Simple Harmonic Motion (S.H.M)

1. Introduction
   → Periodic
   → Harmonic or Oscillatory
2. Displacement
3. Velocity
4. Acceleration
5. Simple Pendulum
6. Horizontal Oscillation
7. Vertical Oscillation
8. Energy
9. Angular Harmonic Motion.

Periodic Motion

→ Any body which repeats its motion in regular interval of time then such motion is known as periodic motion.

For e.g:-
- motion of a planet round the sun
- Switch in a clock
- Rotation of fan

Harmonic Motion

→ A body which moves back & fourth or to and fro is called Harmonic Motion

e.g:- Simple Pendulum, Swing, Piston

* Simple Harmonic motion (SHM)

→ A simplest type of motion having simple frequency and a constant amplitude is known as SHM

→ Motion a body is in a back & fouth position where acceleration is directly proportional to displacement.

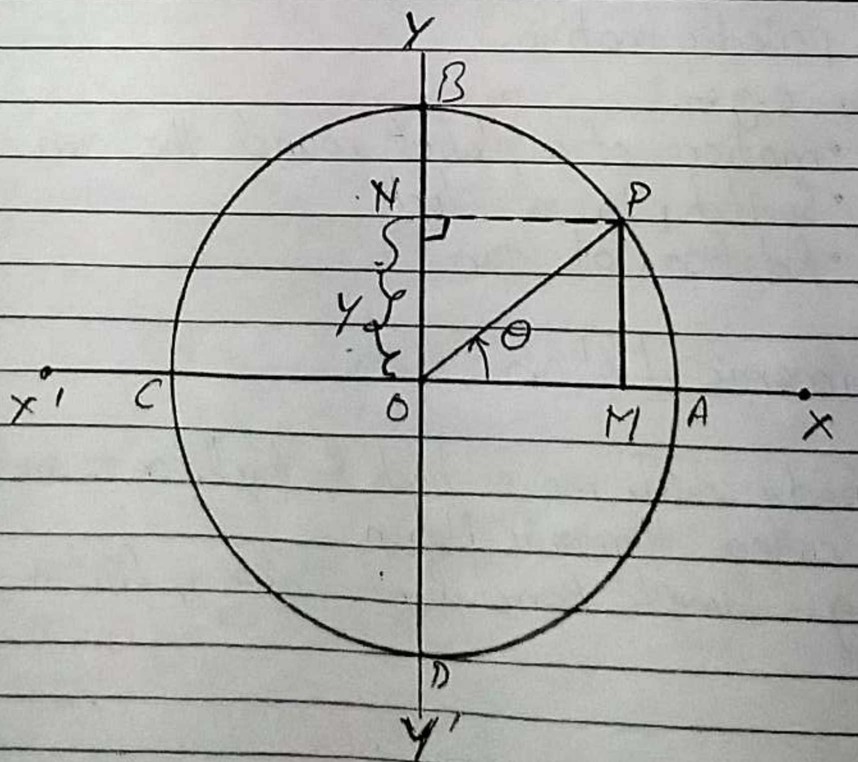
i.e,

acceleration $\propto$ displacement

$a \propto y$

$a = -ky$, where k is a proportionality constant and -ve sign shows that acceleration is in opposite direction of displacement.

* SHM in terms of circular motion



S.H.M in Graphical Representation.

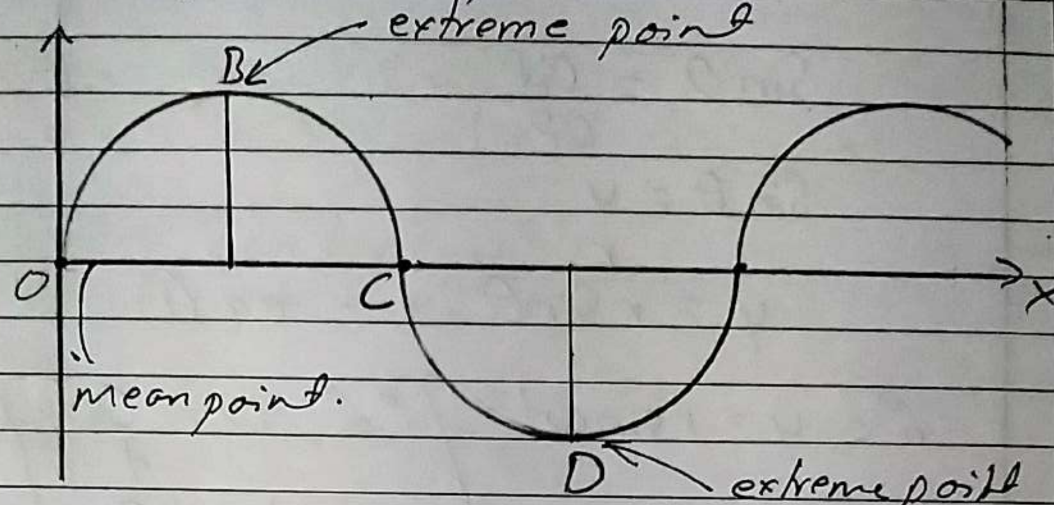


fig :- Sinusoidal Wave

Significance

1. To find out displacement
2. To find out Velocity
3. To find out Acceleration.

Consider a particle is moving in circular motion in anti-clockwise direction Let XOX' and YOY' meet perpendicularly at position O.
If R be the radius of a circle and θ be the angular displacement.
Let ω be the angular velocity of a particle
Initially particle is at mean position ~~Aftim~~ After time (t) particle is at position (P). from where perpendicular is draw.
i.e $PM \perp OX$ and $PN \perp OY$

or

$$ON = y$$
$$OP = r$$

DISPLACEMENT

We know, In $\triangle ONP$

$$\sin\theta = \frac{ON}{OP}$$

$$\sin\theta = \frac{y}{r}$$

$$y = r\sin\theta \text{ ---- eq (i)}$$

$$\therefore y = r\sin\omega t \quad \left[\because \omega = \frac{\theta}{t} \;\&\; \omega t = \theta\right]$$

which is the required equation for displacement of a particle in SHM

* VELOCITY

Velocity is defined as a rate of change of displacement with respect to change of time.
i.e.

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$v = \frac{dy}{dt}$$

$$v = \frac{d r\sin\omega t}{dt}$$

$$v = r\frac{d\sin\omega t}{dt} \times \frac{\omega dt}{dt}$$

$$v = r\cos\omega t\,\omega$$

$$v = r\omega\cos\omega t \text{ ---- eq (ii)}$$

$$v = r\omega\sqrt{1-\sin^2\omega t}$$

$$v = r\omega\sqrt{1-\left(\frac{y}{r}\right)^2}$$

$$v = r\omega\sqrt{\frac{r^2-y^2}{r^2}}$$

$$r = \omega\sqrt{r^2-y^2}$$

where, y = displacement
r = amplitude

VELOCITY

Special Case

1. When particle is at mean position i.e $y=0$

$$v = \omega\sqrt{r^2-y^2}$$

$$v = \omega\sqrt{r^2-0^2}$$

$$v = \omega\sqrt{r^2} = \omega r$$

i.e velocity is maximum

2. When particle is at extreme position i.e $y=r$

$$v = \omega\sqrt{r^2-y^2}$$

$$r = \omega\sqrt{r^2-r^2} = 0$$

i.e velocity is minimum

* ACCELERATION

Rate of change of velocity with respect to time

$$a = \frac{dv}{dt}$$

$$a = \frac{d\, r\omega \sin\omega t}{dt}$$

$$a = r\omega \frac{d \cos\omega t}{dt}$$

$$a = r\omega \frac{d \cos\omega t}{d\omega t} \times \frac{\omega dt}{dt}$$

$$a = r\omega^2(-\sin\omega t)$$

$$a = -r\omega^2 \sin\omega t$$

$$a = -\omega^2 y \quad [y = r\sin\omega t]$$

ACCELERATION

Special Case

1. When particle is at mean position
i.e $y = 0$

$$a = -\omega^2 y$$
$$a = 0$$

i.e acceleration is minimum

2. When particle is at extreme position i.e $y = r$

$$a = -\omega^2 y$$
$$a = -\omega^2 r$$

i.e acceleration is maximum

Graphical Representation of Displacement Velocity & Acceleration in SHM.

