

## Contents

<b>1</b>	<b>Data Structures</b>	<b>2</b>			
1.1	BIT	2	4.1	Basic Math	30
1.2	BIT 2D	2	4.2	BigInt	31
1.3	BIT In Range	2	4.3	Binomial Coefficients	33
1.4	Dynamic Median	3	4.4	Chinese Remainder Theorem	33
1.5	Implicit Treap	3	4.5	Determinant	34
1.6	LiChao Tree	4	4.6	Division Trick	34
1.7	MergeSort Tree	4	4.7	Euler's totient	34
1.8	Policy Based Tree	5	4.8	Extended Euclidean	34
1.9	Queue Query	5	4.9	Fraction	35
1.10	Randomized Heap	5	4.10	FFT	36
1.11	Range Color	6	4.11	Floyd Cycle Finding	37
1.12	RMQ	6	4.12	Function Root Using Newton	37
1.13	Segment Tree	7	4.13	Gauss	37
1.14	Segment Tree 2D	7	4.14	Gauss Xor	38
1.15	Segment Tree Iterative	8	4.15	Gray Code	38
1.16	Segment Tree Lazy	8	4.16	Matrix	38
1.17	Segment Tree Persistent	9	4.17	Modular Arithmetic	39
1.18	Sparse Table	10	4.18	Modular Integer	39
1.19	SQRT Decomposition	10	4.19	Montgomery Multiplication	39
1.20	SQRT Tree	10	4.20	NTT	40
1.21	Stack Query	12	4.21	Prime Number	41
1.22	Treap	12	4.22	Rank Matrix	41
1.23	Union Find	13	4.23	Simpson Integration	42
1.24	Union Find With Rollback	13	4.24	Sieve And Primes	42
1.25	Wavelet Tree	13			
<b>2</b>	<b>Graph Algorithms</b>	<b>14</b>	<b>5</b>	<b>Geometry</b>	<b>43</b>
2.1	2-SAT	14	5.1	Basic Geometry	43
2.2	Arborescence	15	5.2	Circle Area Union	46
2.3	Articulation Point	15	5.3	Circles to Tree	47
2.4	BFS 0-1	16	5.4	Count Lattices	48
2.5	Bridge	16	5.5	Concave Polygon	48
2.6	Centroid	17	5.6	Convex Hull	49
2.7	Centroid Decomposition	17	5.7	Convex Hull Trick	49
2.8	Checking Bipartiteness Online	18	5.8	Convex Polygon	49
2.9	Dinic	18	5.9	Nearest Pair Of Points	50
2.10	Edmond's Blossoms	19	5.10	Point 3D	50
2.11	Eulerian Path	20			
2.12	Flow With Demand	20	<b>6</b>	<b>String Algorithms</b>	<b>52</b>
2.13	Floyd Warshall	20	6.1	Aho Corasick	52
2.14	Hungarian	21	6.2	KMP	53
2.15	Prim	21	6.3	Manacher	53
2.16	HLD	22	6.4	Min Cyclic String	53
2.17	Kuhn	22	6.5	Palindromic Tree	54
2.18	Kruskal	23	6.6	String Hashing	54
2.19	LCA	23	6.7	Suffix Automaton	55
2.20	Link-Cut Tree	24	6.8	Suffix Array	56
2.21	Link-Cut Tree - Edge	25	6.9	Suffix Tree	56
2.22	Link-Cut Tree - Vertex	26	6.10	Trie	57
2.23	Min-Cut	27	6.11	Z Function	58
2.24	Minimum Cost Maximum Flow	27			
2.25	Strongly Connected Component	28	<b>7</b>	<b>Miscellaneous</b>	<b>58</b>
2.26	Topological Sort	28	7.1	Counting Inversions	58
2.27	Tree ID	28	7.2	Histogram	58
2.28	Vertex Cover In Tree	29	7.3	Identify Pattern	58
			7.4	Kadane 1D and 2D	59
<b>3</b>	<b>Dynamic Programming</b>	<b>29</b>	7.5	Longest Increasing Subsequence	59
3.1	Divide and Conquer Optimization	29	7.6	Mo Algorithm	59
3.2	Divide and Conquer Optimization Implementation	29	7.7	Polyominoes	60
3.3	Knuth Optimization	30	7.8	Scheduling Jobs	60
3.4	Knuth Optimization Implementation	30	7.9	Sprague Grundy	60
<b>4</b>	<b>Math</b>	<b>30</b>	<b>8</b>	<b>Theorems and Formulas</b>	<b>61</b>
			8.1	Binomial Coefficients	61
			8.2	Catalan Number	61
			8.3	Euler's Totient	61
			8.4	Formulas	61
			8.5	Graph	62
			8.6	Manhattan Distance	62
			8.7	Primes	62

# 1 Data Structures

## 1.1 BIT

```
#include <bits/stdc++.h>
using namespace std;
class Bit{
private:
    typedef long long t_bit;
    int nBit;
    int nLog;
    vector<t_bit> bit;
public:
    Bit(int n){
        nBit = n;
        nLog = 20;
        bit.resize(nBit + 1, 0);
    }
    //1-indexed
    t_bit get(int i){
        t_bit s = 0;
        for (; i > 0; i -= (i & -i))
            s += bit[i];
        return s;
    }
    //1-indexed [l, r]
    t_bit get(int l, int r){
        return get(r) - get(l - 1);
    }
    //1-indexed
    void add(int i, t_bit value){
        assert(i > 0);
        for (; i <= nBit; i += (i & -i))
            bit[i] += value;
    }
    t_bit lower_bound(t_bit value){
        t_bit sum = 0;
        int pos = 0;
        for (int i = nLog; i >= 0; i--){
            if ((pos + (1 << i) <= nBit) and (sum + bit[pos + (1 << i)] <
                value)){
                sum += bit[pos + (1 << i)];
                pos += (1 << i);
            }
        }
        return pos + 1;
    }
};
```

## 1.2 BIT 2D

```
#include <bits/stdc++.h>
using namespace std;
class Bit2d{
private:
    typedef long long t_bit;
    vector<vector<t_bit>> bit;
    int nBit, mBit;
public:
```

```
Bit2d(int n, int m){
    nBit = n;
    mBit = m;
    bit.resize(nBit + 1, vector<t_bit>(mBit + 1, 0));
}
//1-indexed
t_bit get(int i, int j){
    t_bit sum = 0;
    for (int a = i; a > 0; a -= (a & -a))
        for (int b = j; b > 0; b -= (b & -b))
            sum += bit[a][b];
    return sum;
}
//1-indexed
t_bit get(int a1, int b1, int a2, int b2){
    return get(a2, b2) - get(a2, b1 - 1) - get(a1 - 1, b2) + get(a1 -
        1, b1 - 1);
}
//1-indexed [i, j]
void add(int i, int j, t_bit value){
    for (int a = i; a <= nBit; a += (a & -a))
        for (int b = j; b <= mBit; b += (b & -b))
            bit[a][b] += value;
}
};
```

## 1.3 BIT In Range

```
#include <bits/stdc++.h>
using namespace std;
class BitRange{
private:
    typedef long long t_bit;
    vector<t_bit> bit1, bit2;
    t_bit get(vector<t_bit> &bit, int i){
        t_bit sum = 0;
        for (; i > 0; i -= (i & -i))
            sum += bit[i];
        return sum;
    }
    void add(vector<t_bit> &bit, int i, t_bit value){
        for (; i < (int)bit.size(); i += (i & -i))
            bit[i] += value;
    }
public:
    BitRange(int n){
        bit1.assign(n + 1, 0);
        bit2.assign(n + 1, 0);
    }
    //1-indexed [i, j]
    void add(int i, int j, t_bit v){
        add(bit1, i, v);
        add(bit1, j + 1, -v);
        add(bit2, i, v * (i - 1));
        add(bit2, j + 1, -v * j);
    }
    //1-indexed
    t_bit get(int i){
        return get(bit1, i) * i - get(bit2, i);
    }
};
```

```
//1-indexed [i,j]
t_bit get(int i, int j){
    return get(j) - get(i - 1);
}
};
```

## 1.4 Dynamic Median

```
#include <bits/stdc++.h>
using namespace std;
class DinamicMedian{
    typedef int t_median;
private:
    priority_queue<t_median> mn;
    priority_queue<t_median, vector<t_median>, greater<t_median>> mx;
public:
    double median(){
        if (mn.size() > mx.size())
            return mn.top();
        else
            return (mn.top() + mx.top()) / 2.0;
    }
    void push(t_median x){
        if (mn.size() <= mx.size())
            mn.push(x);
        else
            mx.push(x);
        if ((!mx.empty()) and (!mn.empty())){
            while (mn.top() > mx.top()){
                t_median a = mx.top();
                mx.pop();
                t_median b = mn.top();
                mn.pop();
                mx.push(b);
                mn.push(a);
            }
        }
    }
};
```

## 1.5 Implicit Treap

```
#include <bits/stdc++.h>
using namespace std;
namespace ITreap{
    const int N = 500010;
    typedef long long treap_t;
    treap_t X[N];
    int en = 1, Y[N], sz[N], L[N], R[N], root;

    const treap_t neutral = 0;
    treap_t op_val[N];
    bool rev[N];
    inline treap_t join(treap_t a, treap_t b, treap_t c){
        return a + b + c;
    }
    void calc(int u) { // update node given children info
        sz[u] = sz[L[u]] + 1 + sz[R[u]];
```

```
// code here, no recursion
    op_val[u] = join(op_val[L[u]], X[u], op_val[R[u]]);
}
void unlaze(int u) {
    if(!u) return;
    // code here, no recursion
    if (rev[u]){
        if(L[u])
            rev[L[u]] ^= rev[u];
        if(R[u])
            rev[R[u]] ^= rev[u];
        swap(L[u], R[u]);
        rev[u] = false;
    }
}
void split(int u, int s, int &l, int &r) { // l gets first s, r gets
    remaining
    unlaze(u);
    if(!u) return (void) (l = r = 0);
    if(sz[L[u]] < s) { split(R[u], s - sz[L[u]] - 1, l, r); R[u] = l;
        l = u; }
    else { split(L[u], s, l, r); L[u] = r; r = u; }
    calc(u);
}
int merge(int l, int r) { // els on l <= els on r
    unlaze(l); unlaze(r);
    if(!l || !r) return l + r;
    int u;
    if(Y[l] > Y[r]) { R[l] = merge(R[l], r); u = l; }
    else { L[r] = merge(l, L[r]); u = r; }
    calc(u);
    return u;
}
int new_node(treap_t x){
    X[en] = x;
    op_val[en] = x;
    rev[en] = false;
    return en++;
}
int nth(int u, int idx){
    if(!u)
        return 0;
    unlaze(u);
    if(idx <= sz[L[u]])
        return nth(L[u], idx);
    else if(idx == sz[L[u]] + 1)
        return u;
    else
        return nth(R[u], idx - sz[L[u]] - 1);
}
//Public
void init(int n=N-1) { // call before using other funcs
    //init position 0
    sz[0] = 0;
    op_val[0] = neutral;
    //init Treap
    root = 0;
    for(int i = en = 1; i <= n; i++) { Y[i] = i; sz[i] = 1; L[i] = R[i]
        ] = 0; }
    random_shuffle(Y + 1, Y + n + 1);
}
```

```

//0-indexed
void insert(int idx, int val){
    int a, b;
    split(root, idx, a, b);
    root = merge(merge(a, new_node(val)), b);
}
//0-indexed
void erase(int idx){
    int a, b, c, d;
    split(root, idx, a, b);
    split(b, 1, c, d);
    root = merge(a, d);
}
//0-indexed
treap_t nth(int idx){
    int u = nth(root, idx+1);
    return X[u];
}
//0-indexed [l, r]
treap_t query(int l, int r){
    if(l > r) swap(l, r);
    int a, b, c, d;
    split(root, l, a, d);
    split(d, r - l + 1, b, c);
    treap_t ans = op_val[b];
    root = merge(a, merge(b, c));
    return ans;
}
//0-indexed [l, r]
void reverse(int l, int r){
    if (l > r) swap(l, r);
    int a, b, c, d;
    split(root, l, a, d);
    split(d, r - l + 1, b, c);
    if(b)
        rev[b] ^= 1;
    root = merge(a, merge(b, c));
}
};

```

## 1.6 LiChao Tree

```

#include <bits/stdc++.h>
using namespace std;
const int INF = 0x3f3f3f3f;
class LiChaoTree{
private:
    typedef int t_line;
    struct Line{
        t_line k, b;
        Line() {}
        Line(t_line k, t_line b) : k(k), b(b) {}
    };
    int n_tree, min_x, max_x;
    vector<Line> li_tree;
    t_line f(Line l, int x){
        return l.k * x + l.b;
    }
    void add(Line nw, int v, int l, int r){
        int m = (l + r) / 2;

```

```

        bool lef = f(nw, l) > f(li_tree[v], l);
        bool mid = f(nw, m) > f(li_tree[v], m);
        if (mid)
            swap(li_tree[v], nw);
        if (r - l == 1)
            return;
        else if (lef != mid)
            add(nw, 2 * v, l, m);
        else
            add(nw, 2 * v + 1, m, r);
    }
    int get(int x, int v, int l, int r){
        int m = (l + r) / 2;
        if (r - l == 1)
            return f(li_tree[v], x);
        else if (x < m)
            return max(f(li_tree[v], x), get(x, 2 * v, l, m));
        else
            return max(f(li_tree[v], x), get(x, 2 * v + 1, m, r));
    }
public:
    LiChaoTree(int mn_x, int mx_x){
        min_x = mn_x;
        max_x = mx_x;
        n_tree = max_x - min_x + 5;
        li_tree.resize(4 * n_tree, Line(0, -INF));
    }
    void add(t_line k, t_line b){
        add(Line(k, b), 1, min_x, max_x);
    }
    t_line get(int x){
        return get(x, 1, min_x, max_x);
    }
};

```

## 1.7 MergeSort Tree

```

#include <bits/stdc++.h>
#define all(x) x.begin(),x.end()
using namespace std;
class MergeSortTree{
private:
    typedef vector<int> Node;
    Node neutral;
    vector<Node> st;
    int n;
    inline void join(Node &a, Node &b, Node &ans){
        ans.resize(a.size() + b.size());
        merge(all(a), all(b), ans.begin());
    }
    inline int szEq(int node, int k){
        return upper_bound(all(st[node]), k) - lower_bound(all(st[node]), k);
    }
    inline int szLt(int node, int k){
        return lower_bound(all(st[node]), k) - st[node].begin();
    }
public:
    template <class MyIterator>
    MergeSortTree(MyIterator begin, MyIterator end){

```

```

    int sz = end - begin;
    for (n = 1; n < sz; n <= 1);
    st.assign(n < 1, neutral);
    for (int i = 0; i < sz; i++, begin++)
        st[i + n].assign(1, *begin);
    for (int i = n - 1; i; i--){
        int l = (i < 1);
        join(st[l], st[l+1], st[i]);
    }
}
// 0-indexed
// Counts the number of elements less than k in the range [L..R]
int lt(int l, int r, int k){
    int ans = 0;
    for (l += n, r += n + 1; l < r; l >= 1, r >= 1){
        if (l & 1)
            ans += szLt(l++, k);
        if (r & 1)
            ans += szLt(--r, k);
    }
    return ans;
}
// 0-indexed
// Counts the number of elements equal to k in the range [L..R]
int eq(int l, int r, int k){
    int ans = 0;
    for (l += n, r += n + 1; l < r; l >= 1, r >= 1){
        if (l & 1)
            ans += szEq(l++, k);
        if (r & 1)
            ans += szEq(--r, k);
    }
    return ans;
}
};

```

## 1.8 Policy Based Tree

```

#include <bits/stdc++.h>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
using namespace std;
typedef tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> OrderedSet;
typedef tree<int, int, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> OrderedMap;
//order_of_key(k) : Number of items strictly smaller than k .
//find_by_order(k) : K-th element in a set (counting from zero).

```

## 1.9 Queue Query

```

#include <bits/stdc++.h>
using namespace std;
class QueueQuery{
private:
    typedef long long t_queue;
    stack<pair<t_queue, t_queue>> s1, s2;

```

```

    t_queue cmp(t_queue a, t_queue b){
        return min(a, b);
    }
    void move(){
        if (s2.empty()){
            while (!s1.empty()){
                t_queue element = s1.top().first;
                s1.pop();
                t_queue result = s2.empty() ? element : cmp(element, s2.top().second);
                s2.push({element, result});
            }
        }
    }
public:
    void push(t_queue x){
        t_queue result = s1.empty() ? x : cmp(x, s1.top().second);
        s1.push({x, result});
    }
    void pop(){
        move();
        s2.pop();
    }
    t_queue front(){
        move();
        return s2.top().first;
    }
    t_queue query(){
        if (s1.empty() || s2.empty())
            return s1.empty() ? s2.top().second : s1.top().second;
        else
            return cmp(s1.top().second, s2.top().second);
    }
    t_queue size(){
        return s1.size() + s2.size();
    }
};

```

## 1.10 Randomized Heap

```

#include <bits/stdc++.h>
using namespace std;

typedef int f_type;
struct Node{
    f_type value;
    Node *l, *r;
    Node(f_type x = 0): value(x){
        l = r = nullptr;
    }
};

inline bool heapMin(f_type a, f_type b){
    return a > b;
}

inline bool heapMax(f_type a, f_type b){
    return a < b;
}

struct RandomizedHeap{
    Node *root;
    int sz;

```

```

RandomizedHeap() {
    srand(time(NULL));
    root = nullptr;
    sz = 0;
}
void rdFree(Node *n) {
    if(n == nullptr) return;
    rdFree(n->l); rdFree(n->r);
    delete n;
}
~RandomizedHeap() {
    rdFree(root);
}
Node* merge(Node *t1, Node *t2) {
    if(!t1 || !t2)
        return t1 ? t1 : t2;
    if(heapMin(t1->value, t2->value))
        swap(t1, t2);
    if(rand() & 1)
        swap(t1->l, t1->r);
    t1->l = merge(t1->l, t2);
    return t1;
}
//Can be performed in O(logn) on average.
void merge(RandomizedHeap &oth) {
    root = merge(root, oth.root);
    sz += oth.sz;
    oth.root = nullptr;
}
int top() {
    return (root != nullptr) ? root->value : 0;
}
void pop() {
    if(root == nullptr) return;
    Node *l = root->l;
    Node *r = root->r;
    delete root;
    root = merge(l, r);
    sz--;
}
void push(int x) {
    Node *nw = new Node(x);
    root = merge(root, nw);
    sz++;
}
int size() {
    return sz;
}
};

```

## 1.11 Range Color

```

#include <bits/stdc++.h>
using namespace std;
class RangeColor {
private:
    typedef long long ll;
    struct Node {
        ll l, r;
        int color;
    };

```

```

    Node() {}
    Node(ll l1, ll r1, int color1) : l(l1), r(r1), color(color1) {}
    bool operator<(const Node &oth) const {
        return r < oth.r;
    }
};
std::set<Node> st;
vector<ll> ans;
public:
    RangeColor(ll first, ll last, int maxColor) {
        ans.resize(maxColor + 1);
        ans[0] = last - first + 1LL;
        st.insert(Node(first, last, 0));
    }
    //get color in position x
    int get(ll x) {
        auto p = st.upper_bound(Node(0, x - 1LL, -1));
        return p->color;
    }
    //set newColor in [a, b]
    void set(ll a, ll b, int newColor) {
        auto p = st.upper_bound(Node(0, a - 1LL, -1));
        assert(p != st.end());
        ll l = p->l;
        ll r = p->r;
        int oldColor = p->color;
        ans[oldColor] -= (r - l + 1LL);
        p = st.erase(p);
        if (l < a) {
            ans[oldColor] += (a - l);
            st.insert(Node(l, a - 1LL, oldColor));
        }
        if (b < r) {
            ans[oldColor] += (r - b);
            st.insert(Node(b + 1LL, r, oldColor));
        }
        while ((p != st.end()) and (p->l <= b)) {
            l = p->l;
            r = p->r;
            oldColor = p->color;
            ans[oldColor] -= (r - l + 1LL);
            if (b < r) {
                ans[oldColor] += (r - b);
                st.erase(p);
                st.insert(Node(b + 1LL, r, oldColor));
                break;
            } else {
                p = st.erase(p);
            }
        }
        ans[newColor] += (b - a + 1LL);
        st.insert(Node(a, b, newColor));
    }
    ll countColor(int x) {
        return ans[x];
    }
};

```

## 1.12 RMQ

```
#include <bits/stdc++.h>
using namespace std;
// Source: https://github.com/brunomaletta/Biblioteca
template<typename T> struct RMQ{
    vector<T> v;
    int n; static const int b = 30;
    vector<int> mask, t;
    int op(int x, int y) { return v[x] < v[y] ? x : y; }
    int msb(int x) { return __builtin_clz(1)-__builtin_clz(x); }
    int small(int r, int sz = b) { return r-msb(mask[r]&((1<<sz)-1)); }
    RMQ(const vector<T>& v_) : v(v_), n(v.size()), mask(n), t(n) {
        for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
            at = (at<<1)&((1<<b)-1);
            while (at and op(i, i-msb(at&-at)) == i) at ^= at&-at;
        }
        for (int i = 0; i < n/b; i++) t[i] = small(b*i+b-1);
        for (int j = 1; (1<<j) <= n/b; j++) for (int i = 0; i+(1<<j) <= n/
            b; i++)
            t[n/b*j+i] = op(t[n/b*(j-1)+i], t[n/b*(j-1)+i+(1<<(j-1))]);
    }
    T query(int l, int r) {
        if (r-l+1 <= b) return v[small(r, r-l+1)];
        int ans = op(small(l+b-1), small(r));
        int x = l/b+1, y = r/b-1;
        if (x <= y) {
            int j = msb(y-x+1);
            ans = op(ans, op(t[n/b*j+x], t[n/b*j+y-(1<<j)+1]));
        }
        return v[ans];
    }
    // return ans; // for get position
};
```

## 1.13 Segment Tree

```
#include <bits/stdc++.h>
using namespace std;
class SegTree{
private:
    typedef long long Node;
    Node neutral = 0;
    vector<Node> st;
    vector<int> v;
    int n;
    Node join(Node a, Node b){
        return (a + b);
    }
    void build(int node, int i, int j){
        if (i == j){
            st[node] = v[i];
            return;
        }
        int m = (i + j) / 2;
        int l = (node << 1);
        int r = l + 1;
        build(l, i, m);
        build(r, m + 1, j);
        st[node] = join(st[l], st[r]);
    }
    Node query(int node, int i, int j, int a, int b){
```

```
        if ((i > b) or (j < a))
            return neutral;
        if ((a <= i) and (j <= b))
            return st[node];
        int m = (i + j) / 2;
        int l = (node << 1);
        int r = l + 1;
        return join(query(l, i, m, a, b), query(r, m + 1, j, a, b));
    }
    void update(int node, int i, int j, int idx, Node value){
        if (i == j){
            st[node] = value;
            return;
        }
        int m = (i + j) / 2;
        int l = (node << 1);
        int r = l + 1;
        if (idx <= m)
            update(l, i, m, idx, value);
        else
            update(r, m + 1, j, idx, value);
        st[node] = join(st[l], st[r]);
    }
public:
    template <class MyIterator>
    SegTree(MyIterator begin, MyIterator end){
        n = end - begin;
        v = vector<int>(begin, end);
        st.resize(4 * n + 5);
        build(1, 0, n - 1);
    }
    //0-indexed [a, b]
    Node query(int a, int b){
        return query(1, 0, n - 1, a, b);
    }
    //0-indexed
    void update(int idx, int value){
        update(1, 0, n - 1, idx, value);
    }
};
```

## 1.14 Segment Tree 2D

```
#include <bits/stdc++.h>
using namespace std;
struct SegTree2D{
private:
    int n, m;
    typedef int Node;
    Node neutral = -0x3f3f3f3f;
    vector<vector<Node>> seg;
    Node join(Node a, Node b){
        return max(a, b);
    }
public:
    SegTree2D(int n1, int m1){
        n = n1, m = m1;
        seg.assign(2 * n, vector<Node>(2 * m, 0));
    }
    void update(int x, int y, int val){
```

```

assert(0 <= x && x < n && 0 <= y && y < m);
x += n, y += m;
seg[x][y] = val;
for (int j = y / 2; j > 0; j /= 2)
    seg[x][j] = join(seg[x][2 * j], seg[x][2 * j + 1]);
for (x /= 2; x > 0; x /= 2){
    seg[x][y] = join(seg[2 * x][y], seg[2 * x + 1][y]);
    for (int j = y / 2; j > 0; j /= 2){
        seg[x][j] = join(seg[x][2 * j], seg[x][2 * j + 1]);
    }
}
}
vector<int> getCover(int l, int r, int N){
    l = std::max(0, l);
    r = std::min(N, r);
    vector<int> ans;
    for (l += N, r += N; l < r; l /= 2, r /= 2){
        if (l & 1)
            ans.push_back(l++);
        if (r & 1)
            ans.push_back(--r);
    }
    return ans;
}
Node query(int x1, int y1, int x2, int y2){
    auto c1 = getCover(x1, x2 + 1, n);
    auto c2 = getCover(y1, y2 + 1, m);
    Node ans = neutral;
    for (auto i : c1){
        for (auto j : c2){
            ans = join(ans, seg[i][j]);
        }
    }
    return ans;
}
};

```

## 1.15 Segment Tree Iterative

```

#include <bits/stdc++.h>
using namespace std;
class SegTreeIterative{
private:
    typedef long long Node;
    Node neutral = 0;
    vector<Node> st;
    int n;
    inline Node join(Node a, Node b){
        return a + b;
    }
public:
    template <class MyIterator>
    SegTreeIterative(MyIterator begin, MyIterator end){
        int sz = end - begin;
        for (n = 1; n < sz; n <= 1);
        st.assign(n < 1, neutral);
        for (int i = 0; i < sz; i++, begin++)
            st[i + n] = (*begin);
        for (int i = n - 1; i; i--){
            st[i] = join(st[(i < 1)], st[(i < 1) + 1]);
        }
    }
};

```

```

}
}
//0-indexed
void update(int i, Node x){
    st[i += n] = x;
    for (i >= 1; i; i >= 1)
        st[i] = join(st[i < 1], st[(i < 1) + 1]);
}
//0-indexed [l, r]
Node query(int l, int r){
    Node ansL = neutral, ansR = neutral;
    for (l += n, r += n + 1; l < r; l >= 1, r >= 1){
        if (l & 1)
            ansL = join(ansL, st[l++]);
        if (r & 1)
            ansR = join(st[--r], ansR);
    }
    return join(ansL, ansR);
}
Node lower_bound(int k){
    int no=1, l=0, r=n-1;
    while(l<r){
        int mid = (l+r)>>1;
        int lo = no<<1;
        if(st[lo] >= k){
            no = lo;
            r = mid;
        }else{
            k -= st[lo];
            no = lo + 1;
            l = mid + 1;
        }
    }
    if(st[no] >= k)
        return l;
    else
        return -1;
}
};

```

## 1.16 Segment Tree Lazy

```

#include <bits/stdc++.h>
using namespace std;
class SegTreeLazy{
private:
    typedef long long Node;
    vector<Node> st;
    vector<long long> lazy;
    vector<int> v;
    int n;
    Node neutral = 0;
    inline Node join(Node a, Node b){
        return a + b;
    }
    inline void upLazy(int &node, int &i, int &j){
        if (lazy[node] != 0){
            st[node] += lazy[node] * (j - i + 1);
            //st[node] += lazy[node];
            if (i != j){

