Wave & Optics

Date
Page

Wave

Wave is the contineous transfer of disturbance from one point to another point of medium by successive vibration of particles about their mean position. Wave carries energy without transporting matter.

Types of wave:-

① Mechanical wave (Elastic wave)

The wave which needs material medium to propagate is known as mechanical wave. Eg: sound wave, wave in water, wave in string etc.

② Electromagnetic (Non-mechanical wave)

The wave which doesn't require material medium to propagate is known as electromagnetic wave. Eg: x-ray, microwave, radio waves, etc.

Properties of medium for mechanical wave:-

① Elasticity

→ The property of a material to return to its original state after deformation is applied is called elasticity.

② Inertia

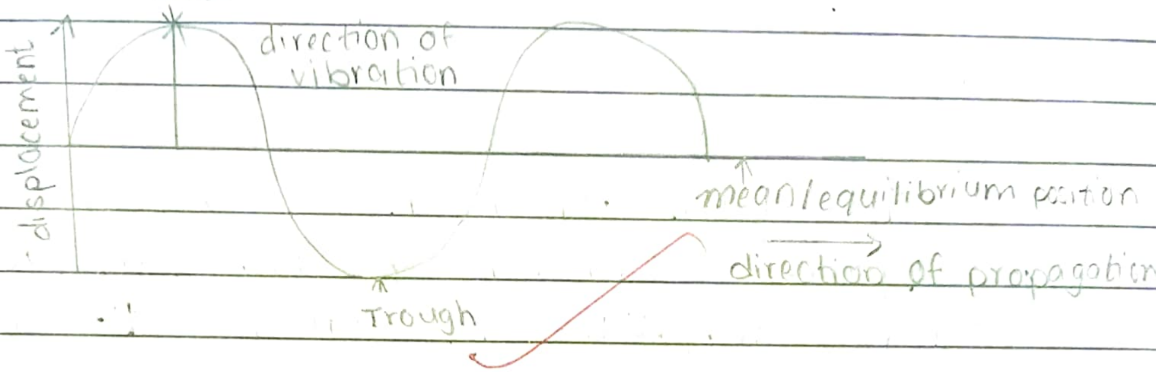
→ Inertia refers to the tendency of resisting any change in motion of the wave particles.

③ Low damping

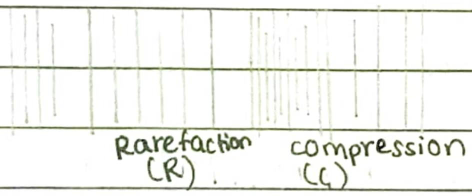
→ The particles oscillates and the wave travels for a sufficiently long time & distance is low damping.

Types of Mechanical wave

1) Transverse wave:- The wave in which direction of vibration of particle & direction of propagation of wave are perpendicular to each other. Eg: wave in string, water wave etc.

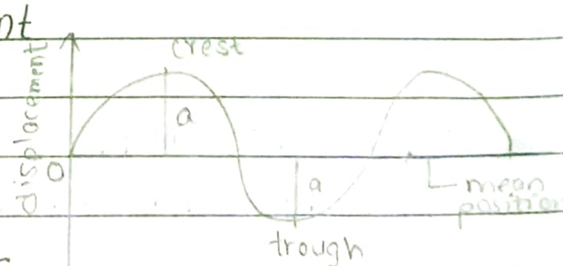


2) Longitudinal wave:- The wave in which direction of vibration of particle & direction of propagation of wave are parallel to each other is called longitudinal wave. Eg: sound wave.



Some important terminologies

1) Crest: The maximum displacement of particle above the mean position is crest.



2) Trough: The maximum displacement of particle below the mean position is trough.

3) Amplitude (a): The maximum displacement of particle from mean position at any instant is amplitude

4) Wave length(λ): The distance travelled by wave to complete 1 cycle is wave length. It is denoted by λ and its unit is m (SI) & cm (CGS).
The distance between two successive crest or trough is known as wave length.

5) Time period (T): The time taken by wave to complete 1 cycle is called time period. Its unit is sec.

6) Frequency(f): The number of cycle made by wave in 1 sec is known as frequency.

