School of Engineering and Applied Science (SEAS), Ahmedabad University

BTech(ICT) Semester IV: Probability and Random Processes (MAT202)

Homework Assignment-1

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1. The given situation can be modelled as
A = Students who join Spanish
B = Students who join French
C = Students who join German

Total number of students = 100
Number of students taking Spanish, $Pr(A) = \frac{28}{100}$
Number of students taking French, $Pr(B) = \frac{26}{100}$
Number of students taking German, $Pr(C) = \frac{16}{100}$
Number of students taking Spanish and French, $Pr(A \cap B) = \frac{12}{100}$
Number of students taking French and German, $Pr(B \cap C) = \frac{6}{100}$
Number of students taking Spanish and German, $Pr(A \cap C) = \frac{4}{100}$
Number of students taking Spanish, French and German, $Pr(A \cap B \cap C) = \frac{2}{100}$

(a) We know that,

$$\begin{aligned} Pr(A \cup B \cup C) = {} & Pr(A) + Pr(B) + Pr(C) - Pr(A \cap B) - Pr(B \cap C) - Pr(A \cap C) \\ & + Pr(A \cap B \cap C) \end{aligned}$$

Therefore,

$$\begin{aligned} Pr(A \cup B \cup C) &= \frac{28}{100} + \frac{26}{100} + \frac{16}{100} - \frac{12}{100} - \frac{6}{100} - \frac{4}{100} + \frac{2}{100} \\ &= \frac{50}{100} \end{aligned}$$

and,

$$\boxed{Pr(A \cup B \cup C)' = 1 - Pr(A \cup B \cup C)}$$

$$\begin{aligned} Pr(A \cup B \cup C)' &= 1 - \frac{50}{100} \\ &= \frac{50}{100} \\ &= \mathbf{0.5000} \end{aligned}$$

(b) $D = \{\text{The student is taking exactly one language class}\}$

$$\begin{aligned}
Pr(A \cap B' \cap C') &= Pr(A) - Pr(A \cap B) - Pr(A \cap C) + Pr(A \cap B \cap C) \\
&= \frac{28}{100} - \frac{12}{100} - \frac{4}{100} + \frac{2}{100} \\
&= \frac{14}{100} \\
&= 0.1400
\end{aligned}$$

$$\begin{aligned}
Pr(A' \cap B \cap C') &= Pr(B) - Pr(A \cap B) - Pr(B \cap C) + Pr(A \cap B \cap C) \\
&= \frac{26}{100} - \frac{12}{100} - \frac{6}{100} + \frac{2}{100} \\
&= \frac{10}{100} \\
&= 0.1000
\end{aligned}$$

$$\begin{aligned}
Pr(A' \cap B' \cap C) &= Pr(C) - Pr(A \cap C) - Pr(B \cap C) + Pr(A \cap B \cap C) \\
&= \frac{16}{100} - \frac{4}{100} - \frac{6}{100} + \frac{2}{100} \\
&= \frac{8}{100} \\
&= 0.0800
\end{aligned}$$

$$\begin{aligned}
Pr(A' \cap B' \cap C) &= Pr(C) - Pr(A \cap C) - Pr(B \cap C) + Pr(A \cap B \cap C) \\
&= \frac{16}{100} - \frac{4}{100} - \frac{6}{100} + \frac{2}{100} \\
&= \frac{8}{100} \\
&= 0.0800
\end{aligned}$$

$$\begin{aligned}
Pr(D) &= Pr(A \cap B' \cap C') + Pr(A' \cap B \cap C') + Pr(A' \cap B' \cap C) \\
&= 0.1400 + 0.1000 + 0.0800 \\
&= \mathbf{0.3200}
\end{aligned}$$

(c) E = {At least one takes a language class}
F = E' = {None take language class}

$$\begin{aligned}
Pr(E) &= 1 - Pr(F) \\
&= 1 - \frac{50}{100} \times \frac{49}{99} \\
&= 1 - 0.2475 \\
&= \mathbf{0.7525}
\end{aligned}$$

2. Let A and D be two events such that,

$A = \{\text{The patient is positive on test A}\}$
$D = \{\text{The patient is diseased}\}$

$$\begin{aligned} Pr(D|A) &= \frac{Pr(A|D)Pr(D)}{Pr(A)} \\ &= \frac{(0.6)(1)}{(1)(0.6) + (0.3)(0.4)} \\ &= \mathbf{0.8333} \end{aligned}$$

Since the probability is greater than 0.8, the doctor should recommend an immediate surgery.