```



```

        lazy[(node << 1)] += lazy[node];
        lazy[(node << 1) + 1] += lazy[node];
    }
    lazy[node] = 0;
}

void build(int node, int i, int j){
    if (i == j){
        st[node] = v[i];
        return;
    }
    int m = (i + j) / 2;
    int l = (node << 1);
    int r = l + 1;
    build(l, i, m);
    build(r, m + 1, j);
    st[node] = join(st[l], st[r]);
}

Node query(int node, int i, int j, int a, int b){
    upLazy(node, i, j);
    if ((i > b) or (j < a))
        return neutral;
    if ((a <= i) and (j <= b)){
        return st[node];
    }
    int m = (i + j) / 2;
    int l = (node << 1);
    int r = l + 1;
    return join(query(l, i, m, a, b), query(r, m + 1, j, a, b));
}

void update(int node, int i, int j, int a, int b, int value){
    upLazy(node, i, j);
    if ((i > j) or (i > b) or (j < a))
        return;
    if ((a <= i) and (j <= b)){
        lazy[node] = value;
        upLazy(node, i, j);
    } else {
        int m = (i + j) / 2;
        int l = (node << 1);
        int r = l + 1;
        update(l, i, m, a, b, value);
        update(r, m + 1, j, a, b, value);
        st[node] = join(st[l], st[r]);
    }
}

public:
template <class MyIterator>
SegTreeLazy(MyIterator begin, MyIterator end){
    n = end - begin;
    v = vector<int>(begin, end);
    st.resize(4 * n + 5);
    lazy.assign(4 * n + 5, 0);
    build(1, 0, n - 1);
}

//0-indexed [a, b]
Node query(int a, int b){
    return query(1, 0, n - 1, a, b);
}

//0-indexed [a, b]
void update(int a, int b, int value){

```

```

        update(1, 0, n - 1, a, b, value);
    }
};

```

## 1.17 Segment Tree Persistent

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 3e4 + 10, UPD = 2e5 + 10, LOG = 20;
const int MAXS = 4 * MAX + UPD * LOG;
namespace PerSegTree{
    typedef long long pst_t;
    pst_t seg[MAXS];
    int T[UPD], L[MAXS], R[MAXS], cnt, t;
    int n, *v;
    pst_t neutral = 0;
    pst_t join(pst_t a, pst_t b){
        return a + b;
    }
    pst_t build(int p, int l, int r){
        if (l == r)
            return seg[p] = v[l];
        L[p] = cnt++, R[p] = cnt++;
        int m = (l + r) / 2;
        return seg[p] = join(build(L[p], l, m), build(R[p], m + 1, r));
    }
    pst_t query(int a, int b, int p, int l, int r){
        if (b < l or r < a)
            return 0;
        if (a <= l and r <= b)
            return seg[p];
        int m = (l + r) / 2;
        return join(query(a, b, L[p], l, m), query(a, b, R[p], m + 1, r));
    }
    pst_t update(int a, int x, int lp, int p, int l, int r){
        if (l == r)
            return seg[p] = x;
        int m = (l + r) / 2;
        if (a <= m)
            return seg[p] = join(update(a, x, L[lp], L[p] = cnt++, l, m),
                                   seg[R[p] = R[lp]]);
        return seg[p] = join(seg[L[p] = L[lp]], update(a, x, R[lp], R[p] =
                                                         cnt++, m + 1, r));
    }
}

//Public:
//O(n)
void build(int n2, int *v2){
    n = n2, v = v2;
    T[0] = cnt++;
    build(0, 0, n - 1);
}

//O(log(n))
pst_t query(int a, int b, int tt){
    return query(a, b, T[tt], 0, n - 1);
}

//O(log(n))
//update: v[idx] = x;
int update(int idx, int x, int tt = t){
    update(idx, x, T[tt], T[tt] = cnt++, 0, n - 1);
    return t;
}

```

```

    }
}; // namespace perseg

```

## 1.18 Sparse Table

```

#include <bits/stdc++.h>
using namespace std;
class SparseTable{
private:
    typedef int t_st;
    vector<vector<t_st>> st;
    vector<int> log2;
    t_st neutral = 0x3f3f3f3f;
    int nLog;
    t_st join(t_st a, t_st b){
        return min(a, b);
    }
public:
    template <class MyIterator>
    SparseTable(MyIterator begin, MyIterator end){
        int n = end - begin;
        nLog = 20;
        log2.resize(n + 1);
        log2[1] = 0;
        for (int i = 2; i <= n; i++)
            log2[i] = log2[i / 2] + 1;
        st.resize(n, vector<t_st>(nLog, neutral));
        for (int i = 0; i < n; i++, begin++){
            st[i][0] = (*begin);
            for (int j = 1; j < nLog; j++)
                for (int i = 0; (i + (1 << (j - 1))) < n; i++)
                    st[i][j] = join(st[i][j - 1], st[i + (1 << (j - 1))][j - 1]);
        }
        //0-indexed [a, b]
        t_st query(int a, int b){
            int d = b - a + 1;
            t_st ans = neutral;
            for (int j = nLog - 1; j >= 0; j--){
                if (d & (1 << j)){
                    ans = join(ans, st[a][j]);
                    a = a + (1 << (j));
                }
            }
            return ans;
        }
        //0-indexed [a, b]
        t_st queryRMQ(int a, int b){
            int j = log2[b - a + 1];
            return join(st[a][j], st[b - (1 << j) + 1][j]);
        }
    };

```

## 1.19 Sqrt Decomposition

```

#include <bits/stdc++.h>
using namespace std;
struct SqrtDecomposition{
    typedef long long t_sqrt;

```

```

    int sqrtLen;
    vector<t_sqrt> block;
    vector<t_sqrt> v;
    template <class MyIterator>
    SqrtDecomposition(MyIterator begin, MyIterator end){
        int n = end - begin;
        sqrtLen = (int)sqrt(n + .0) + 1;
        v.resize(n);
        block.resize(sqrtLen + 5);
        for (int i = 0; i < n; i++, begin++){
            v[i] = (*begin);
            block[i / sqrtLen] += v[i];
        }
    }
    //0-indexed
    void update(int idx, t_sqrt new_value){
        t_sqrt d = new_value - v[idx];
        v[idx] += d;
        block[idx / sqrtLen] += d;
    }
    //0-indexed [l, r]
    t_sqrt query(int l, int r){
        t_sqrt sum = 0;
        int c_l = l / sqrtLen, c_r = r / sqrtLen;
        if (c_l == c_r){
            for (int i = l; i <= r; i++)
                sum += v[i];
        }else{
            for (int i = l, end = (c_l + 1) * sqrtLen - 1; i <= end; i++)
                sum += v[i];
            for (int i = c_l + 1; i <= c_r - 1; i++)
                sum += block[i];
            for (int i = c_r * sqrtLen; i <= r; i++)
                sum += v[i];
        }
        return sum;
    }
};

```

## 1.20 Sqrt Tree

```

#include <bits/stdc++.h>
using namespace std;
class SqrtTree{
private:
    typedef long long t_sqrt;
    t_sqrt op(const t_sqrt &a, const t_sqrt &b){
        return a | b;
    }
    inline int log2Up(int n){
        int res = 0;
        while ((1 << res) < n)
            res++;
        return res;
    }
    int n, lg, indexSz;
    vector<t_sqrt> v;
    vector<int> clz, layers, onLayer;
    vector<vector<t_sqrt>> pref, suf, between;
    inline void buildBlock(int layer, int l, int r){

```

```

    pref[layer][l] = v[l];
    for (int i = l + 1; i < r; i++)
        pref[layer][i] = op(pref[layer][i - 1], v[i]);
    suf[layer][r - 1] = v[r - 1];
    for (int i = r - 2; i >= l; i--)
        suf[layer][i] = op(v[i], suf[layer][i + 1]);
}
inline void buildBetween(int layer, int lBound, int rBound, int
    betweenOffs){
    int bSzLog = (layers[layer] + 1) >> 1;
    int bCntLog = layers[layer] >> 1;
    int bSz = 1 << bSzLog;
    int bCnt = (rBound - lBound + bSz - 1) >> bSzLog;
    for (int i = 0; i < bCnt; i++){
        t_sqrt ans;
        for (int j = i; j < bCnt; j++){
            t_sqrt add = suf[layer][lBound + (j << bSzLog)];
            ans = (i == j) ? add : op(ans, add);
            between[layer - 1][betweenOffs + lBound + (i << bCntLog) + j]
                = ans;
        }
    }
}
inline void buildBetweenZero(){
    int bSzLog = (lg + 1) >> 1;
    for (int i = 0; i < indexSz; i++){
        v[n + i] = suf[0][i << bSzLog];
    }
    build(1, n, n + indexSz, (1 << lg) - n);
}
inline void updateBetweenZero(int bid){
    int bSzLog = (lg + 1) >> 1;
    v[n + bid] = suf[0][bid << bSzLog];
    update(1, n, n + indexSz, (1 << lg) - n, n + bid);
}
void build(int layer, int lBound, int rBound, int betweenOffs){
    if (layer >= (int)layers.size())
        return;
    int bSz = 1 << ((layers[layer] + 1) >> 1);
    for (int l = lBound; l < rBound; l += bSz){
        int r = min(l + bSz, rBound);
        buildBlock(layer, l, r);
        build(layer + 1, l, r, betweenOffs);
    }
    if (layer == 0)
        buildBetweenZero();
    else
        buildBetween(layer, lBound, rBound, betweenOffs);
}
void update(int layer, int lBound, int rBound, int betweenOffs, int
    x){
    if (layer >= (int)layers.size())
        return;
    int bSzLog = (layers[layer] + 1) >> 1;
    int bSz = 1 << bSzLog;
    int blockIdx = (x - lBound) >> bSzLog;
    int l = lBound + (blockIdx << bSzLog);
    int r = min(l + bSz, rBound);
    buildBlock(layer, l, r);
    if (layer == 0)
        updateBetweenZero(blockIdx);
}

```

```

    else
        buildBetween(layer, lBound, rBound, betweenOffs);
    update(layer + 1, l, r, betweenOffs, x);
}
inline t_sqrt query(int l, int r, int betweenOffs, int base){
    if (l == r)
        return v[l];
    if (l + 1 == r)
        return op(v[l], v[r]);
    int layer = onLayer[clz[(l - base) ^ (r - base)]];
    int bSzLog = (layers[layer] + 1) >> 1;
    int bCntLog = layers[layer] >> 1;
    int lBound = ((l - base) >> layers[layer]) << layers[layer] +
        base;
    int lBlock = ((l - lBound) >> bSzLog) + 1;
    int rBlock = ((r - lBound) >> bSzLog) - 1;
    t_sqrt ans = suf[layer][l];
    if (lBlock <= rBlock){
        t_sqrt add;
        if (layer == 0)
            add = query(n + lBlock, n + rBlock, (1 << lg) - n, n);
        else
            add = between[layer - 1][betweenOffs + lBound + (lBlock <<
                bCntLog) + rBlock];
        ans = op(ans, add);
    }
    ans = op(ans, pref[layer][r]);
    return ans;
}
public:
    template <class MyIterator>
    SqrtTree(MyIterator begin, MyIterator end){
        n = end - begin;
        v.resize(n);
        for (int i = 0; i < n; i++, begin++)
            v[i] = (*begin);
        lg = log2Up(n);
        clz.resize(1 << lg);
        onLayer.resize(lg + 1);
        clz[0] = 0;
        for (int i = 1; i < (int)clz.size(); i++)
            clz[i] = clz[i >> 1] + 1;
        int tlg = lg;
        while (tlg > 1){
            onLayer[tlg] = (int)layers.size();
            layers.push_back(tlg);
            tlg = (tlg + 1) >> 1;
        }
        for (int i = lg - 1; i >= 0; i--)
            onLayer[i] = max(onLayer[i], onLayer[i + 1]);
        int betweenLayers = max(0, (int)layers.size() - 1);
        int bSzLog = (lg + 1) >> 1;
        int bSz = 1 << bSzLog;
        indexSz = (n + bSz - 1) >> bSzLog;
        v.resize(n + indexSz);
        pref.assign(layers.size(), vector<t_sqrt>(n + indexSz));
        suf.assign(layers.size(), vector<t_sqrt>(n + indexSz));
        between.assign(betweenLayers, vector<t_sqrt>((1 << lg) + bSz));
        build(0, 0, n, 0);
    }
}
//0-indexed

```

```

inline void update(int x, const t_sqrt &item){
    v[x] = item;
    update(0, 0, n, 0, x);
}
//0-indexed [l, r]
inline t_sqrt query(int l, int r){
    return query(l, r, 0, 0);
}
};

```

## 1.21 Stack Query

```

#include <bits/stdc++.h>
using namespace std;
struct StackQuery{
    typedef int t_stack;
    stack<pair<t_stack, t_stack>> st;
    t_stack cmp(t_stack a, t_stack b){
        return min(a, b);
    }
    void push(t_stack x){
        t_stack new_value = st.empty() ? x : cmp(x, st.top().second);
        st.push({x, new_value});
    }
    void pop(){
        st.pop();
    }
    t_stack top(){
        return st.top().first;
    }
    t_stack query(){
        return st.top().second;
    }
    t_stack size(){
        return st.size();
    }
};

```

## 1.22 Treap

```

#include <bits/stdc++.h>
using namespace std;
namespace Treap{
    const int N = 500010;
    typedef long long treap_t;
    treap_t X[N];
    int en = 1, Y[N], sz[N], L[N], R[N], root;

    const treap_t neutral = 0;
    treap_t op_val[N];
    inline treap_t join(treap_t a, treap_t b, treap_t c){
        return a + b + c;
    }
    void calc(int u) { // update node given children info
        sz[u] = sz[L[u]] + 1 + sz[R[u]];
        // code here, no recursion
        op_val[u] = join(op_val[L[u]], X[u], op_val[R[u]]);
    }
}

```

```

void unlaze(int u) {
    if(!u) return;
    // code here, no recursion
}
void split(int u, treap_t x, int &l, int &r) { // l gets <= x, r
    gets > x
    unlaze(u);
    if(!u) return (void) (l = r = 0);
    if(X[u] <= x) { split(R[u], x, l, r); R[u] = l; l = u; }
    else { split(L[u], x, l, r); L[u] = r; r = u; }
    calc(u);
}
void split_sz(int u, int s, int &l, int &r) { // l gets first s, r
    gets remaining
    unlaze(u);
    if(!u) return (void) (l = r = 0);
    if(sz[L[u]] < s) { split_sz(R[u], s - sz[L[u]] - 1, l, r); R[u] =
        l; l = u; }
    else { split_sz(L[u], s, l, r); L[u] = r; r = u; }
    calc(u);
}
int merge(int l, int r) { // els on l <= els on r
    unlaze(l); unlaze(r);
    if(!l || !r) return l + r;
    int u;
    if(Y[l] > Y[r]) { R[l] = merge(R[l], r); u = l; }
    else { L[r] = merge(l, L[r]); u = r; }
    calc(u);
    return u;
}
int new_node(treap_t x){
    X[en] = x;
    op_val[en] = x;
    return en++;
}
int nth(int u, int idx){
    if(!u)
        return 0;
    unlaze(u);
    if(idx <= sz[L[u]])
        return nth(L[u], idx);
    else if(idx == sz[L[u]] + 1)
        return u;
    else
        return nth(R[u], idx - sz[L[u]] - 1);
}
//Public
void init(int n=N-1) { // call before using other funcs
    //init position 0
    sz[0] = 0;
    op_val[0] = neutral;
    //init Treap
    root = 0;
    for(int i = en = 1; i <= n; i++) { Y[i] = i; sz[i] = 1; L[i] = R[i]
        ] = 0; }
    random_shuffle(Y + 1, Y + n + 1);
}
void insert(treap_t x){
    int a, b;
    split(root, x, a, b);
    root = merge(merge(a, new_node(x)), b);
}

```

```

}
void erase(treap_t x){
    int a, b, c, d;
    split(root, x-1, a, b);
    split(b, x, c, d);
    split_sz(c, 1, b, c);
    root = merge(a, merge(c, d));
}
int count(treap_t x){
    int a, b, c, d;
    split(root, x-1, a, b);
    split(b, x, c, d);
    int ans = sz[c];
    root = merge(a, merge(c, d));
    return ans;
}
int size(){ return sz[root];}
//0-indexed
treap_t nth(int idx){
    int u = nth(root, idx + 1);
    return X[u];
}
//Query in k smallest elements
treap_t query(int k){
    int a, b;
    split_sz(root, k, a, b);
    treap_t ans = op_val[a];
    root = merge(a, b);
    return ans;
}
};

```

## 1.23 Union Find

```

#include <bits/stdc++.h>
using namespace std;
class UnionFind{
private:
    vector<int> p, w, sz;
public:
    UnionFind(int n){
        w.resize(n + 1, 1);
        sz.resize(n + 1, 1);
        p.resize(n + 1);
        for (int i = 0; i <= n; i++)
            p[i] = i;
    }
    int find(int x){
        if (p[x] == x)
            return x;
        return p[x] = find(p[x]);
    }
    bool join(int x, int y){
        x = find(x);
        y = find(y);
        if (x == y)
            return false;
        if (w[x] > w[y])
            swap(x, y);
        p[x] = y;
    }
};

```

```

        sz[y] += sz[x];
        if (w[x] == w[y])
            w[y]++;
        return true;
    }
    bool isSame(int x, int y){
        return find(x) == find(y);
    }
    int size(int x){
        return sz[find(x)];
    }
};

```

## 1.24 Union Find With Rollback

```

#include <bits/stdc++.h>
using namespace std;
struct RollbackUF {
    vector<int> e;
    vector<tuple<int, int, int, int>> st;
    RollbackUF(int n) : e(n, -1) {}
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x] < 0 ? x : find(e[x]); }
    int time() { return st.size(); }
    void rollback(int t) {
        while (st.size() > t){
            auto [a1, v1, a2, v2] = st.back();
            e[a1] = v1; e[a2] = v2;
            st.pop_back();
        }
    }
    bool unite(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b);
        st.push_back({a, e[a], b, e[b]});
        e[a] += e[b]; e[b] = a;
        return true;
    }
};

```

## 1.25 Wavelet Tree

```

#include <bits/stdc++.h>
using namespace std;
namespace WaveletTree{
    const int MAXN = 100010, MAXW = MAXN*30; // MAXN * LOG(maxX-MinX)
    typedef int t_wavelet;
    int last;
    int v[MAXN], aux[MAXN];
    int lo[MAXW], hi[MAXW], l[MAXW], r[MAXW];
    vector<t_wavelet> a[MAXW];
    int stable_partition(int i, int j, t_wavelet mid){
        int pivot=0;
        for(int k=i; k<j; k++){
            aux[k] = v[k], pivot += (v[k]<=mid);
            int i1=i, i2=i+pivot;
            for(int k=i; k<j; k++){

```

```

    if(aux[k]<=mid) v[i1++] = aux[k];
    else v[i2++] = aux[k];
}
return i1;
}
void build(int u, int i, int j, t_wavelet minX, t_wavelet maxX){
    lo[u] = minX, hi[u] = maxX;
    if (lo[u] == hi[u] or i >= j)
        return;
    t_wavelet mid = (minX + maxX - 1)/2;
    a[u].resize(j - i + 1);
    a[u][0] = 0;
    for(int k=i; k<j; k++)
        a[u][k-i+1] = a[u][k-i] + (v[k] <= mid);
    int pivot = stable_partition(i, j, mid);
    l[u] = last++, r[u] = last++;
    build(l[u], i, pivot, minX, mid);
    build(r[u], pivot, j, mid + 1, maxX);
}
inline int b(int u, int i){
    return i - a[u][i];
}
}
//Public
template <class MyIterator>
void init(MyIterator begin, MyIterator end, t_wavelet minX,
    t_wavelet maxX){
    last = 1;
    int n = end-begin;
    for(int i=0; i<n; i++, begin++)
        v[i] = *begin;
    build(last++, 0, n, minX, maxX);
}
//kth smallest element in range [i, j]
//1-indexed
int kth(int i, int j, int k, int u=1){
    if (i > j)
        return 0;
    if (lo[u] == hi[u])
        return lo[u];
    int inLeft = a[u][j] - a[u][i - 1];
    int i1 = a[u][i - 1] + 1, j1 = a[u][j];
    int i2 = b(u, i - 1) + 1, j2 = b(u, j);
    if (k <= inLeft)
        return kth(i1, j1, k, l[u]);
    return kth(i2, j2, k - inLeft, r[u]);
}
//Amount of numbers in the range [i, j] Less than or equal to k
//1-indexed
int lte(int i, int j, int k, int u=1){
    if (i > j or k < lo[u])
        return 0;
    if (hi[u] <= k)
        return j - i + 1;
    int i1 = a[u][i - 1] + 1, j1 = a[u][j];
    int i2 = b(u, i - 1) + 1, j2 = b(u, j);
    return lte(i1, j1, k, l[u]) + lte(i2, j2, k, r[u]);
}
//Amount of numbers in the range [i, j] equal to k
//1-indexed
int count(int i, int j, int k, int u=1){
    if (i > j or k < lo[u] or k > hi[u])

```

```

        return 0;
    if (lo[u] == hi[u])
        return j - i + 1;
    t_wavelet mid = (lo[u] + hi[u] - 1) / 2;
    int i1 = a[u][i - 1] + 1, j1 = a[u][j];
    int i2 = b(u, i - 1) + 1, j2 = b(u, j);
    if (k <= mid)
        return count(i1, j1, k, l[u]);
    return count(i2, j2, k, r[u]);
}
//swap v[i] with v[i+1]
//1-indexed
void swp(int i, int u=1){
    if (lo[u] == hi[u] or a[u].size() <= 2)
        return;
    if (a[u][i - 1] + 1 == a[u][i] and a[u][i] + 1 == a[u][i + 1])
        swp(a[u][i], l[u]);
    else if (b(u, i - 1) + 1 == b(u, i) and b(u, i) + 1 == b(u, i + 1))
        swp(b(u, i), r[u]);
    else if (a[u][i - 1] + 1 == a[u][i])
        a[u][i]--;
    else
        a[u][i]++;
}
}
};

```

## 2 Graph Algorithms

### 2.1 2-SAT

```

#include "strongly_connected_component.h"
using namespace std;
struct SAT{
    typedef pair<int, int> pii;
    vector<pii> edges;
    int n;
    SAT(int size){
        n = 2 * size;
    }
    vector<bool> solve2SAT(){
        vector<bool> vAns(n / 2, false);
        vector<int> comp = SCC::scc(n, edges);
        for (int i = 0; i < n; i += 2){
            if (comp[i] == comp[i + 1])
                return vector<bool>();
            vAns[i / 2] = (comp[i] > comp[i + 1]);
        }
        return vAns;
    }
    int v(int x){
        if (x >= 0)
            return (x << 1);
        x = ~x;
        return (x << 1) ^ 1;
    }
    void add(int a, int b){
        edges.push_back(pii(a, b));
    }
}

```

```

}
void addOr(int a, int b){
    add(v(~a), v(b));
    add(v(~b), v(a));
}
void addImp(int a, int b){
    addOr(~a, b);
}
void addEqual(int a, int b){
    addOr(a, ~b);
    addOr(~a, b);
}
void addDiff(int a, int b){
    addEqual(a, ~b);
}
};

```

## 2.2 Arborescence

```

#include <bits/stdc++.h>
#include "../data_structures/union_find_with_rollback.h"
using ll = long long;
struct Edge { int a, b; ll w; };
struct Node { /// lazy skew heap node
    Edge key;
    Node *l, *r;
    ll delta;
    void prop() {
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    }
    Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
    if (!a || !b) return a ?: b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r)));
    return a;
}
void pop(Node& a) { a->prop(); a = merge(a->l, a->r); }
void free(vector<Node*> &v){
    for(auto &x: v)
        delete x;
}
// O(M * log(N))
// return {sum of weights, vector with parents}
pair<ll, vector<int>> dmst(int n, int r, vector<Edge>& g) {
    RollbackUF uf(n);
    vector<Node*> heap(n);
    vector<Node*> vf;
    for (Edge e : g){
        Node* node = new Node{e};
        vf.push_back(node);
        heap[e.b] = merge(heap[e.b], node);
    }
    ll res = 0;
    vector<int> seen(n, -1), path(n), par(n);

```

```

    seen[r] = r;
    vector<Edge> Q(n), in(n, {-1, -1}), comp;
    deque<tuple<int, int, vector<Edge>>> cys;
    for(int s = 0; s < n; ++s) {
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
            if (!heap[u]){
                free(vf);
                return {-1, {}};
            }
            Edge e = heap[u]->top();
            heap[u]->delta -= e.w, pop(heap[u]);
            Q[qi] = e, path[qi++] = u, seen[u] = s;
            res += e.w, u = uf.find(e.a);
            if (seen[u] == s) { /// found cycle, contract
                Node* cyc = 0;
                int end = qi, time = uf.time();
                do cyc = merge(cyc, heap[w = path[--qi]]);
                while (uf.unite(u, w));
                u = uf.find(u), heap[u] = cyc, seen[u] = -1;
                cys.push_front({u, time, {&Q[qi], &Q[end]}});
            }
        }
        for(int i = 0; i < qi; ++i) in[uf.find(Q[i].b)] = Q[i];
    }
    for (auto& [u, t, c] : cys) { // restore sol (optional)
        uf.rollback(t);
        Edge inEdge = in[u];
        for (auto& e : c) in[uf.find(e.b)] = e;
        in[uf.find(inEdge.b)] = inEdge;
    }
    for(int i = 0; i < n; ++i) par[i] = in[i].a;
    free(vf);
    return {res, par};
}
///Careful with overflow
pair<ll, vector<int>> dmstAnyRoot(int n, vector<Edge> v) {
    ll maxEdge = 1000000010;
    ll INF = n*maxEdge;
    for(int i=0; i<n; i++)
        v.push_back(Edge{{n, i, INF}});
    auto [ans, dad] = dmst(n+1, n, v);
    if(ans >= 0 and ans < 2*INF){
        for(int i=0; i<n; i++){
            if(dad[i] == n)
                dad[i] = -1;
            dad.pop_back();
            return {ans - INF, dad};
        }
    }
    return {-1, {}};
}

```

## 2.3 Articulation Point

```

#include <bits/stdc++.h>

using namespace std;

const int MAXN = 500010;

```

```

typedef pair<int, int> pii;
//Articulation Point
namespace AP{
    vector<int> adj[MAXN];
    vector<bool> visited, isAP;
    vector<int> tin, low;
    int timer, n;
    vector<pii> bridges;
    void init(int n1){
        n = n1;
        for(int i=0; i<n; i++) adj[i].clear();
    }
    void addEdge(int a, int b){
        adj[a].push_back(b);
        adj[b].push_back(a);
    }
    void dfs(int u, int p = -1) {
        visited[u] = true;
        tin[u] = low[u] = timer++;
        int children=0;
        for (int to : adj[u]) {
            if (to == p) continue;
            if (visited[to]) {
                low[u] = min(low[u], tin[to]);
            } else {
                dfs(to, u);
                low[u] = min(low[u], low[to]);
                if (low[to] >= tin[u] && p!=-1)
                    isAP[u] = true;
                ++children;
            }
        }
        if(p == -1 && children > 1)
            isAP[u] = true;
    }
    vector<bool> findArticulationPoint() {
        timer = 0;
        visited.assign(n, false);
        tin.assign(n, -1);
        low.assign(n, -1);
        isAP.assign(n, false);
        for (int i = 0; i < n; i++) {
            if (!visited[i])
                dfs(i);
        }
        return isAP;
    }
};

```

## 2.4 BFS 0-1

