Mechanical wave:

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velocity of sound in medium is given by

$$v = \sqrt{\frac{E}{\rho}} \quad -(i)$$, E = Elasticity of medium
ρ = density of medium

1) Velocity of sound in solid (Thin rod)

$$v = \sqrt{\frac{Y}{\rho}} \quad -(ii)$$, Y = Young's modulus

2) Velocity of sound in liquid & gas

$$v = \sqrt{\frac{B}{\rho}} \quad -(iii)$$, B = Bulk modulus of elasticity

3) Velocity of sound in stretched string.

$$v = \sqrt{\frac{T}{M}} \quad -(iv)$$, T = tension
$M = m/l$ (linear density)

V.IMP Newton's formula for velocity of sound in air.
According to wave mechanics,

$$v = \sqrt{\frac{B}{\rho}} \quad -(i)$$

Newton assumed that, when sound travels through air, temp. of air remains constant. It means ~~that~~ condition of propagation is isothermal.

∴ Under isothermal process, equation of state is given by,

PV = const, where P is pressure & V is volume of air (medium).

∴ diff. both side,

$P \cdot dV + V \cdot dP = 0$

or, $P dV = -V dP$

or, $P = -\frac{V\,dP}{dV}$

or, $P = -\frac{dP}{(dV/V)}$

-ve sign indicates that pressure & volume are in opp. direction

or, $P = B$ $\quad [\because B = -\frac{dP}{(dV/V)}]$

Now,

eq^n (1) becomes

$$\boxed{v = \sqrt{\frac{P}{\rho}}}$$

This is Newton's formula for velocity of sound in air.

$$v = \sqrt{\frac{P}{\rho}}$$

At NTP, $P = 1.013 \times 10^5$ Pa

$\rho = 1.293$ kg/m^3

$$v = \sqrt{\frac{1.013 \times 10^5}{1.293}}$$

$$\approx 280 \text{ m/s}$$

But experimentally, velocity of sound in air observed as 332 m/s. Above calculated value is 16 % less than that of experimental value. The large difference in experimental & theoritical value is due to some discrepancies in Newton's assumption.

Laplace correction:

Laplace pointed out that, Newton's assumption of isothermal process is incorrect. When sound travels in air compression & rarefaction regions occur very rapidly. These regions cannot exchange heat as air is bad conductor of heat. He suggested that condition of propagation of sound in air is adiabatic process.

Under adiabatic process, eq^n of state is

$PV^{\gamma} = \text{const}$, $\gamma = \frac{C_p}{C_v}$

diff. both side

$P \cdot \frac{dV^{\gamma}}{dV} + V^{\gamma} \frac{dP}{dV} = 0$

$\frac{dx^n}{dx} = nx^{n-1}$

or, $P \cdot \gamma V^{\gamma - 1} = -V^{\gamma} \frac{dP}{dV}$

or, $P\gamma = \frac{-V^{\gamma}}{V^{\gamma-1}} \cdot \frac{dP}{dV}$

or, $P\gamma = \frac{-\cancel{V^{\gamma}}}{\cancel{V^{\gamma}} \cdot V^{-1}} \cdot \frac{dP}{dV}$

or, $P\gamma = -V \cdot \frac{dP}{dV}$

or, $P\gamma = -\left(\frac{dP}{dV/V}\right)$

or, $P\gamma = B$

Now eq^n ① becomes

$$\boxed{V = \sqrt{\frac{\gamma P}{\rho}}}$$

At NTP,

$P = 1.013 \times 10^5$ Pa

$\rho = 1.293$ kg/m^3

$\gamma = 1.4$

$V = \sqrt{1.4 \times 280}$

≈ 331 m/s

which is very close to experimental value.))

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Factors affecting velocity of sound in air:

1) Effect of temp.

We know,

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \text{— (i)}$$

$$\text{or, } v = \sqrt{\frac{\gamma P V}{M}} \quad \text{— (ii)}, \quad M = \text{molar mass}$$

For 1-mole of gas

$$PV = RT$$

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$v = k\sqrt{T}$$

$$\boxed{\therefore v \propto \sqrt{T}}$$

$\therefore$ Velocity of sound in air is directly proportional to the sq. root of the temp.