Here,

$x = r\sin(\omega t)$

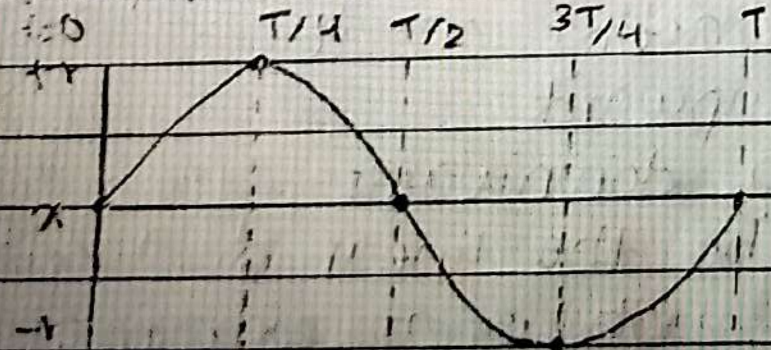
$v = r\omega\cos\omega t$

$a = -r\omega^2\sin(\omega t)$

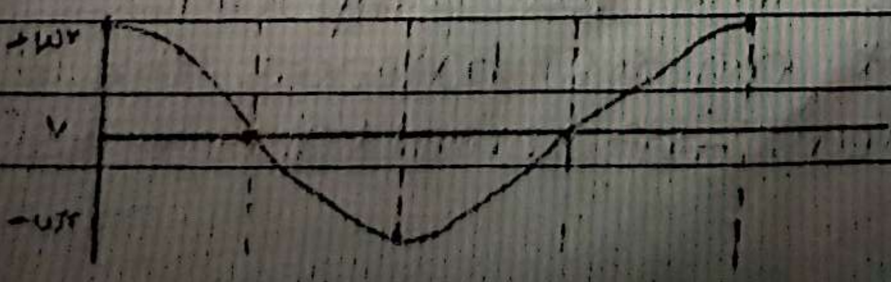
We know,

$\therefore \omega t = \frac{2\pi}{T} \cdot t$

t	0	T/4	T/2	3T/4	T
ωt	0	$\pi/2$	π	$3\pi/2$	2π
x	0	$+r$	0	$-r$	0
v	$r\omega$	0	$-r\omega$	0	$r\omega$
a	0	$-r\omega^2$	0	$r\omega^2$	0



It is sin function.



It is cos function velocity is ahead of displacement by $\pi/2$.

Time Period (T)

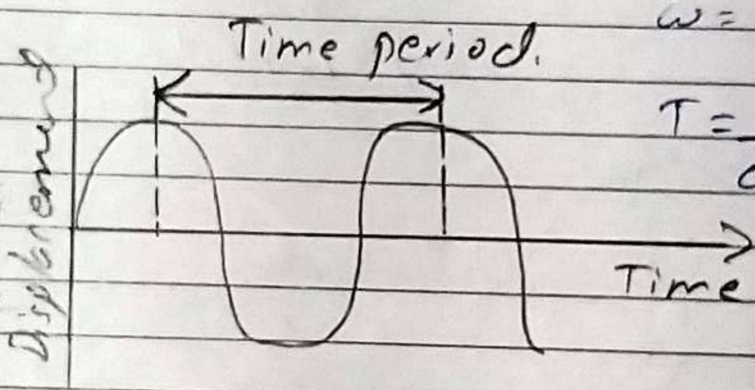
→ The time taken from one complete oscillation or one complete cycle of wave of a point in a wave is called the Time period.

→ It is the time taken for a point to move from one particular position and return to that same position, moving in the same direction.

→ It is measured in units of time eg. seconds

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$



~~Ft~~ Frequency (f)

→ ~~The number of oscillations per~~ The number of complete oscillations per unit time of a point in a wave is called frequency

→ It is measured in ~~pitch~~ Hertz (Hz) where,

1 Hz = One Oscillation per second

e.g.

$1 KHz = 10^3 Hz$, $1 MHz = 10^6 Hz$

$\omega = 2\pi f$

$\frac{\omega}{2\pi} = f$,,

$\therefore Hz = \frac{1}{sec}$,,

The frequency (f) of a wave is the reciprocal of the period (T)
i.e.

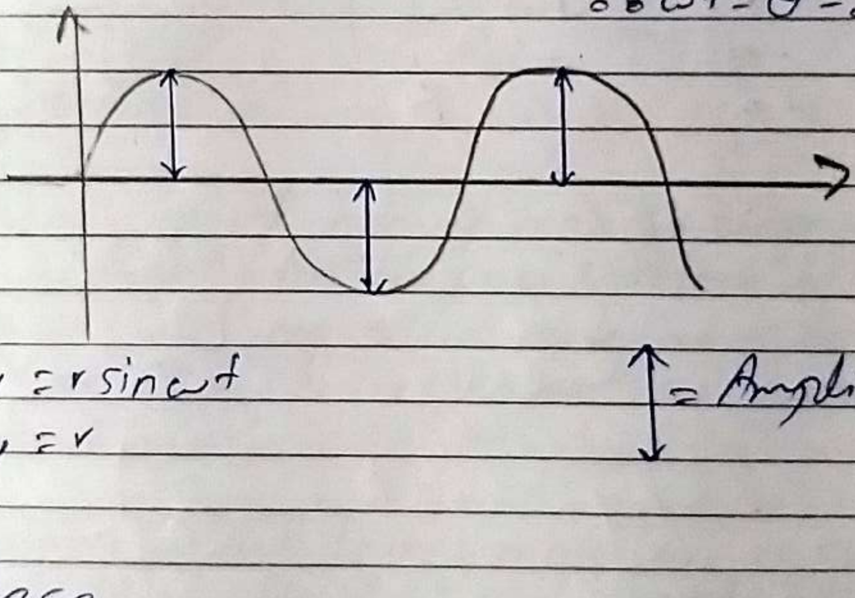
$$\therefore f = \frac{1}{T}$$

frequency (Hz) Time period (s)

* Amplitude, A

→ The maximum displacement of a particle in the wave from its equilibrium or undisturbed position is called amplitude