```

#include <bits/stdc++.h>
using namespace std;
typedef pair<int, int> pii;
const int N = 500010;
const int INF = 0x3f3f3f3f;
namespace BFS01{
    vector<pii> adj[N];
    int n;
    void init(int n1){

```

```

        n = n1;
        for(int i=0; i<n; i++) adj[i].clear();
    }
    //0-indexed
    void addEdge(int u, int to, int w){
        adj[u].emplace_back(to, w);
    }
    vector<int> solve(int s){
        vector<int> d(n, INF);
        d[s] = 0;
        deque<int> q;
        q.push_front(s);
        while (!q.empty()) {
            int u = q.front();
            q.pop_front();
            for (auto edge : adj[u]) {
                int to = edge.first;
                int w = edge.second;
                if (d[u] + w < d[to]) {
                    d[to] = d[u] + w;
                    if (w == 1)
                        q.push_back(to);
                    else
                        q.push_front(to);
                }
            }
        }
        return d;
    }
};

```

## 2.5 Bridge

```

#include <bits/stdc++.h>
using namespace std;
const int MAXN = 500010;
typedef pair<int, int> pii;
namespace Bridge{
    vector<int> adj[MAXN];
    vector<bool> visited;
    vector<int> tin, low;
    int timer, n;
    vector<pii> bridges;
    void init(int n1){
        n = n1;
        for(int i=0; i<n; i++) adj[i].clear();
    }
    void addEdge(int a, int b){
        adj[a].push_back(b);
        adj[b].push_back(a);
    }
    void dfs(int u, int p = -1) {
        visited[u] = true;
        tin[u] = low[u] = timer++;
        for (int to : adj[u]) {
            if (to == p) continue;
            if (visited[to]) {
                low[u] = min(low[u], tin[to]);
            } else {
                dfs(to, u);

```



```

        low[u] = min(low[u], low[to]);
        if (low[to] > tin[u])
            bridges.push_back({u, to});
    }
}
vector<pii> findBridges() {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
    bridges.clear();
    for (int i = 0; i < n; i++) {
        if (!visited[i])
            dfs(i);
    }
    return bridges;
}
};

```

## 2.6 Centroid

```

#include <bits/stdc++.h>
using namespace std;
const int MAXN = 500010;
typedef pair<int, int> pii;
namespace Centroid{
    vector<int> adj[MAXN];
    int sub[MAXN];
    int n;
    void init(int n1){
        n = n1;
        for(int i=0; i<n; i++) adj[i].clear();
    }
    void addEdge(int a, int b){
        adj[a].push_back(b);
        adj[b].push_back(a);
    }
    int dfsS(int u, int p){
        sub[u] = 1;
        for(int to: adj[u]){
            if(to != p)
                sub[u] += dfsS(to, u);
        }
        return sub[u];
    }
    pii dfsC(int u, int p){
        for(int to : adj[u]){
            if(to != p and sub[to] > n/2)
                return dfsC(to, u);
        }
        for(int to : adj[u]){
            if(to != p and (sub[to]*2) == n)
                return pii(u, to);
        }
        return pii(u, u);
    }
    pii findCentroid(){
        dfsS(0, -1);
        return dfsC(0, -1);
    }
}

```

```

    }
}

```

## 2.7 Centroid Decomposition

```

#include <bits/stdc++.h>
using namespace std;
// O(N*log(N))
struct CentroidDecomposition{
    vector<vector<int>> adj;
    vector<int> dad, sub;
    vector<bool> rem;
    int centroidRoot, n;
    void init(int _n){
        n = _n;
        adj.resize(n);
        dad.resize(n);
        sub.resize(n);
        rem.assign(n, false);
    }
    // Return Centroid Decomposition Tree
    vector<vector<int>> build(){
        assert(n > 0);
        centroidRoot = decomp(0, -1);
        vector<vector<int>> ret(n);
        for (int u = 0; u < n; u++){
            if (dad[u] != u)
                ret[dad[u]].push_back(u);
        }
        return ret;
    }
    void addEdge(int a, int b){
        adj[a].push_back(b);
        adj[b].push_back(a);
    }
    int decomp(int u, int p){
        int sz = dfs(u, p);
        int c = centroid(u, p, sz);
        if (p == -1)
            p = c;
        dad[c] = p;
        rem[c] = true;
        for (auto to : adj[c]){
            if (!rem[to])
                decomp(to, c);
        }
        return c;
    }
    int dfs(int u, int p){
        sub[u] = 1;
        for (int to : adj[u]){
            if (!rem[to] and to != p)
                sub[u] += dfs(to, u);
        }
        return sub[u];
    }
    int centroid(int u, int p, int sz){
        for (auto to : adj[u])
            if (!rem[to] and to != p and sub[to] > sz / 2)
                return centroid(to, u, sz);
    }
}

```

```

    return u;
}
int operator[] (int i) {
    return dad[i];
}
};

```

## 2.8 Checking Bipartiteness Online

```

#include <bits/stdc++.h>
using namespace std;
typedef pair<int, int> pii;
const int N = 500010;
pii parent[N];
int rk[N];
int bipartite[N];
void make_set(int v) {
    parent[v] = pii(v, 0);
    rk[v] = 0;
    bipartite[v] = true;
}
pii find_set(int v) {
    if (v != parent[v].first) {
        int parity = parent[v].second;
        parent[v] = find_set(parent[v].first);
        parent[v].second ^= parity;
    }
    return parent[v];
}
void add_edge(int a, int b) {
    pii pa = find_set(a);
    a = pa.first;
    int x = pa.second;
    pair<int, int> pb = find_set(b);
    b = pb.first;
    int y = pb.second;
    if (a == b) {
        if (x == y)
            bipartite[a] = false;
    } else {
        if (rk[a] < rk[b])
            swap(a, b);
        parent[b] = pii(a, x^y^1);
        bipartite[a] ^= bipartite[b];
        if (rk[a] == rk[b])
            ++rk[a];
    }
}
bool is_bipartite(int v) {
    return bipartite[find_set(v).first];
}

```

## 2.9 Dinic

```

#include <bits/stdc++.h>
using namespace std;
//O((V^2)*E): for generic graph.

```

*//O(sqrt(V)\*E): on unit networks. A unit network is a network in which all the edges have unit capacity, and for any vertex except s and t either incoming or outgoing edge is unique. That's exactly the case with the network we build to solve the maximum matching problem with flows.*

```

template <typename flow_t>
struct Dinic{
    struct FlowEdge{
        int from, to, id;
        flow_t cap, flow = 0;
        FlowEdge(int f, int t, flow_t c, int id1) : from(f), to(t), cap(c)
        {
            id = id1;
        }
    };
    const flow_t flow_inf = numeric_limits<flow_t>::max();
    vector<FlowEdge> edges;
    vector<vector<int>> adj;
    int n, m = 0;
    int s, t;
    vector<int> level, ptr;
    queue<int> q;
    bool bfs(){
        while (!q.empty()){
            int u = q.front();
            q.pop();
            for (int id : adj[u]){
                if (edges[id].cap - edges[id].flow < 1)
                    continue;
                if (level[edges[id].to] != -1)
                    continue;
                level[edges[id].to] = level[u] + 1;
                q.push(edges[id].to);
            }
        }
        return level[t] != -1;
    }
    flow_t dfs(int u, flow_t pushed){
        if (pushed == 0)
            return 0;
        if (u == t)
            return pushed;
        for (int &cid = ptr[u]; cid < (int)adj[u].size(); cid++){
            int id = adj[u][cid];
            int to = edges[id].to;
            if (level[u] + 1 != level[to] || edges[id].cap - edges[id].flow < 1)
                continue;
            flow_t tr = dfs(to, min(pushed, edges[id].cap - edges[id].flow));
            edges[id].flow += tr;
            edges[id ^ 1].flow -= tr;
            return tr;
        }
        return 0;
    }
}
//Public:
Dinic() {}
void init(int _n){

```

```

n = _n;
adj.resize(n);
level.resize(n);
ptr.resize(n);
}
void addEdge(int from, int to, flow_t cap, int id=0){
    assert(n>0);
    edges.emplace_back(from, to, cap, id);
    edges.emplace_back(to, from, 0, -id);
    adj[from].push_back(m);
    adj[to].push_back(m + 1);
    m += 2;
}
flow_t maxFlow(int s1, int t1){
    s = s1, t = t1;
    flow_t f = 0;
    //Reset Flow
    for(int i=0; i<m; i++){
        edges[i].flow = 0;
    }
    while (true){
        level.assign(n, -1);
        level[s] = 0;
        q.push(s);
        if (!bfs())
            break;
        ptr.assign(n, 0);
        while (flow_t pushed = dfs(s, flow_inf))
            f += pushed;
    }
    return f;
};
// Returns the minimum cut edge IDs
vector<int> recoverCut(Dinic<int> &d){
    vector<bool> seen(d.n, false);
    queue<int> q;
    q.push(d.s);
    seen[d.s] = true;
    while (!q.empty()){
        int u = q.front();
        q.pop();
        for (int idx : d.adj[u]){
            auto e = d.edges[idx];
            if (e.cap == e.flow)
                continue;
            if (!seen[e.to]){
                q.push(e.to);
                seen[e.to] = true;
            }
        }
    }
    vector<int> ans;
    for(auto e: d.edges){
        if(e.cap > 0 and (e.cap == e.flow) and (seen[e.from] != seen[e.to])){
            if(e.id >= 0) ans.push_back(e.id);
        }
    }
    return ans;
}

```

## 2.10 Edmond's Blossoms

```

#include <bits/stdc++.h>
using namespace std;
const int MAXN = 510;
// Adaptado de: https://github.com/brunomaletta/Biblioteca/blob/master
// Codigo/Grafos/blossom.cpp
// Edmond's Blossoms algorithm give a maximum matching in general
// graphs (non-bipartite)
// O(N^3)
namespace EdmondBlossoms{
    vector<int> adj[MAXN];
    int match[MAXN];
    int n, pai[MAXN], base[MAXN], vis[MAXN];
    queue<int> q;
    void init(int n1){
        n = n1;
        for(int i=0; i<n; i++){
            adj[i].clear();
        }
    }
    void addEdge(int a, int b){
        adj[a].push_back(b);
        adj[b].push_back(a);
    }
    void contract(int u, int v, bool first = 1) {
        static vector<bool> blossom;
        static int l;
        if (first) {
            blossom = vector<bool>(n, 0);
            vector<bool> teve(n, 0);
            int k = u; l = v;
            while (1) {
                teve[k = base[k]] = 1;
                if (match[k] == -1) break;
                k = pai[match[k]];
            }
            while (!teve[l = base[l]]) l = pai[match[l]];
        }
        while (base[u] != l) {
            blossom[base[u]] = blossom[base[match[u]]] = 1;
            pai[u] = v;
            v = match[u];
            u = pai[match[u]];
        }
        if (!first) return;
        contract(v, u, 0);
        for (int i = 0; i < n; i++) if (blossom[base[i]]) {
            base[i] = l;
            if (!vis[i]) q.push(i);
            vis[i] = 1;
        }
    }
    int getpath(int s) {
        for (int i = 0; i < n; i++){
            base[i] = i, pai[i] = -1, vis[i] = 0;
        }
        vis[s] = 1; q = queue<int>(); q.push(s);
        while (q.size()) {
            int u = q.front(); q.pop();
            for (int i : adj[u]) {
                if (base[i] == base[u] or match[u] == i) continue;

```

```

    if (i == s or (match[i] != -1 and pai[match[i]] != -1))
        contract(u, i);
    else if (pai[i] == -1) {
        pai[i] = u;
        if (match[i] == -1) return i;
        i = match[i];
        vis[i] = 1; q.push(i);
    }
}
return -1;
}
typedef pair<int, int> pii;
vector<pii> maximumMatching(){
    vector<pii> ans;
    memset(match, -1, sizeof(match));
    for (int i = 0; i < n; i++) if (match[i] == -1)
        for (int j : adj[i]) if (match[j] == -1) {
            match[i] = j;
            match[j] = i;
            break;
        }
    for (int i = 0; i < n; i++) if (match[i] == -1) {
        int j = getpath(i);
        if (j == -1) continue;
        while (j != -1) {
            int p = pai[j], pp = match[p];
            match[p] = j;
            match[j] = p;
            j = pp;
        }
    }
    for (int i = 0; i < n; i++)
        if (i < match[i])
            ans.emplace_back(i, match[i]);
    return ans;
}
};

```

## 2.11 Eulerian Path

```

#include <bits/stdc++.h>
using namespace std;
typedef pair<int, int> pii;
template<bool directed=false> struct EulerianPath{
    vector<vector<pii>> adj;
    vector<int> ans, pos;
    vector<bool> used;
    int n, m;
    EulerianPath(int n1){
        n = n1; m = 0;
        adj.assign(n, vector<pii>());
    }
    void addEdge(int a, int b) {
        int at = m++;
        adj[a].push_back({b, at});
        if (!directed) adj[b].push_back({a, at});
    }
    void dfs(int u){
        while(pos[u] < adj[u].size()){

```

```

            auto [to, id] = adj[u][pos[u]];
            pos[u]++;
            if(!used[id]){
                used[id] = true;
                dfs(to);
            }
        }
        ans.push_back(u);
    }
    // Remember to call the correct src
    // If you want to check if there is an answer remember to check if
    // all |components| > 1 of the graph are connected
    vector<int> getPath(int src){
        pos.assign(n, 0);
        used.assign(m, false);
        ans.clear();
        dfs(src);
        reverse(ans.begin(), ans.end());
        return ans;
    }
};

```

## 2.12 Flow With Demand

```

#include "dinic.h"
using namespace std;
template <typename flow_t>
struct MaxFlowEdgeDemands{
    Dinic<flow_t> mf;
    vector<flow_t> ind, outd;
    flow_t D;
    int n;
    MaxFlowEdgeDemands(int n) : n(n){
        D = 0;
        mf.init(n + 2);
        ind.assign(n, 0);
        outd.assign(n, 0);
    }
    void addEdge(int a, int b, flow_t cap, flow_t demands){
        mf.addEdge(a, b, cap - demands);
        D += demands;
        ind[b] += demands;
        outd[a] += demands;
    }
    bool solve(int s, int t){
        mf.addEdge(t, s, numeric_limits<flow_t>::max());
        for (int i = 0; i < n; i++){
            if (ind[i]) mf.addEdge(n, i, ind[i]);
            if (outd[i]) mf.addEdge(i, n + 1, outd[i]);
        }
        return mf.maxFlow(n, n + 1) == D;
    }
};

```

## 2.13 Floyd Warshall

```

#include <bits/stdc++.h>
using namespace std;

```

```

typedef long long ll;
const ll INFL = 0x3f3f3f3f3f3f3f3f;
namespace FloydWarshall{
    vector<vector<ll>> dist;
    int n;
    void init(int n1){
        n = n1;
        dist.assign(n, vector<ll>(n, INFL));
        for(int i=0; i<n; i++){
            dist[i][i] = 0LL;
        }
    }
    void addEdge(int a, int b, ll w){
        dist[a][b] = min(dist[a][b], w);
    }
    vector<vector<ll>> solve(){
        for(int k=0; k<n; k++){
            for(int i=0; i<n; i++){
                for(int j=0; j<n; j++){
                    dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);
                }
            }
        }
        return dist;
    }
};

```

## 2.14 Hungarian

```

#include <bits/stdc++.h>
using namespace std;
//input: matrix n x m, n <= m
//return vector p of size n, where p[i] is the match for i
// and minimum cost
// time complexity: O(n^2 * m)
const int ms = 310, INF = 0x3f3f3f3f;
int u[ms], v[ms], p[ms], way[ms], minv[ms];
bool used[ms];
pair<vector<int>, int> solve(const vector<vector<int>> &matrix){
    int n = matrix.size();
    if (n == 0)
        return {vector<int>(), 0};
    int m = matrix[0].size();
    assert(n <= m);
    memset(u, 0, (n + 1) * sizeof(int));
    memset(v, 0, (m + 1) * sizeof(int));
    memset(p, 0, (m + 1) * sizeof(int));
    for (int i = 1; i <= n; i++){
        memset(minv, 0x3f, (m + 1) * sizeof(int));
        memset(way, 0, (m + 1) * sizeof(int));
        for (int j = 0; j <= m; j++){
            used[j] = 0;
        }
        p[0] = i;
        int k0 = 0;
        do{
            used[k0] = 1;
            int i0 = p[k0], delta = INF, k1;
            for (int j = 1; j <= m; j++){
                if (!used[j]){
                    int cur = matrix[i0 - 1][j - 1] - u[i0] - v[j];
                    if (cur < minv[j]){

```

```

                        minv[j] = cur;
                        way[j] = k0;
                    }
                }
            }
            if (minv[j] < delta){
                delta = minv[j];
                k1 = j;
            }
        }
        for (int j = 0; j <= m; j++){
            if (used[j]){
                u[p[j]] += delta;
                v[j] -= delta;
            }
            minv[j] -= delta;
        }
        k0 = k1;
    } while (p[k0]);
    do{
        int k1 = way[k0];
        p[k0] = p[k1];
        k0 = k1;
    } while (k0);
}
vector<int> ans(n, -1);
for (int j = 1; j <= m; j++){
    if (!p[j]) continue;
    ans[p[j] - 1] = j - 1;
}
return {ans, -v[0]};
}

```

## 2.15 Prim

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int, int> pii;
const int MAXN = 500010;
namespace Prim{
    vector<pii> adj[MAXN];
    int weight[MAXN];
    bool seen[MAXN];
    int n;
    void init(int n1){
        n = n1;
        for(int i=0; i<n; i++) adj[i].clear();
    }
    void addEdge(int a, int b, int w){
        adj[a].emplace_back(w, b);
        adj[b].emplace_back(w, a);
    }
    ll solve(){
        for(int i=0; i<n; i++){
            weight[i] = 0x3f3f3f3f;
            seen[i] = 0;
        }
        weight[0] = 0;
        priority_queue<pii, vector<pii>, greater<pii> > st;

```

```

st.push(pii(weight[0], 0));
ll ans = 0;
while(!st.empty()){
    int u = st.top().second;
    st.pop();
    if(seen[u])
        continue;
    seen[u] = true;
    ans += weight[u];
    for(auto [edge, to]: adj[u]){
        if(!seen[to] and (edge < weight[to])){
            weight[to] = edge;
            st.emplace(weight[to], to);
        }
    }
}
return ans;
};

```

## 2.16 HLD

```

#include <bits/stdc++.h>
#include "../data_structures/bit_range.h"
using namespace std;
#define F first
#define S second
using hld_t = long long;
using pii = pair<int, hld_t>;
struct HLD{
    vector<vector<pii>> adj;
    vector<int> sz, h, dad, pos;
    vector<hld_t> val, v;
    int t;
    bool edge;
    //Begin Internal Data Structure
    BitRange *bit;
    hld_t neutral = 0;
    inline hld_t join(hld_t a, hld_t b){
        return a+b;
    }
    inline void update(int a, int b, hld_t x){
        bit->add(a+1, b+1, x);
    }
    inline hld_t query(int a, int b){
        return bit->get(a+1, b+1);
    }
    //End Internal Data Structure
    void init(int n){
        dad.resize(n); pos.resize(n); val.resize(n);
        adj.resize(n); sz.resize(n); h.resize(n);
        bit = new BitRange(n);
    }
    void dfs(int u, int p = -1){
        sz[u] = 1;
        for(pii &to: adj[u]) if(to.F != p){
            if(edge) val[to.F] = to.S;
            dfs(to.F, u);
            sz[u] += sz[to.F];
            if(sz[to.F] > sz[adj[u][0].F] or adj[u][0].F == p)

```

```

                swap(to, adj[u][0]);
            }
        }
    void build_hld(int u, int p=-1){
        dad[u] = p;
        pos[u] = t++;
        v[pos[u]] = val[u];
        for(pii to: adj[u]) if(to.F != p){
            h[to.F] = (to == adj[u][0]) ? h[u] : to.F;
            build_hld(to.F, u);
        }
    }
    void addEdge(int a, int b, hld_t w = 0){
        adj[a].emplace_back(b, w);
        adj[b].emplace_back(a, w);
    }
    void build(int root, bool is_edge){
        assert(!adj.empty());
        edge = is_edge;
        t = 0;
        h[root] = 0;
        dfs(root);
        build_hld(root);
        //Init Internal Data Structure
        for(int i=0; i<t; i++)
            update(i, i, v[i]);
    }
    hld_t query_path(int a, int b) {
        if (edge and a == b) return neutral;
        if (pos[a] < pos[b]) swap(a, b);
        if (h[a] == h[b]) return query(pos[b]+edge, pos[a]);
        return join(query(pos[h[a]], pos[a]), query_path(dad[h[a]], b));
    }
    void update_path(int a, int b, hld_t x) {
        if (edge and a == b) return;
        if (pos[a] < pos[b]) swap(a, b);
        if (h[a] == h[b]) return (void)update(pos[b]+edge, pos[a], x);
        update(pos[h[a]], pos[a], x); update_path(dad[h[a]], b, x);
    }
    hld_t query_subtree(int a) {
        if (edge and sz[a] == 1) return neutral;
        return query(pos[a]+edge, pos[a]+sz[a]-1);
    }
    void update_subtree(int a, hld_t x) {
        if (edge and sz[a] == 1) return;
        update(pos[a] + edge, pos[a]+sz[a]-1, x);
    }
    int lca(int a, int b) {
        if (pos[a] < pos[b]) swap(a, b);
        return h[a] == h[b] ? b : lca(dad[h[a]], b);
    }
};

```

## 2.17 Kuhn

```

#include <bits/stdc++.h>
using namespace std;
mt19937 rng((int)chrono::steady_clock::now().time_since_epoch().count());
namespace Kuhn{

```

```

int na, nb;
vector<vector<int>> adj;
vector<int> vis, ma, mb;
void init(int nal, int nbl){
    na = nal, nb = nbl;
    adj.assign(na, vector<int>());
    vis.assign(na + nb, 0);
    ma.assign(na, -1);
    mb.assign(nb, -1);
}
void addEdge(int a, int b) {
    adj[a].push_back(b);
}
bool dfs(int u) {
    vis[u] = 1;
    for (int to : adj[u]){
        if(vis[na+to])
            continue;
        vis[na+to] = 1;
        if (mb[to] == -1 or dfs(mb[to])) {
            ma[u] = to, mb[to] = u;
            return true;
        }
    }
    return false;
}
int matching() {
    int ans = 0, c = 1;
    for (auto& v: adj)
        shuffle(v.begin(), v.end(), rng);
    while (c) {
        for (int j = 0; j < nb; j++)
            vis[na+j] = 0;
        c = 0;
        for (int i = 0; i < na; i++)
            if (ma[i] == -1 and dfs(i))
                ans++, c = 1;
    }
    return ans;
}
pair<vector<int>, vector<int>> minimumVertexCover() {
    matching();
    for (int i = 0; i < na+nb; i++)
        vis[i] = 0;
    for (int i = 0; i < na; i++)
        if (ma[i] == -1)
            dfs(i);
    vector<int> va, vb;
    for (int i = 0; i < na; i++)
        if (!vis[i])
            va.push_back(i);
    for (int i = 0; i < nb; i++)
        if (vis[na+i])
            vb.push_back(i);
    return {va, vb};
}
vector<int> maximumAntichain(){
    auto [l, r] = minimumVertexCover();
    set<int> L(l.begin(), l.end());
    set<int> R(r.begin(), r.end());
    vector<int> ans;

```

```

        for (int i = 0; i < na; i++)
            if (!L.count(i) and !R.count(i))
                ans.push_back(i);
        return ans;
    }
};

```

## 2.18 Kruskal

```

#include "../data_structures/union_find.h"
typedef long long ll;
struct Edge{
    int u, v; ll w;
    Edge(){}
    Edge(int u, int v, ll w):u(u), v(v), w(w){}
};
ll kruskal(vector<Edge> v, int nVet){
    ll cost = 0;
    UnionFind uf(nVet);
    sort(v.begin(), v.end(), [&](Edge a, Edge b){
        return a.w < b.w;
    });
    for(Edge &e: v){
        if(!uf.isSame(e.u, e.v)){
            cost += e.w;
            uf.join(e.u, e.v);
        }
    }
    return cost;
}

```

## 2.19 LCA

```

#include <bits/stdc++.h>
using namespace std;
namespace LCA{
    typedef long long lca_t;
    typedef pair<int, lca_t> lca_p;
    const lca_t neutral = 0;
    vector<vector<lca_p>> adj;
    vector<int> level;
    vector<vector<lca_t>> D;
    vector<vector<int>> P;
    int n, mxLogN;
    void init(int _n, int _mxLogN = 20){
        n = _n;
        mxLogN = _mxLogN;
        adj.assign(n, vector<lca_p>());
        D.resize(n, vector<lca_t>(mxLogN));
        level.resize(n);
    }
    lca_t join(lca_t a, lca_t b){
        return a + b;
    }
    void addEdge(int a, int b, lca_t w = 1){
        adj[a].emplace_back(b, w);
        adj[b].emplace_back(a, w);
    }
}

```

```

void dfs(int u){
    for (auto to : adj[u]){
        int v = to.first;
        lca_t w = to.second;
        if (v == P[u][0])
            continue;
        P[v][0] = u;
        D[v][0] = w;
        level[v] = level[u] + 1;
        dfs(v);
    }
}

void build(int root = 0){
    P.assign(n, vector<int>(mxLogN, -1));
    level[root] = 0;
    P[root][0] = root;
    D[root][0] = neutral;
    dfs(root);
    for (int j = 1; j < mxLogN; j++){
        for (int i = 0; i < n; i++){
            P[i][j] = P[P[i][j - 1]][j - 1];
            D[i][j] = join(D[P[i][j - 1]][j - 1], D[i][j - 1]);
        }
    }
}

lca_p lca(int u, int v){
    if (level[u] > level[v])
        swap(u, v);
    int d = level[v] - level[u];
    lca_t ans = neutral;
    for (int i = 0; i < mxLogN; i++){
        if (d & (1 << i)){
            ans = join(ans, D[v][i]);
            v = P[v][i];
        }
    }
    if (u == v)
        return lca_p(u, ans);
    for (int i = mxLogN - 1; i >= 0; i--){
        while (P[u][i] != P[v][i]){
            ans = join(ans, D[v][i]);
            ans = join(ans, D[u][i]);
            u = P[u][i];
            v = P[v][i];
        }
    }
    ans = join(ans, D[v][0]);
    ans = join(ans, D[u][0]);
    return lca_p(P[u][0], ans);
}
};

```

## 2.20 Link-Cut Tree

```

#include <bits/stdc++.h>
using namespace std;
// Link-Cut Tree, directed version.
// All operations are O(log(n)) amortized.
//Source: https://github.com/brunomaletha/Biblioteca/
const int MAXN = 200010;
namespace LCT {

