COMP3250

Exam Revision Session (2)

(2022) Question 1

(a) (i) Find the number of distinct words that can be made up using all the letters from the word EXAMINATION

EXAMINATION

11 letters (2 A, 2 I, 2 N)

$$\frac{11!}{(2!)(2!)(2!)} = \mathbf{4989600}$$

(ii) How many words can be made when AA must not occur?

EXMINTIONAA

10 letters (2 I, 2 N)

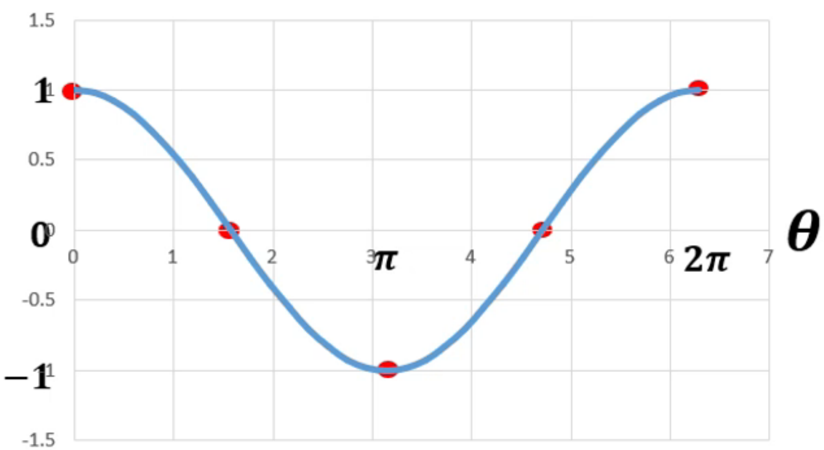
$$\frac{10!}{(2!)(2!)} = 907200$$

EXAMINATION when AA must ***not*** occur

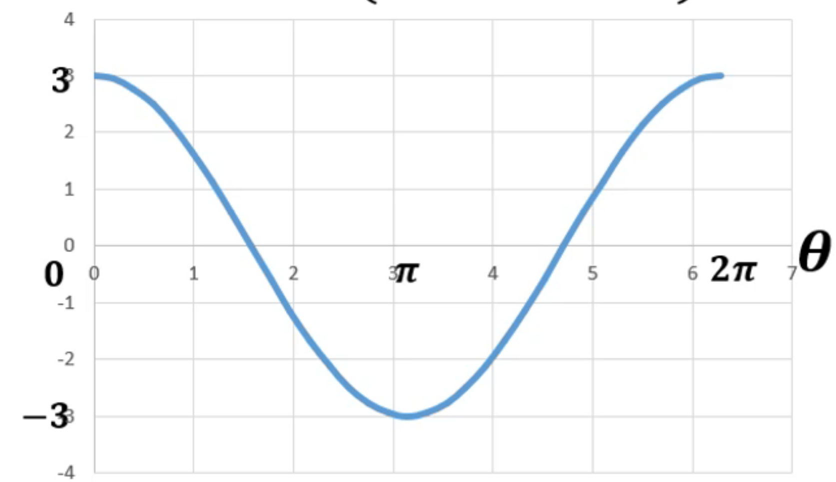