In T sec → 1 cycle
In 1 sec → $\frac{1}{T}$ cycle

$$\boxed{f = \frac{1}{T}}$$

SI unit : Hertz(Hz), cycle/sec

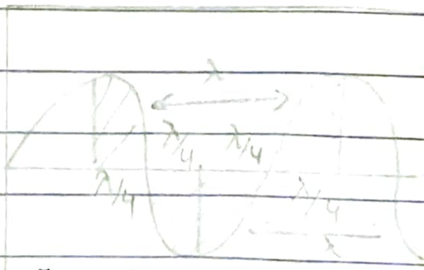
7) Wave velocity: The linear distance travelled by wave in 1 sec is called wave velocity.

$$v = \frac{\text{distance travelled}}{\text{time taken}}$$

$$v = \frac{\text{wave length}}{\text{Time period}}$$

$$v = \frac{\lambda}{T}$$

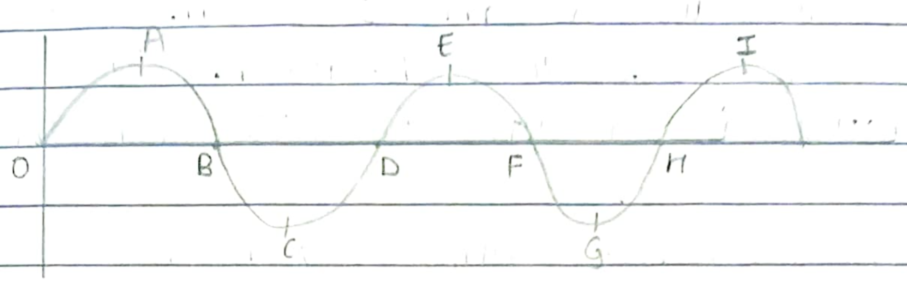
$$\boxed{\therefore v = \lambda f}$$



Four $\lambda/4$ make 1 complete cycle.

8) Phase: The position of particle expressed in angle is called phase. Phase of the particle at any instant

gives position & direction of particle at that instant.



A, E, I
B, D, F, H → are in same phase.
C, G

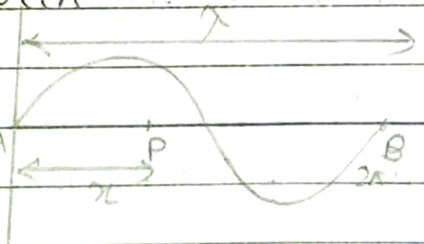
A, B, C are in out of phase.

The particles of medium are said to be at same phase if they are in equal distance & in same direction from the mean position.

IMP

Relation between path difference & phase difference

The linear distance between two point of wave is path difference (x). The difference in angular displacement of particle of wave is called phase difference (ϕ).



Here, path difference λ, phase difference $\phi = 2\pi$

path difference 1, phase difference $\phi = \frac{2\pi}{\lambda}$

path difference x, phase difference $\phi = \frac{2\pi x}{\lambda}$

$$\boxed{\therefore \phi = \frac{2\pi}{\lambda} x}$$

$\therefore$ phase difference $= \frac{2\pi}{\text{wave length}} \times$ path difference

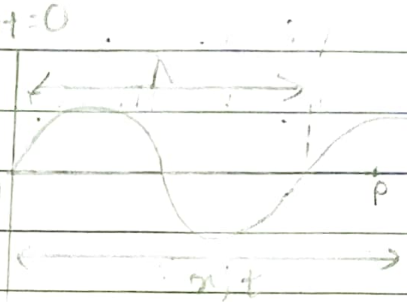
Date
Page

Imp

Progressive wave [Travelling wave]

The wave in which wave profile travels in forward direction with constant amplitude & frequency is called progressive wave. Eg: sound wave, water wave etc.

Consider a progressive wave travelling along +ve x-axis (i.e. left to right). When wave propagates, all the particles start to vibrate about their mean position.



Consider a particle vibrating at origin 'O'. The displacement of particle is

$y_{(x,t)} = a \sin \omega t$ — (I), where a = amplitude, $\omega = 2\pi f$ [angular frequency /velocity]

Suppose, another particle vibrating at point 'P' at distance x from O. ϕ be the phase difference of particle at P.

