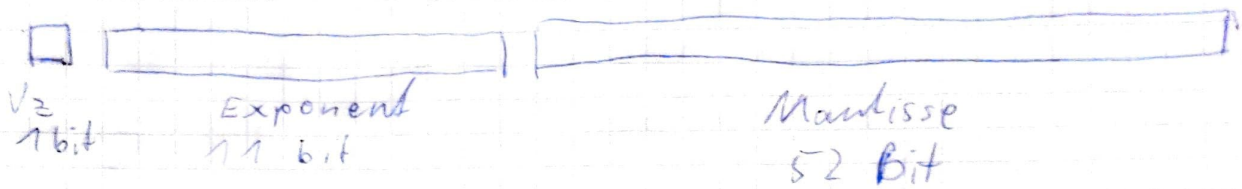
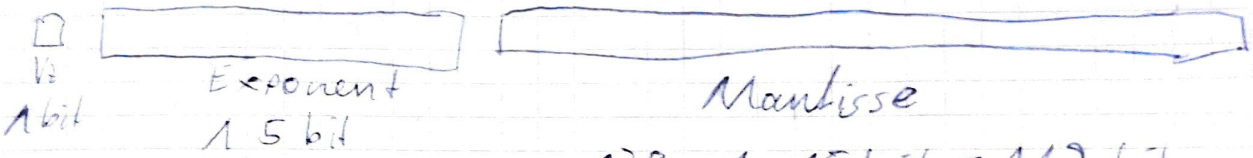
Assignment 1

1.) double precision (64 bit)



quadruple precision



128 − 1 − 15 bit = 112 bit

Machine Epsilon:

$$\varepsilon_{bit} = b^{-(p-1)}$$

b ... Base ≙ 2

p ... precision ≙ 113

$$\varepsilon_{128} = 2^{-(113-1)} = 2^{-112} = 1{,}9259299e{-}34$$

~~Max significant digits (decimal) = 2^{113} = 5.1922974~~

Max significant digits (decimal) = $\pm(2 - 2^{112}) \cdot 2^{16383}$

~~range: Max sig. digits = $\pm(2 - 2^{p-1}) \cdot 2^{offset}$~~

~~$range_{quad} = \pm(2 - 2^{112}) \cdot 2^{16383}$~~

significant digits ≙ $\log_{10}(2^{113}) = 34{,}016389512 \approx 34$

Why is quadruple precision more than twice as accurate?

Precision describes the amount of bits reserved for the ~~mantissa~~ whole floating point word. ~~Doubting the bits means to double the~~ Doubling the precision means to double the AMOUNT of bits. The decimal value of a ~~in~~ bits represented value gets doubled, when adding one bit (e.g.):

$8_{10} = 1000$

$16_{10} = 10000$

Therefore doubling the digits results in a much higher accuracy.

$\varepsilon = b^{-(p-1)}$ $\Rightarrow$ The precision is an exponent for calculating the machine epsilon which describes the error.

2.)

a) $fl(a \text{ op } b) = fl(b \text{ op } a)$

true. ✓

The rounding error of a and b does not change.

b) $fl(a + a) = fl(2 * a)$

true ✓

~~The rounding error that is existing with a gets~~ doubled, no matter what.

"a" before the operation is infinitely precise. ~~It's not~~ It only is rounded after one operation.

c) $fl((a+b)+c) = fl(a+(b+c))$

false. ✗

a+b might produce a different rounding error than b+c. ⇒ leads to a different result.

for a & b: we only have one operation

for c: we have two operations, dependant on each other.

3.)

$$S_n = \sum_{i=1}^{n} x_i$$

$$\hat{s}_1 = fl(x_1) = x_1(1+\delta_1)$$

$$\hat{s}_2 = fl(\hat{s}_1 + x_2) = (\hat{s}_1 + x_2)(1+\delta_2) =$$

$$= (x_1(1+\delta_1) + x_2)(1+\delta_2) =$$

$$= x_1(1+\delta_1)(1+\delta_2) + x_2(1+\delta_2)$$

$$\Downarrow \quad \boxed{1+\delta_i = 1 \pm \delta}$$

$$\hat{s}_2 = x_1(1\pm\delta)^2 + x_2(1\pm\delta)$$

$$\hat{s}_3 = fl(\hat{s}_2 + x_3) = (\hat{s}_2 + x_3)(1\pm\delta) =$$

$$= \underline{x_1(1\pm\delta)^3 + x_2(1\pm\delta)^2 + x_3(1\pm\delta)}$$

Backward-Error

$$\boxed{\prod_{i=1}^{n}(1+\delta_i)^{\rho_i} = 1+\theta_n}$$

↖ rounding errors are independant!

Backward-Error Bound:

$$\boxed{\hat{s}_n = \sum_{i=1}^{n} x_i(1\pm\delta)^{n-i+1}}$$

$$\boxed{\hat{s}_n = \sum_{i=1}^{n} x_i(1+\theta_{n-i+1})}$$

$$\left| \sum_{i=1}^{n} x_i - fl\left(\left(\sum_{i=1}^{n} x_i \right) \right) \right| \leq \gamma_n \sum_{i=1}^{n} |x_i| \leq$$

$$fl(x_1, x_2) = (x_1 + \Delta x_1) + (x_2 + \Delta x_2)$$

$$\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \Delta x_i = \boxed{\sum_{i=1}^{n} x_i + \Delta x_i} \qquad |\Delta x_i| \leq \gamma_n |x_i|$$

forward Error

$$\boxed{\sum_{i=1}^{n} x_i + \gamma_n |x_i|}$$

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 1 & 0 \end{pmatrix}$$