Foundations of Computing 1 - 2023 Exam 07/04/2024

1a. Perform the following Octal calculations without changing base. Show your workings.

i. 47 + 63

$$\begin{array}{r} 47 \\ 63+ \\ \hline 132 \end{array}$$

+1 ~~7~~

ii. 47x63

$7 \times 3 = 21 \longrightarrow 8 \times 2 + 5$

$4 \times 3 \boxed{+2} = 14 \longrightarrow 8 \times 1 + 6$

$\longrightarrow 165$

$7 \times 6 = 42 \longrightarrow 8 \times 5 + 2$

$4 \times 6 \boxed{+5} = 29 \longrightarrow 8 \times 3 + 5$

$\longrightarrow 3520$

$$\begin{array}{r} 165 \\ 3520\ + \\ \hline 3705 \end{array}$$

+1

47x63 = 3705

6. Solve the following:

i. $4x+7=51$

$4x+7=51 \longrightarrow 4x=51-7 \longrightarrow 4x=44 \longrightarrow x=11$

i. $\left.\begin{array}{l} 5x-7y=-3 \\ 2x+3y=22 \end{array}\right\}$

$(5x-7y=-3) \times 2$
$(2x+3y=22) \times 5$ $\longrightarrow$

$$\begin{array}{rcrcl} 10x & - & 14y & = & -6 \\ 10x & + & 15y & = & 110 \quad - \\ \hline & - & 29y & = & -116 \end{array}$$

$-29y=-116$
$29y=116 \longrightarrow y=4$

$2x+3y=22 \longrightarrow 2x+(3\times4)=22 \longrightarrow 2x+12=22$

$2x=22-12 \longrightarrow 2x=10 \longrightarrow x=5$

$x=5, \; y=4$

iii. $5x^2 + 3x - 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ $a = 5,\ b = 3,\ c = -6$

$$x = \frac{-3 \pm \sqrt{3^2 - (4 \times 5 \times -6)}}{2 \times 5} \longrightarrow = \frac{-3 \pm \sqrt{9 + 120}}{10}$$

$$= \frac{-3 \pm \sqrt{129}}{10}$$

$$x = \frac{-3 + \sqrt{129}}{10} = 0.836\ (3dp)$$

$$x = \frac{-3 - \sqrt{129}}{10} = 1.436\ (3dp)$$

2ai. Find the mean and standard deviation for the following set of data.

x	10	20	30	40	50	Total
f	2	3	3	1	1	10
fx	20	60	90	40	50	260
fx^2	200	1200	2700	1600	2500	8200

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{260}{10} = 26$$

$$SD = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

$$= \sqrt{\frac{8200}{10} - \boxed{26^2}} = \boxed{12}$$

ii. A further 20 items with a mean of 25 and a standard deviation of 10 are added to the initial set of data. Find the combined mean and standard deviation for all 30 items.

Previous data set: 10 items, mean = 26, SD = 20.494

New data set: 20 items, mean = 25, SD = 10

Total items: 10 + 20 = 30

Combined mean = $\frac{(10 \times 26) + (20 \times 25)}{30} = 25.333$ (3 dp)

Combined SD = $\sqrt{\frac{f_1 \times SD_1^2 + f_2 \times SD_2^2}{f_1 + f_2}}$

$= \sqrt{\frac{10 \times 20.494^2 + 20 \times 10^2}{30}} = 14.374$ (3dp)

b. Widgets are packed in boxes ready for shipping, with 10 widgets per full box. The mean weight of an individual widget is 12g and the standard deviation is 3g. The corresponding figures for the box are $\mu = 15g$ and $\sigma = 4g$. Find the combined mean and standard deviation of the weight of a full box of widgets.

