add = query(n + lBlock, n + rBlock, (1 << lg) - n, n);
      else
        add = between[layer - 1][betweenOffs + lBound + (lBlock <<
            bCntLog) + rBlock];
      ans = op(ans, add);
    }
    ans = op(ans, pref[layer][r]);
    return ans;
  }
public:
  template <class MyIterator>
  SqrtTree(MyIterator begin, MyIterator end){
    n = end - begin;
    v.resize(n);
    for (int i = 0; i < n; i++, begin++)
      v[i] = (*begin);
    lg = log2Up(n);
    clz.resize(1 << lg);
    onLayer.resize(lg + 1);
    clz[0] = 0;
    for (int i = 1; i < (int)clz.size(); i++)
      clz[i] = clz[i >> 1] + 1;
    int tlg = lg;
    while (tlg > 1){
      onLayer[tlg] = (int)layers.size();
      layers.push_back(tlg);
      tlg = (tlg + 1) >> 1;
    }
    for (int i = lg - 1; i >= 0; i--)
      onLayer[i] = max(onLayer[i], onLayer[i + 1]);
    int betweenLayers = max(0, (int)layers.size() - 1);
    int bSzLog = (lg + 1) >> 1;
    int bSz = 1 << bSzLog;
    indexSz = (n + bSz - 1) >> bSzLog;
    v.resize(n + indexSz);
    pref.assign(layers.size(), vector<t_sqrt>(n + indexSz));
    suf.assign(layers.size(), vector<t_sqrt>(n + indexSz));
    between.assign(betweenLayers, vector<t_sqrt>((1 << lg) + bSz));
    build(0, 0, n, 0);
  }
  //0-indexed
  inline void update(int x, const t_sqrt &item){
    v[x] = item;
    update(0, 0, n, 0, x);
  }
  //0-indexed [l, r]
  inline t_sqrt query(int l, int r){
    return query(l, r, 0, 0);
  }
};
```

1.18 Stack Query

```cpp
#include <bits/stdc++.h>
using namespace std;
struct StackQuery{
  typedef int t_stack;
  stack<pair<t_stack, t_stack>> st;
  t_stack cmp(t_stack a, t_stack b){
    return min(a, b);
  }
  void push(t_stack x){
    t_stack new_value = st.empty() ? x : cmp(x, st.top().second);
    st.push({x, new_value});
  }
  void pop(){
    st.pop();
  }
  t_stack top(){
    return st.top().first;
  }
  t_stack query(){
    return st.top().second;
  }
  t_stack size(){
    return st.size();
  }
};
```

1.19 Treap

```cpp
#include <bits/stdc++.h>
```

```cpp
using namespace std;
class Treap{
private:
  typedef int t_treap;
  struct Node{
    t_treap x, y, size;
    Node *l, *r;
    Node(t_treap _x) : x(_x), y(rand()), size(1), l(NULL), r(NULL) {}
  };
  Node *root;
  int size(Node *t) { return t ? t->size : 0; }
  Node *refresh(Node *t){
    if (!t)
      return t;
    t->size = 1 + size(t->l) + size(t->r);
    return t;
  }
  void split(Node *&t, t_treap k, Node *&a, Node *&b){
    Node *aux;
    if (!t){
      a = b = NULL;
    }else if (t->x < k){
      split(t->r, k, aux, b);
      t->r = aux;
      a = refresh(t);
    }else{
      split(t->l, k, a, aux);
      t->l = aux;
      b = refresh(t);
    }
  }
  Node *merge(Node *a, Node *b){
    if (!a || !b)
      return a ? a : b;
    if (a->y < b->y){
      a->r = merge(a->r, b);
      return refresh(a);
    }else{
      b->l = merge(a, b->l);
      return refresh(b);
    }
  }
  Node *count(Node *t, t_treap k){
    if (!t)
      return NULL;
    else if (k < t->x)
      return count(t->l, k);
    else if (k == t->x)
      return t;
    else
      return count(t->r, k);
  }
  Node *nth(Node *t, int n){
    if (!t)
      return NULL;
    if (n <= size(t->l))
      return nth(t->l, n);
    else if (n == size(t->l) + 1)
      return t;
    else
      return nth(t->r, n - size(t->l) - 1);
  }
  void del(Node *&t){
    if (!t)
      return;
    if (t->l)
      del(t->l);
    if (t->r)
      del(t->r);
    delete t;
    t = NULL;
  }
public:
  Treap() : root(NULL) {}
  ~Treap() { clear(); }
  void clear() { del(root); }
  int size() { return size(root); }
  bool count(t_treap k) { return count(root, k) != NULL; }
  bool insert(t_treap k){
    if (count(k))
      return false;
    Node *a, *b;
    split(root, k, a, b);
    root = merge(merge(a, new Node(k)), b);
    return true;
  }
  bool erase(t_treap k){
    Node *f = count(root, k);
    if (!f)
      return false;
    Node *a, *b, *c, *d;
    split(root, k, a, b);
    split(b, k + 1, c, d);
    root = merge(a, d);
    delete f;
    return true;
  }
  //1-indexed
  t_treap nth(int n){
    Node *ans = nth(root, n);
    return ans ? ans->x : -1;
  }
};
```

1.20 Union Find

```cpp
#include <bits/stdc++.h>
using namespace std;
class UnionFind{
private:
  vector<int> p, w, sz;
public:
  UnionFind(int n){
    w.resize(n + 1, 1);
    sz.resize(n + 1, 1);
    p.resize(n + 1);
    for (int i = 0; i <= n; i++)
      p[i] = i;
  }
  int find(int x){
    if (p[x] == x)
```

```
      return x;
    return p[x] = find(p[x]);
  }
  void join(int x, int y){
    x = find(x);
    y = find(y);
    if (x == y)
      return;
    if (w[x] > w[y])
      swap(x, y);
    p[x] = y;
    sz[y] += sz[x];
    if (w[x] == w[y])
      w[y]++;
  }
  bool isSame(int x, int y){
    return find(x) == find(y);
  }
  int size(int x){
    return sz[find(x)];
  }
};
```

1.21 Wavelet Tree

```
#include <bits/stdc++.h>
using namespace std;
struct WaveletTree{
private:
  typedef int t_wavelet;
  t_wavelet lo, hi;
  WaveletTree *l = nullptr, *r = nullptr;
  vector<t_wavelet> a;
public:
  template <class MyIterator>
  WaveletTree(MyIterator begin, MyIterator end, t_wavelet minX,
      t_wavelet maxX){
    lo = minX, hi = maxX;
    if (lo == hi or begin >= end)
      return;
    t_wavelet mid = (lo + hi - 1) / 2;
    auto f = [mid](int x) {
      return x <= mid;
    };
    a.reserve(end - begin + 2);
    a.push_back(0);
    for (auto it = begin; it != end; it++)
      a.push_back(a.back() + f(*it));
    auto pivot = stable_partition(begin, end, f);
    l = new WaveletTree(begin, pivot, lo, mid);
    r = new WaveletTree(pivot, end, mid + 1, hi);
  }
  inline int b(int i){
    return i - a[i];
  }
  //kth smallest element in range [i, j]
  //1-indexed
  int kth(int i, int j, int k){
    if (i > j)
      return 0;
    if (lo == hi)
      return lo;
    int inLeft = a[j] - a[i - 1];
    int i1 = a[i - 1] + 1, j1 = a[j];
    int i2 = b(i - 1) + 1, j2 = b(j);
    if (k <= inLeft)
      return l->kth(i1, j1, k);
    return r->kth(i2, j2, k - inLeft);
  }
  //Amount of numbers in the range [i, j] Less than or equal to k
  //1-indexed
  int lte(int i, int j, int k){
    if (i > j or k < lo)
      return 0;
    if (hi <= k)
      return j - i + 1;
    int i1 = a[i - 1] + 1, j1 = a[j];
    int i2 = b(i - 1) + 1, j2 = b(j);
    return l->lte(i1, j1, k) + r->lte(i2, j2, k);
  }
  //Amount of numbers in the range [i, j] equal to k
  //1-indexed
  int count(int i, int j, int k){
    if (i > j or k < lo or k > hi)
      return 0;
    if (lo == hi)
      return j - i + 1;
    t_wavelet mid = (lo + hi - 1) / 2;
    int i1 = a[i - 1] + 1, j1 = a[j];
    int i2 = b(i - 1) + 1, j2 = b(j);
    if (k <= mid)
      return l->count(i1, j1, k);
    return r->count(i2, j2, k);
  }
  //swap v[i] with v[i+1]
  //1-indexed
  void swap(int i){
    if (lo == hi or a.size() <= 2)
      return;
    if (a[i - 1] + 1 == a[i] and a[i] + 1 == a[i + 1])
      l->swap(a[i]);
    else if (b(i - 1) + 1 == b(i) and b(i) + 1 == b(i + 1))
      r->swap(b(i));
    else if (a[i - 1] + 1 == a[i])
      a[i]--;
    else
      a[i]++;
  }
  ~WaveletTree(){
    if (l) delete l;
    if (r) delete r;
  }
};
```

2 Graph Algorithms

2.1 2-SAT