```

```

    struct node {
        int p, ch[2];
        node() { p = ch[0] = ch[1] = -1; }
    };
    node t[MAXN];
    bool isRoot(int x) {
        return t[x].p == -1 or (t[t[x].p].ch[0] != x and t[t[x].p].ch[1]
            != x);
    }
    void rotate(int x) {
        int p = t[x].p, pp = t[p].p;
        if (!isRoot(p)) t[pp].ch[t[pp].ch[1] == p] = x;
        bool d = t[p].ch[0] == x;
        t[p].ch[!d] = t[x].ch[d], t[x].ch[d] = p;
        if (t[p].ch[!d]+1) t[t[p].ch[!d]].p = p;
        t[x].p = pp, t[p].p = x;
    }
    void splay(int x) {
        while (!isRoot(x)) {
            int p = t[x].p, pp = t[p].p;
            if (!isRoot(p))
                rotate((t[pp].ch[0] == p)^(t[p].ch[0] == x) ? x : p);
            rotate(x);
        }
    }
    int access(int v) {
        int last = -1;
        for (int w = v; w+1; last = w, splay(v), w = t[v].p)
            splay(w), t[w].ch[1] = (last == -1 ? -1 : v);
        return last;
    }
} // Public:
void init(int n){
    for(int i=0; i<=n; i++)
        t[i] = node();
}
int findRoot(int v) {
    access(v);
    while (t[v].ch[0]+1) v = t[v].ch[0];
    return splay(v), v;
}
// V must be root. W will be the dad of V.
void link(int v, int w) {
    access(v);
    t[v].p = w;
}
// Removes edge (v, dad[v])
void cut(int v) {
    access(v);
    if(t[v].ch[0] == -1)
        return;
    t[v].ch[0] = t[t[v].ch[0]].p = -1;
}
int lca(int v, int w) {
    if(findRoot(v) != findRoot(w))
        return -1;
    access(v);
    return access(w);
}
}

```



## 2.21 Link-Cut Tree - Edge

```
#include <bits/stdc++.h>
using namespace std;
// Link-Cut Tree - Edge, undirected version.
// All operations are O(log(n)) amortized.
// Source: https://github.com/brunomaletta/Biblioteca/
typedef long long ll;
typedef pair<int, int> pii;
const int MAXN = 100010, MAXQ = 100010;
namespace LCT {
    struct node {
        int p, ch[2];
        ll val, sub;
        bool rev;
        int sz, ar;
        ll lazy;
        node() {}
        node(int v, int ar_) :
            p(-1), val(v), sub(v), rev(0), sz(ar_), ar(ar_), lazy(0) {
            ch[0] = ch[1] = -1;
        }
    };
    node t[MAXN + MAXQ]; // MAXN + MAXQ
    map<pii, int> edges;
    int sz;
    void prop(int x) {
        if (t[x].lazy) {
            if (t[x].ar) t[x].val += t[x].lazy;
            t[x].sub += t[x].lazy*t[x].sz;
            if (t[x].ch[0]+1) t[t[x].ch[0]].lazy += t[x].lazy;
            if (t[x].ch[1]+1) t[t[x].ch[1]].lazy += t[x].lazy;
        }
        if (t[x].rev) {
            swap(t[x].ch[0], t[x].ch[1]);
            if (t[x].ch[0]+1) t[t[x].ch[0]].rev ^= 1;
            if (t[x].ch[1]+1) t[t[x].ch[1]].rev ^= 1;
        }
        t[x].lazy = 0, t[x].rev = 0;
    }
    void update(int x) {
        t[x].sz = t[x].ar, t[x].sub = t[x].val;
        for (int i = 0; i < 2; i++) if (t[x].ch[i]+1) {
            prop(t[x].ch[i]);
            t[x].sz += t[t[x].ch[i]].sz;
            t[x].sub += t[t[x].ch[i]].sub;
        }
    }
    bool is_root(int x) {
        return t[x].p == -1 or (t[t[x].p].ch[0] != x and t[t[x].p].ch[1]
            != x);
    }
    void rotate(int x) {
        int p = t[x].p, pp = t[p].p;
        if (!is_root(p)) t[pp].ch[t[pp].ch[1] == p] = x;
        bool d = t[p].ch[0] == x;
        t[p].ch[!d] = t[x].ch[d], t[x].ch[d] = p;
        if (t[p].ch[!d]+1) t[t[p].ch[!d]].p = p;
        t[x].p = pp, t[p].p = x;
        update(p), update(x);
    }
}
```

```

    }
    int splay(int x) {
        while (!is_root(x)) {
            int p = t[x].p, pp = t[p].p;
            if (!is_root(p)) prop(pp);
            prop(p), prop(x);
            if (!is_root(p)) rotate((t[pp].ch[0] == p)^(t[p].ch[0] == x) ? x
                : p);
            rotate(x);
        }
        return prop(x), x;
    }
    int access(int v) {
        int last = -1;
        for (int w = v; w+1; update(last = w), splay(v), w = t[v].p)
            splay(w), t[w].ch[1] = (last == -1 ? -1 : v);
        return last;
    }
    void rootify(int v);
    void link_(int v, int w) {
        rootify(w);
        t[w].p = v;
    }
    void cut_(int v, int w) {
        rootify(w), access(v);
        t[v].ch[0] = t[t[v].ch[0]].p = -1;
    }
    void makeTree(int v, int w=0, int ar=0) {
        t[v] = node(w, ar);
    }
}
// Public:
void init(int n) {
    edges.clear();
    sz = 0;
    for (int i=0; i<=n; i++)
        makeTree(i);
}
int findRoot(int v) {
    access(v), prop(v);
    while (t[v].ch[0]+1) v = t[v].ch[0], prop(v);
    return splay(v);
}
// Checks if v and w are connected
bool connected(int v, int w) {
    access(v), access(w);
    return v == w ? true : t[v].p != -1;
}
// Change v to be root
void rootify(int v) {
    access(v);
    t[v].rev ^= 1;
}
// Sum of the edges in path from v to w
ll query(int v, int w) {
    rootify(w), access(v);
    return t[v].sub;
}
// Sum +x in path from v to w
void update(int v, int w, int x) {
    rootify(w), access(v);
    t[v].lazy += x;
}
```

```

}
// Add edge (v, w) with weight x
void link(int v, int w, int x) {
    int id = MAXN + sz++;
    edges[pai(v, w)] = id;
    makeTree(id, x, 1);
    link_(v, id), link_(id, w);
}
// Remove edge (v, w)
void cut(int v, int w) {
    int id = edges[pai(v, w)];
    cut_(v, id), cut_(id, w);
}
int lca(int v, int w) {
    access(v);
    return access(w);
}
}

```

## 2.22 Link-Cut Tree - Vertex

```

#include <bits/stdc++.h>
using namespace std;
// Link-Cut Tree - Vertex, undirected version.
// All operations are O(log(n)) amortized.
// Source: https://github.com/brunomaletta/Biblioteca/
typedef long long ll;
typedef pair<int, int> pii;
const int MAXN = 200010;
namespace lct {
    struct node {
        int p, ch[2];
        ll val, sub;
        bool rev;
        int sz;
        ll lazy;
        node() {}
        node(int v) : p(-1), val(v), sub(v), rev(0), sz(1), lazy(0) {
            ch[0] = ch[1] = -1;
        }
    };
    node t[MAXN];
    void prop(int x) {
        if (t[x].lazy) {
            t[x].val += t[x].lazy, t[x].sub += t[x].lazy*t[x].sz;
            if (t[x].ch[0] != -1) t[t[x].ch[0]].lazy += t[x].lazy;
            if (t[x].ch[1] != -1) t[t[x].ch[1]].lazy += t[x].lazy;
        }
        if (t[x].rev) {
            swap(t[x].ch[0], t[x].ch[1]);
            if (t[x].ch[0] != -1) t[t[x].ch[0]].rev ^= 1;
            if (t[x].ch[1] != -1) t[t[x].ch[1]].rev ^= 1;
        }
        t[x].lazy = 0, t[x].rev = 0;
    }
    void update(int x) {
        t[x].sz = 1, t[x].sub = t[x].val;
        for (int i = 0; i < 2; i++) if (t[x].ch[i] != -1) {
            prop(t[x].ch[i]);
            t[x].sz += t[t[x].ch[i]].sz;
        }
    }
}

```

```

        t[x].sub += t[t[x].ch[i]].sub;
    }
}
bool is_root(int x) {
    return t[x].p == -1 or (t[t[x].p].ch[0] != x and t[t[x].p].ch[1]
        != x);
}
void rotate(int x) {
    int p = t[x].p, pp = t[p].p;
    if (!is_root(p)) t[pp].ch[t[pp].ch[1] == p] = x;
    bool d = t[p].ch[0] == x;
    t[p].ch[!d] = t[x].ch[d], t[x].ch[d] = p;
    if (t[p].ch[!d] != -1) t[t[p].ch[!d]].p = p;
    t[x].p = pp, t[p].p = x;
    update(p), update(x);
}
int splay(int x) {
    while (!is_root(x)) {
        int p = t[x].p, pp = t[p].p;
        if (!is_root(p)) prop(pp);
        prop(p), prop(x);
        if (!is_root(p)) rotate((t[pp].ch[0] == p) ^ (t[p].ch[0] == x) ? x
            : p);
        rotate(x);
    }
    return prop(x), x;
}
int access(int v) {
    int last = -1;
    for (int w = v; w; w = t[w].p) {
        splay(w), t[w].ch[1] = (last == -1 ? -1 : last);
        return last;
    }
}
// Public:
void makeTree(int v, int w) {
    t[v] = node(w);
}
int findRoot(int v) {
    access(v), prop(v);
    while (t[v].ch[0] != -1) v = t[v].ch[0], prop(v);
    return splay(v);
}
// Checks if v and w are connected
bool connected(int v, int w) {
    access(v), access(w);
    return v == w ? true : t[v].p != -1;
}
// Change v to be root
void rootify(int v) {
    access(v);
    t[v].rev ^= 1;
}
// Sum of the weight in path from v to w
ll query(int v, int w) {
    rootify(w), access(v);
    return t[v].sub;
}
// Sum +x in path from v to w
void update(int v, int w, int x) {
    rootify(w), access(v);
    t[v].lazy += x;
}

```

## 2.23 Min-Cut

```

    }else{
        in_a[sel] = true;
        for(int i=0; i<n; ++i)
            w[i] += g[sel][i];
        prev = sel;
    }
}

return {best_cost, best_cut};
};

```

## 2.24 Minimum Cost Maximum Flow

```
#include <bits/stdc++.h>
using namespace std;
template <class T = int>
class MCMF{
private:
    struct Edge{
        int to;
        T cap, cost;
        Edge(int a, T b, T c) : to(a), cap(b), cost(c) {}
    };
    int n;
    vector<vector<int>> edges;
    vector<Edge> list;
    vector<int> from;
    vector<T> dist, pot;
    vector<bool> visit;
    pair<T, T> augment(int src, int sink){
        pair<T, T> flow = {list[from[sink]].cap, 0};
        for (int v = sink; v != src; v = list[from[v] ^ 1].to){
            flow.first = std::min(flow.first, list[from[v]].cap);
            flow.second += list[from[v]].cost;
        }
        for (int v = sink; v != src; v = list[from[v] ^ 1].to){
            list[from[v]].cap -= flow.first;
            list[from[v] ^ 1].cap += flow.first;
        }
        return flow;
    }
    queue<int> q;
    bool SPFA(int src, int sink){
        T INF = numeric_limits<T>::max();
        dist.assign(n, INF);
        from.assign(n, -1);
        q.push(src);
        dist[src] = 0;
        while (!q.empty()){
            int on = q.front();
            q.pop();
            visit[on] = false;
            for (auto e : edges[on]){
                auto ed = list[e];
                if (ed.cap == 0)
                    continue;
                T toDist = dist[on] + ed.cost + pot[on] - pot[ed.to];
                if (toDist < dist[ed.to]){
                    dist[ed.to] = toDist;

```

```

        from[ed.to] = e;
        if (!visit[ed.to]){
            visit[ed.to] = true;
            q.push(ed.to);
        }
    }
}
return dist[sink] < INF;
}
void fixPot(){
    T INF = numeric_limits<T>::max();
    for (int i = 0; i < n; i++){
        if (dist[i] < INF)
            pot[i] += dist[i];
    }
}
public:
    MCMF(int size){
        n = size;
        edges.resize(n);
        pot.assign(n, 0);
        dist.resize(n);
        visit.assign(n, false);
    }
    pair<T, T> solve(int src, int sink){
        pair<T, T> ans(0, 0);
        // Can use dijkstra to speed up depending on the graph
        if (!SPFA(src, sink))
            return ans;
        fixPot();
        // Can use dijkstra to speed up depending on the graph
        while (SPFA(src, sink)){
            auto flow = augment(src, sink);
            ans.first += flow.first;
            ans.second += flow.first * flow.second;
            fixPot();
        }
        return ans;
    }
    void addEdge(int u, int to, T cap, T cost){
        edges[u].push_back(list.size());
        list.push_back(Edge(to, cap, cost));
        edges[to].push_back(list.size());
        list.push_back(Edge(u, 0, -cost));
    }
};

```

## 2.25 Strongly Connected Component

```

#include "topological_sort.h"
using namespace std;
namespace SCC{
    typedef pair<int, int> pii;
    vector<vector<int>> revAdj;
    vector<int> component;
    void dfs(int u, int c){
        component[u] = c;
        for (int to : revAdj[u]){
            if (component[to] == -1)

```

```

                dfs(to, c);
            }
        }
    }
    vector<int> scc(int n, vector<pii> &edges){
        revAdj.assign(n, vector<int>());
        for (pii p : edges)
            revAdj[p.second].push_back(p.first);
        vector<int> tp = TopologicalSort::order(n, edges);
        component.assign(n, -1);
        int comp = 0;
        for (int u : tp){
            if (component[u] == -1)
                dfs(u, comp++);
        }
        return component;
    }
} // namespace SCC

```

## 2.26 Topological Sort

```

#include <bits/stdc++.h>
using namespace std;
namespace TopologicalSort{
    typedef pair<int, int> pii;
    vector<vector<int>> adj;
    vector<bool> visited;
    vector<int> vAns;
    void dfs(int u){
        visited[u] = true;
        for (int to : adj[u]){
            if (!visited[to])
                dfs(to);
        }
        vAns.push_back(u);
    }
    vector<int> order(int n, vector<pii> &edges){
        adj.assign(n, vector<int>());
        for (pii p : edges)
            adj[p.first].push_back(p.second);
        visited.assign(n, false);
        vAns.clear();
        for (int i = 0; i < n; i++){
            if (!visited[i])
                dfs(i);
        }
        reverse(vAns.begin(), vAns.end());
        return vAns;
    }
} // namespace TopologicalSort

```

## 2.27 Tree ID

```

#include "centroid.h"
#define F first
#define S second
namespace TreeID{
    int id=0;
    map<map<int, int>, int> mpId;

```

```

vector<int> adj[MAXN];
int treeID(int u, int p){
    map<int, int> mp;
    for(int to: adj[u]){
        if(to != p)
            mp[treeID(to, u)]++;
    }
    if(!mpId.count(mp))
        mpId[mp] = ++id;
    return mpId[mp];
}
//Returns a pair of values that represents a tree only. O((N+M)*log(M))
//0-indexed
pii getTreeID(vector<pii> &edges, int n){
    for(int i=0; i<n; i++)
        adj[i].clear();
    Centroid::init(n);
    for(pii e: edges){
        adj[e.F].push_back(e.S);
        adj[e.S].push_back(e.F);
        Centroid::addEdge(e.F, e.S);
    }
    pii c = Centroid::findCentroid();
    pii ans(treeID(c.F, -1), treeID(c.S, -1));
    if(ans.F > ans.S)
        swap(ans.F, ans.S);
    return ans;
}
bool isomorphic(vector<pii> &tree1, vector<pii> &tree2, int n){
    return getTreeID(tree1, n) == getTreeID(tree2, n);
}
};

```

## 2.28 Vertex Cover In Tree

```

#include <bits/stdc++.h>
using namespace std;
const int MAXN = 200010;
int dp[MAXN][2];
vector<int> adj[MAXN];
// vertexCover(node current, free to choose, dad)
int vertexCover(int u, bool color=true, int p=-1){
    if(dp[u][color] != -1)
        return dp[u][color];
    int case1 = 1, case2 = 0;
    for(int to: adj[u]){
        if(to == p) continue;
        case1 += vertexCover(to, true, u);
        case2 += vertexCover(to, false, u);
    }
    if(color)
        return dp[u][color] = min(case1, case2);
    else
        return dp[u][color] = case1;
}

```

## 3 Dynamic Programming

### 3.1 Divide and Conquer Optimization

Reduces the complexity from  $O(n^2k)$  to  $O(nk \log n)$  of PD's in the following ways (and other variants):

$$dp[n][k] = \max_{0 \leq i < n} (dp[i][k-1] + C[i+1][n]), \text{ base case : } dp[0][j], dp[i][0] \quad (1)$$

- $C[i][j]$  = the cost only depends on  $i$  and  $j$ .
- $opt[n][k] = i$  is the optimal value that maximizes  $dp[n][k]$ .

It is necessary that  $opt$  is increasing along each column:  $opt[j][k] \leq opt[j+1][k]$ .

### 3.2 Divide and Conquer Optimization Implementation

```

#include <bits/stdc++.h>
using namespace std;
int C(int i, int j);
const int MAXN = 100010;
const int MAXK = 110;
const int INF = 0x3f3f3f3f;
int dp[MAXN][MAXK];
void calculateDP(int l, int r, int k, int opt_l, int opt_r){
    if(l > r)
        return;
    int mid = (l + r) >> 1;
    int ans = -INF, opt = mid;
    // int ans = dp[mid][k-1], opt=mid; //If you accept empty subsegment
    for(int i = opt_l; i <= min(opt_r, mid-1); i++){
        if(ans < dp[i][k-1] + C(i+1, mid)){
            opt = i;
            ans = dp[i][k-1] + C(i+1, mid);
        }
    }
    dp[mid][k] = ans;
    calculateDP(l, mid-1, k, opt_l, opt);
    calculateDP(mid+1, r, k, opt, opt_r);
}
int solve(int n, int k){
    for(int i = 0; i <= n; i++)
        dp[i][0] = -INF;
    for(int j = 0; j <= k; j++)
        dp[0][j] = -INF;
    dp[0][0] = 0;
    for(int j = 1; j <= k; j++)
        calculateDP(1, n, j, 0, n-1);
    return dp[n][k];
}

```

### 3.3 Knuth Optimization

Reduces the complexity from  $O(n^3)$  to  $O(n^2)$  of PD's in the following ways (and other variants):

$$dp[i][j] = C[i][j] + \min_{i < k < j} (dp[i][k] + dp[k][j]), \text{ caso base : } dp[i][i] \quad (2)$$

$$dp[i][j] = \min_{i < k < j} (dp[i][k] + C[i][k]), \text{ caso base : } dp[i][i] \quad (3)$$

- $C[i][j]$  = the cost only depends on  $i$  and  $j$ .
- $opt[i][j] = k$  is the optimal value that maximizes  $dp[i][j]$ .

The following conditions must be met:

- Foursquare inequality on  $C$ :  $C[a][c] + C[b][d] \leq C[a][d] + C[b][c]$ ,  $a \leq b \leq c \leq d$ .
- Monotonicity on  $C$ :  $C[b][c] \leq C[a][d]$ ,  $a \leq b \leq c \leq d$ .

Or the following condition:

- $opt$  increasing in rows and columns:  $opt[i][j-1] \leq opt[i][j] \leq opt[i+1][j]$ .

### 3.4 Knuth Optimization Implementation

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int MAXN = 1009;
const ll INFLL = 0x3f3f3f3f3f3f3f3f;
ll C(int a, int b);
ll dp[MAXN][MAXN];
int opt[MAXN][MAXN];
ll knuth(int n){
    for (int i = 0; i < n; i++){
        dp[i][i] = 0;
        opt[i][i] = i;
    }
    for (int s = 1; s < n; s++){
        for (int i = 0, j; (i + s) < n; i++){
            j = i + s;
            dp[i][j] = INFLL;
            for (int k = opt[i][j-1]; k < min(j, opt[i+1][j]+1); k++){
                ll cur = dp[i][k] + dp[k+1][j] + C(i, j);
                if (dp[i][j] > cur){
                    dp[i][j] = cur;
                    opt[i][j] = k;
                }
            }
        }
    }
    return dp[0][n-1];
}
```

## 4 Math

### 4.1 Basic Math

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef unsigned long long ull;

ull fastPow(ull base, ull exp, ull mod){
    base %= mod;
    //exp %= phi(mod) if base and mod are relatively prime
    ull ans = 1LL;
    while (exp > 0){
        if (exp & 1LL){
            ans = (ans * (__int128_t)base) % mod;
            base = (base * (__int128_t)base) % mod;
            exp >>= 1;
        }
        return ans;
    }
}

int fastPow(int base, string bigExp, int mod){
    int ans = 1;
    for(char c: bigExp){
        ans = fastPow(ans, 10, mod);
        ans = (ans*1LL*fastPow(base, c-'0', mod))%mod;
    }
    return ans;
}

//\sum_{i=0}^{n-1} floor((a*i+b)/m)
// 0 <= n <= 10^9
// 1 <= m <= 10^9
// 0 <= a, b < m
// O(log(a+b+c+d))
ll floor_sum(ll n, ll m, ll a, ll b){
    ll ans = 0;
    if (a >= m){
        ans += (n-1)*n*(a/m)/2;
        a %= m;
    }
    if (b >= m){
        ans += n*(b/m);
        b %= m;
    }
    ll y_max = (a*n+b)/m, x_max = (y_max*m-b);
    if (y_max == 0) return ans;
    ans += (n-(x_max+a-1)/a)*y_max;
    ans += floor_sum(y_max, a, m, (a-x_max%a)%a);
    return ans;
}

ll gcd(ll a, ll b){ return __gcd(a, b); }
ll lcm(ll a, ll b){ return (a/gcd(a,b))*b; }

void enumeratingAllSubmasks(int mask){
    for (int s = mask; s; s = (s-1)&mask)
        cout << s << endl;
}

//MOD to Hash
namespace ModHash{
```

```

const uint64_t MOD = (1ll<<61) - 1;
uint64_t modmul(uint64_t a, uint64_t b){
    uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (uint32_t)b, h2 = b
        >>32;
    uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
    uint64_t ret = (l&MOD) + (l>>61) + (h << 3) + (m >> 29) + ((m <<
        35) >> 3) + 1;
    ret = (ret & MOD) + (ret>>61);
    ret = (ret & MOD) + (ret>>61);
    return ret-1;
}
};

```

## 4.2 BigInt

```

#include <bits/stdc++.h>
using namespace std;
typedef int32_t intB;
typedef int64_t longB;
typedef vector<intB> vib;
class BigInt{
private:
    vib vb;
    bool neg;
    const int BASE_DIGIT = 9;
    const intB base = 1000000LL*1000; //000LL*1000000LL;
    void fromString(string &s){
        if(s[0] == '-'){
            neg = true;
            s = s.substr(1);
        }else{
            neg = false;
        }
        vb.clear();
        vb.reserve((s.size()+BASE_DIGIT-1)/BASE_DIGIT);
        for(int i=(int)s.length(); i>0; i-=BASE_DIGIT){
            if(i < BASE_DIGIT)
                vb.push_back(stol(s.substr(0, i)));
            else
                vb.push_back(stol(s.substr(i-BASE_DIGIT, BASE_DIGIT)));
        }
        fix(vb);
    }
    void fix(vib &v){
        while(v.size()>1 && v.back()==0)
            v.pop_back();
        if(v.size() == 0)
            neg = false;
    }
    bool comp(vib &a, vib &b){
        fix(a); fix(b);
        if(a.size() != b.size()) return a.size() < b.size();
        for(int i=(int)a.size()-1; i>=0; i--) {
            if(a[i] != b[i]) return a[i] < b[i];
        }
        return false;
    }
    vib sum(vib a, vib b){
        int carry = 0;
        for(size_t i=0; i<max(a.size(), b.size()) or carry; i++){

```

```

            if(i == a.size())
                a.push_back(0);
            a[i] += carry + (i<b.size() ? b[i] : 0);
            carry = (a[i] >= base);
            if(carry) a[i] -= base;
        }
        fix(a);
        return a;
    }
    vib sub(vib a, vib b){
        int carry = 0;
        for(size_t i=0; i<b.size() or carry; i++){
            a[i] -= carry + (i<b.size() ? b[i] : 0);
            carry = a[i] < 0;
            if(carry) a[i] += base;
        }
        fix(a);
        return a;
    }
}

```

```

public:
    BigInt(){}
    BigInt(intB n){
        neg = (n<0);
        vb.push_back(abs(n));
        fix(vb);
    }
    BigInt(string s){
        fromString(s);
    }
    BigInt operator=(BigInt oth){
        this->neg = oth.neg;
        this->vb = oth.vb;
        return *this;
    }
    BigInt operator+(BigInt &oth){
        vib &a = vb, &b = oth.vb;
        BigInt ans;
        if(neg == oth.neg){
            ans.vb = sum(vb, oth.vb);
            ans.neg = neg;
        }else{
            if(comp(a, b)){
                ans.vb = sub(b, a);
                ans.neg = oth.neg;
            }else{
                ans.vb = sub(a, b);
                ans.neg = neg;
            }
        }
        return ans;
    }
    BigInt operator-(BigInt oth){
        oth.neg ^= true;
        return (*this) + oth;
    }
    BigInt operator*(intB b){
        bool negB = false;
        if(b < 0){
            negB = true;
            b = -b;

```

```

    }
    BigInt ans = *this;
    auto &a = ans.vb;
    intB carry = 0;
    for(size_t i=0; i<a.size() or carry; i++){
        if(i == a.size()) a.push_back(0);
        longB cur = carry + a[i] *(longB) b;
        a[i] = intB(cur%base);
        carry = intB(cur/base);
    }
    ans.neg ^= negB;
    fix(ans.vb);
    return ans;
}

BigInt operator *(BigInt &oth){
    BigInt ans;
    auto a = vb, &b = oth.vb, &c = ans.vb;
    c.assign(a.size() + b.size(), 0);
    for(size_t i=0; i<a.size(); i++){
        intB carry=0;
        for(size_t j=0; j<b.size() or carry; j++){
            longB cur = c[i+j] + a[i]*(longB) (j<b.size() ? b[j] : 0);
            cur += carry;
            c[i+j] = intB(cur%base);
            carry = intB(cur/base);
        }
    }
    ans.neg = neg^oth.neg;
    fix(ans.vb);
    return ans;
}

BigInt operator /(intB b){
    bool negB = false;
    if(b < 0){
        negB = true;
        b = -b;
    }
    BigInt ans = *this;
    auto &a = ans.vb;
    intB carry = 0;
    for(int i=(int)a.size()-1; i>=0; i--){
        longB cur = a[i] + (longB)carry * base;
        a[i] = intB(cur/b);
        carry = intB(cur%b);
    }
    ans.neg ^= negB;
    fix(ans.vb);
    return ans;
}

void shiftL(int b){
    vb.resize(vb.size() + b);
    for(int i=(int)vb.size()-1; i>=0; i--) {
        if(i>=b) vb[i] = vb[i-b];
        else vb[i] = 0;
    }
    fix(vb);
}

void shiftR(int b) {
    if((int)vb.size() <= b){
        vb.clear();
        vb.push_back(0);
    }

    return;
}
for(int i=0; i<((int)vb.size() - b); i++)
    vb[i] = vb[i+b];
vb.resize((int)vb.size() - b);
fix(vb);
}

void divide(BigInt a, BigInt b, BigInt &q, BigInt &r){
    BigInt z(0), p(1);
    while(b < a) {
        p.shiftL(max(1, (int)(a.vb.size()-b.vb.size())));
        b.shiftL(max(1, (int)(a.vb.size()-b.vb.size())));
    }
    while(true) {
        while ((a < b) && (z < p)) {
            p = p/10;
            b = b/10;
        }
        if(!(z < p)) break;
        a = a - b;
        q = q + p;
    }
    r = a;
}

BigInt operator /(BigInt &oth){
    BigInt q, r;
    divide(*this, oth, q, r);
    return q;
}

BigInt operator %(BigInt &oth){
    BigInt q, r;
    divide(*this, oth, q, r);
    return r;
}

bool operator <(BigInt &oth){
    BigInt ans = (*this) - oth;
    return ans.neg;
}

bool operator ==(BigInt &oth){
    BigInt ans = (*this) - oth;
    return (ans.vb.size()==1) and (ans.vb.back()==0);
}

friend ostream &operator<<(ostream &out, const BigInt &D){
    if(D.neg)
        out << '-';
    out << (D.vb.empty() ? 0 : D.vb.back());
    for(int i=(int)D.vb.size()-2; i>=0; i--){
        out << setfill('0') << setw(D.BASE_DIGIT) << D.vb[i];
        return out;
    }
}

string to_string(){
    std::stringstream ss;
    ss << (*this);
    return ss.str();
}

friend istream &operator>>(istream &input, BigInt &D) {
    string s;
    input >> s;
    D.fromString(s);
    return input;
}