Now, displacement of particle at 'P'.

$$y = a \sin(\omega t - \phi) \quad \text{— (II)}$$

We know,

phase difference $\phi = \frac{2\pi}{\lambda} x$

$$\text{or, } y = a \sin\left(\omega t - \frac{2\pi}{\lambda} x\right)$$

$$\boxed{y = a \sin(\omega t - kx)}, \quad k = \frac{2\pi}{\lambda}$$

where k is wave number or propagation constant (no of wave per unit length)

$$\text{or, } y = a \sin\left(2\pi f t - \frac{2\pi}{\lambda} x\right)$$

Rough

y at p = y at O
at time t — at time $(t - \frac{x}{v})$

$y = a \sin \omega\left(t - \frac{x}{v}\right)$

$= a \sin\left(\omega t - \frac{\omega x}{v}\right)$

$\omega = 2\pi f$

Date ______
Page ______

$$y = a \sin\left(\frac{2\pi v}{\lambda} t - \frac{2\pi}{\lambda} x\right)$$

$$\boxed{y = a \sin \frac{2\pi}{\lambda} (vt - x)} \quad \text{--- (III)}$$

If progressive wave travelling from right to left,

$$\boxed{y = a \sin \frac{2\pi}{\lambda} (vt + x)} \quad \text{--- (IV)}$$

Neumericals:-

$y = 0.02 \sin(30t - 4x)$ — (1) x in m, t in sec.

Find

i) f, v, λ

ii) eq^n of wave with double amplitude & travelling in opposite direction.

Sol^n: Here,

comparing eq^n (1) with $y = a\sin(\omega t - kx)$, we get,

$a = 0.02$, $\omega = 30$, $k = 4$

Now,

$\omega = 30$

$2\pi f = 30$

$f = \frac{30}{2 \times \frac{22}{7}}$

$\therefore f = 4.774$ Hz

Again,

$k = \frac{2\pi}{\lambda}$

$4 = \frac{2 \times 22/7}{\lambda}$

$\therefore \lambda = 1.57$ m

Now, $v = \lambda f$

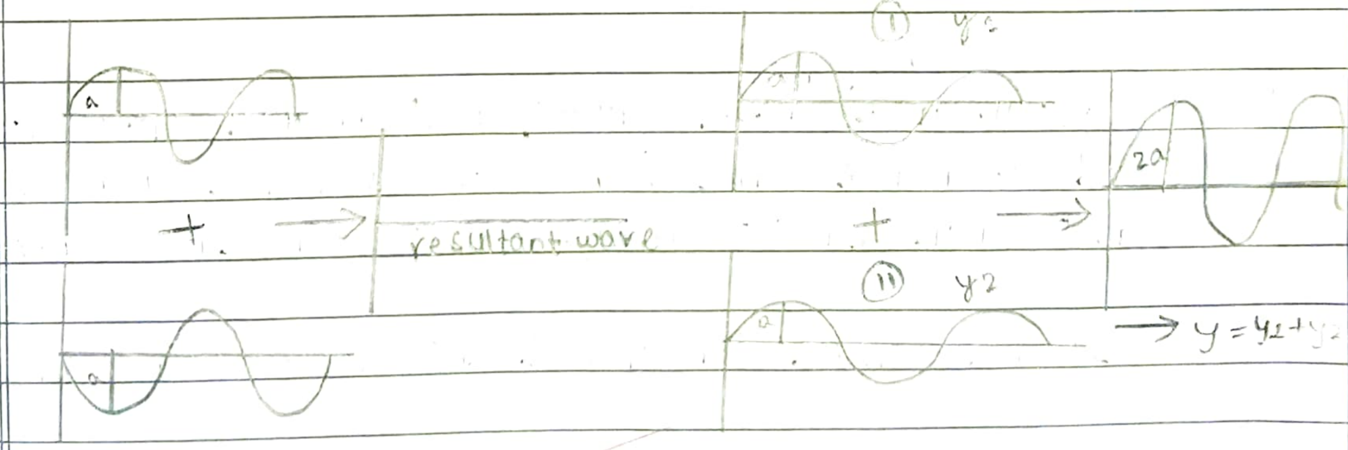
$= 1.57 \times 4.774$

$= 7.49518$ m/s

Then,
When amplitude is double, ~~a~~ $y = 0.04 \sin(30t - 4x)$.
When the wave is travelling in opposite direction,
comparing with $y = a \sin \frac{2\pi}{\lambda}(vt + x)$

$a = 0.04$, $\omega = 30$, $k = 4$
$\therefore y = 0.04 \sin(30t + 4x)$,,
Hence, this is the req. equation.