```
#include "strongly_connected_component.h"
```

```cpp
using namespace std;
struct SAT{
  typedef pair<int, int> pii;
  vector<pii> edges;
  int n;
  SAT(int size){
    n = 2 * size;
  }
  vector<bool> solve2SAT(){
    vector<bool> vAns(n / 2, false);
    vector<int> comp = SCC::scc(n, edges);
    for (int i = 0; i < n; i += 2){
      if (comp[i] == comp[i + 1])
        return vector<bool>();
      vAns[i / 2] = (comp[i] > comp[i + 1]);
    }
    return vAns;
  }
  int v(int x){
    if (x >= 0)
      return (x << 1);
    x = ~x;
    return (x << 1) ^ 1;
  }
  void add(int a, int b){
    edges.push_back(pii(a, b));
  }
  void addOr(int a, int b){
    add(v(~a), v(b));
    add(v(~b), v(a));
  }
  void addImp(int a, int b){
    addOr(~a, b);
  }
  void addEqual(int a, int b){
    addOr(a, ~b);
    addOr(~a, b);
  }
  void addDiff(int a, int b){
    addEqual(a, ~b);
  }
};
```

2.2 Centroid Decomposition

```cpp
#include <bits/stdc++.h>
using namespace std;
// O(N*log(N))
struct CentroidDecomposition{
  vector<vector<int>> adj;
  vector<int> dad, sub;
  vector<bool> rem;
  int centroidRoot, n;
  void init(int _n){
    n = _n;
    adj.resize(n);
    dad.resize(n);
    sub.resize(n);
    rem.assign(n, false);
  }
  // Return Centroid Decomposition Tree
  vector<vector<int>> build(){
    assert(n > 0);
    centroidRoot = decomp(0, -1);
    vector<vector<int>> ret(n);
    for (int u = 0; u < n; u++){
      if (dad[u] != u)
        ret[dad[u]].push_back(u);
    }
    return ret;
  }
  void addEdge(int a, int b){
    adj[a].push_back(b);
    adj[b].push_back(a);
  }
  int decomp(int u, int p){
    int sz = dfs(u, p);
    int c = centroid(u, p, sz);
    if (p == -1)
      p = c;
    dad[c] = p;
    rem[c] = true;
    for (auto to : adj[c]){
      if (!rem[to])
        decomp(to, c);
    }
    return c;
  }
  int dfs(int u, int p){
    sub[u] = 1;
    for (int to : adj[u]){
      if (!rem[to] and to != p)
        sub[u] += dfs(to, u);
    }
    return sub[u];
  }
  int centroid(int u, int p, int sz){
    for (auto to : adj[u])
      if (!rem[to] and to != p and sub[to] > sz / 2)
        return centroid(to, u, sz);
    return u;
  }
  int operator[](int i){
    return dad[i];
  }
};
```

2.3 Dinic

```cpp
#include <bits/stdc++.h>
using namespace std;
template <typename flow_t>
struct Dinic{
  struct FlowEdge{
    int v, u;
    flow_t cap, flow = 0;
    FlowEdge(int v, int u, flow_t cap) : v(v), u(u), cap(cap) {}
  };
  const flow_t flow_inf = numeric_limits<flow_t>::max();
  vector<FlowEdge> edges;
```

```
  vector<vector<int>> adj;
  int n, m = 0;
  int s, t;
  vector<int> level, ptr;
  queue<int> q;
  bool bfs(){
    while (!q.empty()){
      int v = q.front();
      q.pop();
      for (int id : adj[v]){
        if (edges[id].cap - edges[id].flow < 1)
          continue;
        if (level[edges[id].u] != -1)
          continue;
        level[edges[id].u] = level[v] + 1;
        q.push(edges[id].u);
      }
    }
    return level[t] != -1;
  }
  flow_t dfs(int v, flow_t pushed){
    if (pushed == 0)
      return 0;
    if (v == t)
      return pushed;
    for (int &cid = ptr[v]; cid < (int)adj[v].size(); cid++){
      int id = adj[v][cid];
      int u = edges[id].u;
      if (level[v] + 1 != level[u] || edges[id].cap - edges[id].flow < 1)
        continue;
      flow_t tr = dfs(u, min(pushed, edges[id].cap - edges[id].flow));
      if (tr == 0)
        continue;
      edges[id].flow += tr;
      edges[id ^ 1].flow -= tr;
      return tr;
    }
    return 0;
  }
  Dinic(){}
  void init(int _n){
    n = _n;
    adj.resize(n);
    level.resize(n);
    ptr.resize(n);
  }
  void addEdge(int v, int u, flow_t cap){
    assert(n>0);
    edges.push_back(FlowEdge(v, u, cap));
    edges.push_back(FlowEdge(u, v, 0));
    adj[v].push_back(m);
    adj[u].push_back(m + 1);
    m += 2;
  }
  flow_t maxFlow(int s1, int t1){
    s = s1, t = t1;
    flow_t f = 0;
    for(int i=0; i<m; i++)
      edges[i].flow = 0;
    while (true){
      level.assign(n, -1);
      level[s] = 0;
      q.push(s);
      if (!bfs())
        break;
      ptr.assign(n, 0);
      while (flow_t pushed = dfs(s, flow_inf))
        f += pushed;
    }
    return f;
  }
};
typedef pair<int, int> pii;
vector<pii> recoverCut(Dinic<int> &d){
  vector<int> level(d.n, 0);
  vector<pii> rc;
  queue<int> q;
  q.push(d.s);
  level[d.s] = 1;
  while (!q.empty()){
    int v = q.front();
    q.pop();
    for (int id : d.adj[v]){
      if ((id & 1) == 1)
        continue;
      if (d.edges[id].cap == d.edges[id].flow){
        rc.push_back(pii(d.edges[id].v, d.edges[id].u));
      }else{
        if (level[d.edges[id].u] == 0){
          q.push(d.edges[id].u);
          level[d.edges[id].u] = 1;
        }
      }
    }
  }
  vector<pii> ans;
  for (pii p : rc)
    if ((level[p.first] == 0) or (level[p.second] == 0))
      ans.push_back(p);
  return ans;
}
```

2.4 Flow With Demand