$\therefore \omega t = \theta = 90°$



$y = r \sin \omega t$
$y = r$

↕ = Amplitude

* Phase

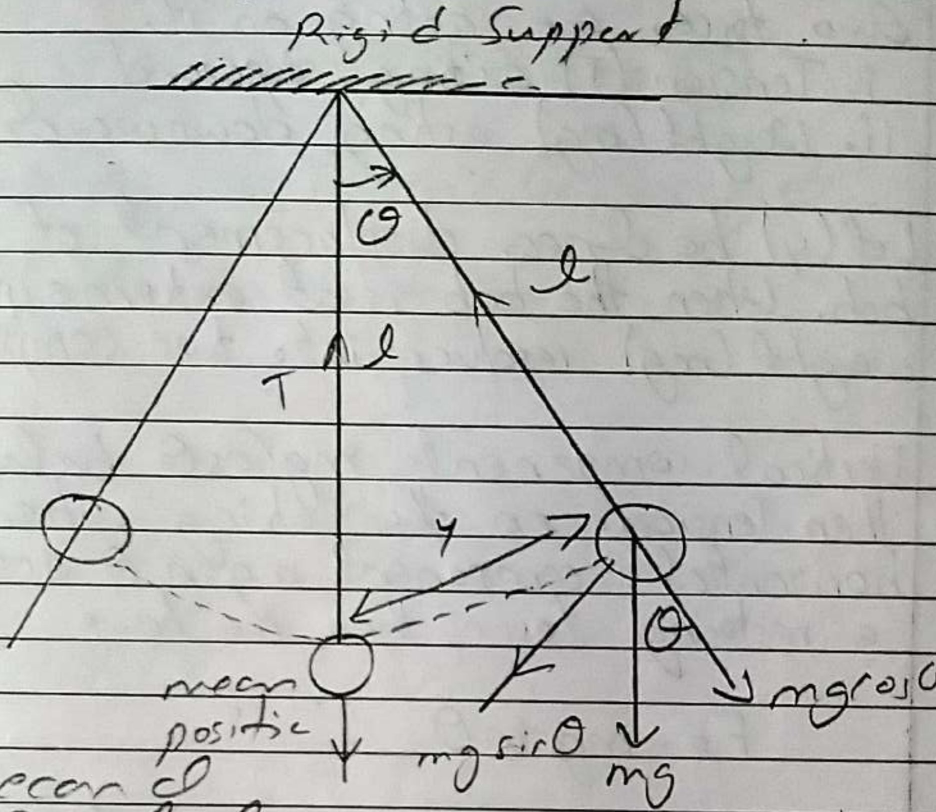
→ The point that an oscillating mass has reached in a complete cycle is called Phase

→ The phase difference tells us how much a point or a wave is in front or behind anoth

Angular displacement of the particle from mean position.

(2π) radian.

Simple Pendulum



second Pendulum

When a point heavy mass is suspended to a light flexible and inextensible string which moves in back and froth position then it is known as simple pendulum

Consider a ball having mass (m) is suspended to a light, flexible, and inextensible string whose one end is attached to rigid support and another end to bob (point heavy mass)

If 'l' be the effective length from rigid support to center of gravity of bob and 'θ' be the angular displacement

SIMPLE PENDULUM

When the bob is in mean position then two forces are acting on it.
i. Tension (T) acting upward
ii. Weight (mg) acting downwards.

Let (y) be linear displacement of a bob. When the bob is at extreme position weight (mg) resolves into two components

Vertical component $mg\cos\theta$ balances then Tension on the string whereas horizontal component $mg\sin\theta$ acts as a restoring force. then we have

$$F = -mg\sin\theta$$

$$ma = -mg\sin\theta \qquad \because \sin\theta \simeq \theta = \frac{y}{l}$$

$$a = -g\theta$$

$$a = -g\frac{y}{l} \quad \text{--- eq (i)}$$

$$a = ky \quad \left(\therefore k = -\frac{g}{l}\right)$$

or, $a \propto y$

Since, acceleration is directly proportional to displacement, It execute Simple Harmonic motion.

Also, we know.

$$a = -\omega^2 y \quad \text{.... eq (ii)}$$

Comparing eq (i) and (ii), we get

$$\cancel{-a} \; -\omega^2 = -\frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

SECOND PENDULUM

* Second Pendulum

If time period for the oscillation of pendulum is 2 seconds. Then it is known as second pendulum

