LSE Department of Statistics

Project ST425: Roulette

Team 5

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I. Outline and Methodology

In the following we compare five different roulette strategies and measure their performance judged by multiple metrics, including the average profit and loss (P/L), the risk measured as standard deviations of P/L, the average game length, as well as the probability of leaving the roulette table with a win.
Our analysis consists of two parts: We compare the strategies using a standard setup and in a second step impose exit conditions on the gambler to investigate if it is possible to optimize the outcomes.

For our standard setup we use the following parameters:

- Number of simulated games: 1000
- Number of spins per game: 500
- Initial bankroll: $5000
- Constant bet size: $10

Note that in the simulation the initial bank balance has been chosen in such a way that it is both realistic and does not impact the results in the standard setup[1] (i.e., the gambler will not run out of money before a game ends).

For part two of the analysis, we restrict the possible values of P/L. In particular, the gambler will leave the table if she either lost $200 ("stop loss") or if she won at least $100 ("stop win"). These limits have been chosen in a way such that they are both restrictive while at the same time allowing the gambler to play all strategies for a while.
Furthermore, the gambler must leave the table when she runs out of money, i.e., she cannot gamble with borrowed money. Especially, should the desired bet exceed her current balance, we require her to bet the entire remaining balance, resulting in a possibly smaller bet size than desired.

Given that our goal is to derive results that are applicable to the real world, we also impose constraints on the bet sizes following the conventions in most European casinos (minimum bet $1 and maximum bet $5000). This constraint is especially relevant for strategies with dynamic bet sizes, like strategy D and strategy E. Should the desired bet of the gambler exceed the maximum bet size (fall short of the minimum bet size) we require her to place the maximum (minimum) bet.

Note that our analysis focuses on a fair European roulette game. Hence for our simulation, we assume a discrete uniform distribution with $P(X = k) = 1/37$ for $k = 0,1, \ldots ,36$.

II. Betting Strategies

II.1 Strategy A: Betting on Red

For this strategy, the gambler places a constant-sized bet on red for each spin for the entire game. If the outcome of the spin is red the gambler gains $10. Otherwise, the entire bet is lost.

II.2 Strategy B: Betting on a Number

The gambler places her bet on a single number instead of a colour. If the outcome of the spin is the selected number, the gambler gains $350. If the ball lands on any other number, the entire bet is lost.

II.3 Strategy C: James Bond Strategy

Strategy C – named after the famous fictional MI6 agent – consists of placing multiple bets at once resulting in the following payoff:

[1]Except for strategy E as explained below.

- Place \$7.00 on numbers 19 through 36 with a potential profit of \$4.00
- Place \$2.50 on numbers 13 through 18 with a potential profit of \$5.00
- Place \$0.50 on zero with a potential profit of \$8.00

II.4 Strategy D: D'Alembert Strategy (Betting on Red)

This strategy is dynamic in the sense that the current bet size depends on earlier outcomes. We implement this betting system in its simplest form, namely as a variation of strategy A:

- The initial bet is \$10
- If $outcome_1$ is red, decrease bet size by \$1. Otherwise, increase bet size by \$1
- In general: $outcome_t \in \{\text{'red'}\} \Rightarrow bet_{t+1} = \max(\$1, bet_t - \$1)$ and $outcome_t \notin \{\text{'red'}\} \Rightarrow bet_{t+1} = \min(\$5000, bet_t + \$1)$

II.5 Strategy E: Martingale (Betting on Red)

Like strategy D, this strategy is dynamic resulting in increasing the bet size by a constant factor (usually 2) if the gambler lost on the previous spin and decreasing the bet size if she won on the previous spin ("Negative Progression System"). Interestingly, the martingale betting system guarantees a profit under the conditions that the gambler has infinite money and there are no maximum bet sizes[2]. For details see among others (Pflaumer, 2019).

However, this strategy crucially relies on being able to recover all cumulative losses with a single win, which due to maximum bets may not always be possible. As a result, the gambler in our simulation will keep playing after a win (if she does not meet any of the exit conditions). The strategy can be summarized as follows:

- The initial bet is \$10 with martingale multiplier $m : m \geq 1$
- If $outcome_1$ is red, divide bet size by m. Otherwise, multiply bet size by m
- In general: $outcome_t \in \{\text{'red'}\} \Rightarrow bet_{t+1} = \max(\$1, bet_t/m)$ and $outcome_t \notin \{\text{'red'}\} \Rightarrow bet_{t+1} = \min(\$5000, bet_t * m)$

Note that for $m = 1$ this strategy is equivalent to strategy A.