```
#include "dinic.h"
using namespace std;
template <typename flow_t>
struct MaxFlowEdgeDemands{
  Dinic<flow_t> mf;
  vector<flow_t> ind, outd;
  flow_t D;
  int n;
  MaxFlowEdgeDemands(int n) : n(n){
    D = 0;
    mf.init(n + 2);
    ind.assign(n, 0);
    outd.assign(n, 0);
  }
  void addEdge(int a, int b, flow_t cap, flow_t demands){
    mf.addEdge(a, b, cap - demands);
```

```
    D += demands;
    ind[b] += demands;
    outd[a] += demands;
  }
  bool solve(int s, int t){
    mf.addEdge(t, s, numeric_limits<flow_t>::max());
    for (int i = 0; i < n; i++){
      if (ind[i]) mf.addEdge(n, i, ind[i]);
      if (outd[i]) mf.addEdge(i, n + 1, outd[i]);
    }
    return mf.maxFlow(n, n + 1) == D;
  }
};
```

2.5 HLD

```
#include <bits/stdc++.h>
#include "../data_structures/bit_range.h"
using namespace std;
#define F first
#define S second
using hld_t = long long;
using pii = pair<int, hld_t>;
struct HLD{
  vector<vector<pii>> adj;
  vector<int> sz, h, dad, pos;
  vector<hld_t> val, v;
  int t;
  bool edge;
  //Begin Internal Data Structure
  BitRange *bit;
  hld_t neutral = 0;
  inline hld_t join(hld_t a, hld_t b){
    return a+b;
  }
  inline void update(int a, int b, hld_t x){
    bit->add(a+1, b+1, x);
  }
  inline hld_t query(int a, int b){
    return bit->get(a+1, b+1);
  }
  //End Internal Data Structure
  void init(int n){
    dad.resize(n); pos.resize(n); val.resize(n); v.resize(n);
    adj.resize(n); sz.resize(n); h.resize(n);
    bit = new BitRange(n);
  }
  void dfs(int u, int p = -1){
    sz[u] = 1;
    for(pii &to: adj[u]) if(to.F != p){
      if(edge) val[to.F] = to.S;
      dfs(to.F, u);
      sz[u] += sz[to.F];
      if(sz[to.F] > sz[adj[u][0].F] or adj[u][0].F == p)
        swap(to, adj[u][0]);
    }
  }
  void build_hld(int u, int p=-1){
    dad[u] = p;
    pos[u] = t++;
    v[pos[u]] = val[u];
    for(pii to: adj[u]) if(to.F != p){
      h[to.F] = (to == adj[u][0]) ? h[u] : to.F;
      build_hld(to.F, u);
    }
  }
  void addEdge(int a, int b, hld_t w = 0){
    adj[a].emplace_back(b, w);
    adj[b].emplace_back(a, w);
  }
  void build(int root, bool is_edge){
    assert(!adj.empty());
    edge = is_edge;
    t = 0;
    h[root] = 0;
    dfs(root);
    build_hld(root);
    //Init Internal Data Structure
    for(int i=0; i<t; i++)
      update(i, i, v[i]);
  }
  hld_t query_path(int a, int b) {
    if (edge and a == b) return neutral;
    if (pos[a] < pos[b]) swap(a, b);
    if (h[a] == h[b]) return query(pos[b]+edge, pos[a]);
    return join(query(pos[h[a]], pos[a]), query_path(dad[h[a]], b));
  }
  void update_path(int a, int b, hld_t x) {
    if (edge and a == b) return;
    if (pos[a] < pos[b]) swap(a, b);
    if (h[a] == h[b]) return (void)update(pos[b]+edge, pos[a], x);
    update(pos[h[a]], pos[a], x); update_path(dad[h[a]], b, x);
  }
  hld_t query_subtree(int a) {
    if (edge and sz[a] == 1) return neutral;
    return query(pos[a]+edge, pos[a]+sz[a]-1);
  }
  void update_subtree(int a, hld_t x) {
    if (edge and sz[a] == 1) return;
    update(pos[a] + edge, pos[a]+sz[a]-1, x);
  }
  int lca(int a, int b) {
    if (pos[a] < pos[b]) swap(a, b);
    return h[a] == h[b] ? b : lca(dad[h[a]], b);
  }
};
```

2.6 Minimum Cost Maximum Flow

```
#include <bits/stdc++.h>
using namespace std;
template <class T = int>
class MCMF{
private:
  struct Edge{
    int to;
    T cap, cost;
    Edge(int a, T b, T c) : to(a), cap(b), cost(c) {}
  };
  int n;
```

```cpp
  vector<vector<int>> edges;
  vector<Edge> list;
  vector<int> from;
  vector<T> dist, pot;
  vector<bool> visit;
  pair<T, T> augment(int src, int sink){
    pair<T, T> flow = {list[from[sink]].cap, 0};
    for (int v = sink; v != src; v = list[from[v] ^ 1].to){
      flow.first = std::min(flow.first, list[from[v]].cap);
      flow.second += list[from[v]].cost;
    }
    for (int v = sink; v != src; v = list[from[v] ^ 1].to){
      list[from[v]].cap -= flow.first;
      list[from[v] ^ 1].cap += flow.first;
    }
    return flow;
  }
  queue<int> q;
  bool SPFA(int src, int sink){
    T INF = numeric_limits<T>::max();
    dist.assign(n, INF);
    from.assign(n, -1);
    q.push(src);
    dist[src] = 0;
    while (!q.empty()){
      int on = q.front();
      q.pop();
      visit[on] = false;
      for (auto e : edges[on]){
        auto ed = list[e];
        if (ed.cap == 0)
          continue;
        T toDist = dist[on] + ed.cost + pot[on] - pot[ed.to];
        if (toDist < dist[ed.to]){
          dist[ed.to] = toDist;
          from[ed.to] = e;
          if (!visit[ed.to]){
            visit[ed.to] = true;
            q.push(ed.to);
          }
        }
      }
    }
    return dist[sink] < INF;
  }
  void fixPot(){
    T INF = numeric_limits<T>::max();
    for (int i = 0; i < n; i++){
      if (dist[i] < INF)
        pot[i] += dist[i];
    }
  }
public:
  MCMF(int size){
    n = size;
    edges.resize(n);
    pot.assign(n, 0);
    dist.resize(n);
    visit.assign(n, false);
  }
  pair<T, T> solve(int src, int sink){
    pair<T, T> ans(0, 0);
    // Can use dijkstra to speed up depending on the graph
    if (!SPFA(src, sink))
      return ans;
    fixPot();
    // Can use dijkstra to speed up depending on the graph
    while (SPFA(src, sink)){
      auto flow = augment(src, sink);
      ans.first += flow.first;
      ans.second += flow.first * flow.second;
      fixPot();
    }
    return ans;
  }
  void addEdge(int from, int to, T cap, T cost){
    edges[from].push_back(list.size());
    list.push_back(Edge(to, cap, cost));
    edges[to].push_back(list.size());
    list.push_back(Edge(from, 0, -cost));
  }
};
/*bool dij(int src, int sink){
  T INF = numeric_limits<T>::max();
  dist.assign(n, INF);
  from.assign(n, -1);
  visit.assign(n, false);
  dist[src] = 0;
  for(int i = 0; i < n; i++){
    int best = -1;
    for(int j = 0; j < n; j++){
      if(visit[j]) continue;
      if(best == -1 || dist[best] > dist[j]) best = j;
    }
    if(dist[best] >= INF) break;
    visit[best] = true;
    for(auto e : edges[best]){
      auto ed = list[e];
      if(ed.cap == 0) continue;
      T toDist = dist[best] + ed.cost + pot[best] - pot[ed.to];
      assert(toDist >= dist[best]);
      if(toDist < dist[ed.to]){
        dist[ed.to] = toDist;
        from[ed.to] = e;
      }
    }
  }
  return dist[sink] < INF;
}*/
```

2.7 Strongly Connected Component