$$4989600 - 907200 = \mathbf{4082400}$$

(b) Sketch the following graphs for $-\pi < \theta < \pi$

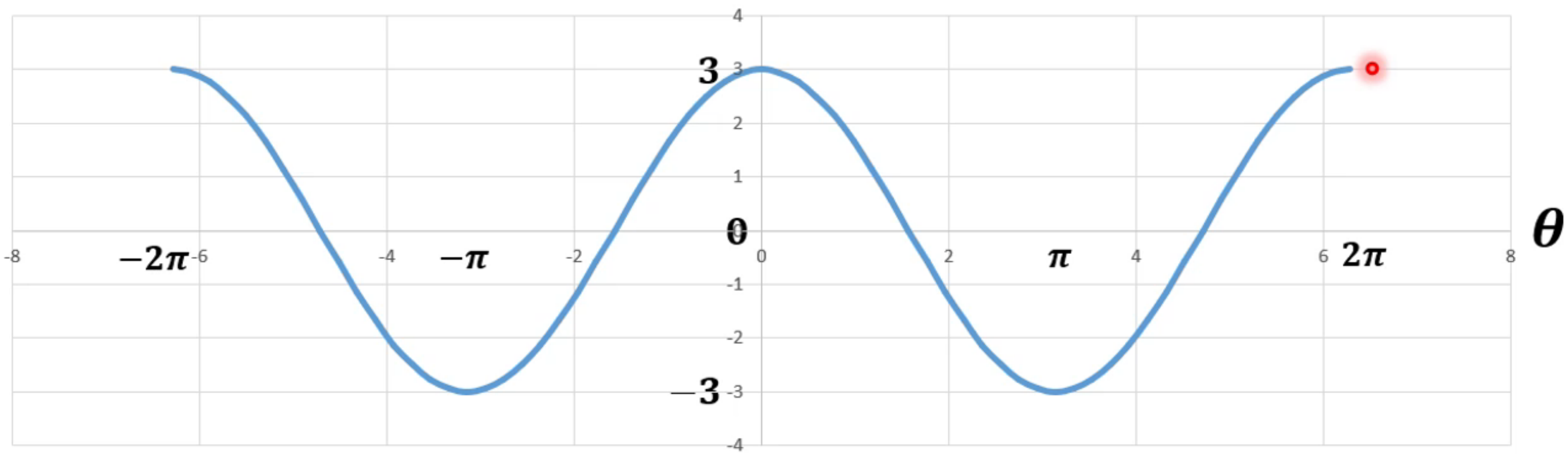
$\boldsymbol{cos\theta}$ $(0 < \theta < 2\pi)$



$\boldsymbol{3cos\theta}$ $(0 < \theta < 2\pi)$



$\boldsymbol{3cos\theta}$
$(-2\pi < \theta < 2\pi)$



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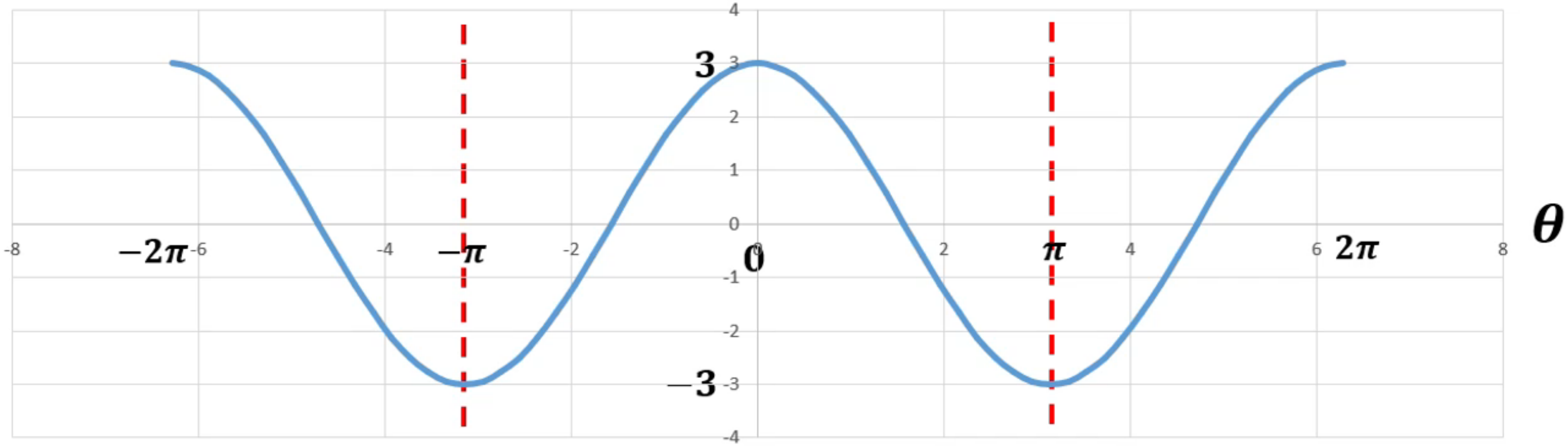
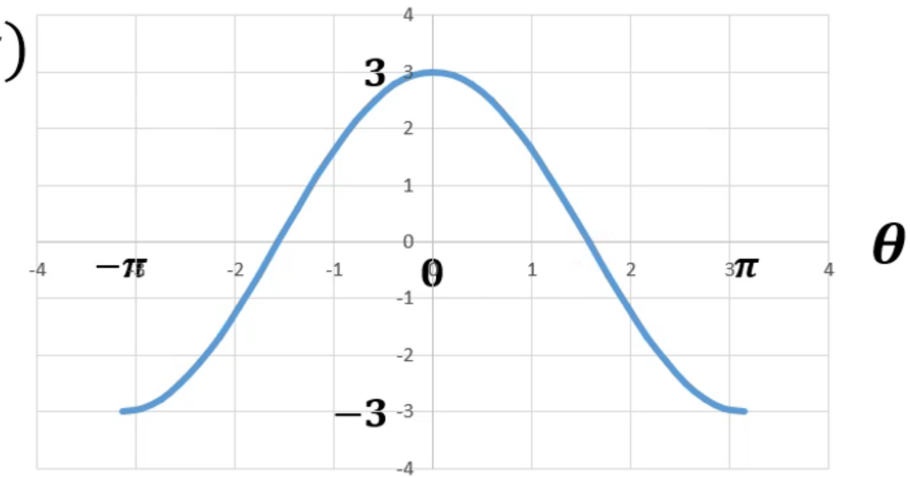
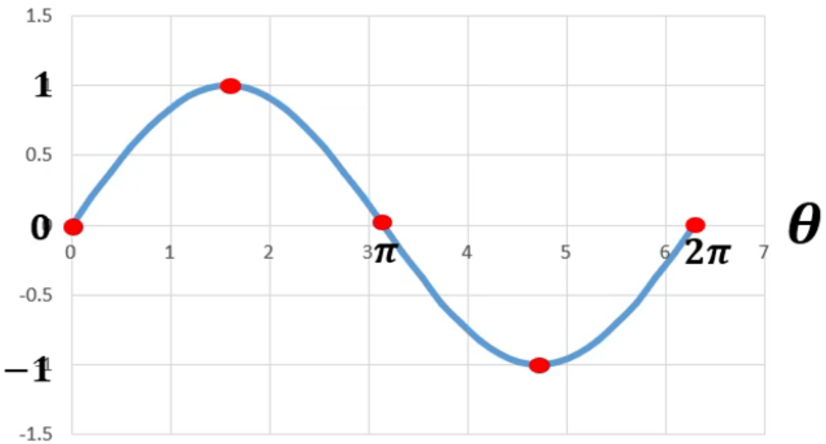