Let v_1 & v_2 be the velocities of sound in air at temp. T_1 & T_2.

$$\boxed{\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}}$$

2) Effect of Pressure

We know,

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \text{— (i)}$$

At const. temp. From Boyle's law,

$$PV = \text{const}$$

$$\text{or, } P\left(\frac{m}{\rho}\right) = \text{const}$$

$$\text{or, } m\left(\frac{P}{\rho}\right) = \text{const}$$

$$\text{or, } \frac{P}{\rho} = \text{const}$$

From eq^n (1)

$v = \text{const}$

Hence, pressure has no effect in velocity of sound in air.

3) Effect of density (only in air)

Let ρ_1 & ρ_2 be the densities of two gases at constant pressure.

$$V_1 = \sqrt{\frac{\gamma P}{\rho_1}} \quad \text{— (1)} \quad , \quad V_2 = \sqrt{\frac{\gamma P}{\rho_2}}$$

$$\text{or, } \frac{V_1}{V_2} = \sqrt{\frac{\cancel{\gamma P}}{\rho_1} \times \frac{\rho_2}{\cancel{\gamma P}}}$$

$$\text{or, } \frac{V_1}{V_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

$$\boxed{\therefore V \propto \frac{1}{\sqrt{\rho}}}$$

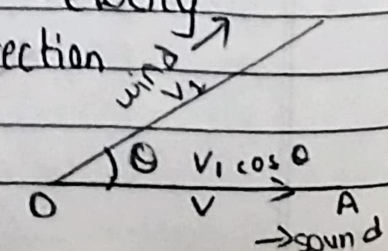
Velocity of sound in air is inversely proportional to the square root of the density.

4) Effect of humidity

Density of water vapour is less than that of dry air. Pressure of water vapour lowers the density of air. As we know, velocity of sound in air is inversely proportional to the square root of the density. So that, sound travels faster in humid air.

5) Effect of direction of wind

Let wind is blowing with velocity V_1 making an angle θ with direction of sound (OA).



Component of velocity of wind along OA = $v_1 \cos\theta$
Resultant velocity of sound along OA

$$v' = v + v_1 \cos\theta$$

Case i) $\theta = 0^\circ$, $\cos 0 = 1$

$$v' = v + v_1 \text{ [max}^m \text{ velocity]}$$

Velocity of sound increases if wind is blowing in the direction of velocity of sound.

Case ii) $\theta = 90^\circ$, $\cos 90^\circ = 0$

$v' = v$, Velocity of sound remains same if wind is blowing perpendicular to the ~~direction~~ velocity of sound.

Case iii) $\theta = 180^\circ$

$v' = v - v_1$, velocity of sound decreases if wind is blowing in opposite direction of velocity of sound.

Numericals:-

1) Calculate the bulk modulus of a liquid in which longitudinal waves with frequency ~~200~~ 250 Hz have the wavelength of 8 m if the density of liquid is 900 kg/m^3.

Soln: Here,

Frequency$(f) = 250$ Hz
Wavelength$(\lambda) = 8$ m
Density of liquid $(\rho) = 900$ kg/m^3

Now,

$v = f\lambda$
$= 250 \times 8$
$= 2000$ m/s

Now,

$v = \sqrt{\frac{B}{\rho}}$

$2000 = \sqrt{\frac{B}{900}}$

$4000000 = \frac{B}{900}$

$\therefore B = 3.6 \times 10^9$ N/m^2

2) A source of frequency 512 Hz emits waves of wavelength 64.5 cm in air at 20°C. What would be the velocity of sound at 0°C?

Soln: Given,

At 20°C,
frequency $(f) = 512$ Hz
Wavelength$(\lambda) = 64.5$ cm $= 64.5 \times 10^{-2}$ m
$v_0 = ?$ $T_0 = 0°C = 273$ K, $T_{20} = 293$ K

We know,

$v_{20} = \lambda \times f$
$= 64.5 \times 10^{-2} \times 512$
$= 330.24$ m/s

Now,

$$\frac{V_0}{V_{20}} = \sqrt{\frac{T_0}{T_{20}}}$$

or, $\frac{V_0}{330.24} = \sqrt{\frac{273}{293}}$

or, $\frac{V_0}{330.24} = 0.965$

or, $V_0 = 0.965 \times 330.24$

$\therefore V_0 = 318.6816$ m/s

3) A source of sound produces a note of ~~50~~ 512 Hz in air at 17°C with wavelength 66.5 cm. Find the ratio of molar heat capacities at constant pressure to constant volume at NTP 0°C.
Density of air & mercury at NTP are 1.293 kg/m³ and 13600 kg/m³ respectively.