III. Comparison

Following the methodology introduced above, the results of our simulations are presented below:

Table III.1 Standard Setup

Strategy	Mean P/L (\$)	Sample Standard Deviation (\$)	Proportion of games won (%)	Maximum Loss (\$)	Maximum Win (\$)	Mean Game Length
A	**- 134**	220	26	- 860	500	500
B	- 139	1315	48	- 4280	6520	500
C	- 144	**149**	17	**- 621**	307	500
D	- 255	663	**49**	- 4700	326	500
E	- 1389	11777	10	- 5000[3]	**68432**	178

[2] After a losing streak of t spins, the bet for spin $t + 1$ would be $\$(10 * 2^t)$. Consequently, after a losing streak of only nine games, the bet would exceed the maximum bet size in most European casinos.

Table III.2 Exit Strategies

Strategy	Mean P/L ($)	Sample Standard Deviation ($)	Proportion of games won (%)	Maximum Loss ($)	Maximum Win ($)	Mean Game Length
A	- 52	146	47	- 200	100	**199**
B	- 3	232	42	- 200	**350**	15
C	- 96	**124**	26	- 200	107	150
D	- 47	157	52	- 200	114	135
E	**-2**	151	**64**	- 200	168	29

The performance metrics differ dramatically for the different strategies. In the standard setup, our simulations suggest that strategy A results in the smallest loss on average, while the modified martingale strategy performs by far the worst judged by this metric. Considering the estimated standard deviation of P/L for the different strategies we can see that strategy C seems to be the least risky whereas strategy E seems very risky compared to the other strategies. Without any exit conditions, the gambler manages to win almost 50% of the games employing strategy D, while only winning roughly 17% following strategy C. Following our results concerning the standard deviation, it is no surprise that the range of simulated cumulative P/L after 500 spins for strategy C is the smallest, whereas it is largest for strategy E by a huge margin.

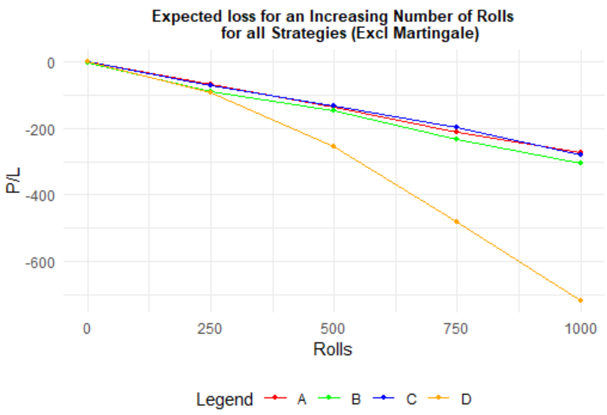
Considering the results of our simulations including exit strategies we see that all metrics improved considerably compared to the scenario without exit strategies, except for the shorter game length and the much lower maximum win. Especially, our simulations suggest that it is possible to reduce the mean loss to approximately $2 (using our exit conditions), reduce the estimated standard deviation to $124, and increase the proportion of games won to approximately 64%. This comes at the cost of not being able to play for as long as before as well as sacrificing the possibility of a huge win.
So, why is this? It is a well-known fact that "the house always wins". This is because the casino has an edge in most games, including roulette. As a result, it is impossible – regardless of the betting strategy – to make money over the long run playing roulette.
Imposing exit conditions on the gambler effectively reduces the game length (and limits the maximum possible variance of outcomes) giving her less opportunity to lose money. Further simulations (as well as mathematics) suggest that the mean loss can be reduced to approximately $0.27 by shortening the game to a single spin.
Evidently, judged by most metrics (except for maximum win and mean game length) the best-case scenario would be not playing at all!