```cpp
#include "topological_sort.h"
using namespace std;
namespace SCC{
  typedef pair<int, int> pii;
  vector<vector<int>> revAdj;
  vector<int> component;
  void dfs(int u, int c){
    component[u] = c;
    for (int to : revAdj[u]){
```

```
    if (component[to] == -1)
      dfs(to, c);
  }
}
vector<int> scc(int n, vector<pii> &edges){
  revAdj.assign(n, vector<int>());
  for (pii p : edges)
    revAdj[p.second].push_back(p.first);
  vector<int> tp = TopologicalSort::order(n, edges);
  component.assign(n, -1);
  int comp = 0;
  for (int u : tp){
    if (component[u] == -1)
      dfs(u, comp++);
  }
  return component;
}
} // namespace SCC
```

2.8 Topological Sort

```
#include <bits/stdc++.h>
using namespace std;
namespace TopologicalSort{
  typedef pair<int, int> pii;
  vector<vector<int>> adj;
  vector<bool> visited;
  vector<int> vAns;
  void dfs(int u){
    visited[u] = true;
    for (int to : adj[u]){
      if (!visited[to])
        dfs(to);
    }
    vAns.push_back(u);
  }
  vector<int> order(int n, vector<pii> &edges){
    adj.assign(n, vector<int>());
    for (pii p : edges)
      adj[p.first].push_back(p.second);
    visited.assign(n, false);
    vAns.clear();
    for (int i = 0; i < n; i++){
      if (!visited[i])
        dfs(i);
    }
    reverse(vAns.begin(), vAns.end());
    return vAns;
  }
}; // namespace TopologicalSort
```

3 Dynamic Programming

3.1 Divide and Conquer Optimization

Reduces the complexity from $O(n^2k)$ to $O(nk \log n)$ of PD's in the following ways (and other variants):

$$dp[n][k] = \max_{0 \le i < n} (dp[i][k-1] + C[i+1][n]),\ base\ case:\ dp[0][j], dp[i][0] \qquad (1)$$

- $C[i][j]$ = the cost only depends on i and j.
- $opt[n][k] = i$ is the optimal value that maximizes $dp[n][k]$.

It is necessary that opt is increasing along each column: $opt[j][k] \le opt[j+1][k]$.

3.2 Divide and Conquer Optimization Implementation

```
#include <bits/stdc++.h>
using namespace std;
int C(int i, int j);
const int MAXN = 100010;
const int MAXK = 110;
const int INF = 0x3f3f3f3f;
int dp[MAXN][MAXK];
void calculateDP(int l, int r, int k, int opt_l, int opt_r){
  if (l > r)
    return;
  int mid = (l + r) >> 1;
  int ans = -INF, opt = mid;
//  int ans = dp[mid][k-1], opt=mid; //If you accept empty subsegment
  for (int i = opt_l; i <= min(opt_r, mid - 1); i++){
    if (ans < dp[i][k - 1] + C(i + 1, mid)){
      opt = i;
      ans = dp[i][k - 1] + C(i + 1, mid);
    }
  }
  dp[mid][k] = ans;
  calculateDP(l, mid - 1, k, opt_l, opt);
  calculateDP(mid + 1, r, k, opt, opt_r);
}
int solve(int n, int k){
  for (int i = 0; i <= n; i++)
    dp[i][0] = -INF;
  for (int j = 0; j <= k; j++)
    dp[0][j] = -INF;
  dp[0][0] = 0;
  for (int j = 1; j <= k; j++)
    calculateDP(1, n, j, 0, n - 1);
  return dp[n][k];
}
```

3.3 Knuth Optimization

Reduces the complexity from $O(n^3)$ to $O(n^2)$ of PD's in the following ways (and other variants):

$$dp[i][j] = C[i][j] + \min_{i<k<j}(dp[i][k] + dp[k][j]),\ caso\ base:\ dp[i][i] \tag{2}$$

$$dp[i][j] = \min_{i<k<j}(dp[i][k] + C[i][k]),\ caso\ base:\ dp[i][i] \tag{3}$$

- $C[i][j]$ = the cost only depends on i and j.
- $opt[i][j] = k$ is the optimal value that maximizes $dp[i][j]$.

The following conditions must be met:

- Foursquare inequality on C: $C[a][c] + C[b][d] \leq C[a][d] + C[b][c]$, $a \leq b \leq c \leq d$.
- Monotonicity on C: $C[b][c] \leq C[a][d]$, $a \leq b \leq c \leq d$.

Or the following condition:

- opt increasing in rows and columns: $opt[i][j-1] \leq opt[i][j] \leq opt[i+1][j]$.

3.4 Knuth Optimization Implementation

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int MAXN = 1009;
const ll INFLL = 0x3f3f3f3f3f3f3f3f;
ll C(int a, int b);
ll dp[MAXN][MAXN];
int opt[MAXN][MAXN];
ll knuth(int n){
  for (int i = 0; i < n; i++){
    dp[i][i] = 0;
    opt[i][i] = i;
  }
  for (int s = 1; s < n; s++){
    for (int i = 0, j; (i + s) < n; i++){
      j = i + s;
      dp[i][j] = INFLL;
      for (int k = opt[i][j - 1]; k < min(j, opt[i + 1][j] + 1); k++){
        ll cur = dp[i][k] + dp[k + 1][j] + C(i, j);
        if (dp[i][j] > cur){
          dp[i][j] = cur;
          opt[i][j] = k;
        }
      }
    }
  }
  return dp[0][n - 1];
}
```

4 Math

4.1 Basic Math

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef unsigned long long ull;

ull fastPow(ull base, ull exp, ull mod){
  base %= mod;
  //exp %= phi(mod) if base and mod are relatively prime
  ull ans = 1LL;
  while (exp > 0){
    if (exp & 1LL)
      ans = (ans * (__int128_t)base) % mod;
    base = (base * (__int128_t)base) % mod;
    exp >>= 1;
  }
  return ans;
}
ll gcd(ll a, ll b){ return __gcd(a, b); }
ll lcm(ll a, ll b){ return (a / gcd(a, b)) * b; }
void enumeratingAllSubmasks(int mask){
  for (int s = mask; s; s = (s - 1) & mask)
    cout << s << endl;
}
//MOD to Hash
namespace ModHash{
  const uint64_t MOD = (1ll<<61) - 1;
  uint64_t modmul(uint64_t a, uint64_t b){
    uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (uint32_t)b, h2 = b
        >>32;
    uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
    uint64_t ret = (l&MOD) + (l>>61) + (h << 3) + (m >> 29) + ((m <<
        35) >> 3) + 1;
    ret = (ret & MOD) + (ret>>61);
    ret = (ret & MOD) + (ret>>61);
    return ret-1;
  }
};
```

4.2 BigInt

```
#include <bits/stdc++.h>
using namespace std;
typedef int32_t intB;
typedef int64_t longB;
typedef vector<intB> vib;
class BigInt{
private:
  vib vb;
  bool neg;
  const int BASE_DIGIT = 9;
  const intB base = 1000000LL*1000;//000LL*1000000LL;
  void fromString(string &s){
    if(s[0] == '-'){
```