(i) $y = 3cos\theta \ (-\pi < \theta < \pi)$

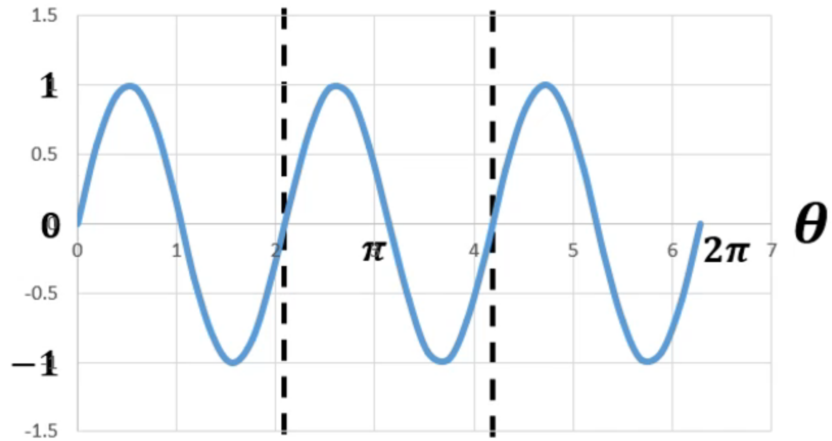


(b) Sketch the following graphs for $-\pi < \theta < \pi$

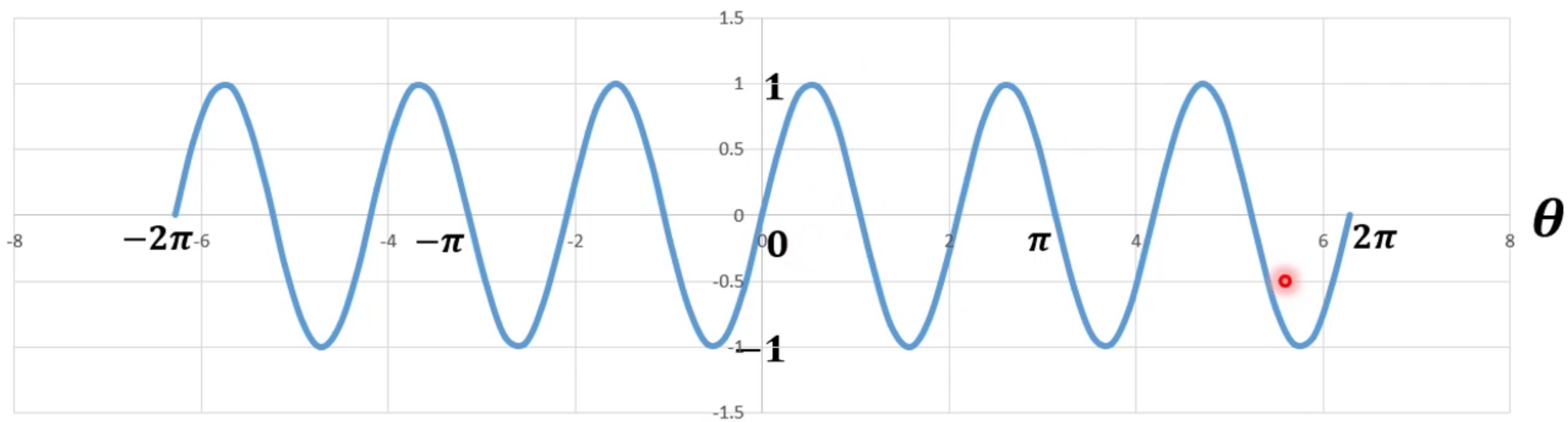
$\boldsymbol{sin\theta}$ $(0 < \theta < 2\pi)$



