

Foundations of Computing 1 - 20% Exam

15/04/2024

- 1a. Solve the following:

$$i. 12x + 17 = 65$$

$$12x + 17 = 65 \rightarrow 12x = 65 - 17 \rightarrow 12x = 48$$

$$x = 4$$

$$ii. \begin{cases} 14x - 13y = 16 \\ 2x + 9y = 24 \end{cases}$$

$$\begin{array}{rcl} 14x - 13y & = & 16 \\ -(2x + 9y = 24) & \times 7 & \rightarrow \\ \hline 14x - 13y & = & 16 \\ 14x + 63y & = & 168 \\ \hline -76y & = & -152 \end{array}$$

$$-76y = -152$$

$$76y = 152 \rightarrow y = 2$$

$$\begin{array}{rcl} 2x + (9 \times 2) & = & 24 \\ 2x + 18 & = & 24 \\ 2x & = & 24 - 18 \\ 2x & = & 6 \\ x & = & 3 \end{array}$$

$$x = 3, y = 2$$

$$\text{iii. } 5x^2 + 7x - 19 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a=5, b=7, c=-19$$

$$= \frac{-7 \pm \sqrt{49 - (4 \times 5 \times -19)}}{2 \times 5}$$

$$= \frac{-7 \pm \sqrt{49 + 380}}{10}$$

$$= \frac{-7 \pm \sqrt{429}}{10}$$

$$x = \frac{-7 + \sqrt{429}}{10} = 1.371 \text{ (3 dp)}$$

$$x = \frac{-7 - \sqrt{429}}{10} = -2.771 \text{ (3 dp)}$$

b. Find the sum to infinity for the series $18 + 12 + 8 + \dots$

$$S_{\infty} = \frac{a}{1-r}$$

$$\begin{array}{ccccccc} 18 & & 12 & & 8 & a=18, & r=\frac{2}{3} \\ \swarrow & \searrow & \swarrow & \searrow & \swarrow & & \\ \times \frac{2}{3} & & \times \frac{2}{3} & & \times \frac{2}{3} & & \end{array}$$

$$= \frac{18}{1-\frac{2}{3}}$$

$$= 54$$

c. Determine the values of a, b, c and d:

$$(1+2x)^4 = 1 + bx + 24x^2 + dx^3 + 16x^4$$

$$\begin{array}{cccccc} & & 1 & & & \\ & 1 & 1 & 1 & 1 & \\ 1 & 1 & 3 & 3 & 1 & \\ & 1 & 6 & 6 & 4 & 1 \end{array}$$

$$= \left[\binom{4}{0} \times 1^4 \times (2x)^0 \right] + \left[\binom{4}{1} \times 1^3 \times (2x)^1 \right] + \left[\binom{4}{2} \times 1^2 \times (2x)^2 \right]$$

$$+ \left[\binom{4}{3} \times 1^1 \times (2x)^3 \right] + \left[\binom{4}{4} \times 1^0 \times (2x)^4 \right]$$

$$= (1 \times 1 \times 1) + (4 \times 1 \times 2x) + (6 \times 1 \times 4x^2) + (4 \times 1 \times 8x^3) + (1 \times 1 \times 16x^4)$$

