COMP3220

Exam Revision Session (1)

(2021) Question 1

(a) Solve the following

(i) $12x + 17 = 65$

$$12x = 65 - 17$$

$$12x = 48$$

$$\boldsymbol{x = 4}$$

3/33

(ii) $\begin{cases} 14x - 13y = 16 & (1) \\ 2x + 9y = 24 & (2) \times 7 \end{cases}$

$$\begin{cases} 14x - 13y = 16 \\ 14x + 63y = 168 \end{cases} \quad (-)$$

$$76y = 152$$

$$\boldsymbol{y = 2} \quad (3)$$

Substitute (3) into (2)

$$2x + 9(2) = 24$$

$$2x + 18 = 24$$

$$2x = 6$$

$$\boldsymbol{x = 3}$$

4/33

(iii) $5x^2 + 7x - 19 = 0$

$$a = 5, \qquad b = 7, \qquad c = -19$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(5)(-19)}}{2(5)} = \mathbf{1.37,\ -2.77}$$

(b) Find the sum to infinity for the series

$$18 + 12 + 8 + \cdots$$

$$r = \frac{T_2}{T_1} = \frac{12}{18} = \frac{2}{3} \qquad\qquad r = \frac{T_3}{T_2} = \frac{8}{12} = \frac{2}{3}$$

constant ratio *r*, so it is a geometric progression

$$r = \frac{2}{3}, \qquad a = 18$$

$$S_\infty = \frac{a}{1-r} = \frac{18}{1-\frac{2}{3}} = \mathbf{54}$$

(c) Determine the values of a, b, c, and d

$$(1+2x)^a = 1 + bx + 24x^c + dx^3 + 16x^4$$

$\boldsymbol{a = 4}$

$1+x$	1 1
$(1+x)^2$	1 2 1
$(1+x)^3$	1 3 3 1
$(1+x)^4$	1 4 6 4 1

$$(1+2x)^4 = 1 + 4\,(2x) + 6\,(2x)^2 + 4\,(2x)^3 + 1\,(2x)^4$$

$$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$$

$\boldsymbol{b = 8,\quad c = 2,\quad d = 32}$

String

Σ^* is the set of all possible strings over an alphabet Σ

the alphabet we use in our language

$$\Sigma = \{a, b, c, d, e, f, g, \cdots x, y, z\}$$

$$\Sigma^* = \{"aaa", "able", "cftty", "city", "gggghh", \cdots\}$$

the words in our language is a subset of Σ^*

the alphabet a computer uses

$$\Sigma = \{0, 1\}$$

$\Sigma^* = \{0, 1\}^*$ the set of all possible binary strings

$$= \{"", "0", "1", "00", "01", "10", "11", "000", \cdots \}$$

empty string

infinite number of possible strings

Concatenation (joint or connect strings)

$$"10" \circ "01" = "1001"$$

$$L = \{\, r \circ \underline{10} \circ r \mid r \in \{0,1\}^* \,\}$$

which of the following strings are in set L?