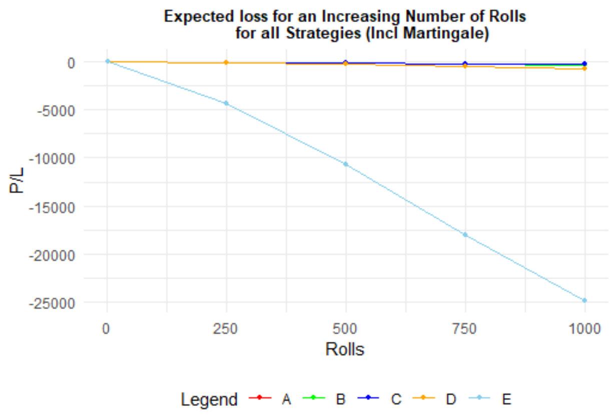
Table III.3 Prohibitive Exit Strategies (max loss -$10, max win $10, 10^5 games)

Strategy	Mean P/L ($)	Sample Standard Deviation ($)	Proportion of games won (%)	Maximum Loss ($)	Maximum Win ($)	Mean Game Length
A	- 0.27	10	49	- 10	10	1
B	- 0.52	58	3	- 10	350	1
C	- 0.98	11	40	- 10	17	7
D	- 0.27	10	49	- 10	10	1
E	- 0.27	10	49	- 10	10	1

[3] For the sake of comparability, we decided to assume an initial bank balance of $5000 for all strategies, even though it is apparent that the bank balance for strategy E is going to hit zero with a probability larger than 0 (technically imposing a stop loss also in the standard setup), resulting in shorter games.

Illustration III.1 and **Illustration III.2** Mean Loss as a function of rolls





IV. Conclusion

To conclude, our analysis shows that the “optimal” strategy depends both on the gambler’s goal as well as whether additional exit strategies – besides a finite bank account – are implemented. Furthermore, ignoring the trivial optimal solution of not playing at all, our simulations suggest that it is highly beneficial to implement some kind of exit strategy when playing roulette.

If no additional exit strategies are implemented and the gambler’s goal is losing as little money as possible on average, it seems to be beneficial to follow any of the strategies A to C. Choosing between these three depends on the gambler’s risk tolerance and her secondary goals. Choosing a less risky strategy results in a lower probability of leaving the table with a win. If the gambler wants to maximise the likelihood of ending the night with a win, however, she might consider choosing strategy D, at the expense of a higher average loss. In contrast, should her goal be maximizing the highest possible pay-out, she might instead choose a very risky strategy such as B or E – again, her choice depends on her individual preferences.
The same applies to the variation with the exit strategies: there is no single "optimal" strategy. The "best" strategy depends on the player's objectives, her risk appetite, and the relative importance of the individual performance indicators.

Limitations and Future Work: Given that we simulated 1,000 games per strategy, we recognized that the performance statistics differed slightly with each simulation. We also acknowledge that in this analysis, we explored only a single configuration of exit conditions. Further work would involve finding better exit conditions for each strategy, which are not necessarily the most restrictive. We could also explore different step sizes for the Martingale strategy, and their respective optimal stopping conditions depending on the gambler’s goals.

V. Appendix

V.1 Illustrations

Illustration V.1 Quantile Plot P/L Strategy A (Standard Setup)

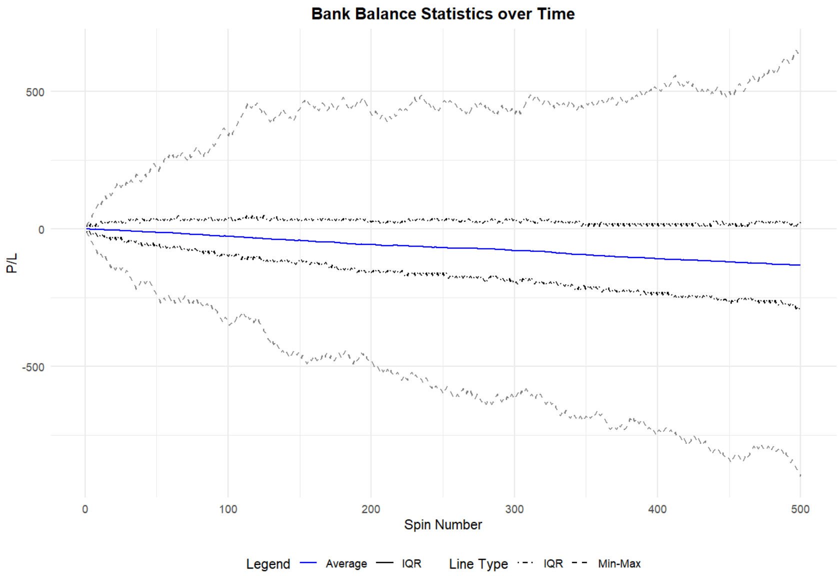


Illustration V.2 Quantile Plot P/L Strategy B (Standard Setup)

