《数学物理方法》第十章《δ 函数》

1. 证明 δ 函数的下列性质：

   (1) $\delta(x) = \delta(-x)$;   (2) $x\delta(x) = 0$;

   (3) $f(x)\delta(x) = f(0)\delta(x)$;   (4) $\delta(ax) = \frac{1}{|a|}\delta(x)$;

   (5) $\delta(x^2 - a^2) = \frac{1}{2|a|}\left[\delta(x-a) + \delta(x+a)\right]$;

   (6) $\delta(x-a)\delta(x-b) = \delta(a-b)\delta(x-a)$.

2. 求解下列微分方程：

   (1) $y'' = -\delta(x - x_0)$, $y(0) = 0$, $y'(1) = 0$;

   (2) $y'' + y = -\delta(x - x_0)$, $y(0) = 0$, $y(\frac{\pi}{2}) = 0$.