```



```
};
```

## 4.3 Binomial Coefficients

```
#include <bits/stdc++.h>
#include "../basic_math.h"
#include "../modular.h"
using namespace std;
typedef long long ll;
//O(k)
ll C1(int n, int k){
    ll res = 1LL;
    for (int i = 1; i <= k; ++i)
        res = (res * (n - k + i)) / i;
    return res;
}
//O(n^2)
vector<vector<ll>> C2(int maxn, int mod){
    vector<vector<ll>> mat(maxn + 1, vector<ll>(maxn + 1, 0));
    mat[0][0] = 1;
    for (int n = 1; n <= maxn; n++){
        mat[n][0] = mat[n][n] = 1;
        for (int k = 1; k < n; k++)
            mat[n][k] = (mat[n - 1][k - 1] + mat[n - 1][k]) % mod;
    }
    return mat;
}
//O(N)
vector<int> factorial, inv_factorial;
void prevC3(int maxn, int mod){
    factorial.resize(maxn + 1);
    factorial[0] = 1;
    for (int i = 1; i <= maxn; i++)
        factorial[i] = (factorial[i - 1] * 1LL * i) % mod;
    inv_factorial.resize(maxn + 1);
    inv_factorial[maxn] = fastPow(factorial[maxn], mod - 2, mod);
    for (int i = maxn - 1; i >= 0; i--)
        inv_factorial[i] = (inv_factorial[i + 1] * 1LL * (i + 1)) % mod;
}
int C3(int n, int k, int mod){
    if (n < k)
        return 0;
    return (((factorial[n] * 1LL * inv_factorial[k]) % mod) * 1LL *
            inv_factorial[n - k]) % mod;
}
//O(P*log(P))
//C4(n, k, p) = Comb(n, k)%p
vector<int> changeBase(int n, int p){
    vector<int> v;
    while (n > 0){
        v.push_back(n % p);
        n /= p;
    }
    return v;
}
int C4(int n, int k, int p){
    auto vn = changeBase(n, p);
    auto vk = changeBase(k, p);
    int mx = max(vn.size(), vk.size());
    vn.resize(mx, 0);
```

```
vk.resize(mx, 0);
prevC3(p - 1, p);
int ans = 1;
for (int i = 0; i < mx; i++)
    ans = (ans * 1LL * C3(vn[i], vk[i], p)) % p;
return ans;
}
//O(P^k)
//C5(n, k, p, pk) = Comb(n, k)%(p^k)
int fat_p(ll n, int p, int pk){
    vector<int> fat1(pk, 1);
    int res = 1;
    for (int i = 1; i < pk; i++){
        if (i % p == 0)
            fat1[i] = fat1[i - 1];
        else
            fat1[i] = (fat1[i - 1] * 1LL * i) % pk;
    }
    while (n > 1){
        res = (res * 1LL * fastPow(fat1[pk - 1], n / pk, pk)) % pk;
        res = (res * 1LL * fat1[n % pk]) % pk;
        n /= p;
    }
    return res;
}
ll cnt(ll n, int p){
    ll ans = 0;
    while (n > 1){
        ans += n / p;
        n /= p;
    }
    return ans;
}
int C5(ll n, ll k, int p, int pk){
    ll exp = cnt(n, p) - cnt(n - k, p) - cnt(k, p);
    int d = (fat_p(n - k, p, pk) * 1LL * fat_p(k, p, pk)) % pk;
    int ans = (fat_p(n, p, pk) * 1LL * inv(d, pk)) % pk;
    return (ans * 1LL * fastPow(p, exp, pk)) % pk;
}
```

## 4.4 Chinese Remainder Theorem

```
#include <bits/stdc++.h>
#include "extended_euclidean.h"
using namespace std;
typedef long long ll;
namespace CRT{
    inline ll normalize(ll x, ll mod){
        x %= mod;
        if (x < 0)
            x += mod;
        return x;
    }
    ll solve(vector<ll> a, vector<ll> m){
        int n = a.size();
        for (int i = 0; i < n; i++)
            normalize(a[i], m[i]);
        ll ans = a[0];
        ll lcm1 = m[0];
        for (int i = 1; i < n; i++){
```

```

    ll x, y;
    ll g = extGcd(lcm1, m[i], x, y);
    if ((a[i] - ans) % g != 0)
        return -1;
    ans = normalize(ans + (((a[i] - ans) / g) * x) % (m[i] / g)) *
        lcm1, (lcm1 / g) * m[i]);
    lcm1 = (lcm1 / g) * m[i]; //lcm(lcm1, m[i]);
}
return ans;
}
// namespace CRT

```

## 4.5 Determinant

```

#include <bits/stdc++.h>
using namespace std;
typedef long double ld;
const ld EPS = 1e-9;
ld determinant(vector<vector<ld>> a) {
    int n = a.size();
    ld det = 1;
    for(int i=0; i<n; i++) {
        int b = i;
        for(int j=i+1; j<n; j++)
            if(abs(a[j][i]) > abs(a[b][i]))
                b = j;
        if(abs(a[b][i]) < EPS)
            return 0;
        swap(a[i], a[b]);
        if(i != b)
            det = -det;
        det *= a[i][i];
        for(int j=i+1; j<n; ++j)
            a[i][j] /= a[i][i];
        for(int j=0; j<n; ++j)
            if(j != i && abs(a[j][i]) > EPS)
                for(int k=i+1; k<n; k++)
                    a[j][k] -= a[i][k] * a[j][i];
    }
    return det;
}

```

## 4.6 Division Trick

```

#include <bits/stdc++.h>
using namespace std;
using ll = long long;
using pll = pair<ll, ll>;
// O(N)
pll bruteForce(ll n){
    ll ans1 = 0, ans2 = 0;
    for(ll i = 1; i <= n; i++){
        ans1 += n/i;
        ans2 += (n/i)*i; // n - (n mod i);
    }
    return pll(ans1, ans2);
}
ll AP(ll a1, ll an){

```

```

    ll n = (an-a1+1);
    return ((a1+an)*n)/2LL;
}
// O(sqrt(N))
pll divisionTrick(ll n){
    ll ans1 = 0, ans2 = 0;
    for(ll l = 1, r; l <= n; l = r + 1) {
        r = n / (n / l);
        // n / i has the same value for l <= i <= r
        ans1 += (n/l)*(r-l+1);
        ans2 += (n/l)*AP(l, r);
    }
    return pll(ans1, ans2);
}

```

## 4.7 Euler's totient

```

#include <bits/stdc++.h>
using namespace std;
int nthPhi(int n){
    int result = n;
    for (int i = 2; i <= n / i; i++){
        if (n % i == 0){
            while (n % i == 0)
                n /= i;
            result -= result / i;
        }
    }
    if (n > 1)
        result -= result / n;
    return result;
}
vector<int> phiFrom1toN(int n){
    vector<int> vPhi(n + 1);
    vPhi[0] = 0;
    vPhi[1] = 1;
    for (int i = 2; i <= n; i++)
        vPhi[i] = i;
    for (int i = 2; i <= n; i++){
        if (vPhi[i] == i){
            for (int j = i; j <= n; j += i)
                vPhi[j] -= vPhi[j] / i;
        }
    }
    return vPhi;
}

```

## 4.8 Extended Euclidean

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
ll extGcd(ll a, ll b, ll &x, ll &y){
    if (b == 0){
        x = 1, y = 0;
        return a;
    }else{
        ll g = extGcd(b, a % b, y, x);

```

```

    y -= (a / b) * x;
    return g;
}
}
//a*x + b*y = g
//a*(x-(b/g)*k) + b*(y+(a/g)*k) = g
bool dioEq(ll a, ll b, ll c, ll &x0, ll &y0, ll &g){
    g = extGcd(abs(a), abs(b), x0, y0);
    if (c % g) return false;
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}
inline void shift(ll &x, ll &y, ll a, ll b, ll cnt){
    x += cnt * b;
    y -= cnt * a;
}
// a1 + m1*x = a2 + m2*y
// Find the first moment that both are equal
ll findMinimum(ll a1, ll m1, ll a2, ll m2){
    ll a = m1, b = -m2, c = a2 - a1;
    ll x, y, g;
    if (!dioEq(a, b, c, x, y, g))
        return -1;
    a /= g;
    b /= g;
    int sa = a > 0 ? +1 : -1;
    int sb = b > 0 ? +1 : -1;
    shift(x, y, a, b, -x/b);
    if(x < 0)
        shift(x, y, a, b, sb);
    if(y < 0){
        shift(x, y, a, b, y/a);
        if(y < 0)
            shift(x, y, a, b, -sa);
        if(x < 0)
            return -1;
    }
    return a*x*g;
}
ll findAllSolutions(ll a, ll b, ll c, ll minx, ll maxx, ll miny, ll
maxy){
    ll x, y, g;
    if(a==0 or b==0){
        if(a==0 and b==0)
            return (c==0)*(maxx-minx+1)*(maxy-miny+1);
        if(a == 0)
            return (c%b == 0)*(maxx-minx+1)*(miny<=c/b and c/b<=maxy);
        return (c%a == 0)*(minx<=c/a and c/a<=maxx)*(maxy-miny+1);
    }
    if (!dioEq(a, b, c, x, y, g))
        return 0;
    a /= g;
    b /= g;
    int sign_a = a > 0 ? +1 : -1;
    int sign_b = b > 0 ? +1 : -1;
    shift(x, y, a, b, (minx - x) / b);
    if (x < minx)
        shift(x, y, a, b, sign_b);

```

```

    if (x > maxx)
        return 0;
    ll lx1 = x;
    shift(x, y, a, b, (maxx - x) / b);
    if (x > maxx)
        shift(x, y, a, b, -sign_b);
    ll rx1 = x;
    shift(x, y, a, b, -(miny - y) / a);
    if (y < miny)
        shift(x, y, a, b, -sign_a);
    if (y > maxy)
        return 0;
    ll lx2 = x;
    shift(x, y, a, b, -(maxy - y) / a);
    if (y > maxy)
        shift(x, y, a, b, sign_a);
    ll rx2 = x;
    if (lx2 > rx2)
        swap(lx2, rx2);
    ll lx = max(lx1, lx2);
    ll rx = min(rx1, rx2);
    if (lx > rx)
        return 0;
    return (rx - lx) / abs(b) + 1;
}

```

## 4.9 Fraction

```

#include <bits/stdc++.h>
using namespace std;
typedef long long f_type;
//Representation of the a/b
struct Fraction {
    f_type a, b;
    Fraction(f_type _a = 0): a(_a), b(1){}
    Fraction(f_type _a, f_type _b) {
        f_type g = __gcd(_a, _b);
        a = _a/g;
        b = _b/g;
        if(b < 0){
            a = -a;
            b = -b;
        }
    }
    Fraction operator+(Fraction oth) {
        return Fraction(a*oth.b + oth.a*b, b*oth.b);
    }
    Fraction operator-(Fraction oth) {
        return Fraction(a*oth.b - oth.a*b, b*oth.b);
    }
    Fraction operator*(Fraction oth) {
        return Fraction(a*oth.a, b*oth.b);
    }
    Fraction operator/(Fraction oth) {
        return Fraction(a*oth.b, b*oth.a);
    }
    bool operator>=(Fraction oth){
        return ((*this) - oth).a >= 0;
    }
    bool operator==(Fraction oth){

```

```

    return a == oth.a and b == oth.b;
}
operator f_type() {return a/b;}
operator double() {return double(a)/b;}
};

```

## 4.10 FFT

```

#include <bits/stdc++.h>
using namespace std;
struct complex_t {
    double a {0.0}, b {0.0};
    complex_t(){}
    complex_t(double na) : a{na}{}
    complex_t(double na, double nb) : a{na}, b{nb} {}
    const complex_t operator+(const complex_t &c) const {
        return complex_t(a + c.a, b + c.b);
    }
    const complex_t operator-(const complex_t &c) const {
        return complex_t(a - c.a, b - c.b);
    }
    const complex_t operator*(const complex_t &c) const {
        return complex_t(a*c.a - b*c.b, a*c.b + b*c.a);
    }
    const complex_t operator/(const int &c) const {
        return complex_t(a/c, b/c);
    }
};
//using cd = complex<double>;
using cd = complex_t;
const double PI = acos(-1);
void fft(vector<cd> &a, bool invert) {
    int n = a.size();
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;
        if (i < j)
            swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len <= 1) {
        double ang = 2 * PI / len * (invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len) {
            cd w(1);
            for (int j = 0; j < len / 2; j++) {
                cd u = a[i+j], v = a[i+j+len/2] * w;
                a[i+j] = u + v;
                a[i+j+len/2] = u - v;
                w = w * wlen;
            }
        }
    }
    if (invert){
        for (cd &x : a)
            x = x / n;
    }
}
typedef long long ll;

```

```

vector<ll> multiply(vector<int> &a, vector<int> &b) {
    vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 1;
    while(n < int(a.size() + b.size() )
        n <= 1;
    fa.resize(n);
    fb.resize(n);
    fft(fa, false);
    fft(fb, false);
    for (int i = 0; i < n; i++)
        fa[i] = fa[i]*fb[i];
    fft(fa, true);
    vector<ll> result(n);
    for (int i = 0; i < n; i++)
        result[i] = ll(fa[i].a + 0.5);
    return result;
}
vector<ll> scalarProdut(vector<int> t, vector<int> p, bool isCyclic=
    false) {
    int nt = t.size();
    int np = p.size();
    t.resize(nt+np, 0);
    reverse(p.begin(), p.end());
    if(isCyclic)
        for(int i=nt; i<nt+np; i++)
            t[i] = t[i%nt];
    vector<ll> ans = multiply(t, p);
    for(int i=0; i<nt; i++)
        ans[i] = ans[np-1+i];
    ans.resize(nt);
    return ans;
}
inline int getID(char c){
    return c - 'a';
}
// Find p in text t. Wildcard character *
vector<bool> stringMatchingWithWildcards(string t, string p){
    int nt = t.size();
    int np = p.size();
    vector<cd> fa(nt), fb(np);
    for(int i=0; i<nt; i++){
        double apha = (2*PI*getID(t[i]))/26;
        fa[i] = cd(cos(apha), sin(apha));
    }
    reverse(p.begin(), p.end());
    int k = 0;
    for(int i=0; i<np; i++){
        if(p[i] != '*'){
            double apha = (2*PI*getID(p[i]))/26;
            fb[i] = cd(cos(apha), -sin(apha));
            k++;
        }else{
            fb[i] = cd(0, 0);
        }
    }
    int n = 1;
    while(n < int(nt + np) )
        n <= 1;
    fa.resize(n);
    fb.resize(n);
    fft(fa, false);

```

```
fft(fb, false);
for (int i = 0; i < n; i++)
    fa[i] = fa[i]*fb[i];
fft(fa, true);
vector<bool> result(nt - np+1);
for (int i = 0; i < (nt - np+1); i++)
    result[i] = (int)(fa[np-1+i].a + 1e-9) == k);
return result;
}
```

## 4.11 Floyd Cycle Finding

```
#include <bits/stdc++.h>
using namespace std;
int f(int x);
typedef pair<int, int> pii;
pii floydCycleFinding(int x0){
    int tortoise = f(x0), hare = f(f(x0));
    while(tortoise != hare){
        tortoise = f(tortoise);
        hare = f(f(hare));
    }
    int mu = 0;
    hare = x0;
    while(tortoise != hare){
        tortoise = f(tortoise);
        hare = f(hare);
        mu++;
    }
    int lambda = 1;
    hare = f(tortoise);
    while(tortoise != hare){
        hare = f(hare);
        lambda++;
    }
    return pii(mu, lambda);
}
```

## 4.12 Function Root Using Newton

```
#include <bits/stdc++.h>
using namespace std;
typedef long double ld;
struct Poly{
    vector<ld> v;
    Poly(vector<ld> &v1):v(v1){}
    //return f(x)
    ld f(ld x){
        ld ans = 0;
        ld e = 1;
        int n = v.size();
        for(int i=0; i<n; i++){
            ans += v[i] * e;
            e *= x;
        }
        return ans;
    }
    //return f'(x)
```

```
ld df(ld x){
    ld ans = 0;
    ld e = 1;
    int n = v.size();
    for(int i=1; i<n; i++){
        ans += i * v[i] * e;
        e *= x;
    }
    return ans;
}
// takes some root of the polynomial
ld root(ld x0=1){
    const ld eps = 1E-10;
    ld x = x0;
    for (;;) {
        ld nx = x - (f(x)/df(x));
        if (abs(x - nx) < eps)
            break;
        x = nx;
    }
    return x;
}
//div f(x) by (x-a)
void div(ld a){
    int g = (int)v.size() - 1;
    vector<ld> aux(g);
    for(int i=g; i>=1; i--){
        aux[i-1] = v[i];
        v[i-1] += a*aux[i-1];
    }
    v = aux;
}
};
```

## 4.13 Gauss

```
#include <bits/stdc++.h>
using namespace std;
const int INF = 0x3f3f3f3f;
typedef long double ld;
const ld EPS = 1e-9;
int gauss(vector<vector<ld>> a, vector<ld> &ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;
    vector<int> where(m, -1);
    for (int col=0, row=0; col<m && row<n; col++) {
        int sel = row;
        for (int i=row; i<n; i++)
            if (abs(a[i][col]) > abs(a[sel][col]))
                sel = i;
        if (abs(a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; i++)
            swap(a[sel][i], a[row][i]);
        where[col] = row;
        for (int i=0; i<n; i++){
            if (i != row) {
                ld c = a[i][col] / a[row][col];
                for (int j=col; j<=m; j++)
                    a[i][j] -= a[row][j] * c;
            }
        }
    }
    ans.resize(n);
    for (int i=0; i<n; i++)
        ans[i] = a[i][m];
    return 1;
}
```

```

    }
}
row++;
}
ans.assign(m, 0);
for (int i=0; i<m; i++)
    if (where[i] != -1)
        ans[i] = a[where[i]][m] / a[where[i]][i];
for (int i=0; i<n; i++) {
    ld sum = 0;
    for (int j=0; j<m; j++)
        sum += ans[j] * a[i][j];
    if (abs (sum - a[i][m]) > EPS)
        return 0;
}
for (int i=0; i<m; i++)
    if (where[i] == -1)
        return INF;
return 1;
}

```

## 4.14 Gauss Xor

```

#include <bits/stdc++.h>
using namespace std;
const int MAXB = 30;
struct GaussXOR {
    int table[MAXB];
    GaussXOR() {
        for(int i = 0; i < MAXB; i++) {
            table[i] = 0;
        }
    }
    int size() {
        int ans = 0;
        for(int i = 0; i < MAXB; i++) {
            if(table[i]) ans++;
        }
        return ans;
    }
    bool isComb(int x) {
        for(int i = MAXB-1; i >= 0; i--) {
            x = std::min(x, x ^ table[i]);
        }
        return x == 0;
    }
    void add(int x) {
        for(int i = MAXB-1; i >= 0; i--) {
            if((table[i] == 0) and ((x>>i) & 1)){
                table[i] = x;
                x = 0;
            } else {
                x = std::min(x, x ^ table[i]);
            }
        }
    }
    int max() {
        int ans = 0;
        for(int i = MAXB-1; i >= 0; i--) {
            ans = std::max(ans, ans ^ table[i]);
        }
    }
}

```

```

    }
    return ans;
}
};

```

## 4.15 Gray Code

```

int grayCode(int nth){
    return nth ^ (nth >> 1);
}
int revGrayCode(int g){
    int nth = 0;
    for (; g > 0; g >= 1)
        nth ^= g;
    return nth;
}

```

## 4.16 Matrix

```

#include <bits/stdc++.h>
#include "modular.h"
using namespace std;
const int D = 3;
struct Matrix{
    int m[D][D];
    Matrix(bool identify = false){
        memset(m, 0, sizeof(m));
        for (int i = 0; i < D; i++)
            m[i][i] = identify;
    }
    Matrix(vector<vector<int>> mat){
        for(int i=0; i<D; i++)
            for(int j=0; j<D; j++)
                m[i][j] = mat[i][j];
    }
    int * operator[] (int pos){
        return m[pos];
    }
    Matrix operator*(Matrix oth){
        Matrix ans;
        for (int i = 0; i < D; i++){
            for (int j = 0; j < D; j++){
                int &sum = ans[i][j];
                for (int k = 0; k < D; k++)
                    sum = modSum(sum, modMul(m[i][k], oth[k][j]));
            }
        }
        return ans;
    }
};
Matrix fastPow(Matrix base, ll exp){
    Matrix ans(true);
    while(exp){
        if(exp&1LL)
            ans = ans * base;
        base = base*base;
        exp>>=1;
    }
}

```

```
    return ans;
}
```

## 4.17 Modular Arithmetic

```
#include <bits/stdc++.h>
#include "extended_euclidean.h"
using namespace std;
const int MOD = 1000000007;
inline int modSum(int a, int b, int mod = MOD) {
    int ans = a+b;
    if(ans >= mod) ans -= mod;
    return ans;
}
inline int modSub(int a, int b, int mod = MOD) {
    int ans = a-b;
    if(ans < 0) ans += mod;
    return ans;
}
inline int modMul(int a, int b, int mod = MOD) {
    return (a*1LL*b)%mod;
}
int inv(int a, int mod=MOD) {
    assert(a > 0);
    ll inv_x, y;
    extGcd(a, mod, inv_x, y);
    return (inv_x%mod + mod)%mod;
}
int modDiv(int a, int b, int mod = MOD) {
    return modMul(a, inv(b, mod));
}
```

## 4.18 Modular Integer

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int MOD = 1e9 + 7;
struct modInt {
    int val;
    modInt(ll v = 0) {
        if (v < 0)
            v = (v % MOD) + MOD;
        if (v >= MOD)
            v %= MOD;
        val = v;
    }
    explicit operator int() const {
        return val;
    }
    modInt operator+(const modInt &oth) {
        int ans = val + oth.val;
        if (ans >= MOD)
            ans -= MOD;
        return modInt(ans);
    }
    modInt operator-(const modInt &oth) {
        int ans = val - oth.val;
```

```
        if (ans < 0) ans += MOD;
        return ans;
    }
    modInt operator*(const modInt &oth) {
        return ((uint64_t) val * oth.val) % MOD;
    }
    modInt operator-() const {
        return (val == 0) ? 0 : MOD - val;
    }
    bool operator==(const modInt &oth) const {
        return val == oth.val;
    }
    bool operator!=(const modInt &oth) const {
        return val != oth.val;
    }
    static int modInv(int a, int m = MOD) {
        int g = m, r = a, x = 0, y = 1;
        while (r != 0) {
            int q = g / r;
            g %= r; swap(g, r);
            x -= q * y; swap(x, y);
        }
        return x < 0 ? x + m : x;
    }
    modInt inv() const {
        return modInv(val);
    }
    modInt operator/(const modInt &oth) {
        return (*this) * oth.inv();
    }
    modInt pow(long long p) const {
        assert(p >= 0);
        modInt a = *this, result = 1;
        while (p > 0) {
            if (p & 1)
                result = result * a;
            a = a * a;
            p >>= 1;
        }
        return result;
    }
};
```

## 4.19 Montgomery Multiplication

```
#include <bits/stdc++.h>
using namespace std;
using u64 = uint64_t;
using u128 = __uint128_t;
using i128 = __int128_t;
struct u256 {
    u128 high, low;
    static u256 mult(u128 x, u128 y) {
        u64 a = x >> 64, b = x;
        u64 c = y >> 64, d = y;
        u128 ac = (u128)a * c;
        u128 ad = (u128)a * d;
        u128 bc = (u128)b * c;
        u128 bd = (u128)b * d;
        u128 carry = (u128)(u64)ad + (u128)(u64)bc + (bd >> 64u);
```

```

    u128 high = ac + (ad >> 64u) + (bc >> 64u) + (carry >> 64u);
    u128 low = (ad << 64u) + (bc << 64u) + bd;
    return {high, low};
}
};
//x_m := x*r mod n
struct Montgomery{
    u128 mod, inv, r2;
    //the N will be an odd number
    Montgomery(u128 n) : mod(n), inv(1), r2(-n % n){
        for (int i = 0; i < 7; i++){
            inv *= 2 - n * inv;
        }
        for (int i = 0; i < 4; i++){
            r2 <<= 1;
            if (r2 >= mod)
                r2 -= mod;
        }
        for (int i = 0; i < 5; i++){
            r2 = mult(r2, r2);
        }
    }
    u128 init(u128 x){
        return mult(x, r2);
    }
    u128 reduce(u256 x){
        u128 q = x.low * inv;
        i128 a = x.high - u256::mult(q, mod).high;
        if (a < 0)
            a += mod;
        return a;
    }
    u128 mult(u128 a, u128 b){
        return reduce(u256::mult(a, b));
    }
};

```

## 4.20 NTT

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int MOD = 998244353;
inline int modMul(int a, int b) {
    return (int) ((a*(ll)b) % MOD);
}
namespace ntt {
    int base = 1;
    vector<int> roots = {0, 1};
    vector<int> rev = {0, 1};
    int max_base = -1;
    int root = -1;
    inline int power(int a, long long b) {
        int res = 1;
        while (b > 0) {
            if (b & 1)
                res = modMul(res, a);
            a = modMul(a, a);
            b >>= 1;
        }
        return res;
    }
}

```

```

inline int inv(int a) {
    a %= MOD;
    if (a < 0) a += MOD;
    int b = MOD, u = 0, v = 1;
    while(a){
        int t = b / a;
        b -= t * a; swap(a, b);
        u -= t * v; swap(u, v);
    }
    assert(b == 1);
    if (u < 0) u += MOD;
    return u;
}
void init() {
    int tmp = MOD - 1;
    max_base = 0;
    while (tmp % 2 == 0) {
        tmp /= 2;
        max_base++;
    }
    root = 2;
    while (true) {
        if (power(root, 1 << max_base) == 1) {
            if (power(root, 1 << (max_base - 1)) != 1) {
                break;
            }
        }
        root++;
    }
}
void ensure_base(int nbase) {
    if (max_base == -1)
        init();
    if (nbase <= base)
        return;
    assert(nbase <= max_base);
    rev.resize(1 << nbase);
    for (int i = 0; i < (1 << nbase); i++)
        rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (nbase - 1));
    roots.resize(1 << nbase);
    while (base < nbase) {
        int z = power(root, 1 << (max_base - 1 - base));
        for (int i = 1 << (base - 1); i < (1 << base); i++) {
            roots[i << 1] = roots[i];
            roots[(i << 1) + 1] = modMul(roots[i], z);
        }
        base++;
    }
}
void fft(vector<int> &a) {
    int n = (int) a.size();
    assert((n & (n - 1)) == 0);
    int zeros = __builtin_ctz(n);
    ensure_base(zeros);
    int shift = base - zeros;
    for (int i = 0; i < n; i++) {
        if (i < (rev[i] >> shift)) {
            swap(a[i], a[rev[i] >> shift]);
        }
    }
    for (int k = 1; k < n; k <<= 1) {

```