Principle of superposition:
Principle of superposition states that "when two or more wave travelling simultaneously through a medium at any instant, the displacement of resultant wave is the vector sum of the (algebric) displacement of individual wave at that instant."



If y_1, y_2 .... are the displacement of the wave then displacement of the resultant wave is given by:
$y = y_1 + y_2 + y_3 + \dots + y_n$

V.V.I

Stationary wave or standing wave

The wave which is formed by the superposition of two progressive wave of same amplitude & frequency travelling in opposite direction with same velocity. Stationary in a sense, the wave seems not to be moving. Further, there is no net flow of energy along the wave. Amplitude of vibration changes with time.

N = Node
A = Antinode

The point of zero amplitude is called node & the point of maxmium amplitude is called antinode.

Stationary wave	SN	Progressive wave
Disturbance doesn't travel forward & hence there is no net transfer of energy.	i)	Disturbance travel forward & hence there is net transfer to energy.
It has fixed length.	ii)	It doesn't have fixed length.
The particals at node are permanently at rest.	iii)	~~The~~ No particle is permanently at rest.
Amplitude of oscillation is same at all positions in the medium.	iv)	Amplitude of vibration on either side of antinode gradually decreases to zero.

Date ______
Page ______

Equation of stationary wave:-

Consider two progressive wave having same frequency and amplitude travelling in opposite direction. Let us suppose a wave pulse travelling along +ve x-axis. The displacement of wave is given by

$$y_1 = a \sin(\omega t - kx) \quad \text{— (i)}$$

Again, displacement of another wave travelling along -ve x-axis.

$$y_2 = a \sin(\omega t + kx) \quad \text{— (ii)}$$

Now, using principle of superposition, displacement of stationary wave is given by,

$$y = y_1 + y_2$$
$$= a\left[\sin(\omega t - kx) + \sin(\omega t + kx)\right]$$
$$= a\left[2 \sin\left(\frac{\omega t - kx + \omega t + kx}{2}\right) \cdot \cos\left(\frac{\omega t - kx - \omega t - kx}{2}\right)\right]$$

A depend upon cos

$$= 2a \sin\frac{\cancel{2}\omega t}{\cancel{2}} \cdot \cos\frac{-\cancel{2}kx}{\cancel{2}} \qquad [\sin C + \sin D]$$

cos (-1, 1 max, 0 - min)

$$= 2a \sin \omega t \cdot \cos(-kx)$$
$$= 2a \sin \omega t \cos kx \qquad [\cos(-\theta) = \cos\theta]$$
$$y = 2a \cos kx \sin \omega t$$
$$y = A \sin \omega t \quad \text{— (iii)}$$

where $A = 2a\cos kx$, is amplitude of stationary wave.

eq^n (iii) represents the eq^n of stationary wave.

Case I: For Maximum Amplitude (Antinode)

We have, $A = 2a \cos kx$

For max^m amplitude $\cos kx = \pm 1$ $\therefore A_{max} = 2a$

Now, $\cos kx = \pm 1$

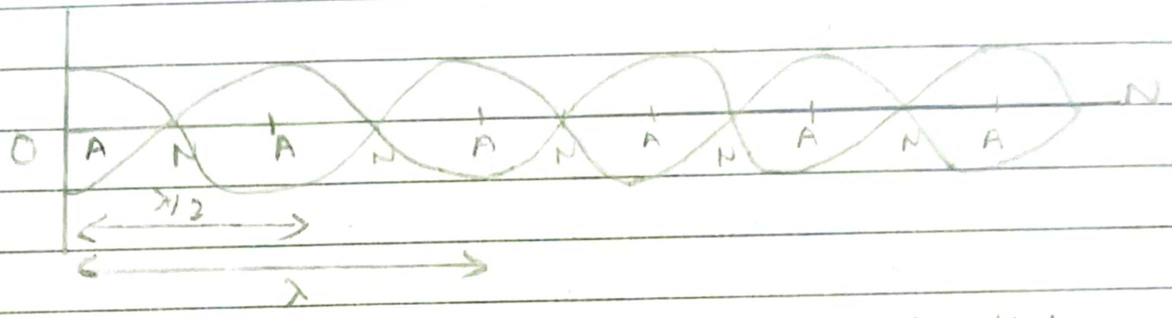
or, $\cos kx = \cos n\pi$, $n = 0, 1, 2, 3, \ldots$

or, $kx = n\pi$

or, $\frac{2\pi}{\lambda} x = n\pi$

or, $x = \frac{n\lambda}{2}$

$\therefore x = 0, \lambda/2, \lambda, 3\lambda/2, 2\lambda \ldots\ldots$



This shows antinodes are occur at distance $0, \lambda/2, \lambda, 3\lambda/2, 2\lambda \ldots\ldots$ from the origin.

