COMP3220

Exam Revision Session (2)

(2022) Question 1

(a) Find the sum to infinity for the series $12 + 6 + 3 + \cdots$

$$r = \frac{T_2}{T_1} = \frac{6}{12} = \frac{1}{2} \qquad\qquad r = \frac{T_3}{T_2} = \frac{3}{6} = \frac{1}{2}$$

constant ratio ***r***, so it is a geometric progression

$$r = \frac{1}{2}, \qquad a = 12$$

$$S_\infty = \frac{a}{1-r} = \frac{12}{1-\frac{1}{2}} = \mathbf{24}$$

(b) Determine the values of a, b, c, and d

$$(2+3x)^a = 16 + bx + 216x^2 + 216x^c + dx^{4}$$

$\boldsymbol{a = 4}$

$$
\begin{array}{lccccccccc}
1+x & & & & 1 & & 1 & & & \\
(1+x)^2 & & & 1 & & 2 & & 1 & & \\
(1+x)^3 & & 1 & & 3 & & 3 & & 1 & \\
(1+x)^4 & 1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

$$(2+3x)^4 = 1(2^4) + 4(2^3)(3x) + 6(2^2)(3x)^2 + 4(2^1)(3x)^3 + 1(3x)^4$$

$$= 16 + 96x + 216x^2 + 216x^3 + 81x^4$$

$\boldsymbol{b = 96,\quad c = 3,\quad d = 81}$

(c) $f(x) = x^2 + 6x - 4$

$g(x) = 4x - x^2$

(i) Find the two values of x for which $f(x)$ and $g(x)$ coincide

$$f(x) = g(x)$$

$$x^2 + 6x - 4 = 4x - x^2$$

$$2x^2 + 2x - 4 = 0$$

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0$$

$$\boldsymbol{x = -2 \ \ or \ \ x = 1}$$

(ii) Find the values of $f(x)$ when this occurs

When $x = -2 \ \ or \ \ x = 1, \ f(x)$ and $g(x)$ coincide

$$f(x) = x^2 + 6x - 4$$

$$f(-2) = (-2)^2 + 6(-2) - 4 = 4 - 12 - 4 = \mathbf{-12}$$

$$f(1) = 1^2 + 6(1) - 4 = 1 + 6 - 4 = \mathbf{3}$$

(2022) Question 2

A series is defined by $S_n = \sum_{r=1}^{n} 5^r$

(a) Find the first 3 terms of the series.

$$T_r = 5^r$$

$$T_1 = 5^1 = \mathbf{5}$$

$$T_2 = 5^2 = \mathbf{25}$$

$$T_3 = 5^3 = \mathbf{125}$$

(b) Use the method of induction to prove that $S_n = \frac{5}{4}(5^n - 1)$

Proof: $T_r = 5^r$, $S_n = \frac{5}{4}(5^n - 1)$

(1) Base case

$T_1 = 5^1 = 5$, $S_1 = \frac{5}{4}(5^1 - 1) = 5$

Therefore, $T_1 = S_1$

(2) Assumption

There is at least one value of k $(1 \leq k < n)$ such that $S_k = \frac{5}{4}(5^k - 1)$