Individual widgets: Mean = 12g, SD = 3g, 10 widgets per box

Individual box: Mean = 15g, SD = 4g

Mean of individual box and 10 widgets = $10 \times 12 + 15 = 135g$

Standard deviation of individual box and 10 widgets:

$$\sigma^2 = \sigma_1^2 \times 10 + \sigma_2^2 \times 1$$

$$= 3^2 \times 10 + 4^2 \times 1$$

$$= 106$$

$$\sigma = \sqrt{106} = 10.295 \text{ (3dp)}$$

3a. Find the sum to infinity for the series 27 + 9 + 3 + ...

$$S_\infty = \frac{a}{1-r}$$

$27 \xrightarrow{\times \frac{1}{3}} 9 \xrightarrow{\times \frac{1}{3}} 3$, $a = 27, r = \frac{1}{3}$

$$= \frac{27}{1-\frac{1}{3}} = 40.5$$

b. Find the values of a, b, c, d and e such that

$$(3+2x)^a = b + 216x + cx^2 + 96x^d + ex^4$$

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$$= \binom{4}{0} \times 3^4 \times (2x)^0 + \binom{4}{1} \times 3^3 \times (2x)^1 + \binom{4}{2} \times 3^2 \times (2x)^2$$

$$+ \binom{4}{3} \times 3^1 \times (2x)^3 + \binom{4}{4} \times (3)^0 \times (2x)^4$$

$$= (1 \times 81) + (4 \times 27 \times 2x) + (6 \times 9 \times 4x^2)$$

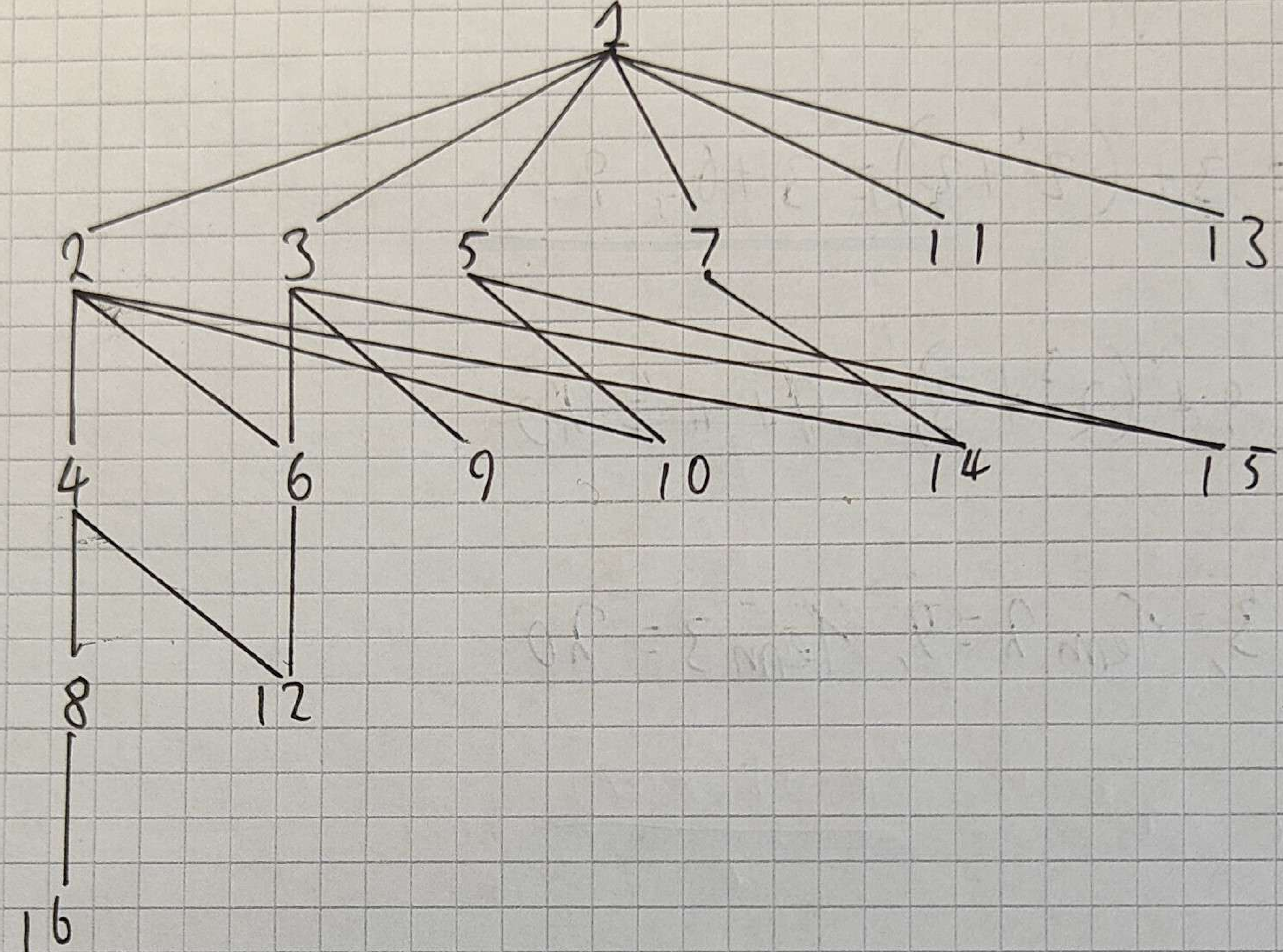
$$+ (4 \times 3 \times 8x^3) + (1 \times 1 \times 16x^4)$$