3. Let the events in this experiment be as below:

R_1 = {Getting a red on the first selection}
G_1 = {Getting a green on the first selection}
W_1 = {Getting a blue on the first selection}
R_2 = {Getting a red on the second selection}
W_2 = {Getting a green on the second selection}
B_2 = {Getting a blue on the second selection}

(a)

$$\begin{aligned} Pr(R_1, B_2) &= \frac{{}^3C_1}{{}^{12}C_1} \times \frac{{}^5C_1}{{}^{11}C_1} \\ &= \mathbf{0.1136} \end{aligned}$$

(b) Replacement makes the two events independent of each other.

$$\begin{aligned} Pr(W_2) &= \frac{{}^4C_1}{{}^{12}C_1} \\ &= \frac{1}{3} \\ &= \mathbf{0.3333} \end{aligned}$$

(c)

$$\begin{aligned} Pr(W_2) &= \sum_{i=1}^{3} Pr(W_2|A_i)Pr(A_i) \\ &= Pr(W_2|R_1)Pr(R_1) + Pr(W_2|W_1)Pr(W_1) + Pr(W_2|B_1)Pr(B_1) \\ &= \frac{{}^3C_1}{{}^{12}C_1} \times \frac{{}^4C_1}{{}^{11}C_1} + \frac{{}^4C_1}{{}^{12}C_1} \times \frac{{}^3C_1}{{}^{11}C_1} + \frac{{}^5C_1}{{}^{12}C_1} \times \frac{{}^4C_1}{{}^{11}C_1} \\ &= \frac{1}{3} \\ &= \mathbf{0.3333} \end{aligned}$$

4. Let the events be described as below:

$A_i = \{\text{i unused balls were chosen during the first trial}\}$
$B = \{\text{None of the balls have been used}\}$

$$Pr(B) = \sum_{i=1}^{n} Pr(B|A_i)Pr(A_i)$$

$$\begin{aligned}
Pr(B) &= Pr(B|A_1)Pr(A_1) + Pr(B|A_2)Pr(A_2) + Pr(B|A_3)Pr(A_3) \\
&= \frac{{}^9C_0 \times^6 C_3}{{}^{15}C_3} \times \frac{{}^9C_3}{{}^{15}C_3} + \frac{{}^9C_1 \times^6 C_2}{{}^{15}C_3} \times \frac{{}^8C_3}{{}^{15}C_3} + \frac{{}^9C_2 \times^6 C_1}{{}^{15}C_3} \times \frac{{}^7C_3}{{}^{15}C_3} + \frac{{}^9C_3 \times^6 C_0}{{}^{15}C_3} \times \frac{{}^6C_3}{{}^{15}C_3} \\
&= \mathbf{0.0893}
\end{aligned}$$

5. Let the events be described as below:
   $A = \{\text{The first selection is defective}\}$
   $B = \{\text{At least one is defective}\}$

$$
\begin{aligned}
Pr(B) &= \sum_{i=1}^{n} Pr(B|A_i)Pr(A_i) \\
Pr(B) &= Pr(B|A)Pr(A) + Pr(B|A')Pr(A') \\
&= 1 \times \frac{{}^{5}C_1}{{}^{30}C_1} + \frac{{}^{5}C_1}{{}^{29}C_1} \times \frac{{}^{25}C_1}{{}^{30}C_1} \\
&= \mathbf{0.3013}
\end{aligned}
$$

6. (a) Let the events be described as below:
$A = \{5 \text{ occours in none of the throws}\}$
$B = \{5 \text{ occours in at least one of the throws}\}$
$C_i = \{5 \text{ occours in the } i^{th} \text{ throw}\}$

$$
\begin{aligned}
Pr(B) &= Pr(C_1) + Pr(C_2) - Pr(C_1 \cap C_2) \\
Pr(B) &= \frac{1}{6} + \frac{1}{6} - \frac{1}{6} \times \frac{1}{6} \\
&= \frac{11}{36} \\
Pr(A) &= 1 - Pr(B) \\
&= 1 - \frac{11}{36} \\
&= \frac{25}{36} \\
&= \mathbf{0.6944}
\end{aligned}
$$

(b) Let A be the event when the sum is 7:
$A = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$$
\begin{aligned}
Pr(A) &= \frac{6}{36} \\
&= \frac{1}{6} \\
&= \mathbf{0.1667}
\end{aligned}
$$

(c) Let A be the event when the 2 dice show (5, 3):
Let B be the event when the 2 dice show (3, 5):

A and B are two mutually exclusive events.

$$
\begin{aligned}
Pr(C) &= Pr(A \cup B) \\
Pr(C) &= Pr(A) + Pr(B) - Pr(A \cap B) \\
&= \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} - 0 \\
&= \frac{2}{36} \\
&= \frac{1}{18} \\
&= \mathbf{0.0556}
\end{aligned}
$$

