第四次作业. 陈启钰. 2300011447.

第三章补充作业.

3.1 均匀分布. $\sigma = const$.

$r \sim dr + r$ 内 $dN = 2\pi r \sigma dr \propto r$

$\theta \sim d\theta + \theta$ 内 $dN = \frac{1}{2}\sigma d\theta$.

$\because$ θ 均匀. r 按照 $f(r) \propto r$ 归一化: $f(r) = 2r$

$\int_0^x g(x)dx' = \int_0^r f(r)dr = r^2 \Rightarrow r = \sqrt{x}$. x 是 $(0,1)$ 间均匀分布的随机变量.

<u>程序见附录</u>

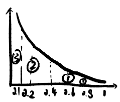
3.2. 一个重要的问题是被积函数在 $x=0$ 是发散的.

为提高效率(以及准确度). 将积分分为三部分:

① $0.4 \le x \le 1.0$

② $0.1 \le x \le 0.4$

③ $0 \le x \le 0.1$



程序见附录. 积分结果为: 1.493

3.3 在 $t \sim t+dt$ 时刻衰变 $f(t)dt = f(t')dt' = \frac{1}{\tau}e^{-\frac{t'}{\tau}}dt'$. $t = \frac{t'}{\sqrt{1-\beta^2}} \Rightarrow f(t) = f(t')\frac{dt'}{dt} = \frac{1}{\tau}e^{-\frac{t\sqrt{1-\beta^2}}{\tau}}\sqrt{1-\beta^2}$.

$\frac{M_K^2c^2}{2m_\lambda} = \frac{M_Kc^2}{\sqrt{1-\beta^2}} \Rightarrow \frac{M_K}{2m_\lambda} = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta = \sqrt{1-\frac{4m_\lambda^2}{M_K^2}} = 0.827$

$f(t)dt = g(z)dz \quad z = \beta ct$

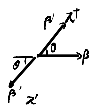
$\frac{1}{\beta c}f(t)$

$\frac{1}{\beta c}\frac{\sqrt{1-\beta^2}}{\tau}e^{-\frac{z\sqrt{1-\beta^2}}{\beta c\tau}}$

质心系中能量守恒,动量守恒:

$M_Kc^2 = 2m_\lambda c^2/\sqrt{1-\beta'^2} \Rightarrow \beta' = \beta = \sqrt{1-\frac{4m_\lambda^2}{M_K^2}}$

质心系中 v_+', v_-' 各向同性. $d\Omega = 2\pi\sin\theta d\theta$.



v_+ 处于 $\theta \sim \theta+d\theta$ 的概率为 $f(\theta)d\theta = \frac{1}{2}\sin\theta d\theta$.

$v_{+x} = \frac{\beta+\beta\cos\theta}{1+\beta^2\cos\theta}c \qquad v_{+y} = \frac{\sqrt{1-\beta^2}\beta\sin\theta}{1+\beta^2\cos\theta}c$

$v_{-x} = \frac{\beta-\beta\cos\theta}{1-\beta^2\cos\theta}c \qquad v_{-y} = \frac{-\sqrt{1-\beta^2}\beta\sin\theta}{1-\beta^2\cos\theta}c$

要求 $\lambda^\pm$ 均击中探测器. $\Delta t = \frac{D-\beta ct}{v_x} \qquad y = \left|\frac{D-\beta ct}{v_x}v_y\right| \le R$

λ^+: $\frac{\sqrt{1-\beta^2}\beta\sin\theta}{\beta(1+\cos\theta)} \le \frac{R}{D-\beta ct}$

λ^-: $\frac{\sqrt{1-\beta^2}\beta\sin\theta}{\beta(1-\cos\theta)} \le \frac{R}{D-\beta ct}$

$\tan\frac{\theta}{2} \le \frac{R}{D-\beta ct}\cdot\frac{1}{\sqrt{1-\beta^2}}$

$\frac{1}{\tan\frac{\theta}{2}} \le \frac{R}{D-\beta ct}\frac{1}{\sqrt{1-\beta^2}}$

$2\arctan\frac{(D-\beta ct)\sqrt{1-\beta^2}}{R} \le \theta \le 2\arctan\frac{R}{D-\beta ct}\frac{1}{\sqrt{1-\beta^2}}$

$\frac{(D-\beta ct)\sqrt{1-\beta^2}}{R} \le \frac{R}{D-\beta ct}\frac{1}{\sqrt{1-\beta^2}} \Leftrightarrow \frac{D}{\beta c} - \frac{R}{\beta\sqrt{1-\beta^2}c} \le t \le \frac{D}{\beta c}$