we have,

$$T = 2\,\text{sec}$$
$$g = 9.8\,\text{m/s}^2$$

Here,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

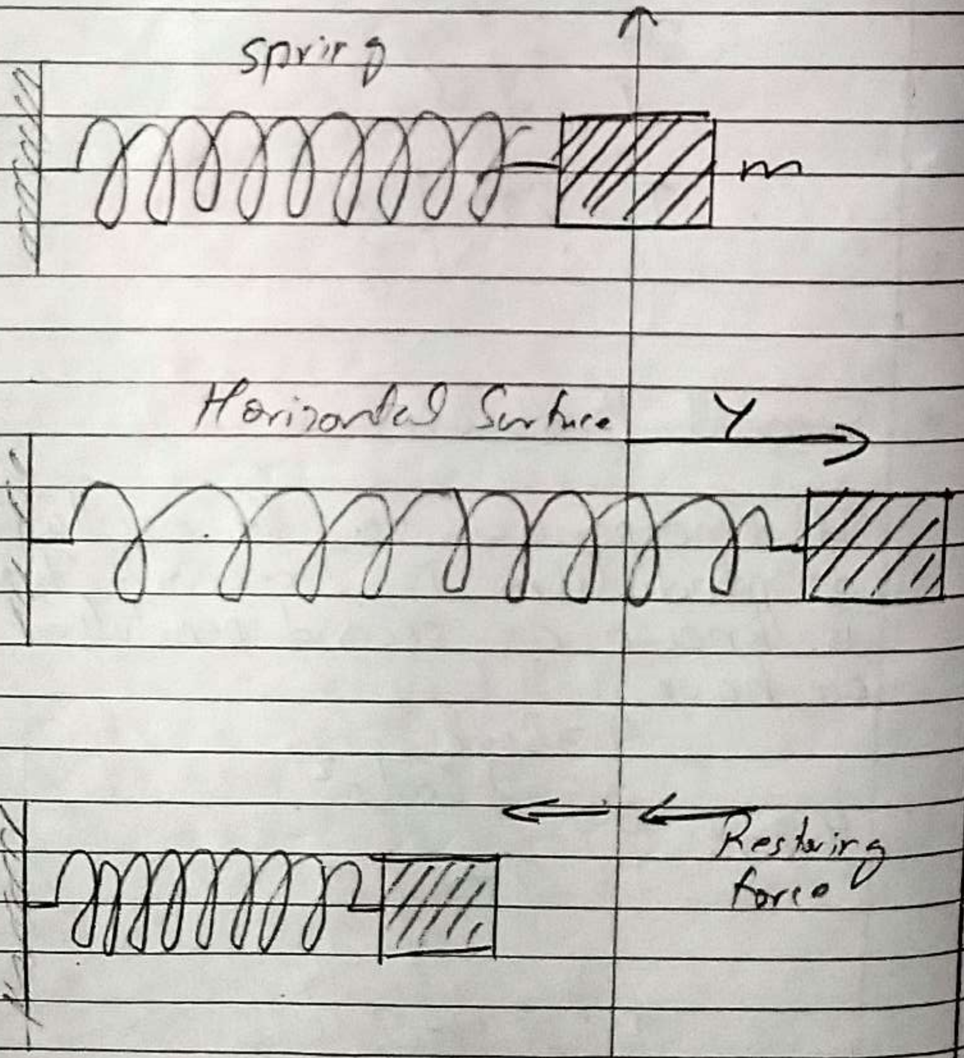
$$T^2 = 4\pi^2\frac{l}{g}$$

$$(2)^2 = 4\pi \frac{l}{9.8}$$

$$l = 0.99\,m \approx 100\,cm$$

Oscillation of a loaded Spring

① Horizontal Oscillation



HORIZONTAL OSCILLATION

$F \propto y$

$F = -ky.$

$ma = -ky$

$a = -\frac{k}{m} y \quad \ldots\ldots eq\ (1)$

$a \propto y$

$a = -\omega^2 y \quad \ldots eq\ (1)$

$-\omega^2 = \frac{-k}{m}$

$\omega = \sqrt{\frac{k}{m}}$

$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

$T = 2\pi \sqrt{\frac{m}{k}}$

Consider, a light flexible and extensible spring which is placed on a smooth horizontal surface where mass(m) is attached to one end and other end is attached to rigid support.

If (l) be the ~~external~~ extension produced on the spring due to external forced applied.

Restoring force and displacement are in opposite direction.

We know, From Hooke's law.

$$F \propto l$$

or, $$F = -kl$$

where, k is a proportonality constant also known as Force or Spring constant and -ve sign represent force acting opposite to the displacement.

$$ma = -kl$$

$$a = \frac{-k}{m} l \quad \ldots \text{eq (i)}$$

$$\therefore a \propto l$$

Since, acceleration is directly porpotional to the displacement. It executes S.H.M.

we know.

$$a = -\omega^2 y \quad \ldots \text{eq (ii)}$$

Now,

equalling eq (i) & (ii) we get

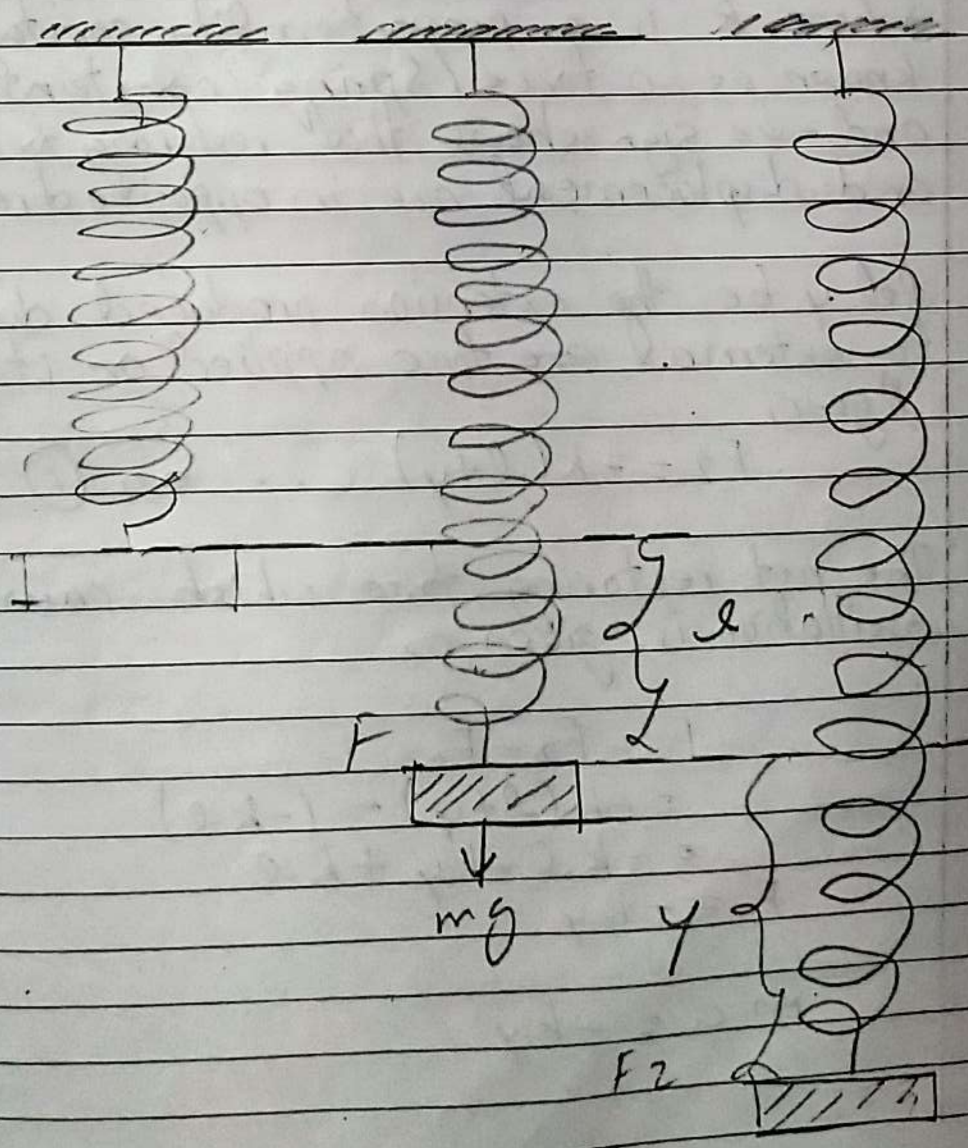
$$-\omega^2 = \frac{-k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$\therefore T = 2\pi\sqrt{\frac{m}{k}}$$

(15) Vertical Oscillation



VERTICAL OSCILLATION

Consider a light, flexible and extensible spring which is placed vertically where one end is attached to a rigid support.

If l be the extension produced on the spring due to suspension of mass
Then,
we know from Hooke's law
i.e.

$$F \propto l$$
$$F_1 = -kl \text{ ---- eq (i)}$$

where k is a proportionality constant known as a force / Spring constant. and -ve sign shows that restoring force and displacement are in opposite direction

let y be the extension produced due to external ~~are~~ force applied on it.
Again,

$$F_2 = -k(l+y) \text{ ---- eq (ii)}$$

The net restoring force which cause oscillation is given as

$$F = F_2 - F_1$$
$$= -k(l+y) - (-kl)$$
$$= -kl - ky + kl$$
$$F = -ky$$

$$ma = -ky$$

$$a = -\frac{k}{m} y \quad \text{--- eq (i)}$$

$$a \propto y$$

Since, acceleration is directly proportional to displacement it execute S.H.M

we know,

$$a = -\omega^2 y \quad \text{--- eq (ii)}$$

Now,

Equating eq (i) & (ii) we get

$$-\omega^2 = \frac{-k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{K}}$$

Energy in Simple Harmonic Motion

Due to the restoring force in a Simple Harmonic Motion (S.H.M), there exist both Kinetic ~~En~~ Energy (KE). and Potential Energy (P.E)

If there is no damping in the system Total energy is sum of kinetic energy & Potential energy
i.e

$$TE = PE + KE$$

Even though, Velocity and acceleration of a oscillating particle changes continuously energy always remain constant.