Soln: Here,

At 17°C,

frequency (f) = 512 Hz

wavelength (λ) = 66.5 cm = 0.665 m

At NTP, $\gamma = ?$, $\rho_a = 1.293$ kg/m³, $\rho_{Hg} = 13600$ kg/m³

We know that,

$V_{17} = \lambda \times f$

$= 0.665 \times 512$

$= 340.48$ m/s

$T_{17} = 273 + 17 = 290$ K

$$\frac{V_0}{V_{17}} = \sqrt{\frac{T_0}{T_{17}}}$$

or, $\frac{V_0}{340.48} = \sqrt{\frac{273}{290}}$

$\therefore V_0 = 330.34$ m/s

Now,

$P = hdg$

$= \frac{760}{1000} \times 13600 \times 9.8$

$= 1.013 \times 10^5$ Pa

Again,

$v_0 = \sqrt{\frac{\gamma P}{\rho_a}}$

$v_0^2 = \frac{\gamma \times 1.013 \times 10^5}{1.293}$

$(330.34)^2 = \gamma \times 78344.934$

$\therefore \gamma = 1.39 \approx 1.4$

4) At what temperature the velocity of sound in air is increased by 50% to that at 27°C.

Soln: Given,

$T_{27} = 300$ K

$V_{27} = V$ (Let)

$T' = ?$

$v' = V + 50\%$ of V

$= V + \frac{V}{2}$

$= 3V/2$

Now,

$\frac{v'}{V_{27}} = \sqrt{\frac{T'}{T_{27}}}$

$\frac{3 \times \cancel{V}}{2 \times \cancel{V}} = \sqrt{\frac{T'}{300}}$

$\left(\frac{3}{2}\right)^2 = \frac{T'}{300}$

$T' = \frac{9}{4} \times 300$

$= 675$ K #

5) What is the difference betⁿ the speed of longitudinal waves in air at 27°c & −13°c? What is the speed at 0°c?

Solⁿ: Here,

$V_{27} - V_{-13} = ?$

$V_0 = ?$

At 0°c,

$P = 1.013 \times 10^5$ pa

$\rho = 1.293$ kg/m³

$V_0 = \sqrt{\frac{\gamma P}{\rho}}$

$= \sqrt{\frac{1.4 \times 1.013 \times 10^5}{1.293}}$

$= 331.184$ m/s

Now,

$\frac{V_0}{V_{27}} = \sqrt{\frac{T_0}{T_{27}}}$

$\frac{331.184}{V_{27}} = \sqrt{\frac{273}{300}}$

$\frac{331.184}{V_{27}} = 0.953$

$\frac{331.184}{0.953} = V_{27}$

$\therefore V_{27} = 347.517$ m/s

Again, $V_{-13} = ?$

Now, $\frac{V_0}{V_{-13}} = \sqrt{\frac{T_0}{T_{-13}}}$

$\frac{331.184}{V_{-13}} = \sqrt{\frac{273}{260}}$

$\frac{331.184}{V_{-13}} = 1.024$

$\therefore V_{-13} = 323.421$ m/s

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Then, $V_{27} - V_{13} = 347.517 - 323.421$
$= 24.096$ m/s $= 24$ m/s

6) In a stormy day, a person observes a lighting flash which is followed by the thunder 3 sec later. How would you estimate the lighting strike from the person?
Velocity of sound(v) = 340 m/s
velocity of light(c) = 3×10^8 m/s