$\therefore$ The distance between two sucessive antinode $= \lambda/2 - 0$

$= \lambda - \lambda/2$

$= \lambda/2$

Case II. For min^m amplitude (Node)

We have, $A = 2a\cos kx$

For min^m amplitude $\cos kx = 0$ $\quad \therefore A_{min} = 0$

Now, $\cos kx = 0$

or, $\cos kx = \cos (2n+1)\frac{\pi}{2}$ $\quad n = 0, 1, 2, 3, \ldots$

or, $kx = (2n+1)\frac{\pi}{2}$

or, $\frac{2\pi}{\lambda} x = (2n+1)\frac{\pi}{2}$

or, $x = (2n+1)\frac{\lambda}{4}$

$\therefore x = \lambda/4, 3\lambda/4, 5\lambda/4, 7\lambda/4 \ldots\ldots$

This shows nodes are occur at distance, $\lambda/4, 3\lambda/4,$ $5\lambda/4, 7\lambda/4 \ldots$ from the origin.

$\therefore$ The distance between two successive nodes $= 3\lambda/4 - \lambda/4$

$$= \frac{3\lambda - \lambda}{4}$$

$$= \frac{2\lambda}{4}$$

$$= \lambda/2$$

$\therefore$ Nodes & antinodes occurs alternatively at the distance of $\lambda/4$ in stationary wave.

MCQ.

1) The distance between two particles in a wave motion vibrating out of phase is

a) $\lambda/4$ ✓ b) $\lambda/2$ c) $3\lambda/4$ d) λ

2) Transverse mechanical wave can propagate

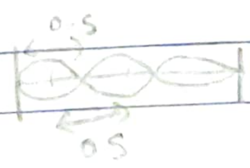
a) both in a gas & in a metal
b) in a gas but not in a metal
c) not in a gas but in a metal ✓
d) neither in a gas nor in a metal

3) A progressive wave equation is $y = 0.25 \sin(100t + 0.25x)$. Frequency of the wave is

a) $50/\pi$ Hz ✓ b) 100 Hz c) $100/\pi$ Hz d) 5 Hz

4) The distance betn two consequtive anti-nodes is 0.5 m. The distance travelled by the wave in half the time period is

a) 0.25 m b) 0.5 m ✓ c) 1 m d) 2 m



Numericals:-

A progressive and stationary simple harmonic wave each have the same frequency of 250 Hz and the same velocity of 30 m/s. Calculate

(i) Phase difference between two particles on the progressive which are 10 cm apart.

(ii) The eqn of motion of progressive wave if its amplitude is 0.03 m.

(iii) The distance betn nodes in the stationary wave

Soln: Here,

Frequency (f) = 250 Hz

Velocity (v) = 30 m/s

Path difference (x) = 10 cm = 0.1 m

Now,

$v = \lambda f$

$30 = \lambda \times 250$

$\therefore \lambda = 0.12$ m

i) Phase difference (ϕ) $= \frac{2\pi}{\lambda} \times x$

$= \frac{2\pi}{0.12} \times 0.1$

$= 5.23$ radian

Again,

Amplitude (a) = 0.03 m

ii) $y = a \sin \omega t$

$= 0.03 \sin 2\pi f \times t$

$=$

$y = a \sin \frac{2\pi}{\lambda}(vt \pm x)$

$= 0.03 \sin \frac{2\pi}{0.12}(30t \pm x)$

$= 0.03 \sin 52.35(30t \pm x)$

$y = 0.03 \sin \frac{50}{3}\pi(30t \pm x)$

Now,

Date
Page

(11) The distance between nodes in the stationary wave

$= \frac{\lambda}{2} = \frac{0.12 \, m}{2} = 0.06 \, m$ //

1) What is principle of superposition?