```
      neg = true;
      s = s.substr(1);
    }else{
      neg = false;
    }
    vb.clear();
    vb.reserve((s.size()+BASE_DIGIT-1)/BASE_DIGIT);
    for(int i=(int)s.length(); i>0; i-=BASE_DIGIT){
      if(i < BASE_DIGIT)
        vb.push_back(stol(s.substr(0, i)));
      else
        vb.push_back(stol(s.substr(i-BASE_DIGIT, BASE_DIGIT)));
    }
    fix(vb);
  }
  void fix(vib &v){
    while(v.size()>1 && v.back()==0)
      v.pop_back();
    if(v.size() == 0)
      neg = false;
  }
  bool comp(vib &a, vib &b){
    fix(a); fix(b);
    if(a.size() != b.size()) return a.size() < b.size();
    for(int i=(int)a.size()-1; i>=0; i--) {
      if(a[i] != b[i]) return a[i] < b[i];
    }
    return false;
  }
  vib sum(vib a, vib b){
    int carry = 0;
    for(size_t i=0; i<max(a.size(), b.size()) or carry; i++){
      if(i == a.size())
        a.push_back(0);
      a[i] += carry + (i<b.size() ? b[i] : 0);
      carry = (a[i] >= base);
      if(carry) a[i] -= base;
    }
    fix(a);
    return a;
  }
  vib sub(vib a, vib b){
    int carry = 0;
    for(size_t i=0; i<b.size() or carry; i++){
      a[i] -= carry + (i<b.size() ? b[i] : 0);
      carry = a[i] < 0;
      if(carry) a[i] += base;
    }
    fix(a);
    return a;
  }

public:
  BigInt(){}
  BigInt(intB n){
    neg = (n<0);
    vb.push_back(abs(n));
    fix(vb);
  }
  BigInt(string s){
    fromString(s);
  }
  BigInt operator =(BigInt oth){
    this->neg = oth.neg;
    this->vb = oth.vb;
    return *this;
  }
  BigInt operator +(BigInt &oth){
    vib &a = vb, &b = oth.vb;
    BigInt ans;
    if(neg == oth.neg){
      ans.vb = sum(vb, oth.vb);
      ans.neg = neg;
    }else{
      if(comp(a, b)){
        ans.vb = sub(b, a);
        ans.neg = oth.neg;
      }else{
        ans.vb = sub(a, b);
        ans.neg = neg;
      }
    }
    return ans;
  }
  BigInt operator -(BigInt oth){
    oth.neg ^= true;
    return (*this) + oth;
  }
  BigInt operator *(intB b){
    bool negB = false;
    if(b < 0){
      negB = true;
      b = -b;
    }
    BigInt ans = *this;
    auto &a = ans.vb;
    intB carry = 0;
    for(size_t i=0; i<a.size() or carry; i++){
      if(i == a.size()) a.push_back(0);
      longB cur = carry + a[i] *(longB) b;
      a[i] = intB(cur%base);
      carry = intB(cur/base);
    }
    ans.neg ^= negB;
    fix(ans.vb);
    return ans;
  }
  BigInt operator *(BigInt &oth){
    BigInt ans;
    auto a = vb, &b = oth.vb, &c = ans.vb;
    c.assign(a.size() + b.size(), 0);
    for(size_t i=0; i<a.size(); i++){
      intB carry=0;
      for(size_t j=0; j<b.size() or carry; j++){
        longB cur = c[i+j] + a[i]*(longB)(j<b.size() ? b[j] : 0);
        cur += carry;
        c[i+j] = intB(cur%base);
        carry = intB(cur/base);
      }
    }
    ans.neg = neg^oth.neg;
    fix(ans.vb);
```

```
  return ans;
}
BigInt operator /(intB b){
  bool negB = false;
  if(b < 0){
    negB = true;
    b = -b;
  }
  BigInt ans = *this;
  auto &a = ans.vb;
  intB carry = 0;
  for(int i=(int)a.size()-1; i>=0; i--){
    longB cur = a[i] + (longB)carry * base;
    a[i] = intB(cur/b);
    carry = intB(cur%b);
  }
  ans.neg ^= negB;
  fix(ans.vb);
  return ans;
}
void shiftL(int b){
  vb.resize(vb.size() + b);
  for(int i=(int)vb.size()-1; i>=0; i--) {
    if(i>=b) vb[i] = vb[i-b];
    else vb[i] = 0;
  }
  fix(vb);
}
void shiftR(int b) {
  if((int)vb.size() <= b){
    vb.clear();
    vb.push_back(0);
    return;
  }
  for(int i=0; i<((int)vb.size() - b); i++)
    vb[i] = vb[i+b];
  vb.resize((int)vb.size() - b);
  fix(vb);
}
void divide(BigInt a, BigInt b, BigInt &q, BigInt &r){
  BigInt z(0), p(1);
  while(b < a) {
    p.shiftL(max(1, int(a.vb.size()-b.vb.size())));
    b.shiftL(max(1, int(a.vb.size()-b.vb.size())));
  }
  while(true) {
    while ((a < b) && (z < p)) {
      p = p/10;
      b = b/10;
    }
    if(!(z < p)) break;
    a = a - b;
    q = q + p;
  }
  r = a;
}
BigInt operator /(BigInt &oth){
  BigInt q, r;
  divide(*this, oth, q, r);
  return q;
}
  BigInt operator %(BigInt &oth){
    BigInt q, r;
    divide(*this, oth, q, r);
    return r;
  }
  bool operator <(BigInt &oth){
    BigInt ans = (*this) - oth;
    return ans.neg;
  }
  bool operator ==(BigInt &oth){
    BigInt ans = (*this) - oth;
    return (ans.vb.size()==1) and (ans.vb.back()==0);
  }
  friend ostream &operator<<(ostream &out, const BigInt &D){
    if(D.neg)
      out << '-';
    out << (D.vb.empty() ? 0 : D.vb.back());
    for(int i=(int)D.vb.size()-2; i>=0; i--)
      out << setfill('0') << setw(D.BASE_DIGIT) << D.vb[i];
    return out;
  }
  string to_string(){
    std::stringstream ss;
    ss << (*this);
    return ss.str();
  }
  friend istream &operator>>(istream  &input, BigInt &D) {
    string s;
    input >> s;
    D.fromString(s);
    return input;
  }
};
```

4.3 Binomial Coefficients

```
#include <bits/stdc++.h>
#include "./basic_math.h"
#include "./modular.h"
using namespace std;
typedef long long ll;
//O(k)
ll C1(int n, int k){
  ll res = 1LL;
  for (int i = 1; i <= k; ++i)
    res = (res * (n - k + i)) / i;
  return res;
}
//O(n^2)
vector<vector<ll>> C2(int maxn, int mod){
  vector<vector<ll>> mat(maxn + 1, vector<ll>(maxn + 1, 0));
  mat[0][0] = 1;
  for (int n = 1; n <= maxn; n++){
    mat[n][0] = mat[n][n] = 1;
    for (int k = 1; k < n; k++)
      mat[n][k] = (mat[n - 1][k - 1] + mat[n - 1][k]) % mod;
  }
  return mat;
}
//O(N)
```

