Foundations of Computing 1 - 2022 Exam 15/04/2024

1a. Find the sum to infinity for the series $12+6+3+\ldots$

$$S_\infty = \frac{a}{1-r} = \frac{12}{1-\frac{1}{2}} = 24$$

12 6 3

$\times\frac{1}{2}$ $\times\frac{1}{2}$

a is first term : 12

b. Determine the values of a, b, c and d:

$$(2+3x)^a = 16 + bx + 216x^2 + 216x^c + dx^4$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$(2+3x)^a = \binom{4}{0} \times 2^4 \times (3x)^0 + \binom{4}{1} \times 2^3 \times (3x)^1$$

$$+ \binom{4}{2} \times 2^2 \times (3x)^2 + \binom{4}{3} \times 2^1 \times (3x)^3 + \binom{4}{4} \times (2)^0 \times (3x)^4$$

$$= (1 \times 16 \times 1) + (4 \times 8 \times 3x) + (6 \times 4 \times 9x^2)$$
$$+ (4 \times 2 \times 27x^3) + (1 \times 1 \times 81x^4)$$

$$= 16 + 96x + 216x^2 + 216x^3 + 81x^4$$

$a = 4$, $b = 96$, $c = 3$, $d = 81$

c. $f(x) = x^2 + 6x - 4$

$g(x) = 4x - x^2$

i. Find the two values of x for which $f(x)$ and $g(x)$ coincide.

Values when $f(x) = g(x)$

$x^2 + 6x - 4 = 4x - x^2$

$x^2 + x^2 + 6x - 4x - 4 \cancel{(x)} = 0 \longrightarrow 2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0 \longrightarrow (x+2)(x-1) = 0$

$x + 2 = 0 \longrightarrow x = -2$, $x - 1 = 0 \longrightarrow x = 1$

ii. Find the values of $f(x)$ when this occurs.

$f(1) = 1^2 + (6 \times 1) - 4 = 1 + 6 - 4 = 3$

$f(-2) = (-2)^2 + (6 \times -2) - 4 = 4 - 12 - 4 = \cancel{4} - 12$

~~when $x = 3$, $x =$~~

Values are 3 and -12

2. A series is defined by $S_n = \sum_{r=1}^{n} 5^r$

a. Find the first three terms of the series.

$T_1 = 5^1 = 5$

$T_2 = 5 + 5^2 = 30$

$T_3 = 30 + 5^3 = 155$

b. Use the method of induction to prove that $S_n = \frac{5}{4}(5^n - 1)$

Base case: $T_1 = S_1$, $r = 1$, $n = 1$

$5^1 = 5$, $\frac{5}{4}(5^1 - 1) = \frac{5}{4} \times 4 = 5$

Assumption: ~~S_{k+1}~~ There is at least one value where k $(1 \leq k < n)$ such that $S_k = T_k$

Statement: If true, then $S_{k+1} = S_k + T_{k+1}$

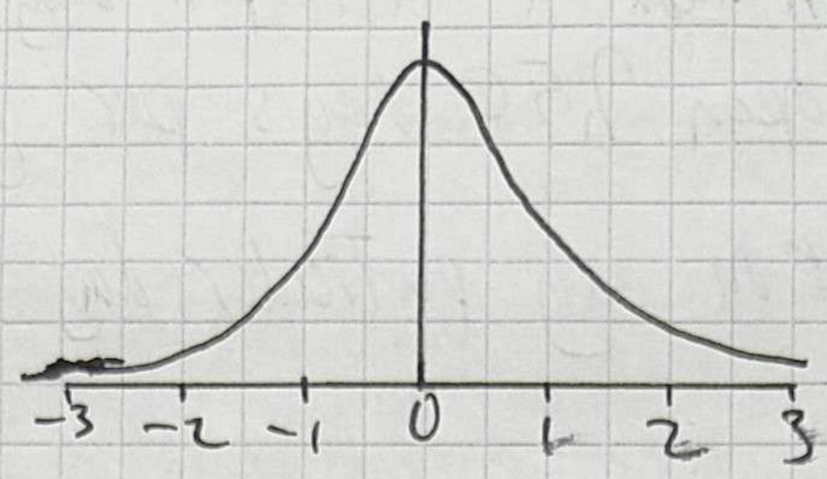
Induction: $S_{k+1} = \frac{5}{4}(5^{k+1} - 1) = \frac{5(5^{k+1} - 1)}{4}$