For Potential Energy

Consider a particle is executing simple harmonic motion where (r) be its amplitude (ω) be its angular velocity
Then, we know,

$$a = -\omega^2 y \text{ ..... eq (i)}$$

where y be the displacement of a particle,
we have

$$F = ma$$

$$F = -m\omega^2 y$$

or.

$$F = -ky \text{ ... eq (ii)} \quad [\because k = \omega^2 m]$$

Let $(d\omega)$ be the small amount of work done for small displacement (dy)

$$d\omega = -F dy$$
$$d\omega = -(-ky) dy$$
$$\therefore \ d\omega = ky \, dy$$

Then,
Total work done can be calculated by Integrating from 0 to y

$$\omega = \int_0^y d\omega$$

$$\omega = \int_0^y ky \, dy$$

$$\omega = k \int_0^y y \, dy$$

$$\omega = k \left(\frac{y^2}{2} \right)_0^y$$

$$\omega = \frac{k}{2} (y^2 - 0)$$

$$\omega = \frac{1}{2} k y^2$$

$$\omega = \frac{1}{2} m \omega^2 y^2$$

Potential Energy $= \frac{1}{2} m \omega^2 y^2$

ENERGY IN SHM

For kinetic Energy

we know,

$$KE = \frac{1}{2} m v^2 \quad \left[v = \omega \sqrt{r^2 - y^2} \right]$$

$$= \frac{1}{2} m \left\{ \omega \sqrt{r^2 - y^2} \right\}^2$$

$$= \frac{1}{2} m \omega^2 (r^2 - y^2)$$

$$KE = \frac{1}{2} m \omega^2 r^2 - \frac{1}{2} m \omega^2 y^2$$

we know

$$\text{Total Energy} = KE + PE$$

$$= \frac{1}{2} m \omega^2 r^2 - \frac{1}{2} m \omega^2 y^2 + \frac{1}{2} m \omega^2 y^2$$

$$= \frac{1}{2} m \omega^2 r^2$$

Special Case

1. When the particle is at mean position i.e $y = 0$

Potential Energy $= \frac{1}{2} m \omega^2 y^2$

$= \frac{1}{2} m\omega^2 \times (0)^2$

$= 0$

Kinetic Energy $= \frac{1}{2} m\omega^2 r^2 - \frac{1}{2} m\omega^2 y^2$

$= \frac{1}{2} m\omega^2 r^2$

$T.E = K.E + P.E$

$= \frac{1}{2} m\omega^2 r^2 + 0$

$T.E = \frac{1}{2} m\omega^2 r^2$

So, at mean position Total energy is equal to kinetic energy

2. When the particle is at extreme posit

i.e $y = r$

$$P.E = \frac{1}{2} m\omega^2 y^2$$

$$= \frac{1}{2} m\omega^2 r^2$$

$$K.E = \frac{1}{2} m\omega^2 r^2 - \frac{1}{2} m\omega^2 y^2$$

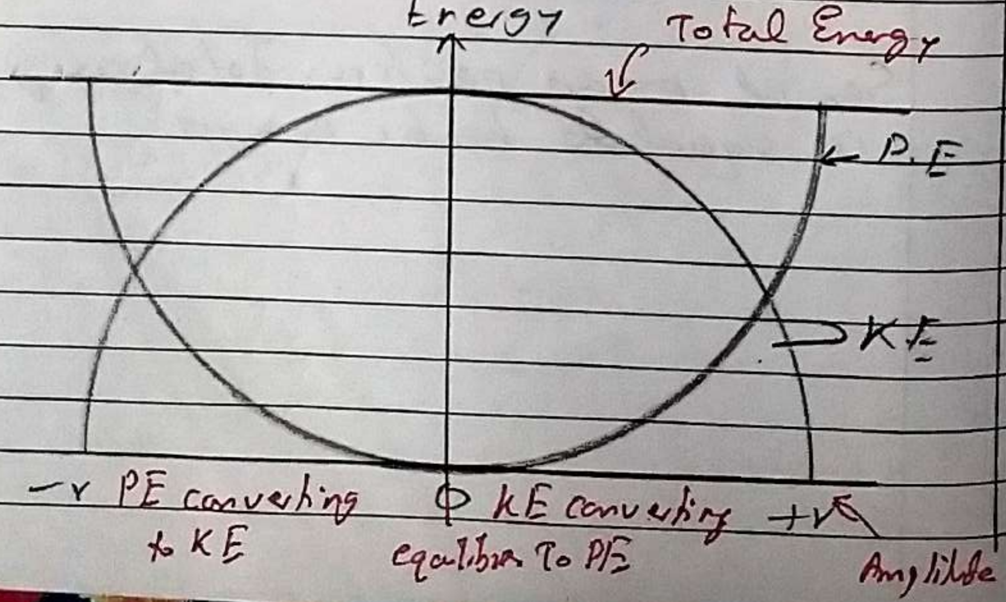
$$= \frac{1}{2} m\omega^2 r^2 - \frac{1}{2} m\omega^2 r^2$$

$$K.E. = 0$$

$$T.E. = KE + PE$$

$$= 0 + \frac{1}{2} m\omega^2 r^2$$

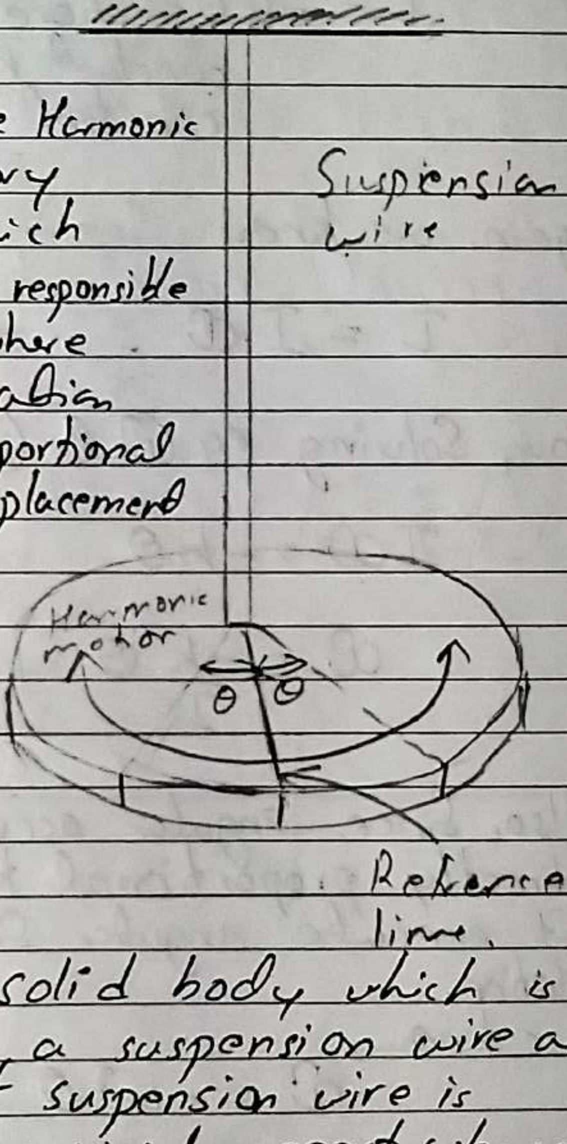
$$T.E = \frac{1}{2} m\omega^2 r^2$$



Angular Simple Harmonic Motion

Angular Simple Harmonic is also oscillatory motion in which Torque (τ) is responsible for the motion where angular acceleration is directly proportional to angular displacement i.e

$$\alpha \propto \theta$$



Consider a solid body which is attached to a suspension wire and either end of suspension wire is attached to a rigid support, where θ be the angular displacement.

where,

I = moment of Inertia of a solid bod

τ = Torque acting.

