Foundations of Computing 1 - 2021 Exam 15/04/2024

1a. Solve the following:

i. $12x + 17 = 65$

$12x + 17 = 65 \longrightarrow 12x = 65 - 17 \longrightarrow 12x = 48$

$x = 4$

ii. $\left.\begin{aligned} 14x - 13y &= 16 \\ 2x + 9y &= 24 \end{aligned}\right\}$

$14x - 13y = 16$

$7(2x + 9y = 24) \times 7 \longrightarrow$

$$\begin{array}{r} 14x - 13y = 16 \\ 14x + 63y = 168 \; - \\ \hline -76y = -152 \end{array}$$

$-76y = -152$

$76y = 152 \longrightarrow y = 2$

$2x + (9 \times 2) = 24 \longrightarrow 2x + 18 = 24 \longrightarrow 2x = 24 - 18$

$2x = 6 \longrightarrow x = 3$

$x = 3, y = 2$

iii. $5x^2 + 7x - 19 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 5, b = 7, c = -19$$

$$= \frac{-7 \pm \sqrt{7^2 - (4 \times 5 \times -19}}{2 \times 5}$$

$$= \frac{-7 \pm \sqrt{49 + 380}}{10}$$

$$= \frac{-7 \pm \sqrt{429}}{10}$$

$$x = \frac{-7 + \sqrt{429}}{10} = 1.371 \ (3dp)$$

$$x = \frac{-7 - \sqrt{429}}{10} = -2.771 \ (3dp)$$

6. Find the sum to infinity for the series $18 + 12 + 8 + \ldots$

$$S_\infty = \frac{a}{1-r}$$

$18 \xrightarrow{\times \frac{2}{3}} 12 \xrightarrow{\times \frac{2}{3}} 8$ $a = 18,\ r = \frac{2}{3}$

$$= \frac{18}{1-\frac{2}{3}}$$

$$= 54$$

c. Determine the values of a, b, c and d:

$$(1+2x)^a = 1 + bx + 24x^c + dx^3 + 16x^4$$

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$$\left[\binom{4}{0} \times 1^4 \times (2x)^0\right] + \left[\binom{4}{1} \times 1^3 \times (2x)^1\right] + \left[\binom{4}{2} \times 1^2 \times (2x)^2\right]$$

$$+\left[\binom{4}{3} \times 1^1 \times (2x)^3\right] + \left[\binom{4}{4} \times 1^0 \times (2x)^3\right]$$

$$=(1 \times 1 \times 1) + (4 \times 1 \times 2x) + (6 \times 1 \times 4x^2) + (4 \times 1 \times 8x^3) + (1 \times 1 \times 16x^4)$$

$$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$$

$a = 4$, $b = 8$, $c = 2$, $d = 32$

2a. Let L be the language $L = \{r \circ 101 \circ r \mid r \in \{0,1\}^*\}$

- State which strings are in L:

i. 110111 No

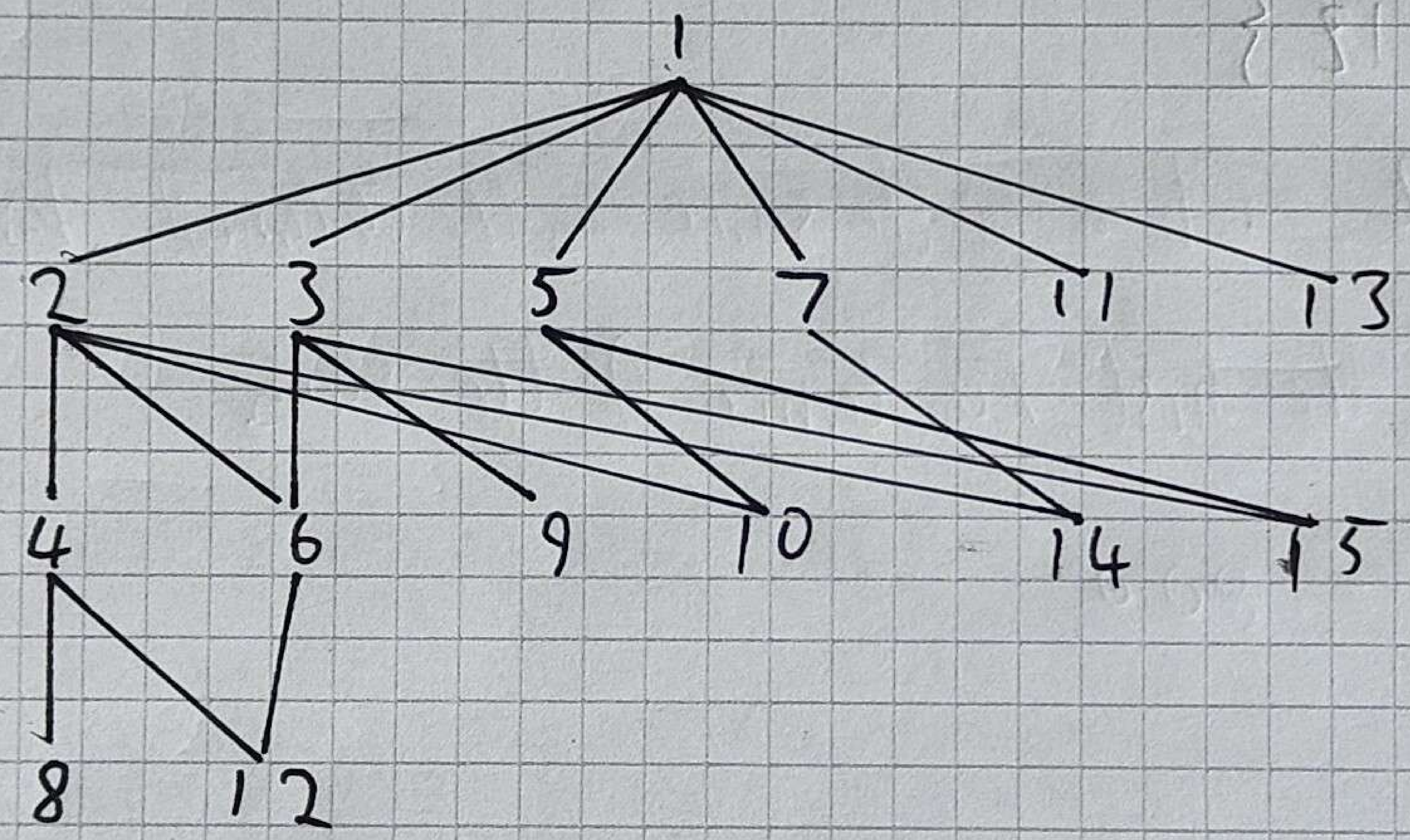
ii. 101 Yes

iii. 0110110 No

iv. 111011 No

v. 1010110 Yes

b. Draw a Hesse diagram to represent the relation "exactly divides" on the natural numbers from 1 to 15 inclusive



c. Enumerate the elements of the following sets:

i. $\{x \mid x = 4p \text{ where } p < 8 \text{ and } p \in \mathbb{N}\}$

$\mathbb{N}$ is all natural numbers, including 0

$P < 8$ means values between 0 to 7

$\{0, 4, 8, 12, 16, 20, 24, 28\}$

ii. $\{x \in \mathbb{N}^+ \mid x = \frac{12}{p} \text{ where } p \in \mathbb{N}\}$

All versions of $\frac{12}{p}$ where the result is a natural number.

$\frac{12}{1} = 12$, $\frac{12}{2} = 6$, $\frac{12}{3} = 4$, $\frac{12}{4} = 3$, $\frac{12}{6} = 2$, $\frac{12}{12} = 1$

$\{1, 2, 3, 4, 6, 12\}$

3. A series is defined by $S_n = \sum_{r=1}^{n} (4r - 2)$

a. Find the first 4 terms of the series

1st term $= 4 \times 1 - 2 = 2$

2nd term $=$ $(4 \times 2 - 2) =$ 6

3rd term $=$ $(4 \times 3 - 2) =$ 10

4th term $=$ $(4 \times 4 - 2) =$ 14

b. Use the method of induction to prove that $S_n = 2n^2$

Base case: $T_1 = S_1$, $r=1$, $n=1$.

$(4\times1-2)=2$, $2\times1^2=2$

Assumption: There is at least one value of k $(1\leq k\leq n)$ such that $S_k = 2k^2$

Statement: If the formula is true, then

$S_{k+1} = 2(k+1)^2 = 2(k^2+2k+1) = 2k^2+4k+2$

Induction: $S_{k+1} = S_k + T_{k+1}$

$2k^2+4k+2 = 2k^2+4(k+1)-2$

$= 2k^2+4k+4-2$

$= 2k^2+4k+2$, which is the same as the statement above.

c. Find the sum of the first 20 terms of the series

$S_{20} = 2\times20^2 = 800$

4. The table summarises the percentage marks obtained by 200 students sitting an examination

Marks	0–	10–	20–	30–	40–	50–	60–	70–	80–	90–	Total
No Students	2	4	4	6	30	70	60	14	6	4	200
Assumed mean	5	15	25	35	45	55	65	75	85	95	
fx	10	60	100	210	1350	3850	3900	1050	510	380	11420
fx^2	~~50~~	~~900~~	~~2500~~	~~7350~~	~~87480~~	~~211750~~	~~253500~~	~~78750~~	~~43350~~	~~36100~~	~~721780~~
cf	2	6	10	16	46	116	176	190	196	200	

a. Find the mean and standard deviation of the marks. State the 695000 assumptions that you make.