$$S_{k+1} = \frac{5}{4}(5^k - 1) + 5^{k+1}$$

$$= \frac{5(5^k - 1) + 4(5^{k+1})}{4} = \frac{1(5^{k+1}) - 5 + 4(5^{k+1})}{4}$$

$$= \frac{5(5^{k+1}) - 5}{4} = \frac{5(5^{k+1} - 1)}{4}$$

3a. Sketch the distribution $N(0, 1)$ and state the 2σ rule for Normal variables



$\sigma = 1$

The 2σ rule states that values between 2σ are within 95% of the mean.

b. State the conditions under which a normal distribution can be used as an approximation to a Poisson distribution. State the values of μ and σ when $\lambda = 49$.

Normal distribution can be used when λ is too high, and when dealing with continuous sets of data.

Values of μ and σ when $\lambda = 49$: $\mu = 49$, $\sigma = 7$

c. The number of visitors to a small tourist attraction follows a Poisson distribution with mean $\lambda = 49$ per day when it is open. The attraction is open 250 days per year.

i. Find the probability that on any particular day there will be more than 55 visitors.

$\lambda = 49$, 250 days per year

$P(X > 55) \rightarrow P(X > 55.5)$

$X \sim N(49, 49)$ variance, $sd = \sqrt{\text{variance}} = 7$

$P(X > 55.5) = 1 - P(X < 55.5)$

$P(Z < z)$ for $Z \sim N(0,1)$

$$z = \frac{x - \mu}{\sigma}$$

$$1 - P\left(Z < \frac{55 - 49}{7}\right) = P(Z < 0.93) = 0.8238$$

On the Normal distribution table at 0.93

$1 - 0.8238 = 0.1762$

ii. How many days per year will there be more than 55 visitors?

250 days $\times$ 0.1762 = 44.05 = 44 days

iii. What is the chance that there will be between 45 and 55 visitors (inclusive) on any particular day?

$$P(45 \leqslant x \leqslant 55) = P(x < 55.5) - P(x < 45.5)$$

$$\rightarrow \frac{55.5-49}{7} - \frac{45.5-49}{7} = 0.93 \text{(2dp)} - (-0.5)$$

$$P(z < 0.93) - P(z < -0.5$$

$$\downarrow$$

$$1 - P(x < 0.5) = 1 - 0.6915 = 0.3085$$

On the Normal distribution table

$$0.8238 - 0.3085 = 0.5153$$

From part i.

4a. L is a language with $L = \{r^0 1 0 1^0 r^0 10\}$. Which of the following strings are contained within L?

i. 10 <u>10</u>1 10 <u>10</u> True

ii. 11 <u>10</u>1 1 <u>10</u> False

iii. 0 <u>1</u>0<u>1</u> 0 <u>10</u> True

iv. <u>10</u>1<u>0</u>1 <u>10</u>1 <u>10</u> True

v. 10 <u>10</u>1 0 1 0 <u>10</u> False

b. Three sets are defined as follows:

$M = \{$natural numbers $< 20\}$

$P = \{$Prime numbers $\leq 20\}$

$Q = \{$odd numbers < 20

$M = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$

$P = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$Q = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

i. List the members of $M \cap P$

$M \cap P = \{2, 3, 5, 7, 11, 13, 17, 19\}$

ii. Find the members of P which are not set in Q

Members of P which are not in Q $= \{2\}$

iii. Find the members of Q which are not in set P

Members of Q not in P $= \{1, 9, 15\}$

c. Create a set comprehension for the set $F = \{1, 2, 4, 8, 16\}$

$F = \{x \mid x = 2^p \text{ where } p \in \mathbb{N}^+ \text{ and } p < 5\}$

5 a i. What is the maximum positive number that can be stored in 8-bit 2's complement form?

Largest positive number is 127

ii. Convert +97 and −58 into 8-bit 2's complement form.