```
vector<int> factorial, inv_factorial;
void prevC3(int maxn, int mod){
  factorial.resize(maxn + 1);
  factorial[0] = 1;
  for (int i = 1; i <= maxn; i++)
    factorial[i] = (factorial[i - 1] * 1LL * i) % mod;
  inv_factorial.resize(maxn + 1);
  inv_factorial[maxn] = fastPow(factorial[maxn], mod - 2, mod);
  for (int i = maxn - 1; i >= 0; i--)
    inv_factorial[i] = (inv_factorial[i + 1] * 1LL * (i + 1)) % mod;
}
int C3(int n, int k, int mod){
  if (n < k)
    return 0;
  return (((factorial[n] * 1LL * inv_factorial[k]) % mod) * 1LL *
      inv_factorial[n - k]) % mod;
}
//O(P*log(P))
//C4(n, k, p) = Comb(n, k)%p
vector<int> changeBase(int n, int p){
  vector<int> v;
  while (n > 0){
    v.push_back(n % p);
    n /= p;
  }
  return v;
}
int C4(int n, int k, int p){
  auto vn = changeBase(n, p);
  auto vk = changeBase(k, p);
  int mx = max(vn.size(), vk.size());
  vn.resize(mx, 0);
  vk.resize(mx, 0);
  prevC3(p - 1, p);
  int ans = 1;
  for (int i = 0; i < mx; i++)
    ans = (ans * 1LL * C3(vn[i], vk[i], p)) % p;
  return ans;
}
//O(P^k)
//C5(n, k, p, pk) = Comb(n, k)%(p^k)
int fat_p(ll n, int p, int pk){
  vector<int> fat1(pk, 1);
    int res = 1;
    for(int i=1; i<pk; i++){
    if(i%p == 0)
      fat1[i] = fat1[i-1];
    else
      fat1[i] = (fat1[i-1]*1LL*i)%pk;
  }
  while(n > 1){
    res = (res*1LL*fastPow(fat1[pk-1], n/pk, pk))%pk;
    res = (res*1LL*fat1[n%pk])%pk;
    n /= p;
  }
  return res;
}
ll cnt(ll n, int p){
  ll ans = 0;
  while(n > 1){
    ans += n/p;
    n/=p;
  }
  return ans;
}
int C5(ll n, ll k, int p, int pk){
  ll exp = cnt(n, p) - cnt(n-k, p) - cnt(k, p);
  int d = (fat_p(n-k, p, pk)*1LL*fat_p(k, p, pk))%pk;
  int ans = (fat_p(n, p, pk)*1LL*inv(d, pk))%pk;
  return (ans*1LL*fastPow(p, exp, pk))%pk;
}
```

4.4 Chinese Remainder Theorem

```
#include <bits/stdc++.h>
#include "extended_euclidean.h"
using namespace std;
typedef long long ll;
namespace CRT{
  inline ll normalize(ll x, ll mod){
    x %= mod;
    if (x < 0)
      x += mod;
    return x;
  }
  ll solve(vector<ll> a, vector<ll> m){
    int n = a.size();
    for (int i = 0; i < n; i++)
      normalize(a[i], m[i]);
    ll ans = a[0];
    ll lcm1 = m[0];
    for (int i = 1; i < n; i++){
      ll x, y;
      ll g = extGcd(lcm1, m[i], x, y);
      if ((a[i] - ans) % g != 0)
        return -1;
      ans = normalize(ans + ((((a[i] - ans) / g) * x) % (m[i] / g)) *
          lcm1, (lcm1 / g) * m[i]);
      lcm1 = (lcm1 / g) * m[i]; //lcm(lcm1, m[i]);
    }
    return ans;
  }
} // namespace CRT
```

4.5 Euler's totient

```
#include <bits/stdc++.h>
using namespace std;
int nthPhi(int n){
  int result = n;
  for (int i = 2; i <= n / i; i++){
    if (n % i == 0){
      while (n % i == 0)
        n /= i;
      result -= result / i;
    }
  }
  if (n > 1)
    result -= result / n;
```

```
  return result;
}
vector<int> phiFrom1toN(int n){
  vector<int> vPhi(n + 1);
  vPhi[0] = 0;
  vPhi[1] = 1;
  for (int i = 2; i <= n; i++)
    vPhi[i] = i;
  for (int i = 2; i <= n; i++){
    if (vPhi[i] == i){
      for (int j = i; j <= n; j += i)
        vPhi[j] -= vPhi[j] / i;
    }
  }
  return vPhi;
}
```

4.6 Extended Euclidean

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
ll extGcd(ll a, ll b, ll &x, ll &y){
  if (b == 0){
    x = 1, y = 0;
    return a;
  }else{
    ll g = extGcd(b, a % b, y, x);
    y -= (a / b) * x;
    return g;
  }
}
//a*x + b*y = g
//a*(x-(b/g)*k) + b*(y+(a/g)*k) = g
bool dioEq(ll a, ll b, ll c, ll &x0, ll &y0, ll &g){
  g = extGcd(abs(a), abs(b), x0, y0);
  if (c % g) return false;
  x0 *= c / g;
  y0 *= c / g;
  if (a < 0) x0 = -x0;
  if (b < 0) y0 = -y0;
  return true;
}
inline void shift_solution(ll &x, ll &y, ll a, ll b, ll cnt){
  x += cnt * b;
  y -= cnt * a;
}
ll findAllSolutions(ll a, ll b, ll c, ll minx, ll maxx, ll miny, ll
    maxy){
  ll x, y, g;
  if(a==0 or b==0){
    if(a==0 and b==0)
      return (c==0)*(maxx-minx+1)*(maxy-miny+1);
    if(a == 0)
      return (c%b == 0)*(maxx-minx+1)*(miny<=c/b and c/b<=maxy);
    return (c%a == 0)*(minx<=c/a and c/a<=maxx)*(maxy-miny+1);
  }
  if (!dioEq(a, b, c, x, y, g))
    return 0;
  a /= g;
  b /= g;
  int sign_a = a > 0 ? +1 : -1;
  int sign_b = b > 0 ? +1 : -1;
  shift_solution(x, y, a, b, (minx - x) / b);
  if (x < minx)
    shift_solution(x, y, a, b, sign_b);
  if (x > maxx)
    return 0;
  ll lx1 = x;
  shift_solution(x, y, a, b, (maxx - x) / b);
  if (x > maxx)
    shift_solution(x, y, a, b, -sign_b);
  ll rx1 = x;
  shift_solution(x, y, a, b, -(miny - y) / a);
  if (y < miny)
    shift_solution(x, y, a, b, -sign_a);
  if (y > maxy)
    return 0;
  ll lx2 = x;
  shift_solution(x, y, a, b, -(maxy - y) / a);
  if (y > maxy)
    shift_solution(x, y, a, b, sign_a);
  ll rx2 = x;
  if (lx2 > rx2)
    swap(lx2, rx2);
  ll lx = max(lx1, lx2);
  ll rx = min(rx1, rx2);
  if (lx > rx)
    return 0;
  return (rx - lx) / abs(b) + 1;
}
```

4.7 Gray Code

```
int grayCode(int nth){
  return nth ^ (nth >> 1);
}
int revGrayCode(int g){
  int nth = 0;
  for (; g > 0; g >>= 1)
    nth ^= g;
  return nth;
}
```

4.8 Matrix

```
#include <bits/stdc++.h>
#include "modular.h"
using namespace std;
const int D = 3;
struct Matrix{
  int m[D][D];
  Matrix(bool identify = false){
    memset(m, 0, sizeof(m));
    for (int i = 0; i < D; i++)
      m[i][i] = identify;
  }
  Matrix(vector<vector<int>> mat){
```

```
    for(int i=0; i<D; i++)
      for(int j=0; j<D; j++)
        m[i][j] = mat[i][j];
  }
  int * operator[](int pos){
    return m[pos];
  }
  Matrix operator*(Matrix oth){
    Matrix ans;
    for (int i = 0; i < D; i++){
      for (int j = 0; j < D; j++){
        int &sum = ans[i][j];
        for (int k = 0; k < D; k++)
          sum = modSum(sum, modMul(m[i][k], oth[k][j]));
      }
    }
    return ans;
  }
};
```

