Study of Wet Scroll

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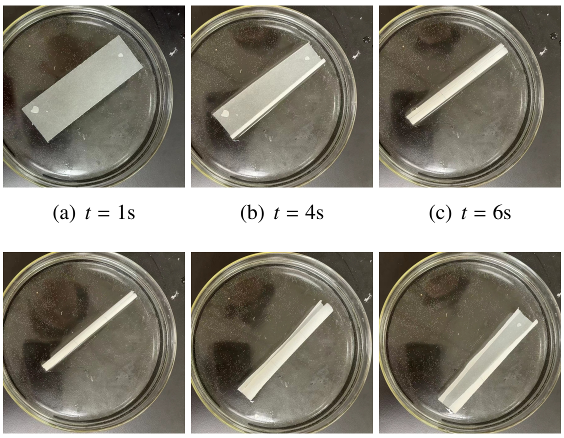
Abstract

When a piece of tracing paper is gently placed on the surface of water, the end of the paper in contact with the water will absorb water and expand, causing it to quickly curl up, and over time, it will slowly unfold. Our paper focuses on the case where the paper strips cannot roll into a cylinder. The movement of paper strips is studied by measuring the change in paper strip width over time using Tracker software. Additionally, a theoretical model is proposed to explain the change in paper width. The experimental and theoretical results agree well in the second half of the curves, but there are differences in the first half.

1. Introduction

When a piece of semi-transparent sulfuric acid paper (Tracing Paper) is gently placed on the surface of water, it may quickly curl up into a roll and then slowly unfold. Sulfuric acid paper(hereinafter referred to as paper for simplicity) is a type of paper that has been specially processed. It is made by immersing high-quality uncut ordinary paper in sulfuric acid for a few seconds, during which some of the cellulose in the paper is converted to starch-like protein by the acid, resulting in increased light transmittance and hydrophobicity.



(a) $t = 1$s (b) $t = 4$s (c) $t = 6$s

(d) $t = 9$s (e) $t = 28$s (f) $t = 34$s

Figure 1: Changes of 12cm × 4cm paper strip over time. The paper strips first curls and then slowly uncurls.

Previous researchers have conducted extensive studies on the properties of paper. Works by Alava and Niskanen (2006) and Van der Reyden et al. (1993) comprehensively explain the properties of paper, such as the paper's microstructure, its interaction with liquids, and the elastic and plastic characteristics of paper as a solid. These works provide a theoretical basis for this experiment, but they do not quantitatively describe these characteristics. Regarding the bending of paper, Mabrouk et al. (2009) proposed a relatively simple theoretical model when studying polymer vesicles (polymersomes). Dano and Bourque (2009) developed a model for the diffusion of water in paper and paperboard and analyzed the energy changes, while Reyssat and Mahadevan (2011) proposed a different model and used it to explain the change in curvature of sulfuric acid paper over time. However, it is difficult to measure the curvature of paper, which increases the complexity of the experiment.

In this paper, the focus lies on cases where paper cannot naturally roll into a cylinder. We have made improvements to Reyssat's method by taking surface tension into account. Also, instead of directly rolling the paper into a cylinder, we investigate changes in the width of paper strips, which are easier to measure compared to Reyssat. we also propose a theoretical framework that describes how the width of the paper strip changes over time. By comparing experimental results with theoretical predictions, the agreement between experiment and theory is strong in the second half of the curves, especially after introducing relaxation time. However, there remains an intriguing mystery: the short-term behavior of paper strips after being placed on the water surface. This aspect remains unexplored, leaving room for further investigation and discovery.

2. Methods

2.1. Experimental Measuring

In this case, the change of the width of paper over time was measured as follows. We used a paper strip with a length $l = 12$cm and width $d_0 = 0.4$cm (the paper specification is 59.4cm × 42.0cm per sheet, 53g per 20 sheets) for the experiment. First, water was filled into a glass dish with a diameter of 12cm. Then, the paper strip was gently placed on the water surface, and a camera was simultaneously turned on to record the changes in the paper strip. The recording was stopped after the paper strip completed one curling-uncurling cycle. Instead of using a laser sheet illuminating the strip and measuring the curvature of paper as Reyssat and Mahadevan (2011), we used Tracker software to analyze and read the changes in the width of the paper strip over time. This approach not only reduces the complexity of the experiment and allows for a more precise examination of the material's behavior.

