Foundations of Computing 2 - 2023 EXAM 16/05/2024

1. $A = \begin{pmatrix} 3 & 9 \\ -2 & 8 \end{pmatrix}$, $B = \begin{pmatrix} a \\ -1 \end{pmatrix}$, $C = \begin{pmatrix} 9 \\ b \end{pmatrix}$

$AB = C$, a and b are constants

a. Find the values of a and b

$$AB = \begin{pmatrix} 3 & 9 \\ -2 & 8 \end{pmatrix} \times \begin{pmatrix} a \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ b \end{pmatrix}$$

$(3 \times a) + (9 \times -1) = 9$

$3a - 9 = 9$

$3a = 9 + 9$

$3a = 18$

$a = 6$

$(-2 \times a) + (8 \times -1) = b$

$-2a - 8 = b$

$(-2 \times 6) - 8 = b$

$-12 - 8 = b$

$b = -20$

b. Find the inverse of A

$$A = \begin{pmatrix} 3 & 9 \\ -2 & 8 \end{pmatrix}$$

Determinant $= (3 \times 8) - (-2 \times 9) = 24 + 18 = 42$

Invest: $\begin{pmatrix} 3 & 9 \\ -2 & 8 \end{pmatrix} \longrightarrow \begin{pmatrix} 8 & 9 \\ -2 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 8 & -9 \\ 2 & 3 \end{pmatrix}$

Divide by the determinant

$$= \begin{pmatrix} 8/42 & -9/42 \\ 2/42 & 3/42 \end{pmatrix} = \begin{pmatrix} 4/21 & -3/14 \\ 1/21 & 1/14 \end{pmatrix}$$

c. $3x+9y=33$
$-2x+8y=20$

Using your answer to part (b), or otherwise, find the values of x and y.

$$\begin{cases} 3x+9y=33 & \times 2 \\ -2x+8y=20 & \times -3 \end{cases}$$

$$\begin{array}{r} 6x+18y=66 \\ 6x-24y=-60 \quad - \\ \hline 42y=126 \end{array}$$

$42y=126$
$y=3$

$3x+9y=33$
$3x+(9\times 3)=33$
$3x+27=33$
$3x=33-27$
$3x=6$
$x=2$

d. Explain why the matrix $D=\begin{pmatrix} 3 & 6 \\ 2 & 4 \end{pmatrix}$ has no inverse.

Find the determinant: $(3\times 4)-(2\times 6)=12-12=0$

The matrix has no inverse because the determinant is 0. In the final step of finding the inverse of a matrix, you have to divide by the determinant. Dividing by 0 would give an error.

2. With respect to a fixed origin, O, the straight lines L_1 and L_2 are:

$L_1: i + 2j + \lambda(i + j)$, $L_2: \frac{x-6}{3} = 3 - y$

a. Find the point of intersection between L_1 and L_2

$L_1: i + 2j + \lambda(i + j)$ $\quad L_2: \frac{x-6}{3} = 3 - y$

$\equiv \frac{x-6}{3} = \frac{y-3}{-1}$

$x = 1 + \lambda$

$y = 2 + \lambda$

$x = 6 + 3\mu$

$y = -3 - \mu$

$1 + \lambda = 6 + 3\mu$

$2 + \lambda = -3 - \mu$ $-$

$-1 = 9 + 4\mu$

$-1 = 9 + 4\mu$

$4\mu = -10$

$\mu = -2.5$

$x = 6 + (3 \times -2.5)$

$x = -1.5$

$y = -3 - (-2.5)$

$y = -0.5$

Point of intersection is $(-1.5, -0.5)$

b. Line L_3 intersects L_1 at the point $7i + 3j$ and it intersects L_2 at the point $9i + 7j$. Find the Vector equation of the line L_3

c. Calculate the area of the triangle enclosed between the three lines.

3ai. The letters of the word DIVIDED are placed at random in a line. How many different orders of the letters are possible?

DIVIDED: Total letters = 7, 7!

Letters in DIVIDED: D = 3, I = 2, V = 1, E = 1

D = 3, I = 2 : 3! 2!