64	32	16	8	4	2	1
1	1	0	0	0	0	1

97 − 64 = 33

33 − 32 = 1 1 − 1 = 0

+97 = 01100001

−128	64	32	16	8	4	2	1
1	1	0	0	0	1	1	0

~~−58~~

−128 + 64 = −64 −64 − 4 = 6 − 60 −60 + 2 = −58

−58 = 11000110

iii. Perform the 2's complement calculations for 97+58 and 97−58, show your working.

58 as 2's complement

−128	64	32	16	8	4	2	1
0	0	1	1	1	0	1	0

58 − 32 = 26 26 − 16 = 10 10 − 8 = 2 2 − 2 = 0

+58 = 00111010

+97 = 01100001

```
 00111010      58
 01100001 +    97
 ---------
1 00011011
```

overflow error

+97
+58
= 00011011 (with an overflow error)

−58 = 11000110

```
 11000110      −58
 01100001 +     97
 ---------
1 00100111
```

+97
+
−58 = 39

00100111 = 39

iv. Using your calculations as examples, show how an overflow can be detected

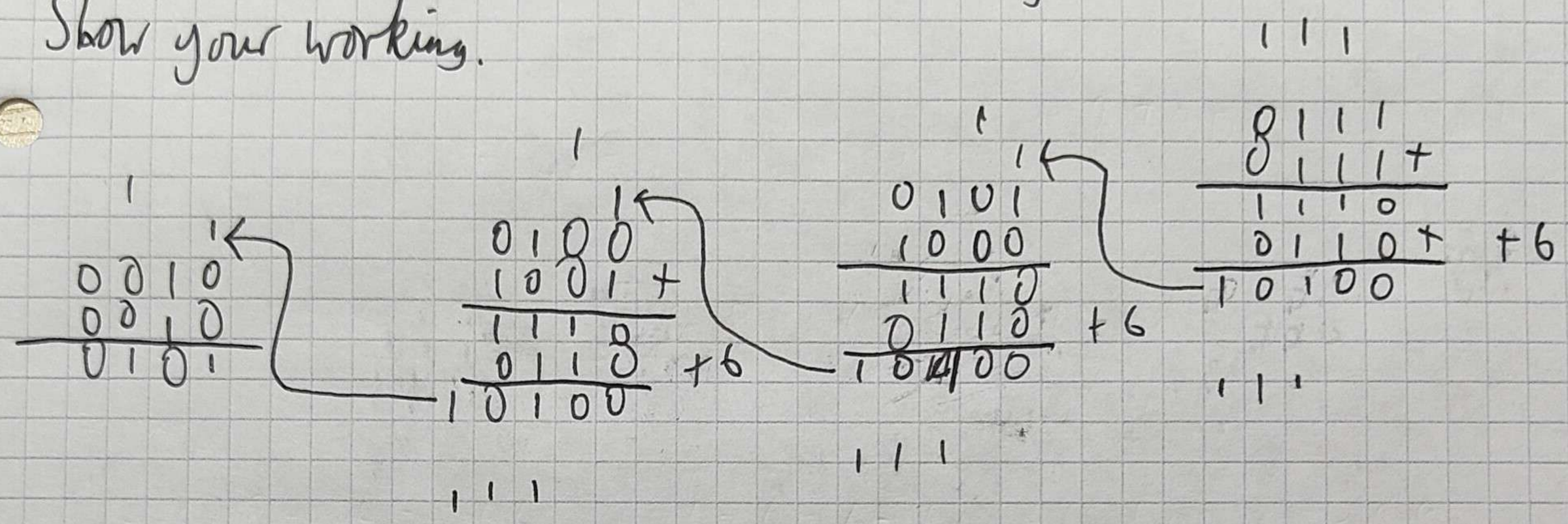
An overflow can be detected when the carryovers are not the same, and therefore the sum isn't in bounds.

bi. Write the numbers 12457 and 2987 in BCD format.

1 2 4 5 7
0001,0010,0100,0101,0111

2 9 8 7
0010,1001,1000,0111

ii. Perform the calculation 12457 + 2987 using BCD arithmetic. Show your working.



Final Answer: 0001,0101,0100,0100,0100

1 5 4 4 4

=15,444