$\boldsymbol{sin3\theta}$ $(0 < \theta < 2\pi)$



$\boldsymbol{sin3\theta}$
$(-2\pi < \theta < 2\pi)$



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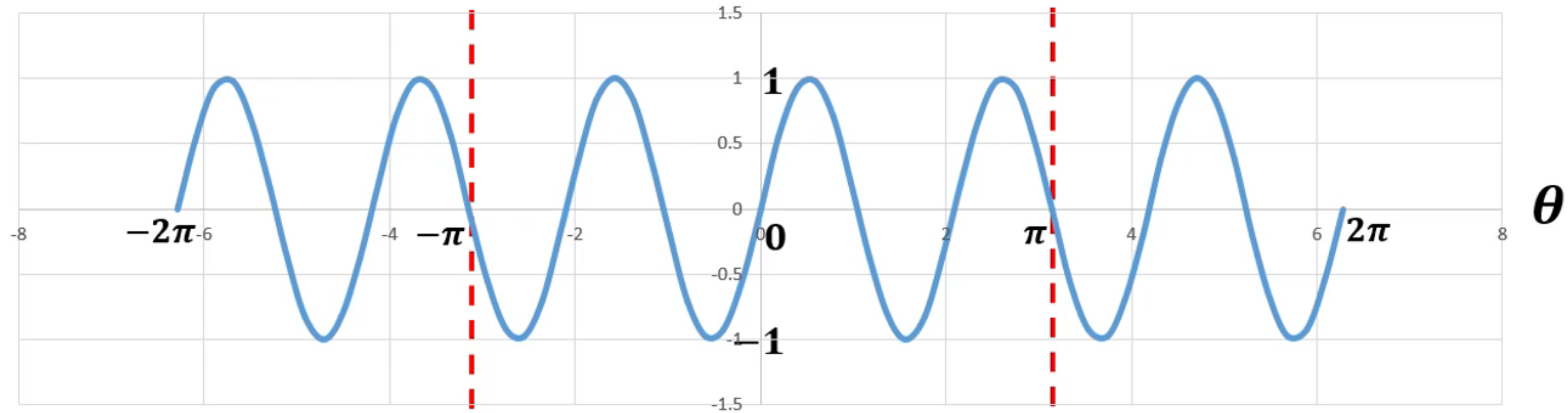
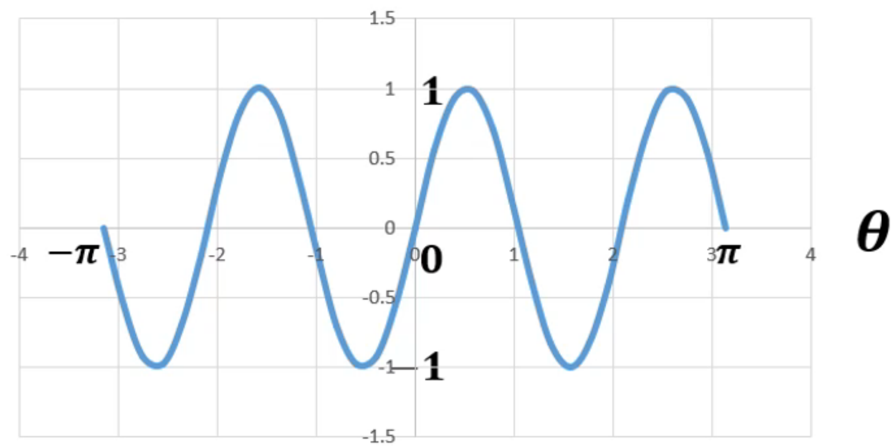
$\boldsymbol{sin3\theta}$

