《数学物理方法（下）》第六章《分离变量法》习题

1. 将下列方程分离变量

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial \phi^2}=0 \tag{1}$$

2. 求解下列本征值问题，证明各题中本征函数的正交性，并计算本征函数的模方：

$$1)\quad \begin{cases} X''+\lambda X=0, \\ X(0)=0,\ X'(l)=0; \end{cases}$$

$$2)\quad \begin{cases} X''+\lambda X=0, \\ \alpha X(0)+\beta X'(0)=0,\ \alpha X(l)+\beta X'(l)=0. \end{cases}$$

3. 求解细杆的导热问题，杆长为l，两端($x=0$及$x=l$）均保持为零度，初始温度分布$u|_{t=0}=bx(l-x)/l^2$。

4. 求解：

$$\begin{cases} \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, \\ u\big|_{x=0}=u_0, \quad u\big|_{x=a}=u_0 y, \\ \frac{\partial u}{\partial y}\Big|_{y=0}=0, \quad \frac{\partial u}{\partial y}\Big|_{y=b}=0. \end{cases}$$

5. 求解：

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}-a^2\frac{\partial^2 u}{\partial x^2}=bx(l-x), \\ u\big|_{x=0}=0, \quad u\big|_{x=l}=0, \\ u\big|_{t=0}=0, \quad \frac{\partial u}{\partial t}\Big|_{t=0}=0. \end{cases}$$