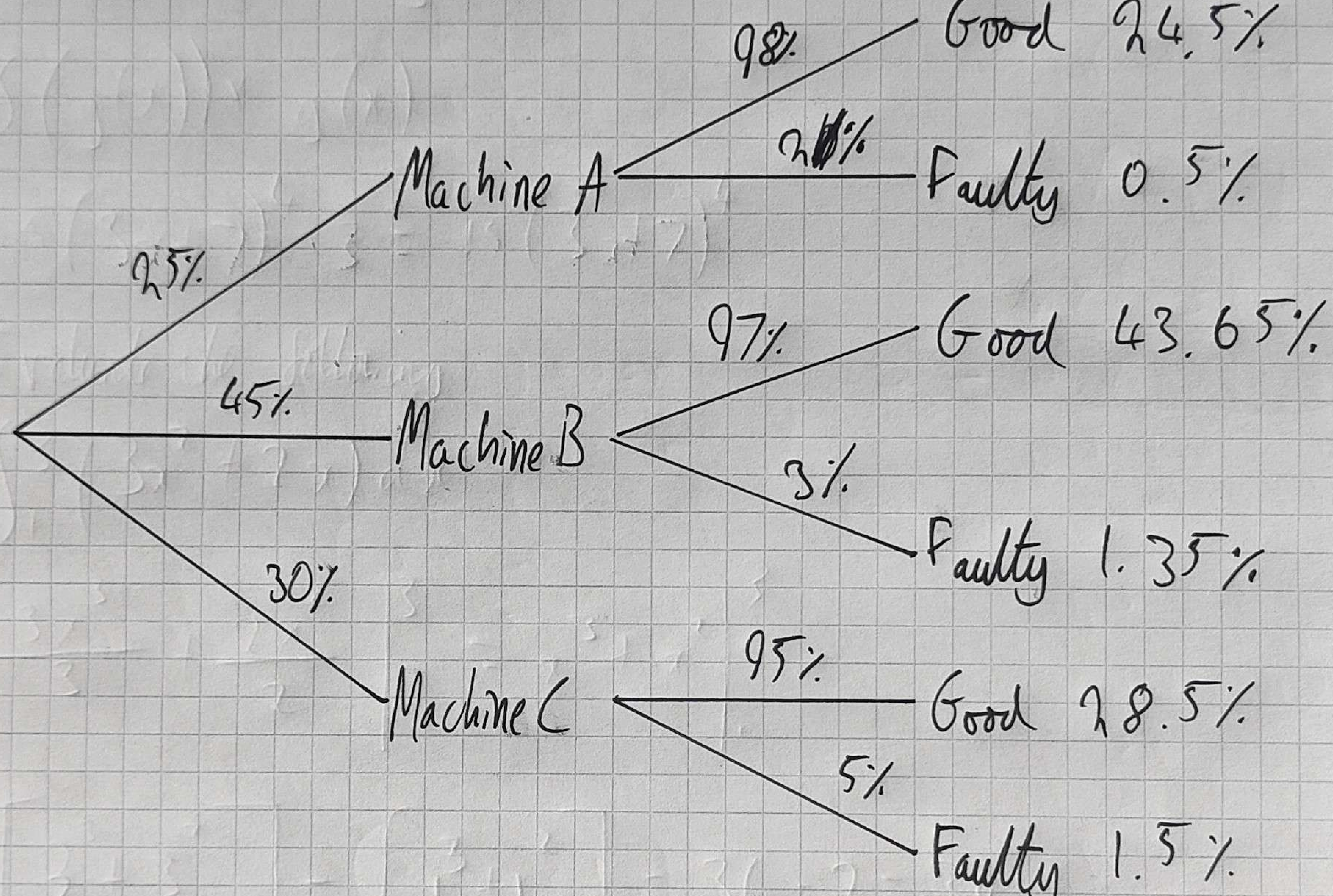
$$\text{Total combinations} = \frac{7!}{3!\,2!} = \frac{5040}{6 \times 2} = 420$$

ii. In how many of the possible orders are the three Ds next to each other?

iii. Find the probability that the first two letters in the order contains at least one D.

b. In a factory a particular component is manufactured on three separate machines. Machine A makes ~~20~~ 25% of the components, machine B produces 45% and machine C makes the remainder. Of the components made by each machine, 2% of those produced by machine A are found to be faulty, with the corresponding figures for machines B and C being 3% and 5% respectively.

i. Draw a tree diagram to represent this information



ii. The components produced by all three machines are merged. Calculate the probability that a component picked at random is faulty.

Probability that random component is faulty =

$(0.25 \times 0.02) + (0.45 \times 0.03) + (0.3 \times 0.05)$

$= 0.005 + 0.0135 + 0.015 = 0.0335$, or 3.35%

iii. Given that the chosen component is faulty, calculate the chance that it was produced by machine B.

P(Produced by machine B | Faulty)

$P(A|B) = \dfrac{P(A \cap B)}{P(B)} \equiv \rightarrow \dfrac{P(\text{Produced by machine B and faulty})}{P(\text{Faulty})}$

$= \dfrac{0.0135}{0.0335} = {}^{27}/_{67}$, or 0.403 (3dp)

4a. Differentiate the following:

i. $y = 4x^3$

$$\frac{dy}{dx} = 3 \times 4x^2 = 12x^2$$

ii. $y = (3x+7)^5$

$f(x) = x^5 \qquad g(x) = 3x+7$

$f'(x) = 5x^4 \qquad g'(x) = 3$

$f'(g(x)) \times g'(x)$

$= 5(3x+7)^4 \times 3 = 15(3x+7)^4$

b. Evaluate the following:

i. $\int_1^3 (3x^2 + 2x)\,dx$

$$= \frac{3x^3}{3} + \frac{2x^2}{2} \Big|_1^3 = x^3 + x^2 \Big|_1^3$$

$$= \left[3^3 + 3^2\right] - \left[1^3 + 1^2\right] = 36 - 2 = 34$$

ii. $$\int_{y=2}^{4} \int_{x=1}^{3} (2x+y)\, dx\, dy$$

$$\int_{x=1}^{3} (2x+y)\,dx = \frac{2x^2}{2} + xy \Big|_1^3 = x^2 + xy$$

$$= \left[3^2 + 3y\right] - \left[1^2 + y\right] = \left[9 + 3y\right] - \left[1 + y\right] = 8 + 2y$$

$$\int_{y=2}^{4} (8+2y)\,dy = 8y + \frac{2y^2}{2} \Big|_2^4 = 8y + y^2 \Big|_2^4 \text{ or } y^2 + 8y \Big|_2^4$$

$$= \left[4^2 + (8 \times 4)\right] - \left[2^2 + (8 \times 2)\right] = \left[16 + 32\right] - \left[4 + 16\right]$$

$$= 48 - 20 = 28$$