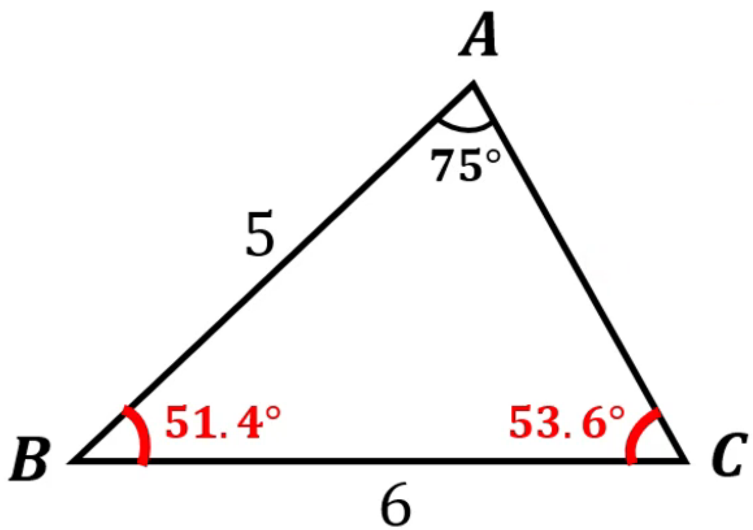
$(-2\pi < \theta < 2\pi)$



(ii) $y = sin3\theta \ (-\pi < \theta < \pi)$



(c) Solve (find all missing lengths and angles) the triangle ABC where $AB = 5cm,\ BC = 6cm,$ and angle $A = 75°$



$$\frac{a}{sinA} = \frac{c}{sinC}$$

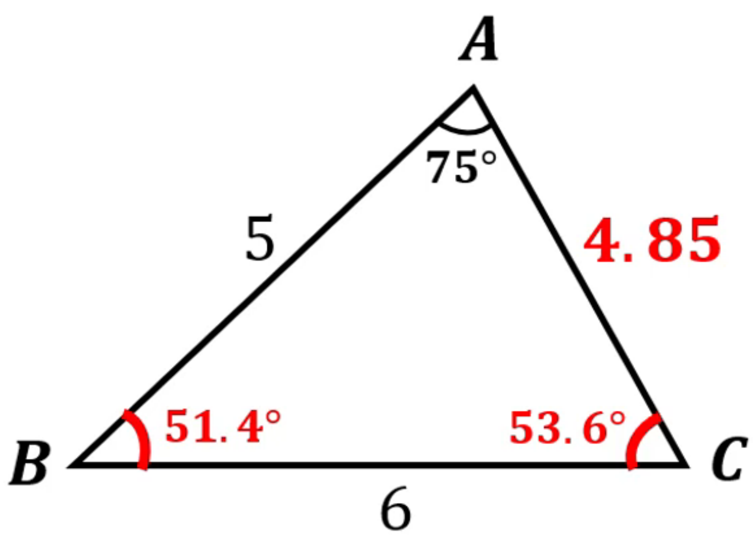
$$\frac{6}{sin75°} = \frac{5}{sinC}$$

$$sinC = \frac{5sin75°}{6}$$

$$\angle C = sin^{-1}\left(\frac{5sin75°}{6}\right) = \mathbf{53.6°}$$

$$\angle B = 180° - 75° - 53.6° = \mathbf{51.4°}$$

(c) Solve (find all missing lengths and angles) the triangle ABC where $AB = 5cm,\ BC = 6cm,$ and angle $A = 75°$



$$\frac{a}{sinA} = \frac{b}{sinB}$$

$$\frac{6}{sin75°} = \frac{b}{sin51.4°}$$

$$b = \frac{6sin51.4°}{sin75°} = \mathbf{4.85(cm)}$$

(2022) Question 2

$\underline{r_1}$: $A(3,2,4)$ $\underline{m} = i + j - k$

$\underline{r_2}$: $A(2,3,1)$ $B(4,4,1)$

(a) Create Vector and Parametric forms of the equations for lines $\underline{r_1}$ and $\underline{r_2}$.