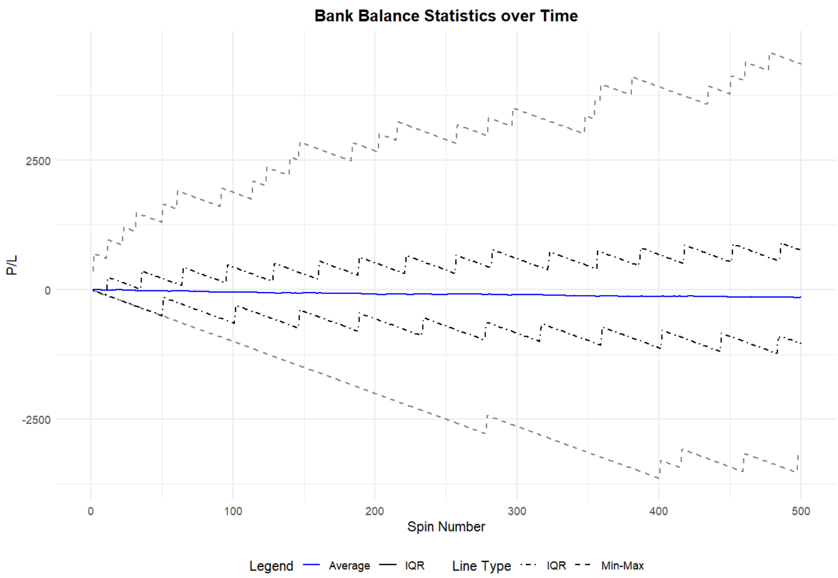


Illustration V.3 Quantile Plot P/L Strategy C (Standard Setup)

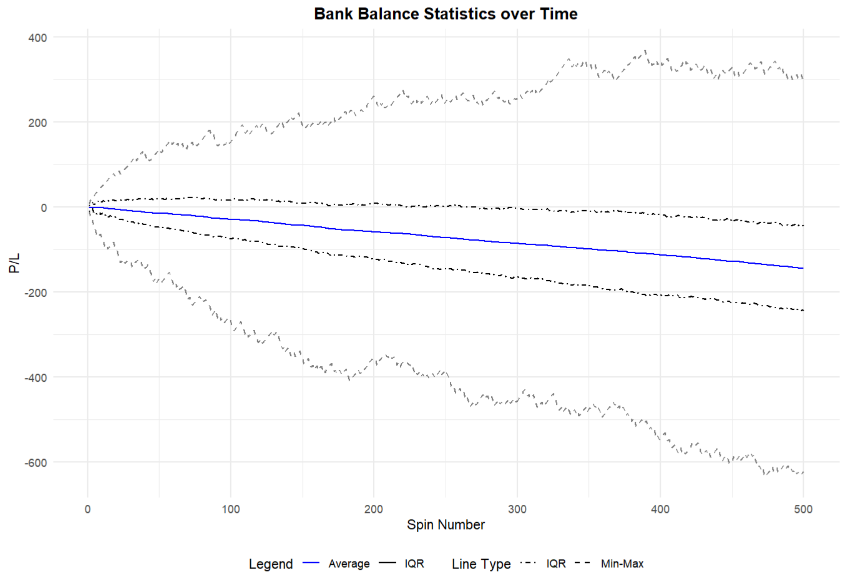


Illustration V.4 Quantile Plot P/L Strategy D (Standard Setup)

