第四次作业 陈启超 230001447

第三章补充作业

3.1 均匀分布 $\sigma = const$

$r \sim dr + r$ 内 $dN = 2\pi r\sigma dr \propto r$

$\theta \sim d\theta + \theta$ 内 $dN = \frac{1}{2}\sigma d\theta$

$\therefore$ θ 均匀，r 按照 $f(r) \propto r$ 归一化：$f(r) = 2r$

$\int_0^x g(x)dx' = \int_0^r f(r)dr = r^2 \Rightarrow r = \sqrt{x}$，$x$ 是 $(0,1)$ 间均匀分布的随机变量

程序见附录

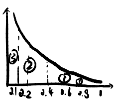
3.2 一个重要的问题是被积函数在 $x=0$ 是发散的

为提高效率（以及准确度），将积分分为三部分：

① $0.4 \le x \le 1.0$

② $0.1 \le x \le 0.4$

③ $0 \le x \le 0.1$



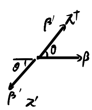
程序见附录，积分结果为：1.493

3.3 在 $t \sim t+dt$ 时刻衰变 $f(t)dt = \frac{1}{\tau}e^{-\frac{t}{\tau}}$，$0 < t \le \frac{D}{\beta c}$

$$\frac{M_K^2 c^2}{2m_\lambda} = \frac{M_K c^2}{\sqrt{1-\beta^2}} \Rightarrow \frac{M_K}{2m_\lambda} = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta = \sqrt{1-\frac{4m_\lambda^2}{M_K^2}} = 0.827$$

质心系中能量守恒，动量守恒：

$$M_K c^2 = 2m_\lambda c^2/\sqrt{1-\beta'^2} \Rightarrow \beta' = \beta = \sqrt{1-\frac{4m_\lambda^2}{M_K^2}}$$



质心系中 v_+', v_-' 各向同性，$d\Omega = 2\pi\sin\theta d\theta$

v_+ 处于 $\theta \sim \theta + d\theta$ 的概率为 $f(\theta)d\theta = \frac{1}{2}\sin\theta d\theta$

$$v_{+x} = \frac{\beta + \beta\cos\theta}{1+\beta^2\cos\theta}c \qquad v_{+y} = \frac{\sqrt{1-\beta^2}\beta\sin\theta}{1+\beta^2\cos\theta}c$$

$$v_{-x} = \frac{\beta - \beta\cos\theta}{1-\beta^2\cos\theta}c \qquad v_{-y} = \frac{-\sqrt{1-\beta^2}\beta\sin\theta}{1-\beta^2\cos\theta}c$$

要求 $\lambda^{\pm}$ 均击中探测器，$\Delta t = \frac{D-\beta ct}{v_x}$ $\quad y = \left|\frac{D-\beta ct}{v_x}\cdot v_y\right| \le R$

λ^+：$\frac{\sqrt{1-\beta^2}\beta\sin\theta}{\beta(1+\cos\theta)} \le \frac{R}{D-\beta ct}$

λ^-：$\frac{\sqrt{1-\beta^2}\beta\sin\theta}{\beta(1-\cos\theta)} \le \frac{R}{D-\beta ct}$

$$\tan\frac{\theta}{2} \le \frac{R}{D-\beta ct}\cdot\frac{1}{\sqrt{1-\beta^2}}$$

$$\frac{1}{\tan\frac{\theta}{2}} \le \frac{R}{D-\beta ct}\frac{1}{\sqrt{1-\beta^2}}$$

$$2\arctan\frac{(D-\beta ct)\sqrt{1-\beta^2}}{R} \le \theta \le 2\arctan\frac{R}{D-\beta ct}\frac{1}{\sqrt{1-\beta^2}}$$

$$\frac{(D-\beta ct)\sqrt{1-\beta^2}}{R} \le \frac{R}{D-\beta ct}\frac{1}{\sqrt{1-\beta^2}} \Leftrightarrow \frac{D}{\beta c} - \frac{R}{\beta\sqrt{1-\beta^2}c} \le t \le \frac{D}{\beta c}$$

$t_{min} = 6.243 \times 10^{-11}s$

$t_{max} = 5.643 \times 10^{-10}s$

$$效率 = \int_{t_{min}}^{t_{max}} \frac{1}{\tau}e^{-\frac{t}{\tau}}dt \int_{\theta_{min}}^{\theta_{max}} \frac{1}{2}\sin\theta d\theta = 0.0977$$

$$\int \frac{1}{\tau}e^{-\frac{t}{\tau}} \cdot \left[\frac{1}{1+\frac{(D-\beta ct)^2(1-\beta^2)}{R^2}} - \frac{1}{1+\frac{R^2}{(D-\beta ct)(1-\beta^2)}}\right]$$

MC计算见附录

3.4.