(d) $A_i = \{\text{Getting an i on the first throw}\}$
$B_i = \{\text{Getting an i on the second throw}\}$

$$\begin{aligned}
Pr(A_5, B_5 \cup B_4) &= Pr(A_5)(Pr(B_5) + Pr(B_4)) \quad (\because B_5 \text{ and } B_4 \text{ are mutually exclusive})\\
&= \frac{1}{6} \times (\frac{1}{6} + \frac{1}{6})\\
&= \frac{1}{18}\\
&= \mathbf{0.0556}
\end{aligned}$$

(e) $A_5 = \{\text{Getting a 5 on the first throw}\}$
$B_5 = \{\text{Getting a 5 on the second throw}\}$
Both the events are independent of each other.

$$\begin{aligned}
Pr(A_5, B_5) &= Pr(A_5) \times Pr(B_5) \qquad (\because A_5 \text{ and } B_5 \text{ are independent events})\\
&= \frac{1}{6} \times \frac{1}{6}\\
&= \frac{1}{36}\\
&= \mathbf{0.0278}
\end{aligned}$$

(f) $A_6 = \{\text{Getting a 6 on the first throw}\}$
$B_6 = \{\text{Getting a 6 on the second throw}\}$

$$\begin{aligned}
Pr(A_6, B_6) &= Pr(A_6) \times Pr(B_6) \qquad (\because A_6 \text{ and } B_6 \text{ are independent events})\\
&= \frac{1}{6} \times \frac{1}{6}\\
&= \frac{1}{36}
\end{aligned}$$

$$\begin{aligned}
Pr(A_6 \cup B_6) &= Pr(A_6) + Pr(B_6) - Pr(A_6, B_6)\\
&= \frac{1}{6} + \frac{1}{6} - \frac{1}{36}\\
&= \frac{11}{36}\\
&= \mathbf{0.3056}
\end{aligned}$$

7. (a) $A = \{\text{First ace is drawn on } 5^{th} \text{ selection}\}$

$$
\begin{aligned}
Pr(A) &= \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{4}C_1}{^{52}C_1} \\
&= \frac{(48)^4 \times 4}{(52)^5} \\
&= \mathbf{0.0558}
\end{aligned}
$$

(b) $A = \{\text{At least 5 cards are drawn before the first ace appears}\}$

$$
\begin{aligned}
Pr(A) &= \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{4}C_1}{^{52}C_1} \\
&+ \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{4}C_1}{^{52}C_1} \\
&+ \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} \times \frac{^{4}C_1}{^{52}C_1} \\
&+ ...\infty \text{ terms} \\
&= \frac{(48)^5 \times 4}{(52)^6} + \frac{(48)^6 \times 4}{(52)^7} + \frac{(48)^7 \times 4}{(52)^8} + ...\infty \text{ terms} \\
&= \frac{(48)^5 \times 4}{(52)^6} \Big[1 + \frac{48}{52} + \Big(\frac{48}{52}\Big)^2 + \Big(\frac{48}{52}\Big)^3 + ...\infty \text{ terms}\Big] \\
&= \Big(\frac{12}{13}\Big)^5 \\
&= \mathbf{0.6702}
\end{aligned}
$$

(c) $A = \{\text{First ace is drawn on } 5^{th} \text{ selection}\}$

$$
\begin{aligned}
Pr(A) &= \frac{^{48}C_4}{^{52}C_4} \times \frac{^{4}C_1}{^{48}C_1} \\
&= \frac{194580}{270725} \times \frac{4}{48} \\
&= \mathbf{0.0599}
\end{aligned}
$$

[Answer to corrsponding part (a)]

$B = \{\text{At least 5 cards are drawn before the first ace appears}\}$
$B' = \{\text{The first ace appears within the first 5 selections}\}$

$$
\begin{aligned}
Pr(B) &= 1 - Pr(B') \\
&= 1 - \Big[\frac{^{4}C_1}{^{52}C_1} + \frac{^{48}C_1}{^{52}C_1} \times \frac{^{4}C_1}{^{51}C_1} + \frac{^{48}C_2}{^{52}C_2} \times \frac{^{4}C_1}{^{50}C_1} + \frac{^{48}C_3}{^{52}C_3} \times \frac{^{4}C_1}{^{49}C_1} + \frac{^{48}C_4}{^{52}C_4} \times \frac{^{4}C_1}{^{48}C_1}\Big] \\
&= \mathbf{0.6588}
\end{aligned}
$$

[Answer to corrsponding part (b)]