4.9 Modular Arithmetic

```
#include <bits/stdc++.h>
#include "extended_euclidean.h"
using namespace std;
const int MOD = 1000000007;
inline int modSum(int a, int b, int mod = MOD){
  int ans = a+b;
  if(ans > mod) ans -= mod;
  return ans;
}
inline int modSub(int a, int b, int mod = MOD){
  int ans = a-b;
  if(ans < 0) ans += mod;
  return ans;
}
inline int modMul(int a, int b, int mod = MOD){
  return (a*1LL*b)%mod;
}
int inv(int a, int mod=MOD){
  ll inv_x, y;
  extGcd(a, mod, inv_x, y);
  return (inv_x%mod + mod)%mod;
}
int modDiv(int a, int b, int mod = MOD){
  return modMul(a, inv(b, mod));
}
```

4.10 Montgomery Multiplication

```
#include <bits/stdc++.h>
using namespace std;
using u64 = uint64_t;
using u128 = __uint128_t;
using i128 = __int128_t;
struct u256{
  u128 high, low;
  static u256 mult(u128 x, u128 y){
    u64 a = x >> 64, b = x;
    u64 c = y >> 64, d = y;
    u128 ac = (u128)a * c;
    u128 ad = (u128)a * d;
    u128 bc = (u128)b * c;
    u128 bd = (u128)b * d;
    u128 carry = (u128)(u64)ad + (u128)(u64)bc + (bd >> 64u);
    u128 high = ac + (ad >> 64u) + (bc >> 64u) + (carry >> 64u);
    u128 low = (ad << 64u) + (bc << 64u) + bd;
    return {high, low};
  }
};
//x_m := x*r mod n
struct Montgomery{
  u128 mod, inv, r2;
  //the N will be an odd number
  Montgomery(u128 n) : mod(n), inv(1), r2(-n % n){
    for (int i = 0; i < 7; i++)
      inv *= 2 - n * inv;
    for (int i = 0; i < 4; i++){
      r2 <<= 1;
      if (r2 >= mod)
        r2 -= mod;
    }
    for (int i = 0; i < 5; i++)
      r2 = mult(r2, r2);
  }
  u128 init(u128 x){
    return mult(x, r2);
  }
  u128 reduce(u256 x){
    u128 q = x.low * inv;
    i128 a = x.high - u256::mult(q, mod).high;
    if (a < 0)
      a += mod;
    return a;
  }
  u128 mult(u128 a, u128 b){
    return reduce(u256::mult(a, b));
  }
};
```

4.11 Prime Number

```
#include <bits/stdc++.h>
#include "basic_math.h"
using namespace std;
typedef unsigned long long ull;
ull modMul(ull a, ull b, ull mod){
  return (a * (__uint128_t)b) % mod;
}
bool checkComposite(ull n, ull a, ull d, int s){
  ull x = fastPow(a, d, n);
  if (x == 1 or x == n - 1)
    return false;
  for (int r = 1; r < s; r++){
    x = modMul(x, x, n);
    if (x == n - 1LL)
      return false;
  }
```

```
    return true;
  };
  bool millerRabin(ull n){
    if (n < 2)
      return false;
    int r = 0;
    ull d = n - 1LL;
    while ((d & 1LL) == 0){
      d >>= 1;
      r++;
    }
    for (ull a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}){
      if (n == a)
        return true;
      if (checkComposite(n, a, d, r))
        return false;
    }
    return true;
  }
  ull pollard(ull n){
    auto f = [n](ull x) { return modMul(x, x, n) + 1; };
    ull x = 0, y = 0, t = 0, prd = 2, i = 1, q;
    while (t++ % 40 || __gcd(prd, n) == 1){
      if (x == y)
        x = ++i, y = f(x);
      if ((q = modMul(prd, max(x, y) - min(x, y), n)))
        prd = q;
      x = f(x), y = f(f(y));
    }
    return __gcd(prd, n);
  }
  vector<ull> factor(ull n){
    if (n == 1)
      return {};
    if (millerRabin(n))
      return {n};
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), r.begin(), r.end());
    return l;
  }
```

5 Geometry

6 String Algorithms

6.1 Min Cyclic String

```
#include <bits/stdc++.h>
using namespace std;
string min_cyclic_string(string s){
  s += s;
  int n = s.size();
  int i = 0, ans = 0;
  while (i < n / 2){
    ans = i;
    int j = i + 1, k = i;
    while (j < n && s[k] <= s[j]){
      if (s[k] < s[j])
        k = i;
      else
        k++;
      j++;
    }
    while (i <= k)
      i += j - k;
  }
  return s.substr(ans, n / 2);
}
```

6.2 Suffix Automaton

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
struct SuffixAutomaton{
  struct state{
    int len, link, first_pos;
    bool is_clone = false;
    map<char, int> next;
  };
  vector<state> st;
  int sz, last;
  SuffixAutomaton(string s){
    st.resize(2 * s.size() + 10);
    st[0].len = 0;
    st[0].link = -1;
    st[0].is_clone = false;
    sz = 1;
    last = 0;
    for (char c : s)
      insert(c);
    preCompute();
  }
  void insert(char c){
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    st[cur].first_pos = st[cur].len - 1;
    st[cur].is_clone = false;
    int p = last;
    while (p != -1 && !st[p].next.count(c)){
      st[p].next[c] = cur;
      p = st[p].link;
    }
    if (p == -1){
      st[cur].link = 0;
    }else{
      int q = st[p].next[c];
      if (st[p].len + 1 == st[q].len){
        st[cur].link = q;
      }else{
        int clone = sz++;
        st[clone].len = st[p].len + 1;
        st[clone].next = st[q].next;
        st[clone].link = st[q].link;
        st[clone].first_pos = st[q].first_pos;
        st[clone].is_clone = true;
```

```
      while (p != -1 && st[p].next[c] == q){
        st[p].next[c] = clone;
        p = st[p].link;
      }
      st[q].link = st[cur].link = clone;
    }
  }
  last = cur;
}
string lcs(string s){
  int v = 0, l = 0, best = 0, bestpos = 0;
  for (int i = 0; i < (int)s.size(); i++){
    while (v and !st[v].next.count(s[i])){
      v = st[v].link;
      l = st[v].len;
    }
    if (st[v].next.count(s[i])){
      v = st[v].next[s[i]];
      l++;
    }
    if (l > best){
      best = l;
      bestpos = i;
    }
  }
  return s.substr(bestpos - best + 1, best);
}
vector<ll> dp;
vector<int> cnt;
ll dfsPre(int s){
  if (dp[s] != -1)
    return dp[s];
  dp[s] = cnt[s]; //Accepts repeated substrings
  //dp[s] = 1; //Does not accept repeated substrings
  for (auto p : st[s].next)
    dp[s] += dfsPre(p.second);
  return dp[s];
}
void preCompute(){
  cnt.assign(sz, 0);
  vector<pair<int, int>> v(sz);
  for (int i = 0; i < sz; i++){
    cnt[i] = !st[i].is_clone;
    v[i] = make_pair(st[i].len, i);
  }
  sort(v.begin(), v.end(), greater<pair<int, int>>());
  for (int i = 0; i < sz - 1; i++)
    cnt[st[v[i].second].link] += cnt[v[i].second];
  dp.assign(sz, -1);
  dfsPre(0);
}
};
```

7 Miscellaneous

7.1 Longest Increasing Subsequence

```
#include <bits/stdc++.h>
using namespace std;
vector<int> lis(vector<int> &v){
  vector<int> st, ans;
  vector<int> pos(v.size()+1), dad(v.size()+1);
  for(int i=0; i < (int)v.size(); i++){
    auto it = lower_bound(st.begin(), st.end(), v[i]); // Do not
        accept repeated values
    //auto it = upper_bound(st.begin(), st.end(), v[i]); //Accept
        repeated values
    int p = it-st.begin();
    if(it==st.end())
      st.push_back(v[i]);
    else
      *it = v[i];
    pos[p] = i;
    dad[i] = (p==0)? -1 : pos[p-1];
  }
  int p = pos[st.size() - 1];
  while(p >= 0){
    ans.push_back(v[p]);
    p=dad[p];
  }
  reverse(ans.begin(), ans.end());
  return ans;
}
```

7.2 Mo Algorithm

