

《数学物理方法（下）》第四章《线性偏微分方程的通解》习题

1. 求下列偏微分方程的通解：

$$(1). \frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial^2 u}{\partial x \partial y} - 3 \frac{\partial^2 u}{\partial y^2} = 0;$$

$$(2). \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} = 0;$$

$$(3). (a^2 - b^2) \frac{\partial^2 u}{\partial x^2} + 2b \frac{\partial^2 u}{\partial x \partial t} - \frac{\partial^2 u}{\partial t^2} = 0, \quad a, b \text{ 为常数, } a \neq 0;$$

$$(4). \frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial^2 u}{\partial x \partial y} + 2 \frac{\partial^2 u}{\partial y^2} = 0.$$

2. 求偏微分方程

$$x^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$$

的通解。