$10\underline{10}1$ ✗

$01\underline{10}01$ ✓

$\underline{10}$ ✓

(2021) Question 2

(a) Let L be the language $L = \{ r \circ 101 \circ r \mid r \in \{0,1\}^* \}$

State which strings are in L

(i) 110111 ✗

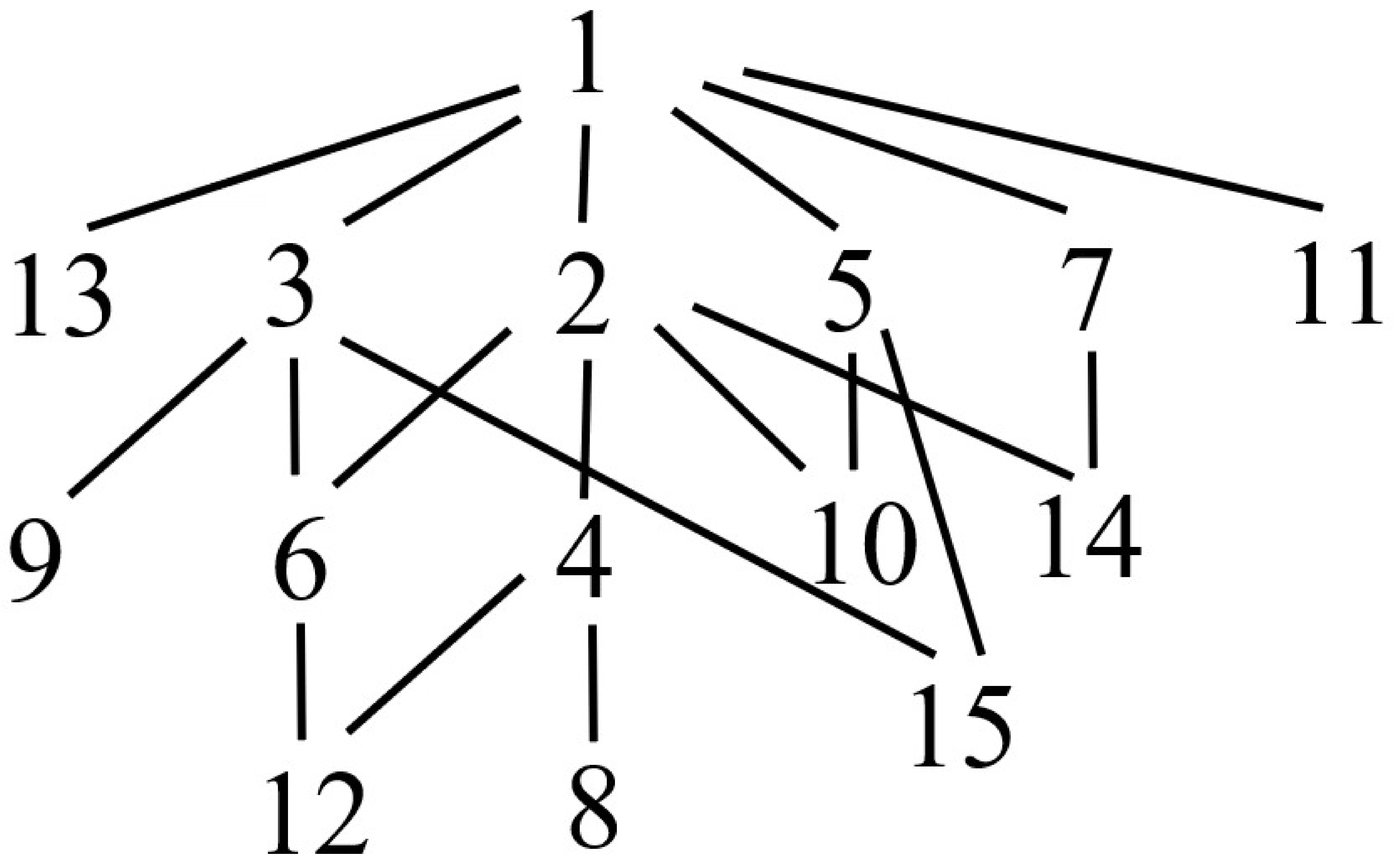
(ii) 101 ✓

(iii) 0110110 ✗

(iv) 111011 ✗

(v) 1010110 ✓

(b) Draw a Hasse diagram to represent the relation "exactly divide" on the natural numbers from 1 to 15 inclusive.



