《数学物理方法（下）》第三章《线性偏微分方程的分类》习题

1. 讨论下述方程的分类，并将它们化为典型形式：

   (1). $\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}-3\frac{\partial^2 u}{\partial y^2}+2\frac{\partial u}{\partial x}+6\frac{\partial u}{\partial y}=0;$

   (2). $\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+5\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0;$

   (3). $\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial y^2}+\frac{1}{2}\frac{\partial u}{\partial y}=0;$

   (4). $(1+x^2)\frac{\partial^2 u}{\partial x^2}+(1+y^2)\frac{\partial^2 u}{\partial y^2}+x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0;$

   (5). $\tan^2 x\frac{\partial^2 u}{\partial x^2}-2y\tan x\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}+y\frac{\partial u}{\partial y}=0.$

2. 求下列各偏微分方程的通解：

   (1). $\frac{\partial^2 u}{\partial x^2}-2\frac{\partial^2 u}{\partial x\partial y}-3\frac{\partial^2 u}{\partial y^2}=0;$

   (2). $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial x\partial y}=0;$

   (3). $(a^2-b^2)\frac{\partial^2 u}{\partial x^2}+2b\frac{\partial^2 u}{\partial x\partial t}-\frac{\partial^2 u}{\partial t^2}=0,\quad a,b$为常数，$a\neq 0;$

   (4). $\frac{\partial^2 u}{\partial x^2}-2\frac{\partial^2 u}{\partial x\partial y}+2\frac{\partial^2 u}{\partial y^2}=0.$