$f_A(x) = \frac{1}{L_A} e^{-\frac{x}{L_A}} \quad f_B(x) = \frac{1}{L_B} e^{-\frac{x}{L_B}} . \Rightarrow g(x)$

运动到 $x \sim x+dx$ 后发生事件的概率 = x之前A,B均不发生 × x处A或B发生.

$$\int_x^{\infty} g(x)dx = \int_x^{\infty} \frac{1}{L_A} e^{-\frac{x}{L_A}} dx \cdot \int_x^{\infty} \frac{1}{L_B} e^{-\frac{x}{L_B}} dx$$

$$= e^{-\frac{x}{L_A} - \frac{x}{L_B}}$$

$$g(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}} \quad \Rightarrow \quad \lambda = \frac{L_A L_B}{L_A + L_B} = \frac{2}{3} cm.$$

MC验证程序见附录.

```python
#############################################################
#week 5 homework  Statistical Methods in Experimental Physics#
#############################################################
#author Qiyu Chen
#date 2024.3.21

import random
import math
import matplotlib.pyplot as plt

'''A plot function to plot hist'''
def plot(list, title, filename):
    plt.hist(list, bins=100, density=True)
    plt.title(title)
    plt.xlabel('Value')
    plt.ylabel('Frequency')
    plt.savefig('Desktop/{0}.jpg'.format(filename))
    plt.cla()

#############################################################
#exer3.1 started
tot = 1000000;  '''1000 is too small to create a satisfying distribution'''

theta=[2*math.pi*(random.random()) for _ in range(tot)]
r=[math.sqrt(random.random()) for _ in range(tot)]

plot(r, 'exer3.1  distribution of r', 'rdistribution')
plot(theta, 'exer3.1  distribution of theta', 'thetadistribution')

rtheta = zip(r, theta); '''rtheta is the list of 100000 sets of coordinate'''
#exer3.1 finished
#############################################################

#############################################################
#exer3.2 started

def f(x):
    return math.exp(-x)/math.sqrt(x)
def integrate(x_min, x_max, tot):
    y_min=0
    y_max=max(f(x_min),f(x_max))
    x=[x_min + (x_max-x_min)*random.random() for _ in range(tot)]
    y=[y_min + (y_max-y_min)*random.random() for _ in range(tot)]
    correct=[]
    for i, j in zip(x, y):
        if j < f(i):
            correct.append(i)
    return len(correct)/tot*(x_max-x_min)*y_max

def part1():
    return integrate(0.4, 1, tot)
def part2():
    return integrate(0.1, 0.4, tot)
def part3():
    return integrate(0.0000001, 0.1, tot)

print(part1()+part2()+part3())

#exer3.2 finished
############################################################

############################################################
#exer3.3 started
D, R, c, beta, tau=0.14, 0.07, 3e8, 0.827, 8.954e-11
all_z=[random.expovariate(1/beta/c/tau) for _ in range(tot)]
costheta=[-1+2*random.random() for _ in range(tot)]
correct=[]
for z, t in zip(all_z, costheta):
    if z<D and math.sqrt(1-beta**2)*math.sqrt(1-t**2)/(1+t)<=R/(D-z)\
    and math.sqrt(1-beta**2)*math.sqrt(1-t**2)/(1-t)<=R/(D-z):
        correct.append(z)
print(len(correct)/tot)
#exer3.3 finished
############################################################
```

```python
###########################################################
#exer3.4 started
a=[random.expovariate(1/2) for _ in range(tot)]
b=[random.expovariate(1) for _ in range(tot)]
correct=[min(i, j) for i, j in zip(a, b)]
print(sum(correct)/len(correct))
#exer3.4 finished
###########################################################

#######
##end##
#######
```