```
#include <bits/stdc++.h>
using namespace std;
const int BLOCK_SIZE = 700;
void remove(int idx);
void add(int idx);
void clearAnswer();
int getAnswer();
struct Query{
  int l, r, idx;
  bool operator<(Query other) const{
    if (l / BLOCK_SIZE != other.l / BLOCK_SIZE)
      return l < other.l;
    return (l / BLOCK_SIZE & 1) ? (r < other.r) : (r > other.r);
  }
};
vector<int> mo_s_algorithm(vector<Query> queries){
  vector<int> answers(queries.size());
  sort(queries.begin(), queries.end());
  clearAnswer();
  int L = 0, R = 0;
  add(0);
  for(Query q : queries){
    while(q.l < L) add(--L);
    while(R < q.r) add(++R);
    while(L < q.l) remove(L++);
    while(q.r < R) remove(R--);
    answers[q.idx] = getAnswer();
  }
  return answers;
}
```

8 Theorems and Formulas

8.1 Binomial Coefficients

$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{k}a^{n-k}b^k + \cdots + \binom{n}{n}b^n$

Pascal's Triangle: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

Symmetry rule: $\binom{n}{k} = \binom{n}{n-k}$

Factoring in: $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$

Sum over k: $\sum_{k=0}^{n}\binom{n}{k} = 2^n$

Sum over n: $\sum_{m=0}^{n}\binom{m}{k} = \binom{n+1}{k+1}$

Sum over n and k: $\sum_{k=0}^{m}\binom{n+k}{k} = \binom{n+m+1}{m}$

Sum of the squares: $\binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$

Weighted sum: $1\binom{n}{1} + 2\binom{n}{2} + \cdots + n\binom{n}{n} = n2^{n-1}$

Connection with the Fibonacci numbers: $\binom{n}{0} + \binom{n-1}{1} + \cdots + \binom{n-k}{k} + \cdots + \binom{0}{n} = F_{n+1}$

More formulas: $\sum_{k=0}^{m}(-1)^k \cdot \binom{n}{k} = (-1)^m \cdot \binom{n-1}{m}$

8.2 Catalan Number

Recursive formula: $C_0 = C_1 = 1$

$C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}, n \geq 2$

Analytical formula: $C_n = \binom{2n}{n} - \binom{2n}{n-1} = \frac{1}{n+1}\binom{2n}{n}, n \geq 0$

The first few numbers Catalan numbers, C_n (starting from zero): $1, 1, 2, 5, 14, 42, 132, 429, 1430, \ldots$

The Catalan number C_n is the solution for:

- Number of correct bracket sequence consisting of n opening and n closing brackets.
- The number of rooted full binary trees with $n+1$ leaves (vertices are not numbered). A rooted binary tree is full if every vertex has either two children or no children.
- The number of ways to completely parenthesize $n+1$ factors.
- The number of triangulations of a convex polygon with $n+2$ sides (i.e. the number of partitions of polygon into disjoint triangles by using the diagonals).
- The number of ways to connect the $2n$ points on a circle to form n disjoint chords.
- The number of non-isomorphic full binary trees with n internal nodes (i.e. nodes having at least one son).
- The number of monotonic lattice paths from point $(0,0)$ to point (n,n) in a square lattice of size $n \times n$, which do not pass above the main diagonal (i.e. connecting $(0,0)$ to (n,n)).
- Number of permutations of length n that can be stack sorted (i.e. it can be shown that the rearrangement is stack sorted if and only if there is no such index $i < j < k$, such that $a_k < a_i < a_j$).
- The number of non-crossing partitions of a set of n elements.
- The number of ways to cover the ladder $1 \ldots n$ using n rectangles (The ladder consists of n columns, where i^{th} column has a height i).

8.3 Euler's Totient

If p is a prime number: $\phi(p) = p - 1$ and $\phi(p^k) = p^k - p^{k-1}$

If a and b are relatively prime, then: $\phi(ab) = \phi(a) \cdot \phi(b)$

In general: $\phi(ab) = \phi(a) \cdot \phi(b) \cdot \dfrac{gcd(a,b)}{\phi(gcd(a,b))}$

This interesting property was established by Gauss: $\sum_{d|n}\phi(d) = n$, Here the sum is over all positive divisors d of n.

Euler's theorem: $a^{\phi(m)} \equiv 1 \pmod{m}$, if a and m are relatively prime.

Generalization: $a^n \equiv a^{\phi(m)+[n \bmod \phi(m)]} \bmod m$, for arbitrary a, m and n $\geq log_2(m)$.

8.4 Formulas

Count the number of ways to partition a set of n labelled objects into k nonempty labelled subsets.

$$f(n,k) = \sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n$$

Stirling Number 2nd: Partitions of an n element set into k not-empty set. Or count the number of ways to partition a set of n labelled objects into k nonempty unlabelled subsets.

$$S_{2nd}(n,k) = \left\{ {n \atop k} \right\} = \frac{1}{k!}\sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n$$

Euler's formula: $f = e - v + 2$

Number of regions in a planar graph: $R = E - V + C + 1$ where C is the number of connected components

Given a and b co-prime, $n = a \cdot x + b \cdot y$ where $x \geq 0$ and $y \geq 0$. You are required to find the least value of n, such that all currency values greater than or equal to n can be made using any number of coins of denomination a and b: $n = (a-1) * (b-1)$

generalization of the above problem, n is multiple of $gcd(a,b)$: $n = lcm(a,b) - a - b + gcd(a,b)$

8.5 Manhattan Distance

Transformation of the manhattan distance to 2 dimensions between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$:

$|x_1 - x_2| + |y_1 - y_2| = max(|A_1 - B_1|, |A_2 - B_2|)$ where $A = (x_1 + y_1, x_1 - y_1)$ e $B = (x_2 + y_2, x_2 - y_2)$

Transformation of the manhattan distance to 3 dimensions between $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$:

$|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2| = max(|A_1 - B_1|, |A_2 - B_2|, |A_3 - B_3|, |A_4 - B_4|)$ where $A = (x_1 + y_1 + z_1, x_1 + y_1 - z_1, x_1 - y_1 + z_1, -x_1 + y_1 + z_1)$ e $B = (x_2 + y_2 + z_2, x_2 + y_2 - z_2, x_2 - y_2 + z_2, -x_2 + y_2 + z_2)$

Transformation of the manhattan distance to D dimensions between P_1 and P_2:

$isSet(i, x) = 1$ if the i-th bit is setted in x and 0 otherwise.

$A[i] = \sum_{j=0}^{d-1} (-1)^{isSet(j,i)} P_1[j]$

$B[i] = \sum_{j=0}^{d-1} (-1)^{isSet(j,i)} P_2[j]$

$$\sum_{i=0}^{d-1} |P_1[i] - P_2[i]| = \max_{i=0}^{2^d-1} |A_i - B_i|$$

8.6 Primes

If $n = p_1^{e_1} \cdot p_2^{e_2} \cdots p_k^{e_k}$, then:

Number of divisors is $d(n) = (e_1 + 1) \cdot (e_2 + 1) \cdots (e_k + 1)$.

Sum of divisors is $\sigma(n) = \frac{p_1^{e_1+1} - 1}{p_1 - 1} \cdot \frac{p_2^{e_2+1} - 1}{p_2 - 1} \cdots \frac{p_k^{e_k+1} - 1}{p_k - 1}$