~~we know,~~

ANGULAR SHM

we know,

$$\tau \propto \theta \quad \text{eq (i)}$$

$\tau = -k\theta$, where k is a proportionality constant known as torsion constant

Again, we know,

$$\tau = I\alpha \text{ ---- eq (ii)}$$

Now, Solving eq (i) & (ii) we get

$$I\alpha = -k\theta$$

$$\alpha = \frac{-k}{I}\theta \text{ ---- eq (iii)}$$

Also, since, angular acceleration is directly proportional to angular displacement It execute angular Simple Harmonic Motion,

we know,

$$\alpha = -\omega^2\theta \text{ ---- eq (iv)}$$

Comparing eq (iii) & (iv)

$$-\omega^2 = \frac{-k}{I}$$

$$\omega = \sqrt{\frac{k}{I}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{I}}$$

$$\therefore T = 2\pi\sqrt{\frac{I}{k}}$$

In terms of frequecy

$$T = 2\pi\sqrt{\frac{I}{k}}$$

$$\frac{1}{f} = 2\pi\sqrt{\frac{I}{k}} \quad \left[\because T = \frac{1}{f}\right]$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$$

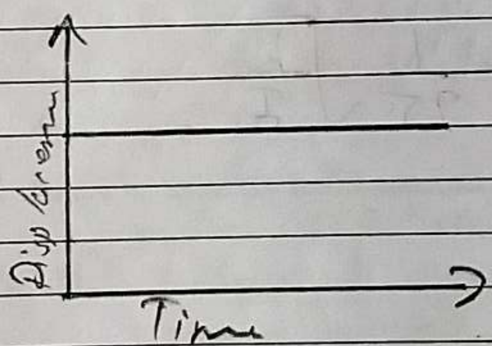
Important Terms.

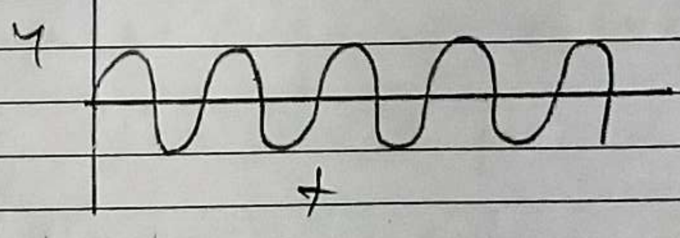
Damping and Undamping Oscillation

Undamped Oscillation.

→ Undamped oscillation is a type of mechanical vibration where there is no opposing force to slow down the motion.

→ The amplitude of the oscillation will stay constant over time, and the period will be the same as well.





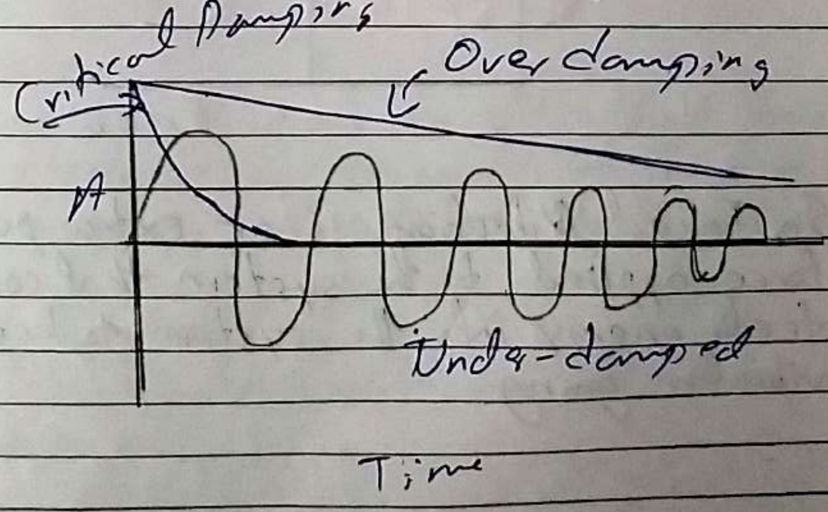
~~True Vibration~~ Damped Oscillation

Dambing is where the energy in an oscillating is lost to the environment, leading to reduced aplitude of Oscillation.
Damping are 3 types ~~of~~ and they are

- Light Damping
  → Also known as under-damping and this where the amplitude gradually decreases by a small amount. eg:- Simple pendulum

- Critical Damping
  → This reduces the amplitude to zero in the shortest possible time (without oscillating) eg:- Shock absorbers in car.

- Heavy Damping
  → Also known as over damping and is where the amplitude reduces slower than with critical damping, but also without any additional oscillations eg:- Door closer

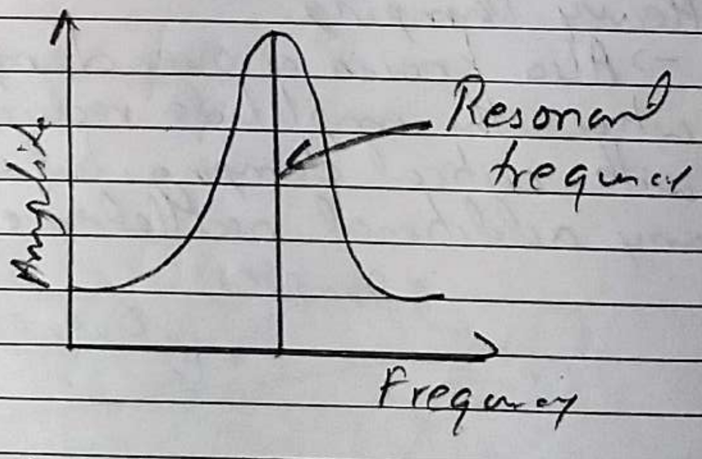


Forced Vibrations.

→ Forced Vibrations are where a system ~~ext~~ experiences an external driving force which causes it to oscillate. the frequency of this driving force ~~which cause~~ known as driving frequency, is significant.

If the driving frequency is equal to the natural frequency of a system (also known as Resonant frequency) then resonance occurs.

As driving frequency approaches the resonant frequency, the amplitude of oscillation will increase,



So, Forced Vibration is an extra periodic force applied to the system that continuously feeds energy into the system to keep the vibration going.

Resonance

-> When the driving frequency applied to an oscillating system is equal to its natural frequency, the amplitude of the resulting oscillation increases significantly is known as Resonance

-> When resonance is achieved, a maximum amplitude of oscillation can be observed.

-> If the driving frequency does not quite match the natural frequency, the amplitude will increase but not to the same extent as when resonance is achieved

-> This is because at resonance, energy is transferred from driver to the oscillating system most efficiently.
So, at resonance, the system will be transferring the maximum kinetic energy possible.

Application
- Radio -> These are tuned so that their electric circuit resonates at the same frequency as the desired broadcast frequency

effects
- Bridges:- Creates a large oscillation due to resonance.