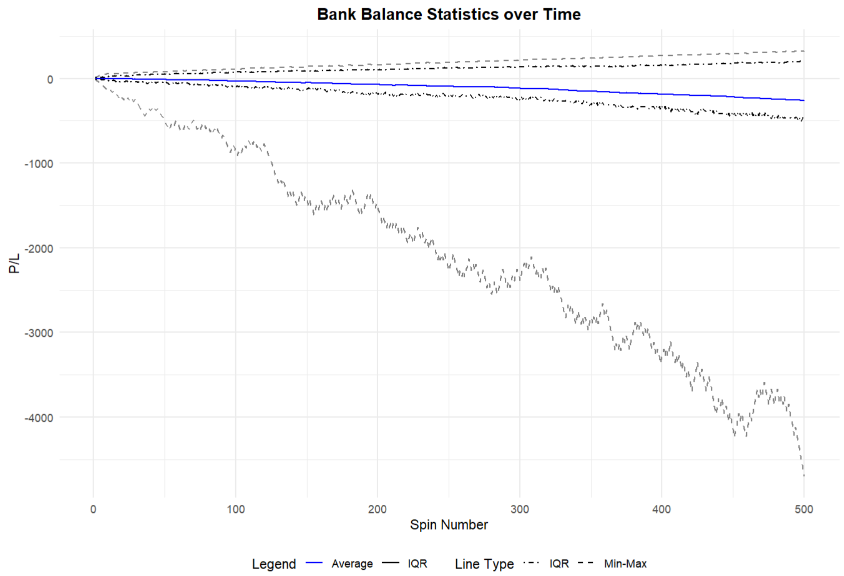


Illustration V.5 Quantile Plot P/L Strategy A (Exit Strategies)

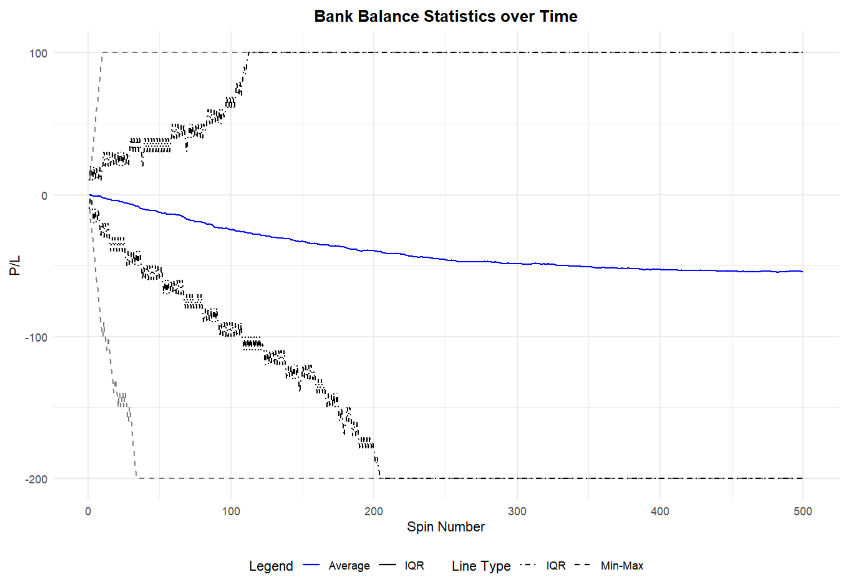


Illustration V.6 Quantile Plot P/L Strategy B (Exit Strategies)