```

    for (int i = 0; i < n; i += 2 * k) {
        for (int j = 0; j < k; j++) {
            int x = a[i + j];
            int y = modMul(a[i + j + k], roots[j + k]);
            a[i + j] = x + y - MOD;
            if (a[i + j] < 0) a[i + j] += MOD;
            a[i + j + k] = x - y + MOD;
            if (a[i + j + k] >= MOD) a[i + j + k] -= MOD;
        }
    }
}

vector<int> multiply(vector<int> a, vector<int> b, int eq = 0) {
    int need = (int) (a.size() + b.size() - 1);
    int nbase = 0;
    while ((1 << nbase) < need) nbase++;
    ensure_base(nbase);
    int sz = 1 << nbase;
    a.resize(sz);
    b.resize(sz);
    fft(a);
    if (eq)
        b = a;
    else
        fft(b);
    int inv_sz = inv(sz);
    for (int i = 0; i < sz; i++)
        a[i] = modMul(modMul(a[i], b[i]), inv_sz);
    reverse(a.begin() + 1, a.end());
    fft(a);
    a.resize(need);
    return a;
}

vector<int> square(vector<int> a) {
    return multiply(a, a, 1);
}

vector<int> pow(vector<int> a, ll e){
    int need = (int) ((a.size()-1)*e + 1);
    int nbase = 0;
    while ((1 << nbase) < need) nbase++;
    ensure_base(nbase);
    int sz = 1 << nbase;
    a.resize(sz);
    fft(a);
    int inv_sz = ntt::inv(sz);
    for (int i = 0; i < sz; i++)
        a[i] = modMul(power(a[i], e), inv_sz);
    reverse(a.begin() + 1, a.end());
    fft(a);
    a.resize(need);
    return a;
}
};

```

## 4.21 Prime Number

```

#include <bits/stdc++.h>
#include "basic_math.h"
using namespace std;
typedef unsigned long long ull;

```

```

ull modMul(ull a, ull b, ull mod){
    return (a * (__uint128_t)b) % mod;
}

bool checkComposite(ull n, ull a, ull d, int s){
    ull x = fastPow(a, d, n);
    if (x == 1 or x == n - 1)
        return false;
    for (int r = 1; r < s; r++){
        x = modMul(x, x, n);
        if (x == n - 1LL)
            return false;
    }
    return true;
};

bool millerRabin(ull n){
    if (n < 2)
        return false;
    int r = 0;
    ull d = n - 1LL;
    while ((d & 1LL) == 0){
        d >>= 1;
        r++;
    }
    for (ull a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}){
        if (n == a)
            return true;
        if (checkComposite(n, a, d, r))
            return false;
    }
    return true;
}

ull pollard(ull n){
    auto f = [n](ull x) { return modMul(x, x, n) + 1; };
    ull x = 0, y = 0, t = 0, prd = 2, i = 1, q;
    while (t++ % 40 || __gcd(prd, n) == 1){
        if (x == y)
            x = ++i, y = f(x);
        if ((q = modMul(prd, max(x, y) - min(x, y), n))
            prd = q;
            x = f(x), y = f(f(y));
        }
        return __gcd(prd, n);
    }
}

vector<ull> factor(ull n){
    if (n == 1)
        return {};
    if (millerRabin(n))
        return {n};
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}

```

## 4.22 Rank Matrix

```

#include <bits/stdc++.h>
using namespace std;
typedef long double ld;
const ld EPS = 1e-9;

```

```

int compute_rank(vector<vector<ld>> A) {
    int n = A.size();
    int m = A[0].size();
    int rank = max(n, m);
    vector<bool> row_selected(n, false);
    for (int i = 0; i < m; ++i) {
        int j;
        for (j = 0; j < n; ++j) {
            if (!row_selected[j] && abs(A[j][i]) > EPS)
                break;
        }
        if (j == n) {
            rank--;
        } else {
            row_selected[j] = true;
            for (int p = i + 1; p < m; p++)
                A[j][p] /= A[j][i];
            for (int k = 0; k < n; k++) {
                if (k != j && abs(A[k][i]) > EPS) {
                    for (int p = i + 1; p < m; p++)
                        A[k][p] -= A[j][p] * A[k][i];
                }
            }
        }
    }
    return rank;
}

```

## 4.23 Simpson Integration

```

#include <bits/stdc++.h>
using namespace std;
double f(double x);
const int N = 1000000;
double simpson_integration(double a, double b) {
    double h = (b - a) / N;
    double s = f(a) + f(b); // a = x_0 and b = x_2n
    for (int i = 1; i <= N - 1; ++i) { // Refer to final Simpson's
        formula
        double x = a + h * i;
        s += f(x) * ((i & 1) ? 4 : 2);
    }
    s *= h / 3;
    return s;
}

```

## 4.24 Sieve And Primes

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
ll ns;
int np;
bitset<10000010> bs;
vector<ll> primes;
void sieve(ll l) {
    ns = l+1;
    bs.set();
}

```

```

primes.clear();
bs[0] = bs[1] = 0;
for (ll i = 2; i < ns; i++) if (bs[i]) {
    for (ll j = i*i; j < ns; j += i)
        bs[j] = 0;
    primes.push_back(i);
}
np = primes.size();
}
bool isPrime(ll n) {
    if (n < ns)
        return bs[n];
    for (ll p: primes) {
        if (p*p > n) break;
        if (n%p == 0)
            return false;
    }
    return true;
}
vector<ll> primeFactors(ll n) {
    vector<ll> factors;
    for (ll p: primes) {
        if (p*p > n) break;
        while (n%p == 0LL) {
            n /= p;
            factors.push_back(p);
        }
    }
    if (n != 1LL) factors.push_back(n);
    return factors;
}
ll numDiv(ll n) {
    ll ans = 1;
    for (ll p: primes) {
        if (p*p > n) break;
        ll f = 0;
        while (n%p == 0LL) {
            n /= p;
            f++;
        }
        ans *= (f+1LL);
    }
    return (n != 1LL) ? 2LL*ans : ans;
}
ll sumDiv(ll n) {
    ll ans = 1;
    for (ll p: primes) {
        if (p*p > n) break;
        ll power = p;
        while (n%p == 0LL) {
            n /= p;
            power *= p;
        }
        ans *= (power - 1LL) / (p - 1LL);
    }
    if (n != 1LL)
        ans *= (n*n - 1LL) / (n - 1LL);
    return ans;
}
int mobius[1000010];
void sieveMobius(ll l) {
}

```

```

sieve(1);
mobius[1] = 1;
for(int i=2; i<=1; i++)
    mobius[i] = 0;
for(ll p: primes){
    if(p > 1) break;
    for(ll j = p; j <= 1; j += p){
        if(mobius[j] != -1){
            mobius[j]++;
            if(j%(p*p) == 0)
                mobius[j] = -1;
        }
    }
}
for(int i=2; i<=1; i++){
    if(mobius[i] == -1)
        mobius[i] = 0;
    else if(mobius[i]%2 == 0)
        mobius[i] = 1;
    else
        mobius[i] = -1;
}
}

```

## 5 Geometry

### 5.1 Basic Geometry

```

#include <bits/stdc++.h>
using namespace std;
#define POINT_DOUBLE
#ifdef POINT_DOUBLE
    // Se necessario, apelar para __float128
    typedef double ftype;
    typedef long double ftLong;
    const double EPS = 1e-9;
    #define eq(a, b) (abs(a - b) < EPS)
    #define lt(a, b) ((a + EPS) < b)
    #define gt(a, b) (a > (b + EPS))
    #define le(a, b) (a < (b + EPS))
    #define ge(a, b) ((a + EPS) > b)
#else
    typedef int32_t ftype;
    typedef int64_t ftLong;
    #define eq(a, b) (a == b)
    #define lt(a, b) (a < b)
    #define gt(a, b) (a > b)
    #define le(a, b) (a <= b)
    #define ge(a, b) (a >= b)
#endif
//Begin Point 2D
struct Point2d{
    ftype x, y;
    Point2d() {}
    Point2d(ftype x1, ftype y1) : x(x1), y(y1) {}
    Point2d operator+(const Point2d &t){
        return Point2d(x + t.x, y + t.y);
    }
}

```

```

Point2d operator-(const Point2d &t){
    return Point2d(x - t.x, y - t.y);
}
Point2d operator*(ftype t){
    return Point2d(x * t, y * t);
}
Point2d operator/(ftype t){
    return Point2d(x / t, y / t);
}
bool operator<(const Point2d &o) const{
    return lt(x, o.x) or (eq(x, o.x) and lt(y, o.y));
}
bool operator==(const Point2d &o) const{
    return eq(x, o.x) and eq(y, o.y);
}
friend std::istream& operator >> (std::istream &is, Point2d &p) {
    return is >> p.x >> p.y;
}
friend std::ostream& operator << (std::ostream &os, const Point2d &p
) {
    return os << p.x << ' ' << p.y;
}
};
ftLong pw2(ftype a){
    return a * (ftLong)a;
}
//Scalar product
ftLong dot(Point2d a, Point2d b){
    return a.x*(ftLong)b.x + a.y*(ftLong)b.y;
}
ftLong norm(Point2d a){
    return dot(a, a);
}
double len(Point2d a){
    return sqrtl(dot(a, a));
}
double dist(Point2d a, Point2d b){
    return len(a - b);
}
//Vector product
ftLong cross(Point2d a, Point2d b){
    return a.x * (ftLong)b.y - a.y * (ftLong)b.x;
}
//Projection size from A to B
double proj(Point2d a, Point2d b){
    return dot(a, b) / len(b);
}
//The angle between A and B
double angle(Point2d a, Point2d b){
    return acos(dot(a, b) / len(a) / len(b));
}
//Left rotation. Angle in radian
Point2d rotateL(Point2d p, double ang){
    return Point2d(p.x * cos(ang) - p.y * sin(ang), p.x * sin(ang) + p.y
        * cos(ang));
}
//90 degree left rotation
Point2d perpL(Point2d a){
    return Point2d(-a.y, a.x);
}
//0-> 1o,2o quadrant, 1-> 3o,4o

```

```

int half(Point2d &p){
    if (gt(p.y, 0) or (eq(p.y, 0) and ge(p.x, 0)))
        return 0;
    else
        return 1;
}
//angle(a) < angle(b)
bool cmpByAngle(Point2d a, Point2d b){
    int ha = half(a), hb = half(b);
    if (ha != hb){
        return ha < hb;
    }else{
        ftLong c = cross(a, b);
        if(eq(c, 0))
            return lt(norm(a), norm(b));
        else
            return gt(c, 0);
    }
}
inline int sgn(ftLong x){
    return ge(x, 0) ? (eq(x, 0) ? 0 : 1) : -1;
}
// -1: angle(a, b) < angle(b, c)
// 0: angle(a, b) = angle(b, c)
// +1: angle(a, b) > angle(b, c)
int cmpAngleBetweenVectors(Point2d a, Point2d b, Point2d c){
    ftLong dotAB = dot(a, b), dotBC = dot(b, c);
    int sgnAB = sgn(dotAB), sgnBC = sgn(dotBC);
    if(sgnAB == sgnBC){
        //Careful with overflow
        ftLong l = pw2(dotAB)*dot(c, c), r = pw2(dotBC)*dot(a, a);
        if(l == r)
            return 0;
        if(sgnAB == 1)
            return gt(l, r)? -1 : +1;
        return lt(l, r)? -1 : +1;
    }else{
        return (sgnAB > sgnBC)? -1 : +1;
    }
}
//Line parameterized: r1 = a1 + d1*t
//This function can be generalized to 3D
Point2d intersect(Point2d a1, Point2d d1, Point2d a2, Point2d d2){
    return a1 + d1 * (cross(a2 - a1, d2) / cross(d1, d2));
}
//Distance between the point(a) and segment(ps1, ps2)
//This function can be generalized to 3D
ftLong distance_point_to_segment(Point2d a, Point2d ps1, Point2d ps2)
{
    if(ps1 == ps2)
        return dist(ps1, a);
    Point2d d = ps2 - ps1;
    ftLong t = max(ftLong(0), min(ftLong(1), ftLong(dot(a-ps1, d)/len(d)
    )));
    Point2d proj = ps1 + Point2d(d.x*t, d.y*t);
    return dist(a, proj);
}
//Distance between the point(a) and line(pl1, pl2)
//This function can be generalized to 3D
double dist(Point2d a, Point2d pl1, Point2d pl2){
    //crs = parallelogram area
    double crs = cross(Point2d(a - pl1), Point2d(pl2 - pl1));
    //h = area/base
    return abs(crs / dist(pl1, pl2));
}
long double area(vector<Point2d> p){
    long double ret = 0;
    for (int i = 2; i < (int)p.size(); i++)
        ret += cross(p[i] - p[0], p[i - 1] - p[0]) / 2.0;
    return abs(ret);
}
ftLong signed_area_parallelogram(Point2d p1, Point2d p2, Point2d p3){
    return cross(p2 - p1, p3 - p2);
}
long double triangle_area(Point2d p1, Point2d p2, Point2d p3){
    return abs(signed_area_parallelogram(p1, p2, p3)) / 2.0;
}
bool pointInTriangle(Point2d a, Point2d b, Point2d c, Point2d p){
    ftLong s1 = abs(cross(b - a, c - a));
    ftLong s2 = abs(cross(a - p, b - p)) + abs(cross(b - p, c - p)) +
        abs(cross(c - p, a - p));
    return eq(s1, s2);
}
bool clockwise(Point2d p1, Point2d p2, Point2d p3){
    return lt(signed_area_parallelogram(p1, p2, p3), 0);
}
bool counter_clockwise(Point2d p1, Point2d p2, Point2d p3){
    return gt(signed_area_parallelogram(p1, p2, p3), 0);
}
//End Point 2D

//Begin Line
ftLong det(ftype a, ftype b, ftype c, ftype d){
    return a * (ftLong)d - b * (ftLong)c;
}
struct Line{
    ftype a, b, c;
    Line() {}
    Line(ftype a1, ftype b1, ftype c1) : a(a1), b(b1), c(c1){
        normalize();
    }
    Line(Point2d p1, Point2d p2){
        a = p1.y - p2.y;
        b = p2.x - p1.x;
        c = -a * p1.x - b * p1.y;
        normalize();
    }
    void normalize(){
#ifdef POINT_DOUBLE
        ftype z = sqrt(pw2(a) + pw2(b));
#else
        ftype z = __gcd(abs(a), __gcd(abs(b), abs(c)));
#endif
        if(eq(z, 0)) return;
        a /= z;
        b /= z;
        c /= z;
        if (lt(a, 0) or (eq(a, 0) and lt(b, 0))){
            a = -a;
            b = -b;
            c = -c;
        }
    }
}

```

```

    }
};
bool intersect(Line m, Line n, Point2d &res){
    ftype zn = det(m.a, m.b, n.a, n.b);
    if (eq(zn, 0))
        return false;
    res.x = -det(m.c, m.b, n.c, n.b) / zn;
    res.y = -det(m.a, m.c, n.a, n.c) / zn;
    return true;
}
bool parallel(Line m, Line n){
    return eq(det(m.a, m.b, n.a, n.b), 0);
}
bool equivalent(Line m, Line n){
    return eq(det(m.a, m.b, n.a, n.b), 0) &&
        eq(det(m.a, m.c, n.a, n.c), 0) &&
        eq(det(m.b, m.c, n.b, n.c), 0);
}
//Distance from a point(x, y) to a line m
double dist(Line m, ftype x, ftype y){
    return abs(m.a * (ftLong)x + m.b * (ftLong)y + m.c) /
        sqrt(m.a * (ftLong)m.a + m.b * (ftLong)m.b);
}
//End Line

//Begin Segment
struct Segment{
    Point2d a, b;
    Segment() {}
    Segment(Point2d a1, Point2d b1) : a(a1), b(b1) {}
};
bool interld(ftype a, ftype b, ftype c, ftype d){
    if (gt(a, b)) swap(a, b);
    if (gt(c, d)) swap(c, d);
    return le(max(a, c), min(b, d));
}
bool check_intersection(Segment s1, Segment s2){
    Point2d a = s1.a, b = s1.b, c = s2.a, d = s2.b;
    if (eq(cross(a - c, d - c), 0) && eq(cross(b - c, d - c), 0))
        return interld(a.x, b.x, c.x, d.x) && interld(a.y, b.y, c.y, d.y);
    return sgn(cross(b - a, c - a)) != sgn(cross(b - a, d - a)) &&
        sgn(cross(d - c, a - c)) != sgn(cross(d - c, b - c));
}
inline bool betw(ftype l, ftype r, ftype x){
    return le(min(l, r), x) and le(x, max(l, r));
}
bool intersect(Segment s1, Segment s2, Segment &ans){
    Point2d a = s1.a, b = s1.b, c = s2.a, d = s2.b;
    if (!interld(a.x, b.x, c.x, d.x) || !interld(a.y, b.y, c.y, d.y))
        return false;
    Line m(a, b);
    Line n(c, d);
    if (parallel(m, n)){
        if (!equivalent(m, n))
            return false;
        if (b < a)
            swap(a, b);
        if (d < c)
            swap(c, d);
        ans = Segment(max(a, c), min(b, d));
        return true;
    }

```

```

    }else{
        Point2d p(0, 0);
        intersect(m, n, p);
        ans = Segment(p, p);
        return betw(a.x, b.x, p.x) && betw(a.y, b.y, p.y) &&
            betw(c.x, d.x, p.x) && betw(c.y, d.y, p.y);
    }
}
//End Segment

//Begin Circle
struct Circle{
    ftype x, y, r;
    Circle() {}
    Circle(ftype x1, ftype y1, ftype r1) : x(x1), y(y1), r(r1){};
};
bool pointInCircle(Circle c, Point2d p){
    return ge(c.r, dist(Point2d(c.x, c.y), p));
}
//CircumCircle of a triangle is a circle that passes through all the
//vertices
Circle circumCircle(Point2d a, Point2d b, Point2d c){
    Point2d u((b - a).y, -((b - a).x));
    Point2d v((c - a).y, -((c - a).x));
    Point2d n = (c - b) * 0.5;
    double t = cross(u, n) / cross(v, u);
    Point2d ct = ((a + c) * 0.5) + (v * t);
    double r = dist(ct, a);
    return Circle(ct.x, ct.y, r);
}
//InCircle is the largest circle contained in the triangle
Circle inCircle(Point2d a, Point2d b, Point2d c){
    double m1 = dist(a, b);
    double m2 = dist(a, c);
    double m3 = dist(b, c);
    Point2d ct = ((c * m1) + (b * m2) + a * (m3)) / (m1 + m2 + m3);
    double sp = 0.5 * (m1 + m2 + m3);
    double r = sqrt(sp * (sp - m1) * (sp - m2) * (sp - m3)) / sp;
    return Circle(ct.x, ct.y, r);
}
//Minimum enclosing circle, O(n)
Circle minimumCircle(vector<Point2d> p){
    random_shuffle(p.begin(), p.end());
    Circle c = Circle(p[0].x, p[0].y, 0.0);
    for (int i = 0; i < (int)p.size(); i++){
        if (pointInCircle(c, p[i]))
            continue;
        c = Circle(p[i].x, p[i].y, 0.0);
        for (int j = 0; j < i; j++){
            if (pointInCircle(c, p[j]))
                continue;
            c = Circle((p[j].x + p[i].x) * 0.5, (p[j].y + p[i].y) * 0.5, 0.5
                * dist(p[j], p[i]));
            for (int k = 0; k < j; k++){
                if (pointInCircle(c, p[k]))
                    continue;
                c = circumCircle(p[j], p[i], p[k]);
            }
        }
    }
    return c;
}

```

```

}
//Return the number of the intersection
int circle_line_intersection(Circle circ, Line line, Point2d &p1,
    Point2d &p2){
    ftLong r = circ.r;
    ftLong a = line.a, b = line.b, c = line.c + line.a * circ.x + line.b
        * circ.y; //take a circle to the (0, 0)
    ftLong x0 = -a * c / (pw2(a) + pw2(b)), y0 = -b * c / (pw2(a) + pw2(
        b)); // (x0, y0) is the shortest distance point of the line
        for (0, 0)
    if (gt(pw2(c), pw2(r) * (pw2(a) + pw2(b)))){
        return 0;
    }
    else if (eq(pw2(c), pw2(r) * (pw2(a) + pw2(b)))){
        p1.x = p2.x = x0 + circ.x;
        p1.y = p2.y = y0 + circ.y;
        return 1;
    }else{
        ftLong d_2 = pw2(r) - pw2(c) / (pw2(a) + pw2(b));
        ftLong mult = sqrt(d_2 / (pw2(a) + pw2(b)));
        p1.x = x0 + b * mult + circ.x;
        p2.x = x0 - b * mult + circ.x;
        p1.y = y0 - a * mult + circ.y;
        p2.y = y0 + a * mult + circ.y;
        return 2;
    }
}
//Return the number of the intersection
int circle_intersection(Circle c1, Circle c2, Point2d &p1, Point2d &p2)
    ){
    if (eq(c1.x, c2.x) and eq(c1.y, c2.y)){
        if (eq(c1.r, c2.r))
            return -1; //INF
        else
            return 0;
    }else{
        Circle circ(0, 0, c1.r);
        Line line;
        line.a = -2 * (c2.x - c1.x);
        line.b = -2 * (c2.y - c1.y);
        line.c = pw2(c2.x - c1.x) + pw2(c2.y - c1.y) + pw2(c1.r) - pw2(c2.
            r);
        int sz = circle_line_intersection(circ, line, p1, p2);
        p1.x += c1.x;
        p2.x += c1.x;
        p1.y += c1.y;
        p2.y += c1.y;
        return sz;
    }
}

bool checkIfTheSegmentIsCompletelyCoveredByCircles(vector<Circle> &vc,
    Segment s){
    Segment s1{
        vector<Point2d> v = {s.a, s.b};
        Line l(s.a, s.b);
        for (Circle c : vc){
            Point2d p1, p2;
            int inter = circle_line_intersection(c, l, p1, p2);
            if (inter >= 1 and betw(s.a.x, s.b.x, p1.x) and betw(s.a.y, s.b.y,
                p1.y))
                v.push_back(p1);

```

```

        if (inter == 2 and betw(s.a.x, s.b.x, p2.x) and betw(s.a.y, s.b.y,
            p2.y))
            v.push_back(p2);
        }
        sort(v.begin(), v.end());
        bool ans = true;
        for (int i = 1; i < (int)v.size(); i++){
            bool has = false;
            for (Circle c : vc){
                if (pointInCircle(c, v[i - 1]) and pointInCircle(c, v[i])){
                    has = true;
                    break;
                }
            }
            ans = ans && has;
        }
        return ans;
    }

void tangents(Point2d c, double r1, double r2, vector<Line> &ans){
    double r = r2 - r1;
    double z = pw2(c.x) + pw2(c.y);
    double d = z - pw2(r);
    if (lt(d, 0))
        return;
    d = sqrt(abs(d));
    Line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
    l.c = r1;
    ans.push_back(l);
}

vector<Line> tangents(Circle a, Circle b){
    vector<Line> ans;
    for (int i = -1; i <= 1; i += 2)
        for (int j = -1; j <= 1; j += 2)
            tangents(Point2d(b.x - a.x, b.y - a.y), a.r * i, b.r * j, ans);
    for (size_t i = 0; i < ans.size(); ++i){
        ans[i].c -= ans[i].a * a.x + ans[i].b * a.y;
        ans[i].normalize();
    }
    return ans;
}
//End Circle

```

## 5.2 Circle Area Union

```

#include "basic_geometry.h"
using namespace std;

const double PI = acos(-1);
pair<double, double> isCC(Circle circ1, Circle circ2){
    Point2d c1(circ1.x, circ1.y), c2(circ2.x, circ2.y);
    double r1 = circ1.r, r2 = circ2.r;
    double d = dist(c1, c2);
    double x1 = c1.x, x2 = c2.x, y1 = c1.y, y2 = c2.y;
    double mid = atan2(y2 - y1, x2 - x1);
    double a = r1, c = r2;
    double t = acos((a * a + d * d - c * c) / (2 * a * d));
    return make_pair(mid - t, mid + t);
}

```

```

}
int testCC(Circle circ1, Circle circ2){
    Point2d c1(circ1.x, circ1.y), c2(circ2.x, circ2.y);
    double r1 = circ1.r, r2 = circ2.r;
    double d = dist(c1, c2);
    if (le(r1 + r2, d))
        return 1; // not intersected or tged
    if (le(r1 + d, r2))
        return 2; // C1 inside C2
    if (le(r2 + d, r1))
        return 3; // C2 inside C1
    return 0; // intersected
}
struct event_t{
    double theta;
    int delta;
    event_t(double t, int d) : theta(t), delta(d) {}
    bool operator<(const event_t &r) const{
        if (fabs(theta - r.theta) < EPS)
            return delta > r.delta;
        return theta < r.theta;
    }
};
vector<event_t> e;
void add(double begin, double end){
    if (begin <= -PI)
        begin += 2 * PI, end += 2 * PI;
    if (end > PI){
        e.push_back(event_t(begin, 1));
        e.push_back(event_t(PI, -1));
        e.push_back(event_t(-PI, 1));
        e.push_back(event_t(end - 2 * PI, -1));
    }else{
        e.push_back(event_t(begin, 1));
        e.push_back(event_t(end, -1));
    }
}
double calc(Point2d c, double r, double a1, double a2){
    double da = a2 - a1;
    double aa = r * r * (da - sin(da)) / 2;
    Point2d p1 = Point2d(cos(a1), sin(a1)) * r + c;
    Point2d p2 = Point2d(cos(a2), sin(a2)) * r + c;
    return cross(p1, p2) / 2 + aa;
}
/* O(n^2 log n), please remove coincided circles first. */
double circle_union(vector<Circle> &vc){
    int n = vc.size();
    for (int i = n - 1; i >= 0; i--){
        if (eq(vc[i].r, 0)){
            swap(vc[i], vc[n - 1]);
            n--;
            continue;
        }
        for (int j = 0; j < i; j++){
            if (eq(vc[i].x, vc[j].x) and eq(vc[i].y, vc[j].y) and eq(vc[i].r,
                vc[j].r)){
                swap(vc[i], vc[n - 1]);
                n--;
            }
        }
    }
}

```

```

if (n == 0)
    return 0;
vc.resize(n);
vector<double> cntarea(2 * n, 0);
for (int c = 0; c < n; c++){
    int cvrcnt = 0;
    e.clear();
    for (int i = 0; i < n; i++){
        if (i != c){
            int r = testCC(vc[c], vc[i]);
            if (r == 2){
                cvrcnt++;
            } else if (r == 0){
                auto paa = isCC(vc[c], vc[i]);
                add(paa.first, paa.second);
            }
        }
    }
    if (e.size() == 0){
        double a = PI * vc[c].r * vc[c].r;
        cntarea[cvrcnt] -= a;
        cntarea[cvrcnt + 1] += a;
    } else {
        e.push_back(event_t(-PI, 1));
        e.push_back(event_t(PI, -2));
        sort(e.begin(), e.end());
        for (int i = 0; i < int(e.size()) - 1; i++){
            cvrcnt += e[i].delta;
            double a = calc(Point2d(vc[c].x, vc[c].y), vc[c].r, e[i].theta,
                e[i + 1].theta);
            cntarea[cvrcnt - 1] -= a;
            cntarea[cvrcnt] += a;
        }
    }
}
double ans = 0;
for(int i=1; i<=n; i++)
    ans += cntarea[i];
return ans;
}