Sol^n: Given,
velocity of sound (V) = 340 m/s
Velocity of light (c) = 3×10^8 m/s

Now, By the question,

$$t_{sound} - t_{light} = 3$$

$$\frac{d}{v} - \frac{d}{c} = 3$$

$$\text{or, } d\left[\frac{1}{v} - \frac{1}{c}\right] = 3$$

$$\text{or, } d\left[\frac{c - v}{v \times c}\right] = 3$$

$$\text{or, } d\left[\frac{3 \times 10^8 - 340}{340 \times 3 \times 10^8}\right] = 3$$

$$\text{or, } d \times 2.941 \times 10^{-3} = 3$$

$$\therefore d = 1020 \text{ m}$$

7) When a detenoter is exploded in a railway line, an observer standing on the rail 2 km away hears 2 sounds. What is the time interval bet^n them?
Young's modulus (Y) = 2×10^{11} N/mts^2
Density of steel = 8000 kg/m^3

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Density of air = 1.4 kg/m^3
$\gamma_{air} = 1.4$
Atmospheric pressure (P) = 10^5 N/m^2
Soln: Given,
$d = 2000$ m
$Y_{st} = 2 \times 10^{11}$ N/m^2
$\rho_{st} = 8000$ kg/m^3
$\rho_{air} = 1.4$ kg/m^3
$\gamma = 1.4$
$P = 10^5$ Pa
$t_a - t_s = ?$

Now,
Through steel,

$$v_s = \sqrt{\frac{Y}{\rho}}$$

$$= \sqrt{\frac{2 \times 10^{11}}{8000}}$$

$$= 5000 \text{ m/s}$$

$$t_s = \frac{d}{v_s} = \frac{2000}{5000} = 0.4 \text{ sec}$$

Through air,

$$v_a = \sqrt{\frac{\gamma P}{\rho}}$$

$$= \sqrt{\frac{1.4 \times 10^5}{1.4}}$$

$$= 316.227 \text{ m/s}$$

$$t_a = \frac{d}{v_a} = \frac{2000}{316.227} = 6.324 \text{ sec}$$

Now, $t_a - t_s$
$= 6.324 - 0.4$
$= 5.924$ sec

8) A man standing at one end of a closed corrider 57 m long blow a short blast on a whisle. He found that the time from the blast to the sixth echo was 2 sec. If the temperature was 17°C what was the velocity of sound at 0°C?

Solⁿ: Given,

$t = 2$ sec

Distance travelled by sound during 6^{th} echo (d) = $2 \times 57 \times 6$

$= 684$

$V_{17} = \frac{684}{2} = 342$ m/s

$V_0 = ?$, $T_{17} = 290$

Now,

$\frac{V_{17}}{V_0} = \sqrt{\frac{290}{273}}$

$\frac{342}{V_0} = 1.0306$

$\therefore V_0 = (332\ \cancel{342\ m/s})\ 331.82$ m/s

Short Question Answers.

1) Velocity of sound in solids is more than in liquids, why?

→ As we know that, velocity of sound in solid, $V = \sqrt{\frac{Y}{\rho}}$ and velocity of sound in liquid, $V = \sqrt{\frac{B}{\rho}}$. Since, the value of Y (Young's Modulus) is greater than B (Bulk's Modulus), so the ratio of $\sqrt{\frac{Y}{\rho}}$ is greater than $\sqrt{\frac{B}{\rho}}$. Hence, velocity of sound in solids is more than in liquids.

2) When sound waves travel through a medium, does the temp. at various points remain constant? Explain.

→ When sound waves travel through a medium, the temp at various points doesn't remain constant. According to Laplace, when a sound wave travels through a medium, it causes compression & rarefaction in a very rapid way so that there is no chance of exchange of heat. That means propagation of sound in a medium is adiabatic process. In adiabatic process temperature changes compulsorily. Temperature increases in compression & it decreases in rarefaction.

3) Speed of sound in humid air is more than that in dry air, why?

→ As we know, velocity of sound in air is inversely proportional to the square root of the density. The density of humid air is greater than the density ~~o~~ in dry air ~~so~~ the velocity of sound in humid air is less than the velocity of sound in dry air. Hence, speed of sound in humid air is more than that in dry air.

4) Although the density of solid is high, the velocity of sound is greater in solid, explain.

→ As we know, velocity of sound in solid is $v = \sqrt{\frac{Y}{\rho}}$. Since, the value of Young's Modulus (Y) is greater, the ratio of $\frac{Y}{\rho}$ also becomes very much larger.

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so, although the density of solid is high, the velocity of sound is greater in solid.

Good