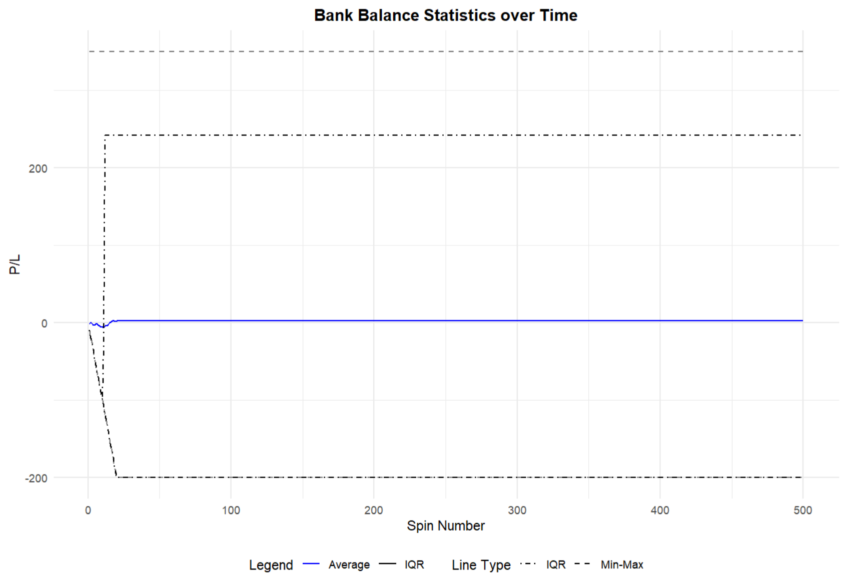


Illustration V.7 Quantile Plot P/L Strategy C (Exit Strategies)

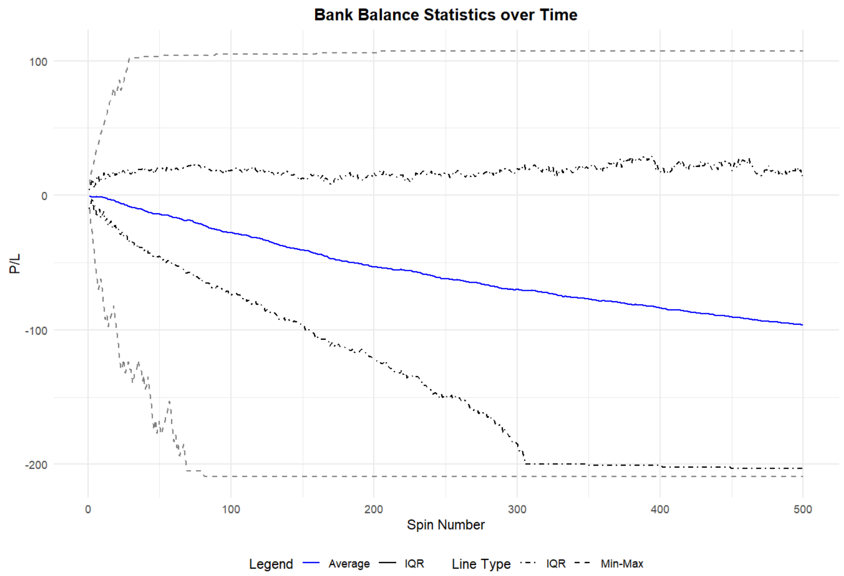
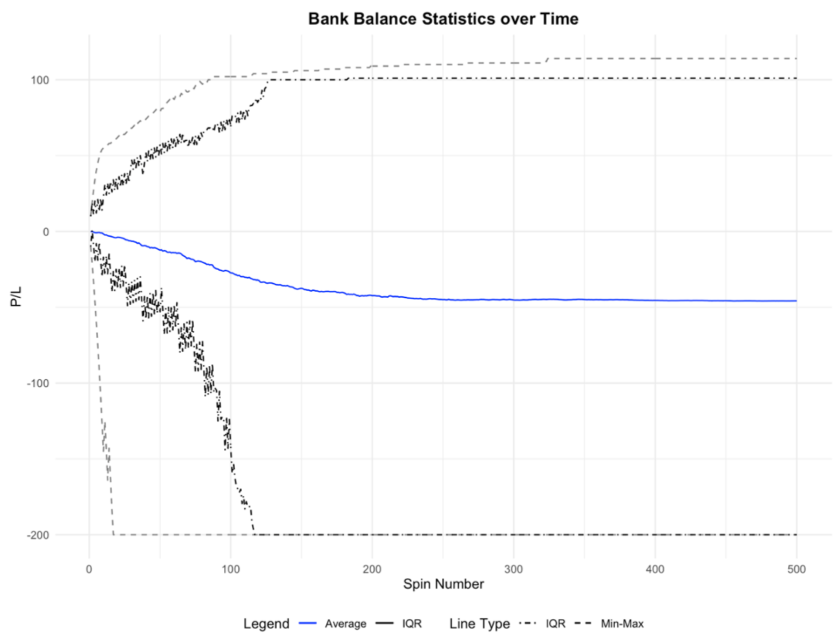


Illustration V.8 Quantile Plot P/L Strategy D (Exit Strategies)