$$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$$

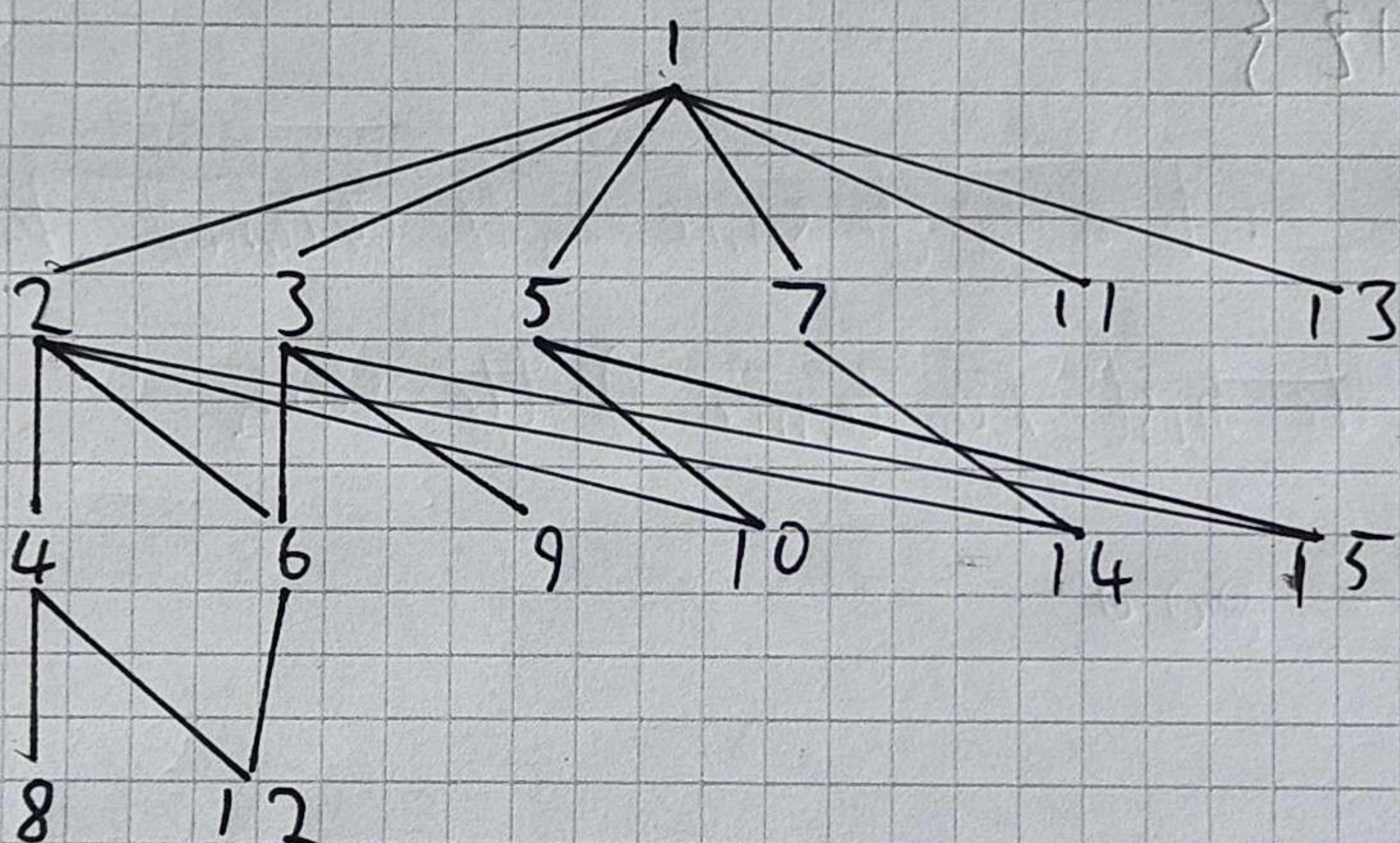
$$a = 1, \quad b = 8, \quad c = 24, \quad d = 32$$

2a. Let L be the language $L = \{r^{\circ}101^{\circ}r \mid r \in \{0,1\}^*\}$

State which strings are in L :

- i. 110111 No
- ii. 101 Yes
- iii. 0110110 No
- iv. 1110111 No
- v. 1010110 Yes

b. Draw a Hasse diagram to represent the relation "exactly divides" on the natural numbers from 1 to 15 inclusive.



c. Enumerate the elements of the following sets:

i. $\{x \mid x = 4p \text{ where } p < 8 \text{ and } p \in \mathbb{N}\}$

\mathbb{N} is all natural numbers, including 0

$p < 8$ means values between 0 to 7

$$\{0, 4, 8, 12, 16, 20, 24, 28\}$$

ii. $\{x \in \mathbb{N}^+ \mid x = \frac{12}{p} \text{ where } p \in \mathbb{N}\}$

All versions of $\frac{12}{p}$ where the result is a natural number.

$$\frac{12}{1} = 12, \frac{12}{2} = 6, \frac{12}{3} = 4, \frac{12}{4} = 3, \frac{12}{6} = 2, \frac{12}{12} = 1$$

$$\{1, 2, 3, 4, 6, 12\}$$

$$3. A \text{ series is defined by } S_n = \sum_{r=1}^n (4r - 2)$$

a. Find the first 4 terms of the series

$$\text{1st term} = 4 \times 1 - 2 = 2$$

$$\text{2nd term} = 2 + (4 \times 2 - 2) = 2 + 8 - 2 = 8$$

$$\text{3rd term} = 8 + (4 \times 3 - 2) = 8 + 12 - 2 = 18$$

$$\text{4th term} = 18 + (4 \times 4 - 2) = 18 + 16 - 2 = 32$$

b. Use the method of induction to prove that $S_n = 2n^2$

Base case: $T_1 = S_1$, $r=1$, $n=1$

$$(4x_1 - 2) = 2, \quad 2x_1^2 = 2$$

Assumption: There is at least one value of k ($1 \leq k \leq n$) such that
 $S_k = 2k^2$

Statement: If the formula is true, then

$$S_{k+1} = 2(k+1)^2 = 2(k^2 + 2k + 1) = 2k^2 + 4k + 2$$

Induction: $S_{k+1} = S_k + T_{k+1}$

$$2k^2 + 4k + 2 = 2k^2 + 4(k+1) - 2$$

$$= 2k^2 + 4k + 4 - 2$$

$= 2k^2 + 4k + 2$, which is the same as the statement above.

