School of Engineering and Applied Science (SEAS), Ahmedabad University

BTech(ICT) Semester IV: Probability and Random Processes (MAT202)

Homework Assignment-1

Due Date: January 26, 2018 (Friday), 11:59 P.M.

1. An elementary school is offering 3 language classes: one in Spanish, one in French, and one in German. The classes are open to any of the 100 students in the school. There are 28 students in the Spanish class, 26 in the French class, and 16 in the German class. There are 12 students that are in both Spanish and French, 4 that are in both Spanish and German, and 6 that are in both French and German. In addition, there are 2 students taking all 3 classes.

   (a) If a student is chosen randomly, what is the probability that he or she is not in any of the language classes?

   (b) If a student is chosen randomly, what is the probability that he or she is taking exactly one language class?

   (c) If 2 students are chosen randomly, what is the probability that at least 1 is taking a language class?

2. Consider a medical practitioner pondering the following dilemma: If I'm at least 80 percent certain that my patient has this disease, then I always recommend surgery, whereas if I'm not quite as certain, then I recommend additional tests that are expensive and sometimes painful. Now, initially I was only 60 percent certain that Jones had the disease, so I ordered the series A test, which always gives a positive result when the patient has the disease and almost never does when he is healthy. The test result was positive, and I was all set to recommend surgery when Jones informed me, for the first time, that he was diabetic. This information complicates matters because, although it doesnt change my original 60 percent estimate of his chances of having the disease in question, it does affect the interpretation of the results of the A test. This is so because the A test, while never yielding a positive result when the patient is healthy, does unfortunately yield a positive result 30 percent of the time in the case of diabetic patients who are not suffering from the disease. Now what do I do? More tests or immediate surgery?

3. Two balls are selected sequentially (without replacement) from a box containing three red, four white, and five blue balls.

   (a) What is the probability that the first is red and the second is blue?.

   (b) What is the probability of selecting a white ball on the second draw if the first ball is replaced before the second is selected?

   (c) What is the probability of selecting a white ball on the second draw if the first ball is not replaced before the second is selected?

4. There are 15 tennis balls in a box, of which 9 have not previously been used. Three of the balls are randomly chosen, played with, and then returned to the box. Later, another 3 balls are randomly chosen from the box. Find the probability that none of these balls has ever been used.

5. A box of 30 diodes is known to contain five defective ones. If two diodes are selected at random without replacement, what is the probability that at least one of these diodes is defective?

6. Two six-sided (balanced) dice are thrown. Find the probabilities of each of the following events:

   (a) a 5 does not occur on either throw.

(b) the sum is 7.

(c) a 5 and a 3 occur in any order.

(d) the first throw is a 5 and the second throw is a 5 or a 4.

(e) both throws are 5.

(f) either throw is a 6.

7. Cards are drawn from a standard 52-card deck until an ace is drawn. After each card is drawn, (if the card is not an ace), it is put back in the deck and the cards are reshuffled so that each card drawn is independent of all others.

   (a) Find the probability that the first ace is drawn on the 5th selection.

   (b) Find the probability that at least 5 cards are drawn before the first ace appears.

   (c) Repeat parts (a) and (b) if the cards are drawn without replacement. That is, after each card is drawn, (if it is not an ace) the card is set aside and not replaced in the deck.