2.2. Theoretical Modeling

First, we consider the transport problem of water in paper. Assume that the coordinate along the thickness direction is z. According to Fick's second law of diffusion proposed by Fick (1855),

$$\frac{\partial \phi(z,t)}{\partial t} = D\frac{\partial^2 \phi(z,t)}{\partial z^2}, \tag{1}$$

where $\phi(z,t)$ represents the water content, defined as the ratio of the mass of water at that location to the mass of the paper when it is dry. The diffusion coefficient, D, is approximately on the order of 10^{-10}. Additionally, we have boundary condition at the water surface ($z = 0$)and at the paper thickness ($z = h = 0.0784$mm),

$$\phi(0,t) = \phi_s, \quad \frac{\partial \phi(h,t)}{\partial z} = 0, \tag{2}$$

where ϕ_s represents the water content at saturation. Then we get

$$\begin{aligned}\phi(z,t) =& \phi_s\left[1 - \sum_{m=1}^{\infty} \frac{4}{(2m-1)\pi} \sin\frac{(2m-1)\pi z}{2h}\right.\\ &\left.\times \exp\frac{-(2m-1)^2\pi^2 D t}{4h^2}\right].\end{aligned} \tag{3}$$

We assume that the variation in paper width with water content is linear. Let paper width is d' after uncurling, then we get

$$d(\phi) = d_0 + (d' - d_0)\frac{\phi}{\phi_s}. \tag{4}$$

And the relative change in width is

$$\varepsilon_\phi(\phi) = \frac{d' - d_0}{d_0}\frac{\phi}{\phi_s}. \tag{5}$$

Next, we perform a mechanical analysis. Let's consider paper as a homogeneous elastic material. When it is dry, its Young's modulus is denoted as E_0. However, when the paper contains water with a moisture content of ϕ, its Young's modulus becomes $E(\phi)$. The relationship between the Young's modulus of wet paper and that of dry paper is described by Zauscher et al. (1996),

$$E(\phi) = E_0 \exp(-\phi). \tag{6}$$

At any moment, the configuration of the paper strip is as shown in Figure 2, where the red line represents the neutral axis. We establish a coordinate system as depicted in the figure, with the s-axis aligned along the neutral axis (not explicitly labeled in the figure). To calculate the forces and moments at the s-section due to elasticity, we define z_0 as the location of the neutral axis,

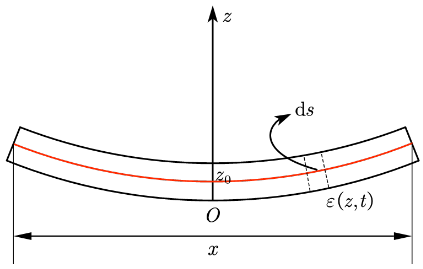


Figure 2: Theoretical modeling diagram when paper strips cannot form a cylinder. The red line represents the neutral axis $z = z_0$ and the s-axis aligns along the neutral axis $z = z_0$.

$\kappa(s)$ as the curvature at s, and $R(s) = 1/\kappa(s)$ as the curvature radius. The total relative elongation at position z is

$$\varepsilon_T(z) = \kappa(s)(z_0 - z). \tag{7}$$

The elastic elongation is then

$$\varepsilon_e(z) = \varepsilon_T - \varepsilon_\phi. \tag{8}$$

For an elemental segment from z to $z + \mathrm{d}z$ along the length l, the force per unit length in the l direction is given by

$$\begin{aligned}\mathrm{d}F &= E(\phi)\varepsilon_e(z)\mathrm{d}z\\ &= E(\phi)\left(\kappa(s)(z_0 - z) - \varepsilon_\phi(\phi)\right)\mathrm{d}z.\end{aligned} \tag{9}$$

The moment at $z = 0$ can be determined from this expression:

$$\begin{aligned}\mathrm{d}M &= E(\phi)\varepsilon_e(z)z\mathrm{d}z\\ &= E(\phi)\left(\kappa(s)(z_0 - z) - \varepsilon_\phi(\phi)\right)z\mathrm{d}z.\end{aligned} \tag{10}$$

Therefore, the effect of forces on each section is equivalent to the combined action of a force $F = \int \mathrm{d}F$ and a moment $M = \int \mathrm{d}M$ acting at $z = 0$.