Note -
-> Resistive force is what opposes the motion of the oscillator and causes damping

-> Restoring force is what bring the oscillator back to the equilibrium position.

Question

1. A simple Pendulum 4m long swings with an amplitude of 0.2m compute the velocity of a pendulum and acceleration at its mean and extreme position
Solution
length (l) = 4m
amplitude (r) = 0.2m
Now,
$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$= 2\pi\sqrt{\frac{4}{10}}$$
$$T = 4s$$

Now at mean position

$$\text{Velocity } (v_m) = \frac{2\pi}{T}\sqrt{r^2 - y^2}$$
$$= \frac{2\pi}{4}\sqrt{(0.2)^2 - (0)^2}$$

$$= \frac{2\pi}{4}\sqrt{(0.2)^2}$$
$$= 0.31 \, m/s$$

Acceleration $(a_m) = -\omega^2 y$
$$= -\omega^2 \times 0$$
$$a_m = 0$$

At extreme position

Velocity $(v_e) = \omega\sqrt{r^2 - y^2}$
$$= \frac{2\pi}{T}\sqrt{r^2 - y^2}$$

$$= \frac{2\pi}{4}\sqrt{r^2 - r^2}$$

$$= 2\pi/4 \times 0$$

$$= 0$$

Acceleration $= -\omega^2 r$
$$= -\left(\frac{2\pi}{T}\right)^2 r$$
$$= \cancel{0.49 \, m/s^2}$$

$$= -\left(\frac{2\pi}{4}\right)^2 \times 0.2m$$
$$= 0.49 \, m/s^2$$

Date
Page

An object is moving with SHM has an amplitude of 0.02m and frequency of 20 Hz. Calculate
i. Period of Oscillation
ii. Acceleration at mean & extreme position
iii. Velocity at mean & extreme position

Solution
Given amplitude $(r) = 0.02$
Frequency $(f) = 20\,Hz$
Now

$$T = \frac{1}{f}$$

$$T = \frac{1}{20}$$

$$T = 0.05s$$

acceleration at mean position $(y = 0)$

$$a = -\omega^2 y$$
$$= -\omega^2 \times 0$$
$$a = 0$$

acceleration at extreme position $(y = r)$

$$a = -\omega^2 y$$
$$= -\omega^2 r$$
$$= -\left(\frac{2\pi}{T}\right)^2 \times 0.02$$
$$= -\left(\frac{2\pi}{0.05}\right)^2 \times 0.02$$
$$a = 315.8\,m/s^2$$

Velocity at mean point

$$v = \omega\sqrt{r^2 - y^2}$$
$$= 2\pi f \sqrt{r^2 - 0}$$
$$= 2\pi \times 20 \sqrt{(0.02)^2}$$
$$= 2\pi \times 20 \times 0.02$$
$$= 2.5 \text{ m/s}$$

Velocity at extreme posit.

$$v = \omega\sqrt{r^2 - y^2}$$
$$= \omega\sqrt{r^2 - r^2}$$
$$= \omega \times 0$$
$$v = 0$$

Q. A small mass of 0.2kg is attached to one end of a helical string and produces an extension of 15mm. A mass is set into vertical oscillation of amplitude 10mm. What is

i. The period of oscillation
ii. The maximum K.E of the mass?
iii. The P.E of the spring when the mass is 5mm below the centre of oscillati.

[$g = 9.8 \text{ m/s}^2$

Solution,

Mass of the body $(m) = 0.2$kg

Extension on the spring $(x) = 15$mm $= 0.015$m

Amplitude of vibration $(r) = 10$mm $= 0.01$m

Now, we know that

$$F = kx$$

$$mg = kx$$

$$\frac{m}{k} = \frac{x}{g}$$

$$\therefore k = \frac{mg}{x} = \frac{0.2 \times 9.8}{0.015} = \frac{1.96}{0.015}$$

$$\therefore k = 130.66$$

Now,

$$\text{Time period } (T) = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{x}{g}}$$

$$= 2\pi\sqrt{\frac{0.015}{9.8}}$$

$$\therefore T = 0.246s$$

$$\text{Kinetic energy } (KE) = \frac{1}{2}kx^2$$

$$= \frac{1}{2} \times 130.66 \times (0.01)^2$$

$$= 6.53 \times 10^{-1} J$$

Potential energy (P.E)

$$E = \frac{1}{2} mgx_e$$

$$= \frac{1}{2} \times 0.2 \times 9.8 \times 5 \times 10^{-3}$$

$$= 4.9 \times 10^{-3} J$$

Q. A body of mass 0.1kg is undergoing S.H.M of amplitude 1m and period 0.2s. If the oscillation is produced by a string, What will be the maximum value of force and force constant of the spring.

Solution,

mass (m) = 0.1kg

amplitude (r) = 1m

period (T) = 0.2s

Now, we know that

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$0.2 = 2\pi\sqrt{\frac{0.1}{k}}$$

$$\frac{0.2}{2\pi} = \sqrt{\frac{0.1}{k}}$$

Squaring

$$\left(\frac{0.2}{2\pi}\right)^2 = \left(\sqrt{\frac{0.1}{k}}\right)^2$$

$$1.013 \times 10^{-3} = \frac{0.1}{k}$$

$$k = \frac{0.1}{1.013 \times 10^{-3}}$$

$$\therefore k = 98.69 \ N/m$$

Again,

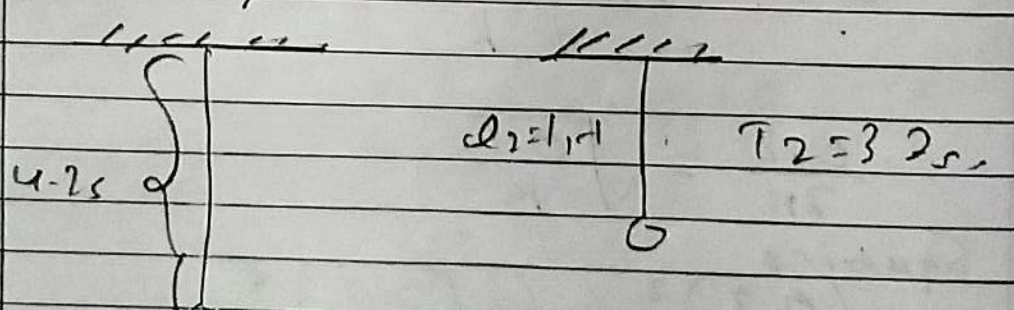
$$F = -m\omega^2 r$$

$$= 0.1 \left(\frac{2\pi}{0.2}\right)^2 \times 1m$$

$$F = 98.69 N$$

Q. A Simple Pendulum has a period of 4.2s, when the pendulum is certain by 1m, the period is 3.7s. From this measurement, Calculate the acceleration of freefall and original length of the pendulum.