Vector form $\underline{r_1} = \overrightarrow{OA} + \lambda \underline{m} = \boldsymbol{3i + 2j + 4k + \lambda(i + j - k)}$

$\underline{r_2} = \overrightarrow{OA} + \mu(\overrightarrow{OB} - \overrightarrow{OA}) = \boldsymbol{2i + 3j + k + \mu(2i + j)}$

Parametric form $\underline{r_1}$: $\boldsymbol{x = 3 + \lambda}$ $\underline{r_2}$: $\boldsymbol{x = 2 + 2\mu}$

$\boldsymbol{y = 2 + \lambda}$ $\boldsymbol{y = 3 + \mu}$

$\boldsymbol{z = 4 - \lambda}$ $\boldsymbol{z = 1}$

(b) Find the point of intersection for the two lines.

$\underline{r_1}$: $x = 3 + \lambda$, $y = 2 + \lambda$, $z = 4 - \lambda$

$\underline{r_2}$: $x = 2 + 2\mu$, $y = 3 + \mu$, $z = 1$

$$\begin{cases} 3 + \lambda = 2 + 2\mu \\ 2 + \lambda = 3 + \mu \end{cases} \quad (-)$$

$$1 = -1 + \mu$$

$$\mu = 2$$

$$x = 2 + 2(2) = 6$$
$$y = 3 + 2 = 5$$
$$z = 1$$

the point of intersection is **(6, 5, 1)**

(c) Find the size of the angle between the two lines.

$$\underline{r_1} = 3i + 2j + 4k + \lambda(i + j - k) \qquad \underline{a} = i + j - k$$

$$\underline{r_2} = 2i + 3j + k + \mu(2i + j) \qquad \underline{b} = 2i + j$$

$$\boldsymbol{\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}| cos\theta}$$

$$1 \times 2 + 1 \times 1 + (-1) \times 0 = \left(\sqrt{1^2 + 1^2 + (-1)^2}\right)\left(\sqrt{2^2 + 1^2}\right) cos\theta$$

$$3 = (\sqrt{3})(\sqrt{5})\, cos\theta$$

$$cos\theta = \frac{3}{(\sqrt{3})(\sqrt{5})}$$

$$\theta = cos^{-1}\left(\frac{3}{(\sqrt{3})\,(\sqrt{5})}\right) = \mathbf{39.23°}$$

(2022) Question 3

(a) Differentiate:

(i) $7x^2 + 14$

$$\begin{aligned}(7x^2 + 14)' &= (7x^2)' + (14)' \\ &= 14x + 0 \\ &= \mathbf{14x}\end{aligned}$$

(ii) $e^x(4x^2+3)$

Product rule $\quad [f(x)\cdot g(x)]' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

Product rule $\quad y = u(x)\cdot v(x) \qquad \dfrac{dy}{dx} = v\dfrac{du}{dx} + u\dfrac{dv}{dx}$

$f(x) = e^x \qquad g(x) = 4x^2+3$

$f'(x) = e^x \qquad g'(x) = 8x$

$$[e^x(4x^2+3)]' = e^x(4x^2+3) + e^x(8x) = \boldsymbol{e^x(4x^2+8x+3)}$$

(b) Integrate:

(i) $\int (4x^3 - 3x^2 + 2x - 1)\, dx$

$$= 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^3}{3}\right) + 2\left(\frac{x^2}{2}\right) - x + c$$

$$= \boldsymbol{x^4 - x^3 + x^2 - x + c}$$

(ii) $\int_{z=-1}^{4}\int_{y=0}^{3}\int_{x=1}^{2}(2xy+z)\,dxdydz$

$$\int_{x=1}^{2}(2xy+z)\,dx = 2y\left(\frac{x^2}{2}\right)+xz$$

$$= x^2y+xz\Big|_{x=1}^{2} = (4y+2z)-(y+z) = 3y+z$$

$$\int_{y=0}^{3}(3y+z)\,dy = 3\left(\frac{y^2}{2}\right)+yz\Big|_{y=0}^{3} = \left(\frac{27}{2}+3z\right)-0 = \frac{27}{2}+3z$$

$$\int_{z=-1}^{4}\left(\frac{27}{2}+3z\right)dz = \frac{27}{2}(z)+3\left(\frac{z^2}{2}\right)\Big|_{z=-1}^{4}$$

$$= \left(\frac{27}{2}(4)+3\left(\frac{4^2}{2}\right)\right)-\left(\frac{27}{2}(-1)+3\left(\frac{(-1)^2}{2}\right)\right) = \mathbf{90}$$

(2022) Question 4

(a) Explain in words the meaning of $P(A|B)$

$P(A|B)$: the chance of event A happening, given that event B has already happened

State the relationship between A and B when:

(i) $P(A|B) = P(A)$

events A and B are independent

(ii) $P(A|B) = 0$

events A and B are mutually exclusive

(b) The events A and B are such that $P(A) = {}^4/_9$, $P(A|B) = {}^1/_2$,
$P(A \cup B) = {}^2/_3$