Now consider a small segment of paper strip. Apart from elastic forces and moments, this segment is also subject to surface tension and gravity. Given the dimensions of the paper, we can easily estimate that gravity is much smaller than surface tension, so we can neglect it. The force analysis for the elemental segment yields,

$$F(s) = F(s + \mathrm{d}s) = -\gamma, \tag{11}$$

where γ is coefficient of surface tension. Applying torque balance equation,

$$M(s) = M(s + \mathrm{d}s). \tag{12}$$

We observe that both $F(s)$ and $M(s)$ are constants, and at the free end, there is no moment ($M(s) \equiv 0$). Since $F(s)$ is independent of s, we can infer from Equation 9 that κ is also independent of s, resulting in uniform curvature throughout the paper strip at time t. From the given equations, we obtain the following expressions:

$$-\gamma = \int_0^h E(\phi)(\kappa(z_0 - z) - \varepsilon_\phi(\phi))\mathrm{d}z, \tag{13}$$

$$0 = \int_0^h E(\phi)(\kappa(z_0 - z) - \varepsilon_\phi(\phi))z\mathrm{d}z. \tag{14}$$

Using Equation 6, divide both sides by E_0 and solve for κ and z_0, we get

$$\frac{1}{\kappa} = \frac{i_1^2 - i_0 i_2}{i_0 i_{02} - i_1 i_{01} + i_1 \frac{\gamma}{E_0}}, \tag{15}$$

$$z_0 = \frac{\left(i_{01} - \frac{\gamma}{E_0}\right) i_2 - i_{02} i_1}{\left(i_{01} - \frac{\gamma}{E_0}\right) i_1 - i_{02} i_0}. \tag{16}$$

Here, the following integrals are defined as

$$i_n = \int_0^h E(\phi(z,t)) z^n \mathrm{d}z, \tag{17}$$

$$i_{0n} = \int_0^h E(\phi(z,t)) z^{n-1} \varepsilon_\phi(\phi(z,t)) \mathrm{d}z. \tag{18}$$

The width of the paper, denoted as x, satisfies

$$\begin{cases} x = 2R \sin \frac{d_0(1+\varepsilon_\phi)}{2R}, & \text{if } \frac{d_0(1+\varepsilon_\phi)}{2R} < \frac{\pi}{2} \\ x = 2R, & \text{if } \frac{d_0(1+\varepsilon_\phi)}{2R} \geq \frac{\pi}{2} \end{cases}. \tag{19}$$

By substituting the relevant data, we can derive the relationship between the width of the paper strip and time.

3. Results

3.1. Experimental Results

By analyzing the data from cameras and using Tracker software, we have plotted the changes in the paper strip width over time, as shown in the Figure 3. Observing Figure 3, we can see that the width of the paper initially decreases with time as paper curls, reaches a minimum value, and then gradually increases as paper uncurls. The rate of increase is slightly slower than the rate of decrease. Eventually, the width of the paper exceeds its original width. This phenomenon occurs because the paper expands after absorbing water. From this figure, we also obtain the value of paper width after uncurling

$$d' = 0.44\text{cm}. \tag{20}$$

3.2. Comparison Between Experimental And Theoretical Results

Using Mathematica for numerical calculations, we take the Young's modulus of dry paper as 1GPa according to Alava and Niskanen (2006). The surface tension coefficient of the paper is $\gamma = 7.2 \times 10^{-2}$N/m. Experimental measurements yield the thickness of the paper h as 0.0784mm and paper width after uncurling d' as 0.44cm. Substituting these values into the equation, we compare the theoretical results with experimental data in Figure 4(a). Notably, there is a significant deviation between the theoretical curve and the experimental values in Figure 4(a).

For the experimental data, the width of the paper changes slowly upon initial contact with water, and it only accelerates after approximately 5s. To account for this behavior, we introduce a relaxation time $\tau = 5$s to characterize the time it takes for

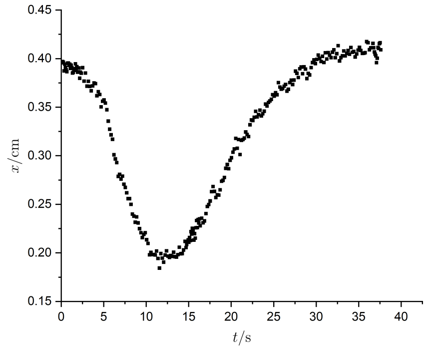


Figure 3: Changes in paper width(x) over time(t) in water: experimental results. Initially, the width of the paper decreases as paper curls, then it increases as paper uncurls, and finally, the width becomes slightly larger than the initial width, which is a result from expanding after absorbing water.

the paper to curl after being placed in water. Shifting the time axis of the theoretical curve by τ, we compare the modified theoretical results with the experimental data in Figure 4(b). While the latter half of Figure 4(b) aligns with the experimental data, there are still discrepancies in the first half.