(c) Enumerate the elements of the following sets

(i) $\{x \mid x = 4p \text{ where } p < 8 \text{ and } p \in N\}$

$\mathbf{\{0, 4, 8, 12, 16, 20, 24, 28\}}$

(ii) $\{x \in N^{+} \mid x = \dfrac{12}{p} \text{ where } p \in N\}$

$\mathbf{\{12, 6, 4, 3, 2, 1\}}$

Induction

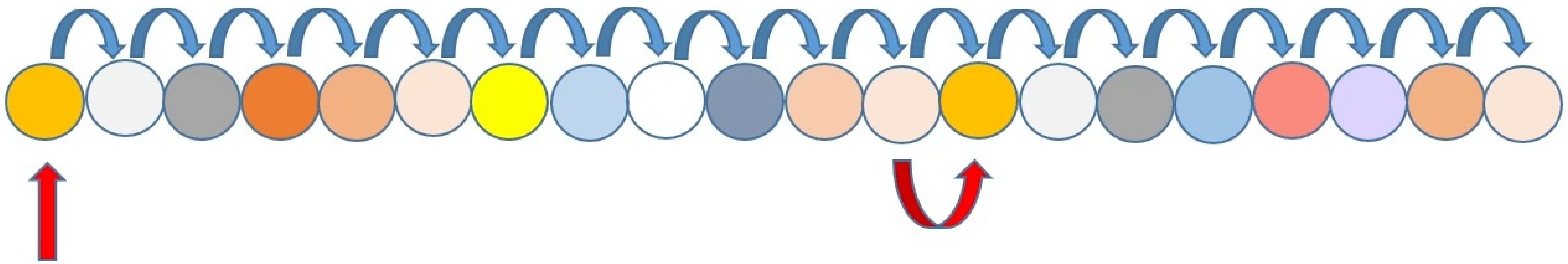
to prove "sums of series"

Prove: $$\sum_{r=1}^{n}(4r-2)=2n^2$$

need to prove this formula is true for *all* values of n (for *all* natural numbers)

Idea of "proof by induction method"

Prove "**all** students in my class understand Normal Distribution"



Method #1: ask ***each*** student to see if they understand Normal Distribution

Method #2: use induction

(1) Show the 1st student understands Normal Distribution

(2) Show for any two adjacent students, if one understands, then the other one will also understand.

$$\sum_{r=1}^{n}(4r-2) = 2n^2$$

$$S_n = \sum_{r=1}^{n}(4r-2) \qquad\qquad S_n = 2n^2$$

$$S_1 = T_1 = \sum_{r=1}^{1}(4r-2) = 2 \qquad\qquad S_1 = 2(1^2) = 2$$

$$S_2 = T_1 + T_2 = \sum_{r=1}^{2}(4r-2) = 2 + 6 = 8 \qquad\qquad S_2 = 2(2^2) = 8$$

$$S_3 = T_1 + T_2 + T_3 = \sum_{r=1}^{3}(4r-2) = 2 + 6 + 10 = 18 \qquad\qquad S_3 = 2(3^2) = 18$$

Prove $S_n = \sum_{r=1}^{n}(4r-2) = 2n^2$ is true for all values of n

$n = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ \ldots\ldots k,\ k+1\ldots\ldots\ 1000,\ 1001,\ \ldots\ldots$

Method #1: show for ***each*** value of n, the formula is true **not possible**

Method #2: use induction

(1) Show the formula is true for $n = 1$ **(base case)** $T_1 = S_1$

(2) Show for any two adjacent n values, k and $k+1$,

if the formula is true for $n = k$, **(assumption)** $S_k = \sum_{r=1}^{k}(4r-2) = 2k^2$

then the formula will also be true for $n = k+1$

Prove $S_n = \sum_{r=1}^{n}(4r-2) = 2n^2$ is true for all values of n

$n = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ \ldots\ldots k,\ k+1 \ldots\ldots 1000,\ 1001,\ \ldots\ldots$