→ Principle of superpostion is "when two or more than two waves travelling simultaneously through a medium at any instant, the displacement of resultant wave is the vector sum of the displacement of individual wave at that instant."

2) Define wave motion.

→ Wave motion is the contineous transfer of disturbance from one point to another point of medium by successive vibration of particles about their mean position.

3) How stationary waves are formed? Prove that the distance between any two consecutive nodes in stationary wave is $\lambda/2$.

→ stationary waves are formed by the superposition of two progressive wave of same amplitude & frequency travelling in opposite direction with same velocity.

Consider two progressive wave having same frequency & amplitude travelling in opposite direction. Let us suppose a wave pulse travelling along +ve x-axis. The displacement of wave is given by

$$y_1 = a \sin(\omega t - kx) \quad \text{—} (1)$$

Again, displacement of another wave travelling along -ve x-axis,

$$y_2 = a\sin(\omega t + kx) \quad \text{— (ii)}$$

Now, using principle of superposition, displacement of stationary wave is given by,

$$y = y_1 + y_2$$
$$= a[\sin(\omega t - kx) + \sin(\omega t + kx)]$$
$$= a\left[2\sin\frac{\omega t - \cancel{kx} + \omega t + \cancel{kx}}{2} \cdot \cos\frac{\cancel{\omega t} - kx - \cancel{\omega t} - kx}{2}\right]$$
$$= a\left[2\sin\frac{\cancel{2}\omega t}{\cancel{2}} \cdot \cos\frac{-\cancel{2}kx}{\cancel{2}}\right]$$
$$= 2a\sin\omega t \cdot \cos -kx$$
$$= 2a\cos kx \cdot \sin\omega t \qquad [\because \cos(-\theta) = \cos\theta]$$
$$y = \cancel{2a}\, A\sin\omega t \quad \text{— (iii)}$$

where $A = 2a\cos kx$, is amplitude of stationary wave.

eq^n (iii) represents the eq^n of stationary wave.

For Minimum amplitude (Node)
We have, $A = 2a\cos kx$
For min^m amplitude $\cos kx = 0$ $\qquad \therefore A_{min} = 0$
Now,

$$\cos kx = 0$$
$$\text{or, } \cos kx = \cos(2n+1)\frac{\pi}{2} \qquad n = 0, 1, 2, 3, 4 \ldots$$
$$\text{or, } kx = (2n+1)\frac{\pi}{2}$$
$$\text{or, } \frac{2\pi}{\lambda}x = (2n+1)\frac{\pi}{2}$$
$$\text{or, } x = (2n+1)\frac{\lambda}{4}$$

Date ______
Page ______

$\therefore x = \lambda/4, 3\lambda/4, 5\lambda/4, 7\lambda/4, \ldots$

This shows nodes are occur at distance, $\lambda/4, 3\lambda/4, 5\lambda/4, 7\lambda/4 \ldots$ from the origin.

$\therefore$ The distance between two successive nodes $= 3\lambda/4 - \lambda/4$

$$= \frac{3\lambda - \lambda}{4}$$

$$= \lambda/2$$

4) A standing wave having 3 nodes & 2 antinodes is formed between two atoms having a distance of 1.21×10^{-10} m between them. Calculate the above length of standing wave.

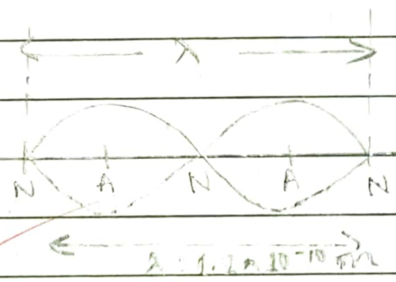
Here,

3 nodes

3 antinodes

& distance $(x) = 1.2 \times 10^{-10}$ m

Thus, $\lambda = 1.2 \times 10^{-10}$ m



5) Differentiable between lightwave & soundwave.

Light wave	Sound wave
Light wave is a non-mechanical wave which does not require any mechanical medium to transfer from one place to another.	1) Sound wave is a mechanical wave which requires mechanical medium to transfer from one place to another.
It is a transverse wave.	ii) It is a longitudinal wave.
It can be polarized.	iii) It cannot be polarized.