```

### 5.3 Circles to Tree

```

#include <bits/stdc++.h>
using namespace std;
typedef pair<int, int> pii;
struct Circle{
    int x, y, r, id;
    Circle(){}
    Circle(int x1, int y1, int r1, int id1): x(x1), y(y1), r(r1), id(id1)
    {}
};
// a^2 + b^2 == c^2
double findB(double a, double c){
    return sqrt(c*c - a*a);
}
//- There is no intersection between the circles
//- The parent of circle i will be the smallest circle that includes i
namespace CirclesToTree{
    int X = 0;
}

```

```

int n;
vector<Circle> vc;
vector<int> p;
struct SetElement{
    int id;
    int side; //Up:1, Down:-1
    SetElement(int id1, int side1): id(id1), side(side1){};
    double getY(int x = X) const{
        return vc[id].y + side*findB(vc[id].x - x, vc[id].r);
    }
    bool operator <(const SetElement &o) const{
        auto l = getY(), r = o.getY();
        if(abs(l-r)<1e-9)
            return vc[id].r*side < vc[o.id].r*o.side;
        else
            return l < r;
    }
};
long long pw2(int a){
    return a*1LL*a;
}
bool contains(int big, int small){
    if(big == -1 or small == -1) return false;
    Circle &s = vc[small], &b = vc[big];
    if(s.r > b.r) return false;
    return pw2(s.x-b.x) + pw2(s.y-b.y) <= pw2(b.r-s.r);
}
void updateParent(int id, int par){
    if(par != -1 and p[id] == -1) p[id] = par;
}
//Public
vector<vector<int>> solve(vector<Circle> circles){
    vc = circles; n = vc.size();
    p.assign(n, -1);
    vector<vector<int>> adj(n, vector<int>());
    vector<pii> events;
    for(auto c: vc){
        events.emplace_back(c.x-c.r, ~c.id);
        events.emplace_back(c.x+c.r, c.id);
    }
    sort(events.begin(), events.end());
    set<SetElement> st;
    for(auto e: events){
        X = e.first;
        int id = e.second;
        if(id < 0){
            id = ~id;
            auto it = st.lower_bound(SetElement(id, -2));
            if(it != st.end()){
                int id2 = it->id;
                if(contains(id2, id)) updateParent(id, id2);
                if(contains(p[id2], id)) updateParent(id, p[id2]);
            }
            if(it != st.begin()){
                it--;
                int id2 = it->id;
                if(contains(id2, id)) updateParent(id, id2);
                if(contains(p[id2], id)) updateParent(id, p[id2]);
            }
            st.emplace(id, 1);
            st.emplace(id, -1);
        }
    }
}

```

```

        if(p[id] != -1){
            adj[p[id]].push_back(id);
        }
    }else{
        st.erase(SetElement(id, 1));
        st.erase(SetElement(id, -1));
    }
}
return adj;
}
};

```

## 5.4 Count Lattices

```

#include "../code/math/fraction.h"
Fraction f_1 = 1;
//Calculates number of integer points (x,y) such for 0<=x<n and 0<y<=
//floor(k*x+b)
//O(log(N)*log(MAXV))
f_type count_lattices(Fraction k, Fraction b, f_type n) {
    auto fk = (f_type)k;
    auto fb = (f_type)b;
    auto cnt = 0LL;

    if (k >= f_1 || b >= f_1) {
        cnt += (fk * (n - 1) + 2 * fb) * n / 2;
        k = k - Fraction(fk, 1);
        b = b - Fraction(fb, 1);
    }
    auto t = k * Fraction(n, 1) + b;
    auto ft = (f_type)t;
    if (ft >= 1) {
        cnt += count_lattices(f_1 / k, (t - Fraction((f_type)t, 1)) / k, (
            f_type)t);
    }
    return cnt;
}

```

## 5.5 Concave Polygon

```

#include "basic_geometry.h"
const int INSIDE=-1, BOUNDARY=0, OUTSIDE=1;
struct ConcavePolygon{
    vector<Point2d> vp;
    ConcavePolygon(vector<Point2d> aux){
        vp = aux;
    }
    // -1 inside, 0 boundary, 1 outside
    int pointInPolygon(Point2d pt) {
        int n = vp.size(), w = 0;
        for(int i=0; i<n; i++){
            if(pt == vp[i])
                return 0;
            int j = (i+1==n?0:i+1);
            if(vp[i].y == pt.y and vp[j].y == pt.y) {
                if (min(vp[i].x, vp[j].x) <= pt.x and pt.x <= max(vp[i].x, vp[
                    j].x))
                    return 0;
            }
        }
    }
}

```



```

    }else{
        bool below = vp[i].y < pt.y;
        if (below != (vp[j].y < pt.y)) {
            auto orientation = cross(pt-vp[i], vp[j]-vp[i]);
            if (orientation == 0) return 0;
            if (below == (orientation > 0))
                w += below ? 1 : -1;
        }
    }
}
return (w==0?1:-1);
};

```

## 5.6 Convex Hull

```

#include "basic_geometry.h"
using namespace std;
//If accept collinear points then change for <=
bool cw(Point2d a, Point2d b, Point2d c) {
    return lt(cross(b - a, c - b), 0);
}
//If accept collinear points then change for >=
bool ccw(Point2d a, Point2d b, Point2d c) {
    return gt(cross(b - a, c - b), 0);
}
// Returns the points clockwise
vector<Point2d> convex_hull(vector<Point2d> a){
    if (a.size() == 1)
        return a;
    sort(a.begin(), a.end());
    a.erase(unique(a.begin(), a.end()), a.end());
    vector<Point2d> up, down;
    Point2d p1 = a[0], p2 = a.back();
    up.push_back(p1);
    down.push_back(p1);
    for (int i = 1; i < (int)a.size(); i++){
        if ((i == int(a.size() - 1)) || cw(p1, a[i], p2)){
            while (up.size() >= 2 && !cw(up[up.size() - 2], up[up.size() - 1], a[i]))
                up.pop_back();
            up.push_back(a[i]);
        }
        if ((i == int(a.size() - 1)) || ccw(p1, a[i], p2)){
            while (down.size() >= 2 && !ccw(down[down.size() - 2], down[down.size() - 1], a[i]))
                down.pop_back();
            down.push_back(a[i]);
        }
    }
    a.clear();
    for (int i = 0; i < (int)up.size(); i++)
        a.push_back(up[i]);
    for (int i = down.size() - 2; i > 0; i--)
        a.push_back(down[i]);
    return a;
}

```

## 5.7 Convex Hull Trick

```

#include "basic_geometry.h"
using namespace std;
struct LineCHT{
    ftype k, b;
    int id;
    LineCHT() {}
    LineCHT(ftype k, ftype b, int id=-1): k(k), b(b), id(id) {}
};
struct ConvexHullTrick{
    vector<Point2d> hull, vecs;
    ConvexHullTrick(){}
    ConvexHullTrick(vector<LineCHT> v){
        sort(v.begin(), v.end(), [&](LineCHT a, LineCHT b){
            return lt(a.k, b.k);
        });
        for(auto l: v)
            add_line(l.k, l.b);
    }
    //Here we will assume that when linear functions are added, their k
    //only increases and we want to find minimum values.
    void add_line(ftype k, ftype b) {
        Point2d nw(k, b);
        while(!vecs.empty() && lt(dot(vecs.back(), nw - hull.back()), 0))
        {
            hull.pop_back();
            vecs.pop_back();
        }
        if(!hull.empty())
            vecs.push_back(perpL(nw - hull.back()));
        hull.push_back(nw);
    }
    //Find minimum value
    ftLong get(ftype x) {
        Point2d query(x, 1);
        auto it = lower_bound(vecs.begin(), vecs.end(), query, [](Point2d
            a, Point2d b) {
                return gt(cross(a, b), 0);
            });
        return dot(query, hull[it - vecs.begin()]);
    }
};

```

## 5.8 Convex Polygon

```

#include "convex_hull.h"
using namespace std;
//Checks if the point P belongs to the segment AB
bool pointInSegment(Point2d &a, Point2d &b, Point2d &p) {
    if(!eq(cross(a-p, b-p), 0))
        return false;
    return betw(a.x, b.x, p.x) && betw(a.y, b.y, p.y);
}
struct ConvexPolygon{
    vector<Point2d> vp;
    ConvexPolygon(vector<Point2d> aux){
        //The points have to be clockwise
    }
};

```

```

    vp = convex_hull(aux);
}
//O(log(N))
//Accepts points on the edge
bool pointInPolygon(Point2d point){
    if(vp.size() < 3)
        return pointInSegment(vp[0], vp[1], point);
    if(!eq(cross(vp[1]-vp[0], point-vp[0]), 0) and sgn(cross(vp[1]-vp
        [0], point-vp[0])) != sgn(cross(vp[1]-vp[0], vp.back()-vp[0]))
        )
        return false;
    if(!eq(cross(vp.back()-vp[0], point-vp[0]), 0) and sgn(cross(vp.
        back()-vp[0], point-vp[0])) != sgn(cross(vp.back() - vp[0], vp
        [1]-vp[0])) )
        return false;
    if(eq(cross(vp[1]-vp[0], point-vp[0]), 0))
        return ge(norm(vp[1]-vp[0]), norm(point-vp[0]));
    int pos = 1, l = 1, r = vp.size() - 2;
    while(l <= r){
        int mid = (l + r)/2;
        if(le(cross(vp[mid] - vp[0], point - vp[0]), 0)){
            pos = mid;
            l = mid+1;
        }else{
            r = mid-1;
        }
    }
    return pointInTriangle(vp[0], vp[pos], vp[pos+1], point);
}
};

```

## 5.9 Nearest Pair Of Points

```

#include <bits/stdc++.h>
using namespace std;
struct pt {
    long long x, y, id;
    pt(){}
    pt(int _x, int _y, int _id=-1):x(_x), y(_y), id(_id){}
};
namespace NearestPairOfPoints{
    struct cmp_x {
        bool operator()(const pt & a, const pt & b) const {
            return a.x < b.x || (a.x == b.x && a.y < b.y);
        }
    };
    struct cmp_y {
        bool operator()(const pt & a, const pt & b) const {
            return a.y < b.y;
        }
    };
    int n;
    vector<pt> v;
    vector<pt> t;
    double mindist;
    pair<int, int> best_pair;
    void upd_ans(const pt & a, const pt & b) {
        double dist = sqrt((a.x - b.x)*(a.x - b.x) + (a.y - b.y)*(a.y - b.
            y));
        if (dist < mindist) {

```

```

            mindist = dist;
            best_pair = {a.id, b.id};
        }
    }
    void rec(int l, int r) {
        if (r - l <= 3) {
            for (int i = l; i < r; ++i) {
                for (int j = i + 1; j < r; ++j) {
                    upd_ans(v[i], v[j]);
                }
            }
            sort(v.begin() + l, v.begin() + r, cmp_y());
            return;
        }
        int m = (l + r) >> 1;
        int midx = v[m].x;
        rec(l, m);
        rec(m, r);
        merge(v.begin() + l, v.begin() + m, v.begin() + m, v.begin() + r,
            t.begin(), cmp_y());
        copy(t.begin(), t.begin() + r - l, v.begin() + l);
        int tsz = 0;
        for (int i = l; i < r; ++i) {
            if (abs(v[i].x - midx) < mindist) {
                for (int j = tsz - 1; j >= 0 && v[i].y - t[j].y < mindist; --j
                    )
                    upd_ans(v[i], t[j]);
                t[tsz++] = v[i];
            }
        }
    }
    pair<int, int> solve(vector<pt> _v){
        v = _v;
        n = v.size();
        t.resize(n);
        sort(v.begin(), v.end(), cmp_x());
        mindist = 1E20;
        rec(0, n);
        return best_pair;
    }
};

```

## 5.10 Point 3D

```

#include <bits/stdc++.h>
using namespace std;
//#define POINT_DOUBLE
#ifdef POINT_DOUBLE
    typedef double ftype;
    typedef long double ftLong;
    const double EPS = 1e-9;
    #define eq(a, b) (abs(a-b)<EPS)
    #define lt(a, b) ((a+EPS)<b)
    #define gt(a, b) (a>(b+EPS))
    #define le(a, b) (a<(b+EPS))
    #define ge(a, b) ((a+EPS)>b)
#else
    typedef int32_t ftype;
    typedef int64_t ftLong;
    #define eq(a, b) (a==b)

```

```

#define lt(a, b) (a<b)
#define gt(a, b) (a>b)
#define le(a, b) (a<=b)
#define ge(a, b) (a>=b)
#endif
//Point3D
struct Point3d{
    ftype x, y, z;
    Point3d() {}
    Point3d(ftype x, ftype y, ftype z) : x(x), y(y), z(z) {}
    Point3d operator+(Point3d t){
        return Point3d(x + t.x, y + t.y, z + t.z);
    }
    Point3d operator-(Point3d t){
        return Point3d(x - t.x, y - t.y, z - t.z);
    }
    Point3d operator*(ftype t){
        return Point3d(x * t, y * t, z * t);
    }
    Point3d operator/(ftype t){
        return Point3d(x / t, y / t, z / t);
    }
};
ftLong dot(Point3d a, Point3d b){
    return a.x * (ftLong)b.x + a.y * (ftLong)b.y + a.z * (ftLong)b.z;
}
double len(Point3d a){
    return sqrt(dot(a, a));
}
double dist(Point3d a, Point3d b){
    return len(a-b);
}
double proj(Point3d a, Point3d b){
    return dot(a, b) / len(b);
}
//theta -> XY; phi -> ZY;
Point3d toVetor(double theta, double phi, double r){
    return Point3d(r*cos(theta)*sin(phi), r*sin(theta)*sin(phi), r*cos(phi));
}
double getAngleTheta(Point3d p){
    return atan2(p.y, p.x);
}
double getAnglePhi(Point3d p){
    return acos(p.z/len(p));
}
Point3d rotateX(Point3d p, double ang){
    return Point3d(p.x, p.y*cos(ang)-p.z*sin(ang), p.y*sin(ang)+p.z*cos(ang));
}
Point3d rotateY(Point3d p, double ang){
    return Point3d(p.x*cos(ang)+p.z*sin(ang), p.y, -p.x*sin(ang)+p.z*cos(ang));
}
Point3d rotateZ(Point3d p, double ang){
    return Point3d(p.x*cos(ang)-p.y*sin(ang), p.x*sin(ang)+p.y*cos(ang), p.z);
}
//Rotation in relation to the normal axis
Point3d rotateNormal(Point3d v, Point3d n, double ang){
    double theta = getAngleTheta(n);

```

```

double phi = getAnglePhi(n);
v = rotateZ(v, -theta);
v = rotateY(v, -phi);
v = rotateZ(v, ang);
v = rotateY(v, phi);
v = rotateZ(v, theta);
return v;
}
Point3d cross(Point3d a, Point3d b){
    return Point3d(a.y * b.z - a.z * b.y,
        a.z * b.x - a.x * b.z,
        a.x * b.y - a.y * b.x);
}
ftLong triple(Point3d a, Point3d b, Point3d c){
    return dot(a, cross(b, c));
}
Point3d planeIntersect(Point3d a1, Point3d n1, Point3d a2, Point3d n2,
    Point3d a3, Point3d n3){
    Point3d x(n1.x, n2.x, n3.x);
    Point3d y(n1.y, n2.y, n3.y);
    Point3d z(n1.z, n2.z, n3.z);
    Point3d d(dot(a1, n1), dot(a2, n2), dot(a3, n3));
    return Point3d(triple(d, y, z),
        triple(x, d, z),
        triple(x, y, d)) / triple(n1, n2, n3);
}
struct Sphere{
    ftype x, y, z, r;
    Sphere(){}
    Sphere(ftype x, ftype y, ftype z, ftype r):x(x), y(y), z(z), r(r){}
};
//Minimum enclosing Sphere, O(n*70000)
//It is also possible to do with ternary search in the 3 dimensions
Sphere minimumSphere(vector<Point3d> vp){
    Point3d ans(0, 0, 0);
    int n = vp.size();
    for(Point3d p: vp)
        ans = ans + p;
    ans = ans/n;
    double P = 0.1;
    double d = 0, e = 0;
    for(int i = 0; i < 70000; i++){
        int f = 0;
        d = dist(ans, vp[0]);
        for (int j = 1; j < n; j++) {
            e = dist(ans, vp[j]);
            if (d < e) {
                d = e;
                f = j;
            }
        }
        ans = ans + (vp[f]-ans)*P;
        P *= 0.998;
    }
    return Sphere(ans.x, ans.y, ans.z, d);
}

```

## 6 String Algorithms

### 6.1 Aho Corasick

```
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
const int K = 26;
inline int getID(char c){
    return c-'a';
}
namespace Aho{
    struct Vertex {
        int next[K], go[K];
        int leaf = -1; // CAUTION with repeated strings!
        int p = -1, sz, match=-1;
        char pch;
        int suff_link = -1;
        int end_link = -1;
        Vertex(int pl=-1, char chl='$', int szl=0) : p(pl), pch(chl){
            fill(begin(next), end(next), -1);
            fill(begin(go), end(go), -1);
            sz = szl;
        }
    };
    vector<Vertex> trie;
    void init(){
        trie.clear();
        trie.emplace_back();
    }
    int add_string(string const& s, int id=1) {
        int v = 0;
        for (char ch : s) {
            int c = getID(ch);
            if (trie[v].next[c] == -1) {
                trie[v].next[c] = trie.size();
                trie.emplace_back(v, ch, trie[v].sz+1);
            }
            v = trie[v].next[c];
        }
        trie[v].leaf = id;
        return v;
    }
    int go(int v, char ch);
    int get_suff_link(int v) {
        if (trie[v].suff_link == -1) {
            if (v == 0 || trie[v].p == 0)
                trie[v].suff_link = 0;
            else
                trie[v].suff_link = go(get_suff_link(trie[v].p), trie[v].pch);
        }
        return trie[v].suff_link;
    }
    int get_end_link(int v) {
        if (trie[v].end_link == -1) {
            if (v == 0 || trie[v].p == 0){
                trie[v].end_link = 0;
```

```
            }else{
                int suff_link = get_suff_link(v);
                if(trie[suff_link].leaf != -1)
                    trie[v].end_link = suff_link;
                else
                    trie[v].end_link = get_end_link(suff_link);
            }
        }
        return trie[v].end_link;
    }
    int go(int v, char ch) {
        int c = getID(ch);
        if (trie[v].go[c] == -1) {
            if (trie[v].next[c] != -1)
                trie[v].go[c] = trie[v].next[c];
            else
                trie[v].go[c] = (v == 0) ? 0 : go(get_suff_link(v), ch);
        }
        return trie[v].go[c];
    }
};
//Aplication:
typedef pair<int, int> pii;
void addMatch(vector<pii> &ans, int v, int i){
    // This runs at most sqrt(N) times:1+2+3+4+...+sqrt(N)=N
    while(v != 0){
        // The string id is Aho::trie[v].leaf
        ans.emplace_back(i - Aho::trie[v].sz + 1, i);
        v = Aho::get_end_link(v);
    }
}
//Get match positions: O(answer) = O(N * sqrt(N))
vector<pii> whatMatch(string t){
    int state = 0;
    int i=0;
    vector<pii> ans;
    for(char c : t){
        state = Aho::go(state, c);
        if(Aho::trie[state].leaf != -1)
            addMatch(ans, state, i);
        else
            addMatch(ans, Aho::get_end_link(state), i);
        i++;
    }
    sort(ans.begin(), ans.end());
    return ans;
}

int countMatch(int v){
    if(Aho::trie[v].match == -1) {
        if (v == 0 || Aho::trie[v].p == 0){
            if(Aho::trie[v].leaf != -1)
                Aho::trie[v].match = 1;
            else
                Aho::trie[v].match = 0;
        }else{
            if(Aho::trie[v].leaf != -1)
                Aho::trie[v].match = 1 + countMatch(Aho::get_end_link(v));
            else
                Aho::trie[v].match = countMatch(Aho::get_end_link(v));
        }
    }
```

```

    }
    return Aho::trie[v].match;
}
//Get match amount: O(t)
long long matchAmount(string t){
    int state = 0;
    long long ans = 0;
    for(char c : t){
        state = Aho::go(state, c);
        ans += countMatch(state);
    }
    return ans;
}

```

## 6.2 KMP

```

#include <bits/stdc++.h>
using namespace std;
// "abcbcd" is [0,0,0,1,2,3,0]
// "aabaaab" is [0,1,0,1,2,2,3]
vector<int> kmp(string s) {
    int n = (int)s.length();
    // pi[i] is the length of the longest proper prefix of the substring
    // s[0..i] which is also a suffix of this substring.
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i-1];
        while (j > 0 and s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}
//The ans[i] count the amount of occurrence of the prefix s[0..i] in s
vector<int> prefixOccurrences(string &s){
    auto pi = kmp(s);
    int n = pi.size();
    vector<int> ans(n + 1);
    for (int i = 0; i < n; i++)
        ans[pi[i]]++;
    for (int i = n-1; i > 0; i--)
        ans[pi[i-1]] += ans[i];
    for (int i = 1; i <= n; i++)
        ans[i-1] = ans[i] + 1;
    ans.pop_back();
    return ans;
}
int K = 26;
inline int getID(char c){
    return c-'a';
}
vector<vector<int>> computeAutomaton(string s) {
    s += '#';
    int n = s.size();
    vector<int> pi = kmp(s);
    vector<vector<int>> aut(n, vector<int>(26));
    for(int i = 0; i < n; i++){
        for(int c = 0; c < K; c++){

```

```

            if(i > 0 and c != getID(s[i]))
                aut[i][c] = aut[pi[i-1]][c];
            else
                aut[i][c] = i + (c == getID(s[i]));
        }
    }
    return aut;
}

```

## 6.3 Manacher

```

#include <bits/stdc++.h>
using namespace std;
// source: https://github.com/brunomaletta/Biblioteca/blob/master/
//     Codigo/Strings/manacher.cpp
// ret[2*i] = larger size palindrome centered on i
// ret[2*i+1] = larger size palindrome centered on i and i + 1
vector<int> manacher(const string &s) {
    int l = 0, r = -1, n = s.size();
    vector<int> d1(n), d2(n);
    for (int i = 0; i < n; i++) {
        int k = i > r ? 1 : min(d1[l+r-i], r-i);
        while (i+k < n && i-k >= 0 && s[i+k] == s[i-k]) k++;
        d1[i] = k--;
        if (i+k > r) l = i-k, r = i+k;
    }
    l = 0, r = -1;
    for (int i = 0; i < n; i++) {
        int k = i > r ? 0 : min(d2[l+r-i+1], r-i+1); k++;
        while (i+k <= n && i-k >= 0 && s[i+k-1] == s[i-k-1]) k++;
        d2[i] = --k;
        if (i+k-1 > r) l = i-k, r = i+k-1;
    }
    vector<int> ret(2*n-1);
    for (int i = 0; i < n; i++) ret[2*i] = 2*d1[i]-1;
    for (int i = 0; i < n-1; i++) ret[2*i+1] = 2*d2[i+1];
    return ret;
}
struct Palindrome {
    vector<int> man;
    Palindrome(const string &s) : man(manacher(s)) {}
    bool isPalindrome(int i, int j) {
        return man[i+j] >= j-i+1;
    }
};

```

## 6.4 Min Cyclic String

```

#include <bits/stdc++.h>
using namespace std;
string min_cyclic_string(string s){
    s += s;
    int n = s.size();
    int i = 0, ans = 0;
    while (i < n / 2){
        ans = i;
        int j = i + 1, k = i;
        while (j < n && s[k] <= s[j]){

```

```

    if (s[k] < s[j])
        k = i;
    else
        k++;
    j++;
}
while (i <= k)
    i += j - k;
}
return s.substr(ans, n / 2);
}

```

## 6.5 Palindromic Tree

```

#include <bits/stdc++.h>
using namespace std;
const int MAXN = 100010;
typedef long long ll;
namespace eertree{
    struct Node {
        int i, j;
        int sz, suf;
        int to[26]; //Can change to vector<pii>
    };
    Node tree[MAXN];
    int f[MAXN], cnt[MAXN], p[MAXN];
    int currNode, n, len;
    char s[MAXN];
    int newNode(int l, int r){
        Node &no = tree[++n];
        f[n] = p[n] = 0;
        no.i = l, no.j = r;
        no.sz = r-l+1;
        memset(no.to, 0, sizeof(no.to));
        return n;
    }
    void init(){
        n = len = 0;
        newNode(0, -2);
        tree[1].suf = 1;
        newNode(0, -1);
        tree[2].suf = 1;
        currNode = 1;
    }
    int getId(char c){
        return c-'a';
    }
    // O(1) amortized
    void add(char c){
        int tmp = currNode, idx = len++, idC = getId(c);
        s[idx] = c;
        while (true) {
            int sz = tree[tmp].sz;
            if (idx - sz >= 1 and s[idx] == s[idx-sz-1])
                break;
            tmp = tree[tmp].suf;
        }
        if (tree[tmp].to[idC] != 0) {
            currNode = tree[tmp].to[idC];
        } else{

```

```

            currNode = newNode(idx - (tree[tmp].sz + 2) + 1, idx);
            tree[tmp].to[idC] = currNode;
            tmp = tree[tmp].suf;
            if (tree[currNode].sz == 1) {
                tree[currNode].suf = 2;
            } else{
                while (true) {
                    int sz = tree[tmp].sz;
                    if (idx-sz >= 1 and s[idx] == s[idx-sz-1])
                        break;
                    tmp = tree[tmp].suf;
                }
                tree[currNode].suf = tree[tmp].to[idC];
            }
            p[currNode] = p[tree[currNode].suf] + 1;
        }
        f[currNode]++;
    }
    //Returns the total of distinct palindrome substrings
    int size(){
        return n - 2;
    }
    //Returns the number of the suffix that is palindrome. Online.
    int countSuffix(){
        return p[currNode];
    }
    // Calculates the number of equal palindromes and saves in cnt
    // Returns the total of palindrome substrings
    ll precompute(){
        ll ans = 0;
        for(int i=0; i<=n; i++) cnt[i] = f[i];
        for(int i=n; i>=3; i--){
            ans += cnt[i];
            cnt[tree[i].suf] += cnt[i];
        }
        return ans;
    }
    // Call precompute before
    int count(int id){
        return cnt[id];
    }
    //O(N^2)
    /*void show(){
        ll ans = precompute();
        cout << "Total Palindrome Substrings: " << ans << endl;
        cout << "Total of distinct palindrome substrings: " << size() << endl;
        for(int i=3; i <= n; i++)
            cout << s.substr(tree[i].i, tree[i].sz) << ": " << cnt[i] << endl;
    } */
};

```

## 6.6 String Hashing

```

#include <bits/stdc++.h>
using namespace std;
struct StringHashing{
    const uint64_t MOD = (1LL<<61) - 1;
    const int base = 31;

```

```

uint64_t modMul(uint64_t a, uint64_t b){
    uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (uint32_t)b, h2 = b
    >>32;
    uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
    uint64_t ret = (l&MOD) + (l>>61) + (h << 3) + (m >> 29) + ((m <<
    35) >> 3) + 1;
    ret = (ret & MOD) + (ret>>61);
    ret = (ret & MOD) + (ret>>61);
    return ret-1;
}
int getInt(char c){
    return c-'a'+1;
}
vector<uint64_t> hs, p;
//Public:
StringHashing(string s){
    int n = s.size();
    hs.resize(n); p.resize(n);
    p[0] = 1;
    hs[0] = getInt(s[0]);
    for(int i=1; i<n; i++){
        p[i] = modMul(p[i-1], base);
        hs[i] = (modMul(hs[i-1], base) + getInt(s[i]))%MOD;
    }
}
uint64_t getValue(int l, int r){
    if(l > r) return -1;
    uint64_t res = hs[r];
    if(l > 0) res = (res + MOD - modMul(p[r-l+1], hs[l-1]))%MOD;
    return res;
}
};