$\sum fx^2 = 695000$

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{11420}{200} = 57.1$$

$$SD = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} = \sqrt{\frac{695000}{200} - 57.1^2} = 14.649 \text{ (3 dp)}$$

I assumed that the mid-points for the data were ~~between~~ the ~~mid point~~ mean between adjacent columns (e.g. $(0+10)\div 2 = 5$, $(10+20)\div 2 = 15$, etc). In addition, I assumed that the marks ended at 100, given that the previous spacing between marks was 10, and that 90 was the final column.

b. Use the method of interpolation to determine the median for the data.

$$Q_2 = Lcb + \frac{(v - \text{prev. cf})}{f} \times (ucb - Lcb)$$

$v = (200 + 1) \div 2 = 100.5$

The 100.5^{th} position is between 50 and 60

$Lcb = 50$, $ucb = 60$, frequency at $50 = 70$, prev. $cf = 46$

$$50 + \frac{(100.5 - 46)}{70} \times (10) = 57.786 \text{ (3dp)}$$

c. Two further sets of data have the following properties:

Set 1 : $\bar{x}_1 = 16$, $\sigma_1 = 3$, $n = 24$

Set 2 : $\bar{x}_2 = 13$, $\sigma_2 = 5$, $n = 20$

i. Find the combined mean for the data

$$\text{combined mean} = \frac{(16 \times 24) + (13 \times 20)}{44} = 14.636 \text{ (3dp)}$$

ii. Find the scaling formula that maps Set 2 onto Set 1.

$y = ax + b$

Find a: $\sigma_1 = a \times \sigma_2 \longrightarrow 3 = a \times 5 \longrightarrow 3 = 5a \longrightarrow a = \frac{3}{5}$

Find b: $\bar{x}_1 = \frac{3}{5} \times \bar{x}_2 + b \longrightarrow 16 = \left(\frac{3}{5} \times 13\right) + b \longrightarrow b = 8.2$

$y = 0.6x + 8.2$

5a. Use BCD arithmetic to perform the calculations
14326 + 92378.
Show all your working.

14326 in BCD:

0001, 0100, 0011, 0010, 0110

92378 in BCD:

1001, 0010, 0011, 0111, 1000

Add them together:

$$\begin{array}{r} 11 \\ 1 \\ 0011 \\ +\,0011 \\ \hline 0111 \end{array} \qquad \begin{array}{rl} 111 & \\ 1 & \\ 0010 & \\ +\,0111 & \\ \hline 1010 & \\ 0110 & \text{Add 6} \\ \hline 10000 & \\ 11 & \end{array} \qquad \begin{array}{rl} 0110 & \\ +\,1000 & \\ \hline 1110 & \\ 0110 & \text{Add 6} \\ \hline 10100 & \\ 11 & \end{array}$$

$$\begin{array}{r} 1 \\ 0000 \\ \hline 1 \end{array} \qquad \begin{array}{rl} 1 & \\ 0001 & \\ +\,1001 & \\ \hline 1010 & \\ 0110 & \text{Add 6} \\ \hline 10000 & \\ 111 & \end{array} \qquad \begin{array}{r} 0100 \\ +\,0010 \\ \hline 0110 \end{array}$$

Final result: 0001, 0000, 0110, 0111, 0000, 0100

1 0 6 7 0 4

= 106,704

b. In a particular computer, numbers are stored in IEEE standard 754 floating point format, with an 8-bit mantissa and a 4-bit ~~component~~ exponent.

i. Show how 43.125 would be stored

Convert 43.125: $43.125 = 43\frac{1}{8}$

$= 101011.001$

Normalise: $1\underbrace{01011}_{5}.001 = 1.01011001 \times 2^{5}$

Mantissa 8-bit: 1.01011001 becomes 01011001×2^{5}

Convert exponent: $2^{(4-1)} - 1 = 7$

Add bits to power: $7 + 5 = 12 \rightarrow 1100$

Add sign bit: ~~1~~0

Final answer: 0 01011001 1100

ii. What number is stored as 1 11001010 1011

1 11001010 1011

Negative

11

$11 = 7 - \boxed{4}$

Add 1 to the front $\rightarrow 1.\underbrace{1100}_{4}.1010 \longrightarrow 11100.1010 = 28.625$

Negative $\rightarrow -28.625$