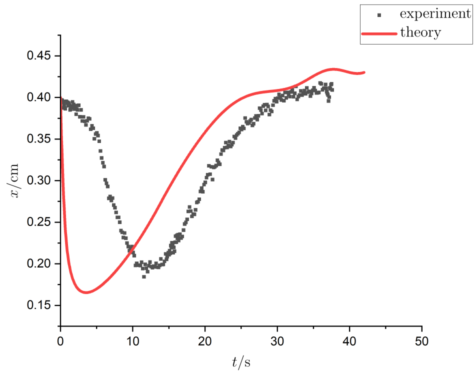
4. Conclusion And Discussions

In our experiment, we introduced a new method to measure the motion of paper strips, which is easier than Reyssat and Mahadevan (2011). Additionally, we proposed improvements to Reyssat and Mahadevan (2011)'s theory by considering the impact of surface tension on paper curling. Finally, we compared the experimental and theoretical results. After introducing a relaxation time, the theoretical and experimental curves aligned well in the latter half.

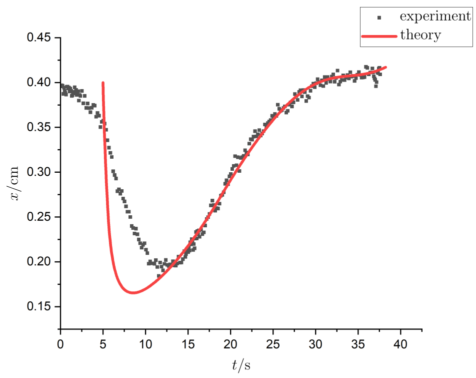
4.1. Relaxation time needs explanation.

Short-term behavior of paper and relaxation time remain unknown. In Figure 4(b), after the paper's width reaches its minimum value, the theoretical and experimental data fit well. However, in the first half of the curve, only the trend is correct. The discrepancy arises from the behavior of the paper just after being placed on the water. The theoretical assumption is that the lower surface of the paper is saturated at $t = 0$. However, in reality, the paper is not saturated immediately upon contact with the water, leading to the differences observed in Figure 4(a). When introducing a relaxation time τ, the revised assumption becomes that the lower surface of the paper reaches saturation between $t = 0$ and $t = \tau$. However, even this assumption is not accurate.

Compared to Reyssat and Mahadevan (2011), we consider the effect of surface tension in our experiments, and the conditions are considered more comprehensively. There is a high probability that the strip of paper will not be able to curl due to surface tension in the short-term period immediately after it is placed on water surface, resulting in the so-called relaxation time.



(a) original



(b) modified

Figure 4: (a)Changes in paper width(x) over time(t) in water: comparison of theoretical results and experimental results(original). (b)Changes in paper width(x) over time(t) in water: comparison of theoretical results and experimental results(modified). Figure(b) is obtained by shifting the red curve(theoretical results) in Figure(a) to the right by 5 seconds. In the second half of the curves, the theory and experiments agree well. But there are still differences in the short period after the paper strip is placed on the water surface.

Currently, there is no satisfactory theoretical solution for the short-term behavior of paper upon water contact. Future studies should mainly focus on this short-term behavior of paper and explore what happens just after the paper is placed on the water surface.

4.2. Another case remains unsolved.

In this paper, we only focus on the case where paper strips cannot roll into a cylinder. The case where paper strips can roll into a cylinder is more challenging to explain theoretically. That is because during the process of cylindrical formation, the uncurled portion of the paper simultaneously experiences elongation. At this point, analyzing the forces acting on the entire paper strip becomes highly complex. Additionally, due to water adhering to the lower surface of the paper after forming the cylinder, there exists a water film between multiple layers of cylinders, further complicating the mechanical situation.

4.3. Problems occur during the experiment.

A slight problem occurred in the experiment. During our experiment, we also observed that paper strips exhibited slow movement on the water surface due to environmental disturbances during the curling process. This movement may potentially impact the curling behavior, but the results from multiple experiments indicate that its influence can be neglected.

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