To draw a Venn diagram, we need to know $P(A)$, $P(B)$, $P(A \cap B)$, $P(A \cup B)$. $P(A)$ and $P(A \cup B)$ are already given.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{2} P(B)$$

$$P(A \cap B) = \frac{1}{2} \times \frac{4}{9} = \mathbf{\frac{2}{9}}$$

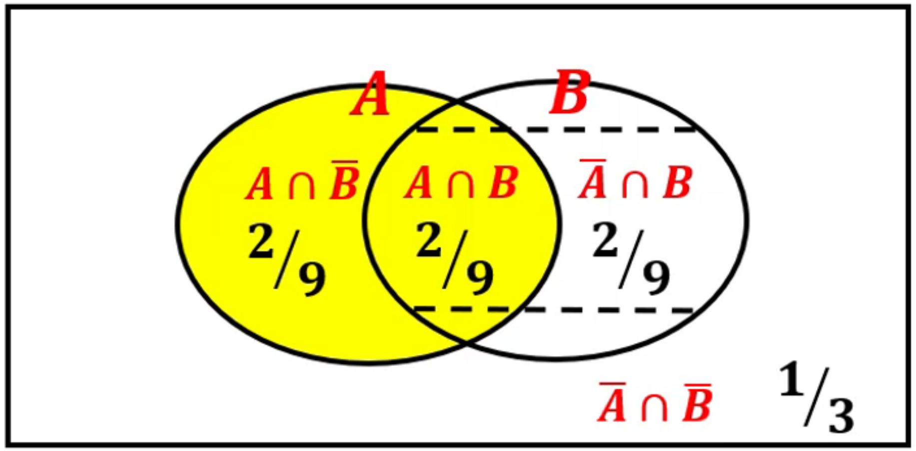
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{2}{3} = \frac{4}{9} + P(B) - \frac{1}{2} P(B)$$

$$P(B) = \mathbf{\frac{4}{9}}$$

(i) Draw a Venn diagram to represent the information

$P(A) = {}^4/_9 \quad P(B) = {}^4/_9 \quad P(A \cap B) = {}^2/_9 \quad P(A \cup B) = {}^2/_3$



$P(A \cap \bar{B}) = P(A) - P(A \cap B) = {}^2/_9$

$P(\bar{A} \cap B) = P(B) - P(A \cap B) = {}^2/_9$

$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = {}^1/_3$

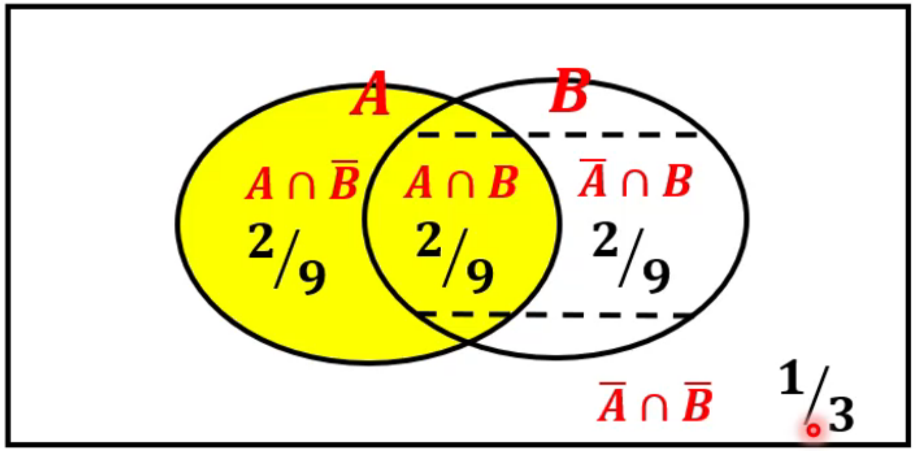
Calculate:

(ii) $P(B) = \frac{4}{9}$

(iii) $P(A \cap B) = \frac{2}{9}$

(iii) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{{}^2/_9}{{}^4/_9} = \frac{1}{2}$

$$P(A) = {}^{4}/_{9} \quad P(B) = {}^{4}/_{9} \quad P(A \cap B) = {}^{2}/_{9} \quad P(A \cup B) = {}^{2}/_{3}$$



$$P(A \cap \bar{B}) = P(A) - P(A \cap B) = {}^{2}/_{9}$$

$$P(\bar{A} \cap B) = P(B) - P(A \cap B) = {}^{2}/_{9}$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = {}^{1}/_{3}$$

Calculate:

(iv) $P\left(A \cup \overline{B}\right) = P(A) + P\left(\overline{B}\right) - P(A \cap \overline{B})$

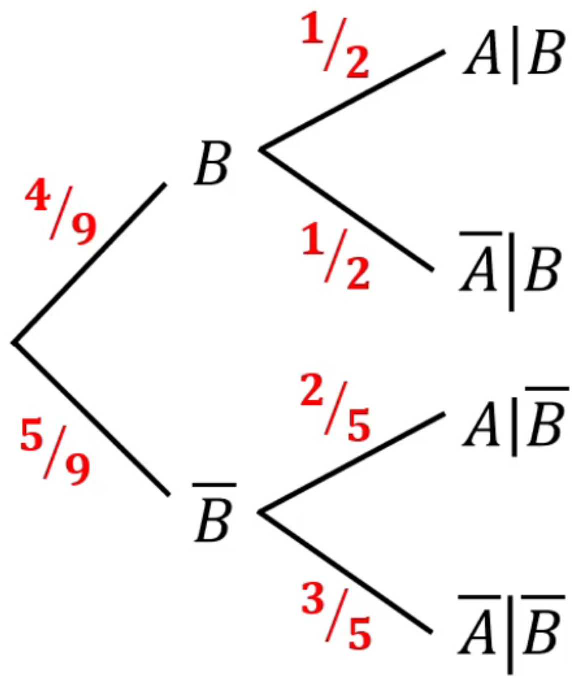
$$= \frac{4}{9} + \left(1 - \frac{4}{9}\right) - \frac{2}{9} = \mathbf{\frac{7}{9}}$$

(v) $P\left(\overline{A} \middle| \overline{B}\right) = \dfrac{P(\overline{A} \cap \overline{B})}{P(\overline{B})}$

$$= \frac{{}^{1}/_{3}}{1 - {}^{4}/_{9}} = \mathbf{\frac{3}{5}}$$

(vi) Represent the information as a tree diagram with B preceding A

$P(B) = {}^4/_9$ $\quad P(A|B) = {}^1/_2$ $\quad P(\overline{A}|\overline{B}) = {}^3/_5$



(2022) Question 5

(a) Solve the simultaneous equations:

$$\begin{cases} 4x + 2y = 20 & (1) \\ 5x - 2y = 7 & (2) \end{cases} \quad (+)$$

$$9x = 27$$

$$\boldsymbol{x = 3} \quad (3)$$

Substitute (3) into (1)

$$4(3) + 2y = 20$$

$$12 + 2y = 20$$

$$2y = 8$$

$$\boldsymbol{y = 4}$$

(b) $M = \begin{pmatrix} 4 & 2 \\ 5 & -2 \end{pmatrix}$ $\quad N = \begin{pmatrix} 20 \\ 7 \end{pmatrix}$

Find:

(i) M^{-1}

$$|M| = (4)(-2) - 5(2) = -8 - 10 = -18$$

$$M^{-1} = \frac{1}{-18}\begin{pmatrix} -2 & -2 \\ -5 & 4 \end{pmatrix} = \frac{1}{18}\begin{pmatrix} 2 & 2 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} \mathbf{1/9} & \mathbf{1/9} \\ \mathbf{5/18} & \mathbf{-2/9} \end{pmatrix}$$

(ii) $M^{-1}N$

$$M^{-1}N = \frac{1}{18}\begin{pmatrix} 2 & 2 \\ 5 & -4 \end{pmatrix}\begin{pmatrix} 20 \\ 7 \end{pmatrix} = \frac{1}{18}\begin{pmatrix} 40 + 14 \\ 100 - 28 \end{pmatrix} = \frac{1}{18}\begin{pmatrix} 54 \\ 72 \end{pmatrix} = \begin{pmatrix} \mathbf{3} \\ \mathbf{4} \end{pmatrix}$$

(c) $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

Find AB and explain why BA cannot be found.

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}_{3\times 2} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}_{2\times 2} = \begin{pmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \\ 5+18 & 10+24 \end{pmatrix} = \begin{pmatrix} \mathbf{7} & \mathbf{10} \\ \mathbf{15} & \mathbf{22} \\ \mathbf{23} & \mathbf{34} \end{pmatrix}$$

$$BA = \underset{r_1 \times c_1}{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}_{2\times \mathbf{2}}} \underset{r_2 \times c_2}{\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}_{\mathbf{3}\times 2}}$$

$\boldsymbol{c_1 = 2, r_2 = 3, \textit{that is } c_1 \neq r_2}$

so BA can not be found