$$= 81 + 216x + 216x^2 + 96x^3 + 16x^4$$

$a = 4$, $b = 81$, $c = 216$, $d = 3$, $e = 16$

c. Draw a Hasse diagram to represent the relation "exactly divides" on the natural numbers from 1 to 16 inclusive.

Numbers 1 to 16:



4. A series is defined by $S_n = \sum_{r=1}^{n} (2^r + r)$

a. Find the first three terms of the series

First three terms: $n=1$, $n=2$, $n=3$

$$\sum_{r=1}^{1} = (2^1 + 1) = 3$$

$$\sum_{r=1}^{2} = 3 + (2^2 + 2) = 3 + 6 = 9$$

$$\sum_{r=1}^{3} = 9 + (2^3 + 3) = 9 + 11 = 20$$

Term 1 = 3, Term 2 = 9, Term 3 = 20

b. Use the method of induction to prove that

$$S_n = 2(2^n - 1) + \frac{n}{2}(n+1)$$

Base Case: $S_1 = T_1$, $r=1$, $n=1$

$(2^1 + 1) = 3$, $2(2^1 - 1) + \frac{1}{2}(1+1) = 2+1 = 3$

Assumption: There is at least one value in which T & S hold the same value: $1 \leq k < n$

Statement: If true then we can check by substituting $k+1$ into the formula

$$S_{k+1} = 2(2^{k+1} - 1) + \frac{k+1}{2}(k+1+1)$$

$$= 2(2^{k+1} - 1) + \frac{(k+1)(k+2)}{2}$$

Induction:

$$S_{k+1} = S_k + T_{k+1}$$

$$2(2^{k+1}-1) + \frac{(k+1)(k+2)}{2} = 2(2^k-1) + \frac{k}{2}(k+1) + 2^{k+1} + k+1$$

$$= 2(2^k-1) + \frac{k}{2}(k+1) + 2^{k+1} + k+1$$

$$= 2(2^k-1) + 2^{k+1} + \frac{k(k+1)}{2} + \frac{k+1}{1}$$

$$= 2(2^k-1) + 2^{k+1} + \frac{k(k+1)+2(k+1)}{2}$$

$$= 2(2^k-1) + 2^{k+1} + \frac{k^2+k+2k+2}{2}$$

$$= 2(2^k-1) + 2^{k+1} + \frac{(k+1)(k+2)}{2}$$

$$= 2^{k+1} - 2 + 2^{k+1} + \frac{(k+1)(k+2)}{2}$$

$$= 2(2^{k+1}-1) + \frac{(k+1)(k+2)}{2}$$