$t_{min} = 6.243\times10^{-11}s$

$t_{max} = 5.643\times10^{-10}s$

$$效率 = \int_{t_{min}}^{t_{max}}\frac{\sqrt{1-\beta^2}}{\tau}e^{-\frac{t\sqrt{1-\beta^2}}{\tau}}dt\int_{\theta_{min}}^{\theta_{max}}\frac{1}{2}\sin\theta d\theta = 0.202$$

$$\int \frac{1}{\tau}e^{-\frac{t}{\tau}}\cdot\left[\frac{1}{1+\frac{(D-\beta ct)^2(1-\beta^2)}{R^2}} - \frac{1}{1+\frac{R^2}{(D-\beta ct)(1-\beta^2)}}\right]$$

MC计算见附录.

```
############################################################
#week 5 homework  Statistical Methods in Experimental Physics#
############################################################
#author Qiyu Chen
#date 2024.3.21

import random
import math
import matplotlib.pyplot as plt

'''A plot function to plot hist'''
def plot(list, title, filename):
    plt.hist(list, bins=100, density=True)
    plt.title(title)
    plt.xlabel('Value')
    plt.ylabel('Frequency')
    plt.savefig('Desktop/{0}.jpg'.format(filename))
    plt.cla()

############################################################
#exer3.1 started
tot = 1000000;  '''1000 is too small to create a satisfying distribution'''

theta=[2*math.pi*(random.random()) for _ in range(tot)]
r=[math.sqrt(random.random()) for _ in range(tot)]

plot(r, 'exer3.1  distribution of r', 'rdistribution')
plot(theta, 'exer3.1  distribution of theta', 'thetadistribution')

rtheta = zip(r, theta); '''rtheta is the list of 100000 sets of coordinate'''
#exer3.1 finished
############################################################

############################################################
#exer3.2 started

def f(x):
    return math.exp(-x)/math.sqrt(x)
def integrate(x_min, x_max, tot):
    y_min=0
    y_max=max(f(x_min),f(x_max))
    x=[x_min + (x_max-x_min)*random.random() for _ in range(tot)]
    y=[y_min + (y_max-y_min)*random.random() for _ in range(tot)]
    correct=[]
    for i, j in zip(x, y):
        if j < f(i):
            correct.append(i)
    return len(correct)/tot*(x_max-x_min)*y_max

def part1():
    return integrate(0.4, 1, tot)
def part2():
    return integrate(0.1, 0.4, tot)
def part3():
    return integrate(0.0000001, 0.1, tot)

print(part1()+part2()+part3())

#exer3.2 finished
############################################################

############################################################
#exer3.3 started
D, R, c, beta, tau=0.14, 0.07, 3e8, 0.826968, 8.954e-11
all_z=[random.expovariate(1/beta/c/tau*math.sqrt(1-beta**2)) for _ in range(tot)]
costheta=[-1+2*random.random() for _ in range(tot)]
correct=[]
for z, t in zip(all_z, costheta):
    if z<D and math.sqrt(1-beta**2)*math.sqrt(1-t**2)/(1+t)<=R/(D-z)\
    and math.sqrt(1-beta**2)*math.sqrt(1-t**2)/(1-t)<=R/(D-z):
        correct.append(z)
print(len(correct)/tot)
#exer3.3 finished
############################################################
```

```
############################################################
#exer3.4 started
a=[random.expovariate(1/2) for _ in range(tot)]
b=[random.expovariate(1) for _ in range(tot)]
correct=[min(i, j) for i, j in zip(a, b)]
print(sum(correct)/len(correct))
#exer3.4 finished
############################################################

#######
##end##
#######
```

3.4.

$f_A = \frac{1}{L_A} e^{-\frac{x}{L_A}}$ $f_B = \frac{1}{L_B} e^{-\frac{x}{L_B}}$ $\Rightarrow g(x)$

运动到 $x \sim x+dx$ 后发生事件的概率 = x之前A, B均不发生 × x处A或B发生.

$$\int_x^\infty g(x)dx = \int_x^\infty \frac{1}{L_A} e^{-\frac{x}{L_A}} dx \cdot \int_x^\infty \frac{1}{L_B} e^{-\frac{x}{L_B}} dx$$

$$= e^{-\frac{x}{L_A}-\frac{x}{L_B}}$$

$g(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}} \Rightarrow \lambda = \frac{L_A L_B}{L_A + L_B} = \frac{2}{3}\,cm$

MC验证程序见附录.