Method #1: show for ***each*** value of n, the formula is true **not possible**

Method #2: use induction

(1) Show the formula is true for $n = 1$ **(base case)** $T_1 = S_1$

(2) Show for any two adjacent n values, k and $k+1$,

if the formula is true for $n = k$, **(assumption)** $S_k \qquad = 2k^2$

then the formula will also be true for $n = k + 1$ **(statement and induction)**

$$S_{k+1} = \sum_{r=1}^{k+1}(4r-2) = 2(k+1)^2$$

(2021) Question 3

A series is defined by $S_n = \sum_{r=1}^{n}(4r - 2)$

(a) Find the first 4 terms of the series.

$$T_r = 4r - 2$$

$$T_1 = 4(1) - 2 = \mathbf{2}$$

$$T_2 = 4(2) - 2 = \mathbf{6}$$

$$T_3 = 4(3) - 2 = \mathbf{10}$$

$$T_4 = 4(4) - 2 = \mathbf{14}$$

(b) Use the method of induction to prove that $S_n = 2n^2$

Proof: $T_r = 4r - 2$, $S_n = 2n^2$

(1) Base case

$T_1 = 4(1) - 2 = 2$, $S_1 = 2(1^2) = 2$

Therefore, $T_1 = S_1$

(2) Assumption

~~There is at least one value of k $(1 \le k < n)$ such that~~ $S_k = 2k^2$

(3) Statement

If the formula is true, then $S_{k+1} = 2(k+1)^2$

(4) Induction

$$S_{k+1} = S_k + T_{k+1}$$
$$= 2k^2 + 4(k+1) - 2$$
$$= 2k^2 + 4k + 4 - 2$$
$$= 2k^2 + 4k + 2$$
$$= 2(k^2 + 2k + 1)$$
$$= 2(k+1)^2$$

$$\boxed{T_r = 4r - 2, \quad S_n = 2n^2}$$

$$\overset{T_1}{2} + \overset{T_2}{6} + \overset{T_3}{10} + \overset{T_4}{14} + \cdots$$

$$T_1 + T_2 + T_3 = S_3$$
$$T_1 + T_2 + T_3 + T_4 = S_4$$
$$S_3 + T_4 = S_4$$

$$T_1 + T_2 + T_3 + \cdots + T_k = S_k$$
$$T_1 + T_2 + T_3 + \cdots + T_k + T_{k+1} = S_{k+1}$$
$$\boldsymbol{S_k + T_{k+1} = S_{k+1}}$$

(c) Find the sum of the first 20 terms of the series

$$S_n = 2n^2$$

$$S_{20} = 2(20^2) = 2(400) = \mathbf{800}$$

(2021) Question 4

The table summarises the marks of 200 students sitting an examination

Marks (x)	0 –	10 –	20 –	30 –	40 –	50 –	60 –	70 –	80 –	90 –
No. students (f)	2	4	4	6	30	70	60	14	6	4

(a) Find the mean and standard deviation for the marks. State the assumption that you make

mid-x	5	15	25	35	45	55	65	75	85	95
f	2	4	4	6	30	70	60	14	6	4

Mean $\bar{x} = \mathbf{57.1}$, Standard deviation $\sigma = \mathbf{14.65}$

Assumption: we assume that the middle point $(mid - x)$ of each group can represent the data points in that group.

Marks (x)	0 –	10 –	20 –	30 –	40 –	50 –	60 –	70 –	80 –	90 –
No. students (f)	2	4	4	6	30	70	60	14	6	4

(b) Use the method of interpolation to determine the median for the data.

Q_2

ucb	10	20	30	40	50	60	70	80	90	100
cf	2	6	10	16	46	116	176	190	196	200

$V_2 = 100.5$

$$V_2 = \frac{n+1}{2} = \frac{200+1}{2} = 100.5$$

$$Q_2 = 50 + \left(\frac{100.5 - 46}{116 - 46}\right) \times (60 - 50) = \mathbf{57.79}$$

(c) Two further sets of data have the following properties:

Set 1: $\bar{x}_1 = 16, \sigma_1 = 3, n_1 = 24$

Set 2: $\bar{x}_2 = 13, \sigma_2 = 5, n_2 = 20$

(i) find the combined mean for the data

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$= \frac{(24)(16) + (20)(13)}{24 + 20} = \mathbf{14.64}$$

Set 1: $\bar{x}_1 = 16, \sigma_1 = 3, n_1 = 24$

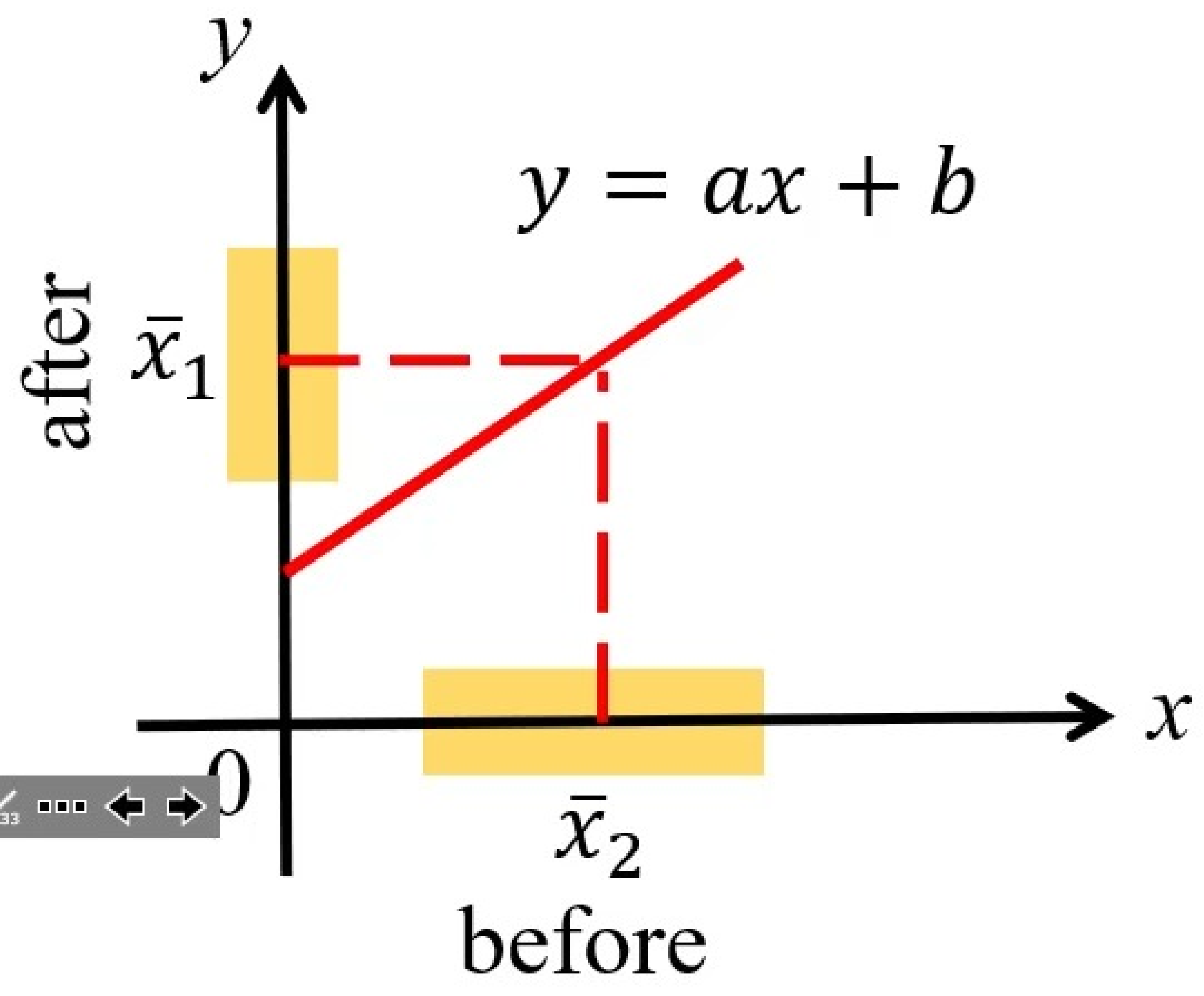
Set 2: $\bar{x}_2 = 13, \sigma_2 = 5, n_2 = 20$