V.2 R Code

For simplicity's sake we present only an excerpt of the code used in this simulation. The full code is available upon request.

```
### ------ Setup  ------

# fixed parameters given game design:
red = c(1,3,5,7,9,12,14,16,18,19,21,23,25,27,30,32,34,36)
black = c(2,4,6,8,10,11,13,15,17,20,22,24,26,28,29,31,33,35)
green = 0

# only strategy B: choose number
number = 27

# only strategy C: define bets
bet1_C = 19:36
bet2_C = 13:18
bet3_C = 0
nobet_C = 1:12

# variable parameters

#game params
n_rolls = 500
n_games = 1000

#bet sizes
bet = 10
max_bet = 5000 #standard in UK
min_bet = 1 #standard in UK

#bet strategy
multiplier = 1
bank = 5000
step_size_D = 0.1*bet

#exit strategy
max_win = 1e+08 #For standard case (w/o max_win) use very high max_win
max_loss = -bank


### ------ Simulation  ------

#simulate n_games * n_rolls outcomes
set.seed(123)
sim = sample(x = 0:36, replace = TRUE, size = n_games * n_rolls)

#create data frame filled with the simulated outcomes
outcomes = as.data.frame(matrix(sim,nrow = n_games,ncol = n_rolls))
rownames(outcomes) = paste("Game",1:n_games)
colnames(outcomes) = paste("Outcome Spin", 1:n_rolls)

#check histogram
```

```
plot(table(unname(unlist(outcomes[1,])))/n_rolls,main   =   "Histogram   of
outcomes for the first game",
     xlab = "Outcome",ylab = "Proportion")

### ------ strategy A ------


# Create a data frame to store pnl per spin
pnl_A = as.data.frame(matrix(0, nrow = n_games, ncol = n_rolls))
rownames(pnl_A) = paste("Game", 1:n_games)
colnames(pnl_A) = paste("P/L Spin", 1:n_rolls)

bank_A = numeric(n_games)

# Create a function to calculate pnl for each game
calculate_pnl_A = function(game_outcome, current_bet) {
  red_hits = game_outcome %in% red
  black_hits = game_outcome %in% black
  green_hits = game_outcome %in% green

  pnl = 0
  pnl[red_hits] = current_bet
  pnl[black_hits] = -current_bet
  pnl[green_hits] = -current_bet

  return(pnl)
}

len_A = numeric(n_games)

#loop through all games and all spins to obtain P/L for A
for (game in 1:n_games) {
  current_bet = bet
  current_pnl = 0
  current_bank = bank

  for (spin in 1:n_rolls) {
    if (current_pnl >= max_win || current_pnl <= max_loss) {#exit strategy
      len_A[game] = spin - 1
      break
    } else {
      pnl_A[game,spin] = calculate_pnl_A(outcomes[game, spin], current_bet)
      if (pnl_A[game, spin] > 0) {
        current_bet = current_bet / multiplier #martingale
      } else {
        current_bet = current_bet * multiplier #martingale
      }
      if (current_bet > max_bet) {
        current_bet = max_bet #maximum bet constraint
      } else if (current_bet < min_bet) {
        current_bet = min_bet #minimum bet constraint
      }
      current_pnl = current_pnl + pnl_A[game, spin] #cumulative P/L
      current_bank = bank + current_pnl #current balance
      if (current_bet > current_bank) {
        current_bet = current_bank #no credit constraint
      }
```

```
      if (spin == n_rolls) {
        len_A[game] = n_rolls
      }
    }
  }
  bank_A[game] = bank + current_pnl #final P/L for game
}


#find mean path
pnl_A_paths = t(apply(pnl_A, 1, cumsum))
pnl_A_mean = apply(pnl_A_paths,2,mean)


#select for which game you want to see the cumulative P/L
no_game = 1
plot(cumsum(unname(unlist(pnl_A[no_game,]))),main    =    paste("P/L    for
game",no_game,"and strategy A"),
     xlab = "Spin",ylab = "Cumulative P/L",type="l")
abline(h=0,col="grey",lty=2)
```

V.3 References

Pflaumer, P. (2019) "A Statistical Analysis of the Roulette Martingale System: Examples, Formulas and Simulations with R", *Conference: 17th International Conference on Gambling & Risk Taking. May 27-30, 2019, at Las Vegas, NV*