(3) Statement

If the formula is true, then $S_{k+1} = \frac{5}{4}(5^{k+1} - 1)$

$$T_r = 5^r, \qquad S_n = \frac{5}{4}(5^n - 1)$$

(4) Induction

$$S_{k+1} = S_k + T_{k+1}$$

$$= \frac{5}{4}(5^k - 1) + 5^{k+1}$$

$$= \frac{5(5^k - 1)}{4} + \frac{4(5^{k+1})}{4}$$

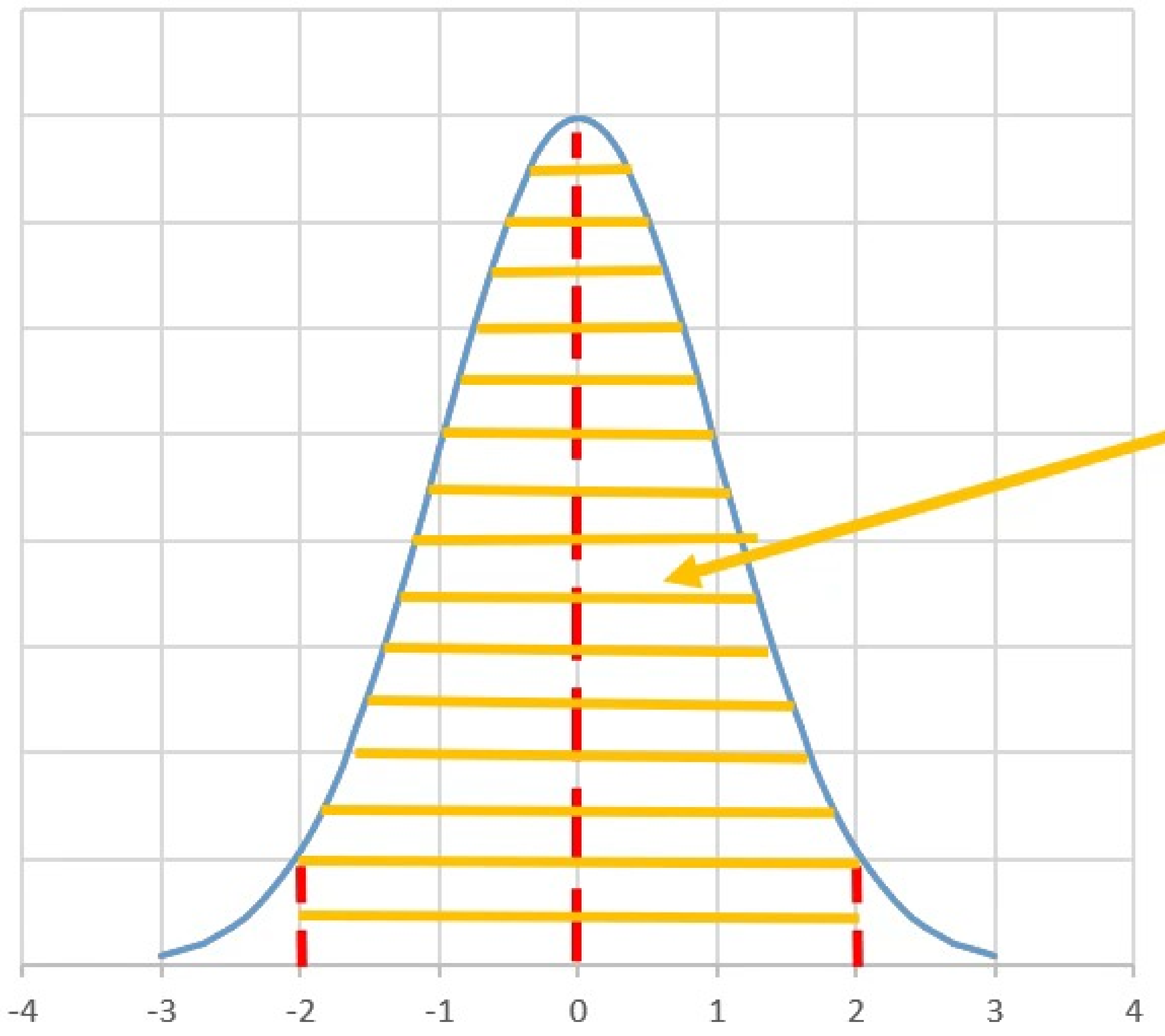
$$= \frac{5(5^k - 1) + 4(5^{k+1})}{4} = \frac{(5^{k+1}) - 5 + 4(5^{k+1})}{4}$$

$$= \frac{5(5^{k+1}) - 5}{4} = \frac{5(5^{k+1} - 1)}{4} = \frac{5}{4}(5^{k+1} - 1)$$

(2022) Question 3

(a) Sketch the distribution $N(0, 1)$ and state the 2σ rule for Normal variables.