```

## 6.7 Suffix Automaton

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
struct SuffixAutomaton{
    struct state{
        int len, link, first_pos;
        bool is_clone = false;
        map<char, int> next;
    };
    vector<state> st;
    int sz, last;
    SuffixAutomaton(string s){
        st.resize(2 * s.size() + 10);
        st[0].len = 0;
        st[0].link = -1;
        st[0].is_clone = false;
        sz = 1;
        last = 0;
        for (char c : s)
            insert(c);
        preCompute();
    }
    void insert(char c){
        int cur = sz++;
        st[cur].len = st[last].len + 1;

```

```

        st[cur].first_pos = st[cur].len - 1;
        st[cur].is_clone = false;
        int p = last;
        while (p != -1 && !st[p].next.count(c)){
            st[p].next[c] = cur;
            p = st[p].link;
        }
        if (p == -1){
            st[cur].link = 0;
        }else{
            int q = st[p].next[c];
            if (st[p].len + 1 == st[q].len){
                st[cur].link = q;
            }else{
                int clone = sz++;
                st[clone].len = st[p].len + 1;
                st[clone].next = st[q].next;
                st[clone].link = st[q].link;
                st[clone].first_pos = st[q].first_pos;
                st[clone].is_clone = true;
                while (p != -1 && st[p].next[c] == q){
                    st[p].next[c] = clone;
                    p = st[p].link;
                }
                st[q].link = st[cur].link = clone;
            }
        }
        last = cur;
    }
    string lcs(string s){
        int v = 0, l = 0, best = 0, bestpos = 0;
        for (int i = 0; i < (int)s.size(); i++){
            while (v and !st[v].next.count(s[i])){
                v = st[v].link;
                l = st[v].len;
            }
            if (st[v].next.count(s[i])){
                v = st[v].next[s[i]];
                l++;
            }
            if (l > best){
                best = l;
                bestpos = i;
            }
        }
        return s.substr(bestpos - best + 1, best);
    }
    vector<ll> dp;
    vector<int> cnt;
    ll dfsPre(int s){
        if (dp[s] != -1)
            return dp[s];
        dp[s] = cnt[s]; //Accepts repeated substrings
        //dp[s] = 1; //Does not accept repeated substrings
        for (auto p : st[s].next)
            dp[s] += dfsPre(p.second);
        return dp[s];
    }
    void preCompute(){
        cnt.assign(sz, 0);
        vector<pair<int, int>> v(sz);

```

```

for (int i = 0; i < sz; i++){
    cnt[i] = !st[i].is_clone;
    v[i] = make_pair(st[i].len, i);
}
sort(v.begin(), v.end(), greater<pair<int, int>>());
for (int i = 0; i < sz - 1; i++)
    cnt[st[v[i].second].link] += cnt[v[i].second];
dp.assign(sz, -1);
dfsPre(0);
}
};

```

## 6.8 Suffix Array

```

#include <bits/stdc++.h>
using namespace std;
typedef pair<int, int> pii;
vector<int> sort_cyclic_shifts(string const& s) {
    int n = s.size();
    const int alphabet = 256;
    vector<int> p(n), c(n), cnt(max(alphabet, n), 0);
    for(int i = 0; i < n; i++)
        cnt[s[i]]++;
    for(int i = 1; i < alphabet; i++)
        cnt[i] += cnt[i-1];
    for(int i = 0; i < n; i++)
        p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    int classes = 1;
    for(int i = 1; i < n; i++) {
        if(s[p[i]] != s[p[i-1]])
            classes++;
        c[p[i]] = classes - 1;
    }
    vector<int> pn(n), cn(n);
    for(int h = 0; (1 << h) < n; ++h) {
        //Ordenando pelo second no RadixSort
        int h2 = (1 << h);
        for(int i = 0; i < n; i++){
            pn[i] = p[i] - h2;
            if(pn[i] < 0) pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin() + classes, 0);
        for(int i = 0; i < n; i++)
            cnt[c[p[i]]]++;
        for(int i = 1; i < classes; i++)
            cnt[i] += cnt[i-1];
        for(int i = n-1; i >= 0; i--)
            p[--cnt[c[pn[i]]]] = pn[i];
        cn[p[0]] = 0;
        classes = 1;
        for(int i = 1; i < n; i++){
            pii cur(c[p[i]], c[(p[i] + h2) % n]);
            pii prev(c[p[i-1]], c[(p[i-1] + h2) % n]);
            if(cur != prev)
                ++classes;
            cn[p[i]] = classes - 1;
        }
        c.swap(cn);
    }
}

```

```

return p;
}
// O(N*log(N))
vector<int> sa_construction(string s) {
    s += "$";
    vector<int> sorted_shifts = sort_cyclic_shifts(s);
    sorted_shifts.erase(sorted_shifts.begin());
    return sorted_shifts;
}
// Kasai's algorithm: O(N)
vector<int> lcp_construction(string const& s, vector<int> const& suf)
{
    int n = s.size();
    vector<int> rank(n, 0);
    for(int i = 0; i < n; i++)
        rank[suf[i]] = i;
    int k = 0;
    vector<int> lcp(n-1, 0);
    for(int i = 0; i < n; i++){
        if (rank[i] == n - 1) {
            k = 0; continue;
        }
        int j = suf[rank[i] + 1];
        while (i + k < n && j + k < n && s[i+k] == s[j+k])
            k++;
        lcp[rank[i]] = k;
        if (k) k--;
    }
    return lcp;
}

```

## 6.9 Suffix Tree

```

#include <bits/stdc++.h>
typedef long long ll;
using namespace std;
namespace SuffixTree {
    const int NS = 60; //Number of strings
    const int MAXN = 100010; //Number of letters
    int cn, cd, ns, en = 1, lst;
    string S[NS]; int lastS = -1;
    /* sufn[si][i] no do sufixo S[si][i...] */
    vector<int> sufn[NS];
    struct Node {
        int l, r, si=0;
        int p, suf=0;
        map<char, int> adj;
        Node() : l(0), r(-1){ suf = p = 0; }
        Node(int l1, int r1, int s1, int p1) : l(l1), r(r1), si(s1), p(p1) {}
    };
    inline int len() { return r - l + 1; }
    inline int operator[](int i) { return S[si][l + i]; }
    inline int& operator()(char c) { return adj[c]; }
};
Node t[2*MAXN];
inline int new_node(int l, int r, int s, int p) {
    t[en] = Node(l, r, s, p);
    return en++;
}
void init() {

```



```

t[0] = Node();
cn=0, cd=0, ns=0, en = 1, lst=0;
lastS = -1;
}
//The strings are inserted independently
void add_string(string s, char id='$') {
    assert(id < 'A');
    s += id;
    S[++lastS] = s;
    sufn[lastS].resize(s.size() + 1);
    cn = cd = 0;
    int i = 0; const int n = s.size();
    for(int j = 0; j < n; j++){
        for(; i <= j; i++) {
            if(cd == t[cn].len() && t[cn](s[j]))
                cn = t[cn](s[j]), cd = 0;
            if(cd < t[cn].len() && t[cn][cd] == s[j]) {
                cd++;
                if(j < (int)s.size() - 1) break;
            } else {
                if(i) t[lst].suf = cn;
                for(; i <= j; i++) {
                    sufn[lastS][i] = cn;
                    cn = t[cn].suf;
                }
            }
        } else if(cd == t[cn].len()) {
            sufn[lastS][i] = en;
            if(i) t[lst].suf = en;
            lst = en;
            t[cn](s[j]) = new_node(j, n - 1, lastS, cn);
            cn = t[cn].suf;
            cd = t[cn].len();
        } else {
            int mid = new_node(t[cn].l, t[cn].l + cd - 1, t[cn].si, t[cn].
                p);
            t[t[cn].p](t[cn][0]) = mid;
            if(ns) t[ns].suf = mid;
            if(i) t[lst].suf = en;
            lst = en;
            sufn[lastS][i] = en;
            t[mid](s[j]) = new_node(j, n - 1, lastS, mid);
            t[mid](t[cn][cd]) = cn;
            t[cn].p = mid; t[cn].l += cd;
            cn = t[mid].p;
            int g = cn? j - cd : i + 1;
            cn = t[cn].suf;
            while(g < j && g + t[t[cn](S[lastS][g])].len() <= j)
                cn = t[cn](S[lastS][g]), g += t[cn].len();
            if(g == j)
                ns = 0, t[mid].suf = cn, cd = t[cn].len();
            else
                ns = mid, cn = t[cn](S[lastS][g]), cd = j - g;
        }
    }
}

bool match(string &s, int i=0, int no=0, int iEdge=0){
    if(i == (int)s.size())
        return true;
    if(iEdge == t[no].len()){ //I arrived at the Node

```

```

        if(t[no].adj.count(s[i]))
            return match(s, i+1, t[no].adj[s[i]], 1);
        else
            return false;
    }
    if(t[no][iEdge] == s[i])
        return match(s, i+1, no, iEdge+1);
    return false;
}
};

```

## 6.10 Trie

```

#include <bits/stdc++.h>
using namespace std;
const int K = 26;
inline int getId(char c){
    return c - 'a';
}

struct Vertex {
    int next[K];
    int leaf;
    int count;
    Vertex() {
        fill(begin(next), end(next), -1);
        leaf = 0;
        count = 0;
    }
};

struct Trie{
    vector<Vertex> trie;
    Trie(){
        trie.emplace_back();
    }
    void add(string const& s) {
        int v = 0;
        trie[v].count++;
        for(char ch: s) {
            int c = getId(ch);
            if (trie[v].next[c] == -1) {
                trie[v].next[c] = trie.size();
                trie.emplace_back();
            }
            v = trie[v].next[c];
            trie[v].count++;
        }
        trie[v].leaf++;
    }
    int countStr(string const& s) {
        int v = 0;
        for (char ch : s) {
            int c = getId(ch);
            if (trie[v].next[c] == -1)
                return 0;
            v = trie[v].next[c];
        }
        return trie[v].leaf;
    }
    int countPre(string const& s) {
        int v = 0;

```

```

for (char ch : s) {
    int c = getId(ch);
    if (trie[v].next[c] == -1)
        return 0;
    v = trie[v].next[c];
}
return trie[v].count;
}
bool remove(string const& s) {
    vector<int> rm;
    int v = 0;
    rm.push_back(v);
    for(char ch: s) {
        int c = getId(ch);
        if (trie[v].next[c] == -1)
            return false;
        v = trie[v].next[c];
        rm.push_back(v);
    }
    if(trie[v].leaf > 0){
        trie[v].leaf--;
        for(int x: rm)
            trie[x].count--;
        return true;
    }
    return false;
}
};

```

## 6.11 Z Function

```

#include <bits/stdc++.h>
using namespace std;
// z[i] is the length of the longest common prefix between s[0..(n-1)]
// and the suffix of s[i..(n-1)].
// z[0] is generally not well defined.
// "aaabaab" - [0,2,1,0,2,1,0]
// "abacaba" - [0,0,1,0,3,0,1]
vector<int> z_function(string s) {
    int n = (int) s.length();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; i++){
        if (i <= r)
            z[i] = min (r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            z[i]++;
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
    return z;
}

```

## 7 Miscellaneous

### 7.1 Counting Inversions

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int INF = 0x3f3f3f3f;
// Counting Inversions: O(N*log(N))
ll ci(vector<int> &v){
    int n = v.size();
    ll inv = 0LL;
    if(n==1)
        return 0;
    vector<int> u1, u2;
    for(int i=0; i < n/2; i++)
        u1.push_back(v[i]);
    for(int i=n/2; i < n; i++)
        u2.push_back(v[i]);
    inv += ci(u1);
    inv += ci(u2);
    u1.push_back(INF);
    u2.push_back(INF);
    int ini1=0, ini2=0;
    for(int i=0; i < n; i++){
        if(u1[ini1] <= u2[ini2]){
            v[i] = u1[ini1++];
        }else{
            v[i] = u2[ini2++];
            inv += u1.size() - ini1 - 1;
        }
    }
    return inv;
}

```

### 7.2 Histogram

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
// Largest Rectangular Area in a Histogram
ll histogram(vector<int> v){
    int n = v.size();
    v.push_back(0);
    ll ans = 0;
    stack<int> st;
    for(int i = 0; i<=n; i++){
        while(st.size() && v[st.top()] >= v[i]){
            int idx = st.top(); st.pop();
            int L = st.size() ? st.top() : -1;
            ans = max(ans, (i-L-1) * (ll)v[idx]);
        }
        st.push(i);
    }
    return ans;
}

```

### 7.3 Identify Pattern

```

#include <bits/stdc++.h>
using namespace std;
typedef pair<int, int> pii;

```

```
// Return the pattern of vector in O(N): pair<cycle start, cycle size>
pii identifyPattern(vector<int> v){
    int n = v.size();
    reverse(v.begin(), v.end());
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i-1];
        while (j > 0 and v[i] != v[j])
            j = pi[j-1];
        if (v[i] == v[j])
            j++;
        pi[i] = j;
    }
    tuple<int, int, int> ans(n, 1, n-1);
    for(int i=1; i<=n; i++){
        int p = i - pi[i-1];
        if(p == 0)
            continue;
        int idx = n-i;
        ans = min(ans, {idx+p, p, idx});
    }
    auto [sum, p, idx] = ans;
    return pii(idx, p);
}
```

## 7.4 Kadane 1D and 2D

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
// Largest Sum Contiguous Subarray: O(N)
ll kadane(vector<ll> &v){
    ll ans = 0, bigger = 0;
    for(int i=0; i < (int)v.size(); i++){
        bigger = max(0LL, bigger + v[i]);
        ans = max(ans, bigger);
    }
    return ans;
}
// Largest Sum Submatrix: O(N^3)
ll kadane2d(vector<vector<int>> &mat){
    if(mat.size() == 0) return 0;
    int n = mat.size(), m = mat[0].size();
    ll ans = 0;
    vector<ll> v(m);
    for(int a=0; a<n; a++){
        fill(v.begin(), v.end(), 0);
        for(int b=a; b<n; b++){
            for(int k=0; k<m; k++){
                v[k] += mat[b][k];
            }
            ans = max(ans, kadane(v));
        }
    }
    return ans;
}
ll circularKadane(vector<ll> v){
    ll ans1 = kadane(v);
    ll sum = 0;
    for(int i=0; i < (int)v.size(); i++){
        sum += v[i];
    }
    return ans1 - kadane(v);
}
```

```
v[i] = -v[i];
}
return max(ans1, sum + kadane(v));
}
```

## 7.5 Longest Increasing Subsequence

```
#include <bits/stdc++.h>
using namespace std;
vector<int> lis(vector<int> &v){
    vector<int> st, ans;
    vector<int> pos(v.size()+1), dad(v.size()+1);
    for(int i=0; i < (int)v.size(); i++){
        auto it = lower_bound(st.begin(), st.end(), v[i]); // Do not
        // accept repeated values
        //auto it = upper_bound(st.begin(), st.end(), v[i]); //Accept
        // repeated values
        int p = it-st.begin();
        if(it==st.end())
            st.push_back(v[i]);
        else
            *it = v[i];
        pos[p] = i;
        dad[i] = (p==0)? -1 : pos[p-1];
    }
    int p = pos[st.size() - 1];
    while(p >= 0){
        ans.push_back(v[p]);
        p=dad[p];
    }
    reverse(ans.begin(), ans.end());
    return ans;
}
```

## 7.6 Mo Algorithm

```
#include <bits/stdc++.h>
using namespace std;
const int BLOCK_SIZE = 700;
void remove(int idx);
void add(int idx);
void clearAnswer();
int getAnswer();
struct Query{
    int l, r, idx;
    bool operator<(Query other) const{
        if (l / BLOCK_SIZE != other.l / BLOCK_SIZE)
            return l < other.l;
        return (l / BLOCK_SIZE & 1) ? (r < other.r) : (r > other.r);
    }
};
vector<int> mo_s_algorithm(vector<Query> queries){
    vector<int> answers(queries.size());
    sort(queries.begin(), queries.end());
    clearAnswer();
    int L = 0, R = 0;
    add(0);
    for(Query q : queries){
        if(q.l < L)
            remove(L);
        if(q.r > R)
            add(R);
        answers[q.idx] = getAnswer();
        if(q.l > L)
            add(L);
        L = q.l, R = q.r;
    }
    return answers;
}
```

```

    while(q.l < L) add(--L);
    while(R < q.r) add(++R);
    while(L < q.l) remove(L++);
    while(q.r < R) remove(R--);
    answers[q.idx] = getAnswer();
}
return answers;
}

```

## 7.7 Polyominoes

```

#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
const int MAXP = 10;
typedef pair<int, int> pii;
//This implementation considers the rotations as distinct
//      0, 10, 10+9, 10+9+8...
int pos[11] = {0, 10, 19, 27, 34, 40, 45, 49, 52, 54, 55};
struct Polyominoes{
    pii v[MAXP];
    int64_t id;
    int n;
    Polyominoes(){
        n = 1;
        v[0] = {0, 0};
        normalize();
    }
    pii& operator[](int i){
        return v[i];
    }
    bool add(int a, int b){
        for(int i=0; i<n; i++){
            if(v[i].F == a and v[i].S == b)
                return false;
        }
        v[n++] = pii(a, b);
        normalize();
        return true;
    }
    void normalize(){
        int mnx=100, mny=100;
        for(int i=0; i<n; i++){
            mnx = min(mnx, v[i].F), mny = min(mny, v[i].S);
            id = 0;
            for(int i=0; i<n; i++){
                v[i].F -= mnx, v[i].S -= mny;
                id |= (1LL<<(pos[v[i].F] + v[i].S));
            }
        }
    };
    vector<Polyominoes> polyominoes[MAXP+1];
    int dx[] = {0, 0, -1, 1};
    int dy[] = {-1, 1, 0, 0};
    void buildPolyominoes(int mxN=10){
        for(int i=0; i<=mxN; i++){
            polyominoes[i].clear();
            Polyominoes init;
            queue<Polyominoes> q;
            unordered_set<int64_t> used;

```

```

q.push(init);
used.insert(init.id);
while(!q.empty()){
    Polyominoes u = q.front(); q.pop();
    polyominoes[u.n].push_back(u);
    if(u.n == mxN)
        continue;
    for(int i=0; i<u.n; i++){
        for(int j=0; j<4; j++){
            Polyominoes to = u;
            bool ok = to.add(to[i].F + dx[j], to[i].S + dy[j]);
            if(ok and !used.count(to.id)){
                q.push(to);
                used.insert(to.id);
            }
        }
    }
}
}
}
}

```

## 7.8 Scheduling Jobs

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
struct Job {
    int t, c, idx;
    Job(int t1=0, int c1=0, int i=0):t(t1), c(c1), idx(i){}
};
//Penalty functions fi(t) = c[i]*t
bool cmp1(Job a, Job b){
    return a.c*(ll)b.t > b.c*(ll)a.t;
}
//Penalty functions fi(t) = c[i]*e^(alfa*t)
const double alfa = 2;
const double EPS = 1e-9;
bool cmp2(Job a, Job b){
    return (1 - exp(alfa*a.t))/a.c > (1 - exp(alfa*b.t))/b.c + EPS;
}

```

## 7.9 Sprague Grundy

```

#include <bits/stdc++.h>
using namespace std;
const int MAXN = 1010;
int version;
int used[MAXN];
int mex() {
    for(int i=0; ; ++i)
        if(used[i] != version)
            return i;
}
int g[MAXN];
// Can remove 1, 2 and 3
void grundy() {
    //Base case depends on the problem
    g[0] = 0;
    g[1] = 1;

```

```

g[2] = 2;
//Inductive case
for(int i=3; i<MAXN; i++){
    version++;
    used[g[i-1]] = version;
    used[g[i-2]] = version;
    used[g[i-3]] = version;
    g[i] = mex();
}
}
string solve(vector<int> v){
    grundy();
    int ans = 0;
    for(int x: v)
        ans ^= g[x];
    return (ans != 0) ? "First" : "Second";
}

```

## 8 Theorems and Formulas

### 8.1 Binomial Coefficients

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{k}a^{n-k}b^k + \dots + \binom{n}{n}b^n$$

Pascal's Triangle:  $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

Symmetry rule:  $\binom{n}{k} = \binom{n}{n-k}$

Factoring in:  $\binom{n}{k} = \frac{n}{k} \binom{n-1}{k-1}$

Sum over  $k$ :  $\sum_{k=0}^n \binom{n}{k} = 2^n$

Sum over  $n$ :  $\sum_{m=0}^n \binom{m}{k} = \binom{n+1}{k+1}$

Sum over  $n$  and  $k$ :  $\sum_{k=0}^m \binom{n+k}{k} = \binom{n+m+1}{m}$

Sum of the squares:  $\binom{n}{0}^2 + \binom{n}{1}^2 + \dots + \binom{n}{n}^2 = \binom{2n}{n}$

Weighted sum:  $1\binom{n}{1} + 2\binom{n}{2} + \dots + n\binom{n}{n} = n2^{n-1}$

Connection with the Fibonacci numbers:  $\binom{n}{0} + \binom{n-1}{1} + \dots + \binom{n-k}{k} + \dots + \binom{0}{n} = F_{n+1}$

More formulas:  $\sum_{k=0}^m (-1)^k \cdot \binom{n}{k} = (-1)^m \cdot \binom{n-1}{m}$

### 8.2 Catalan Number

Recursive formula:  $C_0 = C_1 = 1$

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}, n \geq 2$$

Analytical formula:  $C_n = \binom{2n}{n} - \binom{2n}{n-1} = \frac{1}{n+1} \binom{2n}{n}, n \geq 0$

The first few numbers Catalan numbers,  $C_n$  (starting from zero):  
1, 1, 2, 5, 14, 42, 132, 429, 1430, ...

The Catalan number  $C_n$  is the solution for:

- Number of correct bracket sequence consisting of  $n$  opening and  $n$  closing brackets.
- The number of rooted full binary trees with  $n+1$  leaves (vertices are not numbered). A rooted binary tree is full if every vertex has either two children or no children.

- The number of ways to completely parenthesize  $n+1$  factors.
- The number of triangulations of a convex polygon with  $n+2$  sides (i.e. the number of partitions of polygon into disjoint triangles by using the diagonals).
- The number of ways to connect the  $2n$  points on a circle to form  $n$  disjoint chords.
- The number of non-isomorphic full binary trees with  $n$  internal nodes (i.e. nodes having at least one son).
- The number of monotonic lattice paths from point  $(0,0)$  to point  $(n,n)$  in a square lattice of size  $n \times n$ , which do not pass above the main diagonal (i.e. connecting  $(0,0)$  to  $(n,n)$ ).
- Number of permutations of length  $n$  that can be stack sorted (i.e. it can be shown that the rearrangement is stack sorted if and only if there is no such index  $i < j < k$ , such that  $a_k < a_i < a_j$ ).
- The number of non-crossing partitions of a set of  $n$  elements.
- The number of ways to cover the ladder  $1 \dots n$  using  $n$  rectangles (The ladder consists of  $n$  columns, where  $i^{th}$  column has a height  $i$ ).

### 8.3 Euler's Totient

If  $p$  is a prime number:  $\phi(p) = p-1$  and  $\phi(p^k) = p^k - p^{k-1}$

If  $a$  and  $b$  are relatively prime, then:  $\phi(ab) = \phi(a) \cdot \phi(b)$

In general:  $\phi(ab) = \phi(a) \cdot \phi(b) \cdot \frac{\gcd(a,b)}{\phi(\gcd(a,b))}$

This interesting property was established by Gauss:  $\sum_{d|n} \phi(d) = n$ , Here the sum is over all positive divisors  $d$  of  $n$ .

Euler's theorem:  $a^{\phi(m)} \equiv 1 \pmod{m}$ , if  $a$  and  $m$  are relatively prime.

Generalization:  $a^n \equiv a^{\phi(m) + [n \bmod \phi(m)]} \pmod{m}$ , for arbitrary  $a$ ,  $m$  and  $n \geq \log_2(m)$ .

### 8.4 Formulas

Count the number of ways to partition a set of  $n$  labelled objects into  $k$  nonempty labelled subsets.

$$f(n, k) = \sum_{i=0}^k (-1)^i \binom{k}{i} (k-i)^n$$

Stirling Number 2nd: Partitions of an  $n$  element set into  $k$  not-empty set. Or count the number of ways to partition a set of  $n$  labelled objects into  $k$  nonempty unlabelled subsets.

$$S_{2nd}(n, k) = \left\{ \begin{matrix} n \\ k \end{matrix} \right\} = \frac{1}{k!} \sum_{i=0}^k (-1)^i \binom{k}{i} (k-i)^n$$

Euler's formula:  $f = e - v + 2$

Euler's formula to  $n$  Lines or Segment if there is no three lines/segments that contains the same point:  $R = intersects + component - n$

Number of regions in a planar graph:  $R = E - V + C + 1$  where  $C$  is the number of connected components

Given  $a$  and  $b$  co-prime,  $n = a \cdot x + b \cdot y$  where  $x \geq 0$  and  $y \geq 0$ . You are required to find the least value of  $n$ , such that all currency values greater than or equal to  $n$  can be made using any number of coins of denomination  $a$  and  $b$ :  $n = (a-1) * (b-1)$

generalization of the above problem,  $n$  is multiple of  $gcd(a, b)$ :  $n = lcm(a, b) - a - b + gcd(a, b)$

---

## 8.5 Graph

---

## 8.6 Manhattan Distance

Transformation of the manhattan distance to 2 dimensions between  $P_1 = (x_1, y_1)$  and  $P_2 = (x_2, y_2)$ :

$|x_1 - x_2| + |y_1 - y_2| = \max(|A_1 - B_1|, |A_2 - B_2|)$  where  $A = (x_1 + y_1, x_1 - y_1)$  e  $B = (x_2 + y_2, x_2 - y_2)$

Transformation of the manhattan distance to 3 dimensions between  $P_1 = (x_1, y_1, z_1)$  and  $P_2 = (x_2, y_2, z_2)$ :

$|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2| = \max(|A_1 - B_1|, |A_2 - B_2|, |A_3 - B_3|, |A_4 - B_4|)$  where  $A = (x_1 + y_1 + z_1, x_1 + y_1 - z_1, x_1 - y_1 + z_1, -x_1 + y_1 + z_1)$  e  $B = (x_2 + y_2 + z_2, x_2 + y_2 - z_2, x_2 - y_2 + z_2, -x_2 + y_2 + z_2)$

Transformation of the manhattan distance to  $D$  dimensions between  $P_1$  and  $P_2$ :

$isSet(i, x) = 1$  if the  $i$ -th bit is setted in  $x$  and 0 otherwise.

$$A[i] = \sum_{j=0}^{d-1} (-1)^{isSet(j,i)} P_1[j]$$

$$B[i] = \sum_{j=0}^{d-1} (-1)^{isSet(j,i)} P_2[j]$$

$$\sum_{i=0}^{d-1} |P_1[i] - P_2[i]| = \max_{i=0}^{2^d-1} |A_i - B_i|$$


---

## 8.7 Primes

If  $n = p_1^{e_1} \cdot p_2^{e_2} \cdot \dots \cdot p_k^{e_k}$ , then:

Number of divisors is  $d(n) = (e_1 + 1) \cdot (e_2 + 1) \cdot \dots \cdot (e_k + 1)$ .

Sum of divisors is  $\sigma(n) = \frac{p_1^{e_1+1}-1}{p_1-1} \cdot \frac{p_2^{e_2+1}-1}{p_2-1} \cdot \dots \cdot \frac{p_k^{e_k+1}-1}{p_k-1}$

---