Solution,



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We know,

$$T_1 = 2\pi\sqrt{\frac{l_1}{g}} \quad \text{.... eq (i)}$$

$$T_2 = 2\pi\sqrt{\frac{l_2}{g}} \quad \text{.... eq (ii)}$$

Now, dividing eq (i) by (ii), we get

$$\frac{T_1}{T_2} = \frac{2\pi}{2\pi}\sqrt{\frac{l_1/g}{l_1-1/g}}$$

$$\frac{4.2}{3.7} = \sqrt{\frac{l_1}{\cancel{g}} \times \frac{\cancel{g}}{l_1-1}}$$

$$\left(\frac{4.2}{3.7}\right)^2 = \frac{l_1}{(l_1-1)}$$

$$(l_1-1)\,1764 = l_1\,1369$$

$$1764 l_1 - 1764 = 1369 l_1$$

$$1764 l_1 - 1369 l_1 = 1764$$

$$l_1 = \frac{1764}{395}$$

$$l_1 = 4.47m$$

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Q. A second pendulum taken to moon. If the time period on the surface of moon is 4.9 sec. What will be the acceleration due to gravity of the moon? Also, prove that acceleration due to gravity of the moon is 1/6 th of the earth.

Solution,

For a second pendulum on the surface of earth,

$$T_1 = 2\text{ sec}, \quad l = 1\text{ m}$$

When it is taken to the surface of moon, then,

$$T_m = 4.90\text{ secc}$$
$$g_m = ?$$
$$l = 1\text{ m}$$

Since, $T_m = 2\pi\sqrt{\dfrac{l}{g_m}}$

$$T_m^2 = 4\pi^2 \cdot \frac{l}{g_m}$$

$$g_m = \frac{4\pi^2 l}{T_m^2}$$

$$g_m = 4 \times (3.14)^2 \times \frac{1}{(4.90)^2}$$

$$g_m = 1.64\text{ m/s}^2$$

Now,

$$\frac{T_E}{T_m} = \sqrt{\frac{g_m}{g_E}}$$

$$\frac{2}{4.90} = \sqrt{\frac{g_m}{g_e}}$$

$$\left(\frac{2}{4.90}\right)^2 = \frac{g_m}{g_e}$$

$$\frac{g_m}{g_E} = 0.167$$

$$\therefore \frac{g_m}{g_E} = \frac{1}{6} \text{ proved}$$

An object moving with S.H.M ~~having~~ having amplitude of 0.02m of frequency of 20Hz. Calculate
a) The period of oscillate
b) The acceleration of the middle and end of an oscillation
c) ~~Vertebrates~~ Velocities at the corresponding instant.

Solution,
Amplitude $(r) = 0.02m$
frequency $(f) = 20\ Hz$

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Now,

a) Period of oscillation (T) $= \frac{1}{f}$

$$T = \frac{1}{20}$$

$$\therefore T = 0.05\,s$$

b) Acceleration at middle position i.e $y=0$

$$a = \omega^2 y$$
$$= \omega^2 \times 0$$
$$\therefore a = 0$$

Acceleration at end position i.e $y = r$

$$a = \omega^2 y$$
$$= \omega^2 r$$
$$= (2\pi \times 20)^2 \times 0.02$$
$$a = 315.83\,m/s^2$$

c) Velocity at mean position

$$v = \omega \sqrt{r^2 - y^2}$$
$$= 2\pi f \sqrt{r^2}$$
$$= 2\pi \times 20 \sqrt{0.02)^2}$$
$$= 2.5\,m/s$$

At extreme position $y = r$

$$v = \omega \sqrt{r^2 - y^2}$$
$$= \omega \sqrt{r^2 - r^2}$$
$$= \omega \times 0$$
$$v = 0$$

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A spring of force constant (k) of $5 Nm^{-1}$ is placed horizontally on smooth table is one end of spring is fixed and a mass of 0.20kg is attached to a free end. Mass is displaced at a distance of 4mm along the table and then released. So that the motion of mass is S.H.M Calculate.

a. The period
b. Maximum acceleration
c. Maximum Kinetic Energy
d. Maximum Potential Energy

Solution,
Force Constant (k) $= 5 Nm^{-1}$
mass (m) = 0.20kg
Displaced (y) = 4mm $= 4 \times 10^{-3} m$
Now.

a) The period $(T) = 2\pi \sqrt{\frac{m}{k}}$

$$= 2\pi \sqrt{\frac{0.20}{5}}$$

$$T = \frac{2\pi}{5} = 1.25 s$$

b) Acceleration at maximum

$$a = \omega^2 y$$
$$= \left(\frac{2\pi}{T}\right)^2 r$$

$$= \left(\frac{2\pi}{2/5\pi}\right)^2 4\times10^{-3}$$

$$= 0.1\ m/s^2$$

c) Kinetic Energy of maximum

$$K.E = \frac{1}{2} m\omega^2 (r^2 - y^2)$$

$$= \frac{1}{2} m \omega^2 (r^2 - 0)$$

$$= \frac{1}{2} m (2\pi f)^2 (r^2)$$

$$= \frac{1}{2} \times 0.20 \left(2\pi \times \frac{1}{1.25}\right)^2 (4\times10^{-3})^2$$

$$= 4.04 \times 10^{-5}\ J$$

d) Potential Energy at maximum

$$PE = \frac{1}{2} m\omega^2 y^2$$

$$= \frac{1}{2} \times 0.20 \times \left(\frac{2\pi}{1.25}\right)^2 (4\times10-3)^2$$

$$= 4.04 \times 10^{-5}\ J$$

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A mass (x) of 0.1 kg is attached to a free end of vertical helical spring whose upper end is fixed and the spring extends by 0.04m. x is now pulled down by 0.02m and then released. Find

- Its period
- Its maximum force during oscillation
- Kinetic Energye at mean position

Solution,

Given, mass (m) = 0.1 kg

Displacement (y) = 0.04 m

Extension (l) = 0.02 m

Now,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{0.04}{9.8}}$$

$$T = 0.4 s$$

Then,

Maximum force,

$$F = ma$$

$$= m(\omega^2 y)$$

$$= 0.1\left\{\left(\frac{2\pi}{0.4}\right)^2 \times 0.02\right\}$$

$$F = 0.49 N$$

Then

K.E at mean position $y = 0$

$$KE = \frac{1}{2} m\omega^2(r^2 - y^2)$$

$$= \frac{1}{2} \times 0.1 \times (2\pi/0.4)^2 \{(0.02)^2 - 0)$$

$$KE = 4.9 \times 10^{-3} J$$

A helical spring gives a displacement of 5cm for a load of 500gm & find maximum displacement produced when mass of 80g is dropped from a height of 10cm onto a light pan scale to the spring

Solution

$x = 5cm$

$m = 500gm$

Now,

$F = kx$

$mg = kx$

$\frac{mg}{x} = k$

$k = \frac{500 \times 10^{-3} \times 10}{5 \times 10^{-3}}$

$k = 1000 N/m$

An object moves in SHM with a period of 0.500sec. The object's maximum acceleration is $6.4 m/s^2$. What is its maximum speed?

Solution

Period (T) = 0.500sec

$a_{max} = 6.4 m/s^2$ $v_{max} = ?$

Since,

$a_{max} = \omega^2 r$

$a_{max} = \left(\frac{2\pi}{T}\right)^2 r$

$r = a_{max}\left(\frac{T}{2\pi}\right)^2$

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$$= 6.4 \times \left(\frac{0.5}{2\pi}\right)^2$$

$\therefore r = 0.041\,m$

Again,

$$V_{max} = r\omega$$
$$= r\left(\frac{2\pi}{T}\right)$$
$$= 0.041 \times \left(\frac{2\pi}{0.5}\right)$$
$$= 0.51\,m/s^2$$