$N(0, 1)$



$\mu = 0, \quad \sigma = 1$

2σ rule

$\mu \pm 2\sigma$

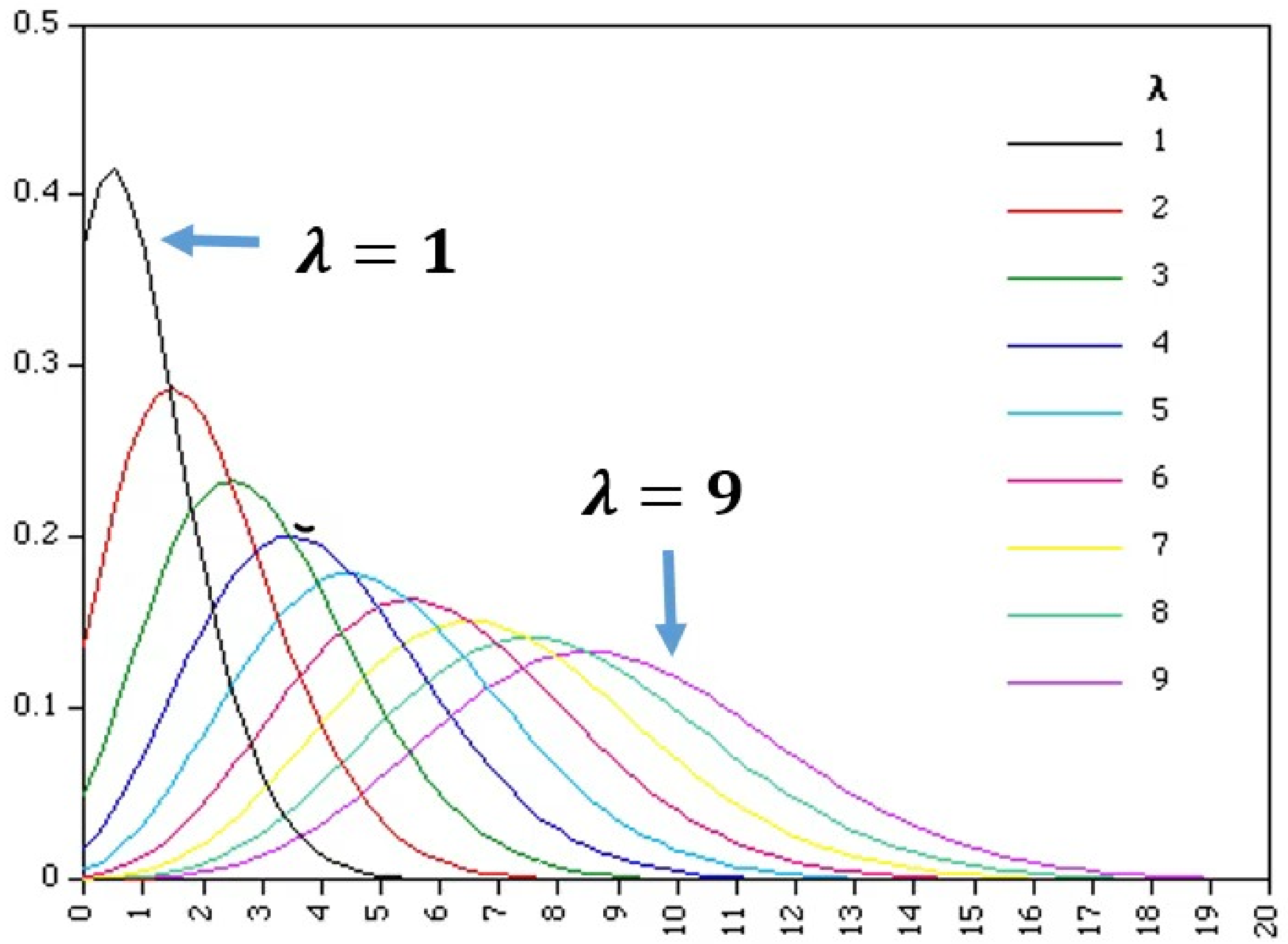
95%

$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.95$

(b) State the condition under which a Normal distribution can be used as an approximation to a Poisson distribution.

State the values of μ and σ when $\lambda = 49$

Condition: $\boldsymbol{\lambda > 10}$



When $\lambda = 49$

$\mu = \lambda = \mathbf{49}$ $\quad\quad$ $\sigma = \sqrt{\lambda} = \sqrt{49} = \mathbf{7}$

(c) The number of visitors to a small tourist attraction follows a Poisson distribution with mean $\lambda = 49$ per day when it is open. The attraction opens 250 days per year.