c. Find the sum of the first 20 terms of the series

$$S_{20} = 2 \times 20^2 = 800$$

4. The table summarises the percentage marks obtained by 200 students sitting an examination.

Marks	0-	10-	20-	30-	40-	50-	60-	70-	80-	90-	Total
No Students	2	4	4	6	30	70	60	14	6	4	200
Assumption	5	15	25	35	45	55	65	75	85	95	
f_x	10	60	100	210	1350	3850	3900	1050	510	380	11420
f_x^2	50	900	2500	7350	87450	211750	253500	78750	43350	36100	721780
cf	2	6	10	16	46	116	176	190	196	200	

a. Find the mean and standard deviation of the marks. State the assumptions that you make.

$$\text{Mean} = \frac{\sum f_x}{\sum f} = \frac{11420}{200} = 57.1$$

$$\sum f_x^2 = 695000$$

$$SD = \sqrt{\frac{\sum f_x^2}{\sum f} - \bar{x}^2} = \sqrt{\frac{695000}{200} - 57.1^2} = 14.649(3dp)$$

I assumed that the mid-points for the data were between the midpoint mean between adjacent columns (e.g. $(0+10)/2 = 5$, $(10+20)/2 = 15$, etc).

In addition, I assumed that the marks ended at 100, given that the previous spacing between marks was 10, and that 90 was the final column.

b. Use the method of interpolation to determine the median for the data.

$$Q_2 = L_{cb} + \frac{(V - \text{prev. Cf})}{f} \times (u_{cb} - (cb))$$

$$V = (200 + 1) \div 2 = 100.5$$

The 100.5th position is between 50 and 60

$L_{cb} = 50$, $u_{cb} = 60$, frequency at 50 = 70, prev. Cf = 46

$$50 + \frac{(100.5 - 46)}{70} \times (10) = 57.786 \text{ (3 dp)}$$

c. Two further sets of data have the following properties:

Set 1: $\bar{x}_1 = 16$, $\sigma_1 = 3$, $n = 24$

Set 2: $\bar{x}_2 = 13$, $\sigma_2 = 5$, $n = 20$

i. Find the combined mean for the data

$$\text{combined mean} = \frac{(16 \times 24) + (13 \times 20)}{44} = 14.636 \text{ (3dp)}$$

ii. Find the scaling formula that maps Set 2 onto Set 1.

$$y = ax + b$$

$$\text{Find } a: \sigma_1 = a \times \sigma_2 \rightarrow 3 = a \times 5 \rightarrow 3 = 5a \rightarrow a = \frac{3}{5}$$

$$\text{Find } b: \bar{x}_1 = \frac{3}{5} \times \bar{x}_2 + b \rightarrow 16 = \left(\frac{3}{5} \times 13\right) + b \rightarrow b = 8.2$$

$$y = 0.6x + 8.2$$

5a. Use BCD arithmetic to perform the calculations

$$14326 + 92378$$

Show all your working.

14326 in BCD:

$$0001, 0100, 0011, 0010, 0110$$

92378 in BCD:

$$1001, 0010, 0011, 0111, 1000$$

Add them together:

$$\begin{array}{r} \text{1111} \\ \text{0011} \\ + \text{0011} \\ \hline \text{0111} \end{array}$$
$$\begin{array}{r} \text{1111} \\ \text{0010} \\ + \text{0111} \\ \hline \text{1010} \\ \text{0110} \\ + \text{1000} \\ \hline \text{10110} \\ \text{0110} \\ + \text{0110} \\ \hline \text{10110} \end{array} \text{Add 6}$$
$$\begin{array}{r} \text{0110} \\ + \text{1000} \\ \hline \text{1110} \\ \text{0110} \\ + \text{1000} \\ \hline \text{10110} \end{array} \text{Add 6}$$

$$\begin{array}{r} \text{0000} \\ + \text{0001} \\ \hline \text{0001} \end{array}$$
$$\begin{array}{r} \text{0001} \\ + \text{1001} \\ \hline \text{1001} \\ \text{0110} \\ + \text{0010} \\ \hline \text{0110} \end{array} \text{Add 6}$$
$$\begin{array}{r} \text{0100} \\ + \text{0010} \\ \hline \text{0110} \end{array}$$

Final result: 0001, 0000, 0110, 0111, 0000, 0100
1 0 6 7 0 4

$$= 106704$$

b. In a particular computer, numbers are stored in IEEE standard 754 floating point format, with an 8-bit mantissa and a 4-bit exponent.

i. Show how 43.125 would be stored

Convert 43.125: $43.125 = 43\frac{1}{8}$
 $= 101011.001$

Normalize: $\underbrace{101011.001}_5 = 1.01011001 \times 2^5$

Mantissa 8-bit: 1.01011001 becomes 01011001×2^5

Convert exponent: $2^{(4-1)-1} = 7$

Add bits to power: $7+5=12 \rightarrow 1100$

Add sign bit: 10

Final answer: 0 01011001 1100

ii. What number is stored as 1 11001010 1011

1 11001010 1011
↑
T
Negative

$$11 = 7 - \boxed{4}$$

Add 1 to $\rightarrow 1.\underbrace{1100}_{4}.1010 \rightarrow 11100.1010 = 28.625$
the front

Negative $\rightarrow -28.625$