(ii) find the scaling formula that maps Set 2 onto Set 1

$y = ax + b$

<u>before</u>: $\bar{x}_2 = 13 \qquad \sigma_2 = 5$

<u>after</u>: $\bar{x}_1 = 16 \qquad \sigma_1 = 3$



$\sigma_1 = a\sigma_2$

$3 = a(5)$

$a = {}^{3}/_{5} = 0.6$

$\bar{x}_1 = a\bar{x}_2 + b$

$16 = (0.6)(13) + b$

$b = 16 - (0.6)(13) = 8.2$

$\mathbf{y = 0.6x + 8.2}$

BCD (binary coded decimal)

each decimal digit ⟶ 4-bit binary string

$$\begin{array}{r} 23 \\ +\ 14 \\ \hline 37 \end{array}$$

$$\begin{array}{cccc} 8 & 4 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 1 & 1 \end{array} \qquad \begin{array}{cccc} 8 & 4 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 1 & 1 \end{array}$$

$$\begin{array}{r} 8 \\ +\ 5 \\ \hline 13 \end{array}$$

$$\begin{array}{ccccc} & 8 & 4 & 2 & 1 \\ & 1 & 0 & 0 & 0 \\ & 0 & 1 & 0 & 1 \\ \hline & 1 & 1 & 0 & 1 \\ & 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 1 & 1 \end{array}$$

≥ 1010 $+0110$

carry over

(2021) Question 5

(a) Use BCD arithmetic to perform the calculation

$$14326 + 92378$$

8 4 2 1	8 4 2 1	8 4 2 1	8 4 2 1	8 4 2 1
0 0 0 1	0 1 0 0	0 0 1 1	0 0 1 0	0 1 1 0
1 0 0 1	0 0 1 0	0 0 1 1	0 1 1 1	1 0 0 0
1 0 1 0	0 1 1 0	1	1	1 1 1 0
0 1 1 0		0 1 1 1	1 0 1 0	0 1 1 0
1 0 0 0 0			0 1 1 0	1 0 1 0 0
			1 0 0 0 0	
1 0	**6**	**7**	**0**	**4**

(b) In a particular computer, numbers are stored in IEEE standard 754 floating point format, with an 8-bit mantissa and a 4-bit exponent.

(i) Show how 43.125 would be stored

43.125

32	16	8	4	2	1	.	0.5	0.25	0.125
1	0	1	0	1	1	.	0	0	1

convert to binary 101011.001

normalise $\boxed{}.\underline{01011001} \times 2^5$

8-bit mantissa $01011001 \times 2^{\boxed{5}}$

convert exponent (4-bit) $2^{(4-1)} - 1 = 7$ (bias)

$7 + \boxed{5} = 12 \longrightarrow 1100$

add sign bit **0 01011001 1100**

4 bits

0000	0
0001	1
⋮	⋮
1111	15

0 | **15**

7

negative exponent ← 7 → positive exponent

(ii) What number is stored as 1   11001010   1011 ?

-28.625

16	8	4	2	1 .	0.5	0.25	0.125	0.0625
1	1	1	0	0 .	1	0	1	0

$16 + 8 + 4 + 0.5 + 0.125 = 28.625$

convert to binary   11100.1010

normalise $1.\underline{11001010} \times 2^4$

8-bit mantissa 11001010×2^4

convert exponent (4-bit) $11 - 7\ (\text{bias}) = 4$

$11 \leftarrow \mathbf{1011}$

add sign bit   **1 11001010 1011**