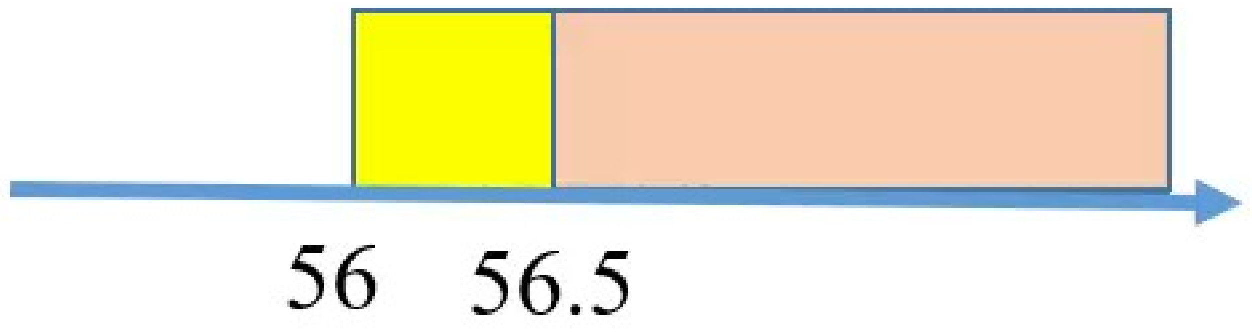
(i) Find the probability that on any particular day, there will be more than 55 visitors.

$\lambda = 49$ per day $\qquad \mu = 49, \qquad \sigma = \sqrt{49} = 7$

$P(X > 55) = P(X \geq 56)$ **Poisson Distribution**

$= P(X > 55.5)$ **Normal Distribution** (continuity correction)

$$= P\left(Z > \frac{55.5 - 49}{7}\right) = P(Z > 0.93)$$



$= 1 - P(Z < 0.93) = 1 - 0.8238 = \mathbf{0.1762}$

(ii) On how many days per year will there be more than 55 visitors?

$$0.1762 \times 250 = 44.05 \approx \mathbf{44\ (days)}$$

(iii) What is the chance that there will be between 45 and 55 visitors (inclusive) on any particular day?

$\lambda = 49$ per day $\qquad \mu = 49, \qquad \sigma = \sqrt{49} = 7$

Poisson Distribution

$P(45 \leq X \leq 55)$

Normal Distribution (continuity correction)

$= P(44.5 < X < 55.5)$

$= P(X < 55.5) - P(X < 44.5)$

$$= P\left(Z < \frac{55.5 - 49}{7}\right) - P\left(Z < \frac{44.5 - 49}{7}\right)$$

$= P(Z < 0.93) - P(Z < -0.64)$

$= 0.8238 - (1 - P(Z < 0.64))$

$= 0.8238 - (1 - 0.7389) = \mathbf{0.5627}$

z	0	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861

(2022) Question 4

(a) L is a language with $L = \{\, r \circ 101 \circ r \circ 10 \mid r \in \{0,1\}^* \,\}$

Which of the following strings are contained within L?

(i) 101011010 ✓

(ii) 11101110 ✗

(iii) 0101010 ✓

(iv) 10110 ✓

(v) 1010101010 ✗

(b) Three sets are defined as follows:

$M = \{natural\ numbers < 20\}$ $\{1, 2, 3 \ldots 19\}$

$\underline{P} = \{prime\ numbers \leq \underline{20}\}$ $\{2, \cancel{3}, \cancel{5}, \cancel{7}, \cancel{11}, 13, \cancel{17}, \cancel{19}\}$

$Q = \{odd\ numbers < 20\}$ $\{\underline{1}, \cancel{3}, \cancel{5}, \cancel{7}, \underline{9}, \cancel{11}, \cancel{13}, \underline{15}, \cancel{17}, \cancel{19}\}$

(i) List the members of $M \cap P$

$\boldsymbol{M \cap P = P = \{2, 3, 5, 7, 11, 13, 17, 19\}}$

(ii) Find the members of P which are not in set Q

$\boldsymbol{\{2\}}$

(iii) Find the members of Q which are not in set P

$\boldsymbol{\{1, 9, 15\}}$

(c) Create a set comprehension for the set $F = \{1, 2, 4, 8, 16\}$

$$1, \quad 2, \quad 4, \quad 8, \quad 16$$

$$2^0, \quad 2^1, \quad 2^2, \quad 2^3, \quad 2^4$$

$$\boldsymbol{F = \{x \mid x = 2^n \text{, } where\ n < 5\ and\ n \in N\}}$$

OR

$$\boldsymbol{F = \{x \in N^+ \mid x = \frac{16}{p} \text{, } where\ p \in N^+\}}$$

(2022) Question 5

(a) (i) What is the maximum positive number that can be stored in 8-bit 2's complement form? **127**

8-bit 2's complement to store *positive* and *negative* integers

0	0	0	0	0	0	0	0	0	positive integers
0	0	0	0	0	0	0	1	1	
0	0	0	0	0	0	1	0	2	
			⋮					⋮	
0	1	1	1	1	1	1	1	127	
1	0	0	0	0	0	0	0	- 128	negative integers
1	0	0	0	0	0	0	1	- 127	
			⋮					⋮	
1	1	1	1	1	1	1	1	- 1	

(ii) Convert + 97 and − 58 into 8-bit 2’s complement form

+ 97

	128	64	32	16	8	4	2	1	
	0	**1**	**1**	**0**	**0**	**0**	**0**	**1**	+ 97

− 58

	128	64	32	16	8	4	2	1	
	0	0	1	1	1	0	1	0	+ 58
Flip bits	1	1	0	0	0	1	0	1	
Add 1	**1**	**1**	**0**	**0**	**0**	**1**	**1**	**0**	− 58

(iii) Perform the 2's complement calculations for 97 + 58 and 97 − 58. Show your working.

97 + 58

	128	64	32	16	8	4	2	1	
	0	1	1	0	0	0	0	1	$+97$
+	0	0	1	1	1	0	1	0	$+58$
	1	**0**	**0**	**1**	**1**	**0**	**1**	**1**	$overflow$

97 − 58

	128	64	32	16	8	4	2	1	
	0	1	1	0	0	0	0	1	$+97$
+	1	1	0	0	0	1	1	0	-58
$\cancel{1}$	**0**	**0**	**1**	**0**	**0**	**1**	**1**	**1**	$+39$

(iv) Using your calculations as examples, show how an overflow can be detected

$$
\begin{array}{cccccccccc}
 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & +97 \\
+ & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & +58 \\
\hline
 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & overflow \\
\hline
\mathbf{0} & \mathbf{1} & & & & & & & &
\end{array}
$$

The carryovers are NOT the same, so the sum is out of bounds – overflow error

$$
\begin{array}{cccccccccc}
 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & +97 \\
+ & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & -58 \\
\hline
1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & +39 \\
\hline
\mathbf{1} & \mathbf{1} & & & & & & & &
\end{array}
$$

The carryovers are the same, so the sum is within bounds

(b) (i) Write the numbers 12457 and 2987 in BCD format

12457

8 4 2 1	8 4 2 1	8 4 2 1	8 4 2 1	8 4 2 1
0 0 0 1	**0 0 1 0**	**0 1 0 0**	**0 1 0 1**	**0 1 1 1**

2987

8 4 2 1	8 4 2 1	8 4 2 1	8 4 2 1
0 0 1 0	**1 0 0 1**	**1 0 0 0**	**0 1 1 1**

(ii) Perform the calculation 12457 + 2987 using BCD arithmetic.
Show your working.

12457 + 2987

8 4 2 1	8 4 2 1	8 4 2 1	8 4 2 1	8 4 2 1
0 0 0 1	0 0 1 0	0 1 0 0	0 1 0 1	0 1 1 1
	0 0 1 0	1 0 0 1	1 0 0 0	0 1 1 1
	1	1	1	
	0 1 0 1	1 1 1 0	1 1 1 0	1 1 1 0
		0 1 1 0	0 1 1 0	0 1 1 0
		1 0 1 0 0	1 0 1 0 0	1 0 1 0 0
1	**5**	**4**	**4**	**4**