一.

1. $\vec{R} = p_1\vec{i} + p_2\vec{j} + p_3\vec{k}$ $\quad\quad$ $\vec{L}_0 = p_3 b\vec{i} + p_1 c\vec{j} + p_2 a\vec{k}$

① 当 $\vec{R}$ 与 $\vec{L}_0$ 可化简为合力 即 $\vec{R}\cdot\vec{L}_0 = 0$ 有

$$p_1p_3b + p_2p_1c + p_3p_2a = 0$$

② 当 $\vec{R} // \vec{L}_0$ 有通过原点中心线 $\vec{R}\times\vec{L}_0 = 0$ 有

$$\begin{cases} p_2^2 a - p_1p_3 c = 0 \\ p_3^2 b - p_1p_2 a = 0 \\ p_1^2 c - p_2p_3 b = 0 \end{cases}$$

$\boxed{\text{如果写成 } \frac{bp_3}{p_1} = \frac{cp_1}{p_2} = \frac{ap_2}{p_3} \text{ 要考虑 } p_1, p_2, p_3 \text{ 为0的情况}}$

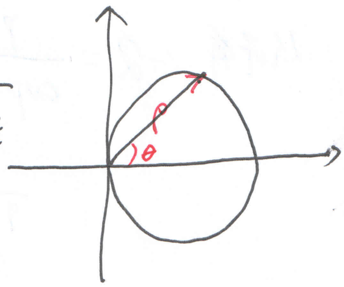
2. ① $x(t) = 2a\cos^2(kt/2) = \cancel{a + a\cos t}$
$a + a\cos kt$

$y(t) = a\sin(kt)$ $\quad$ 有轨道方程 $(x-a)^2 + y^2 = a^2$

② $ds = \sqrt{dx^2+dy^2} = \sqrt{a^2k^2\sin^2 kt + a^2k^2\cos^2 kt} = ak\,dt$

$t=0$ 时 $s=0$ $\quad$ 则有 $s(t) = akt$

③ $\rho = \sqrt{x^2+y^2} = \sqrt{a^2 + 2a^2\cos kt + a^2\cos^2 kt + a^2\sin^2 kt}$



$= \sqrt{2a^2(1+\cos kt)}$

$= \sqrt{4a^2\cos\frac{kt}{2}} = 2a\left|\cos\frac{kt}{2}\right|$. $\quad$ (带绝对值. 注意ρ的范围.

$\theta = \arctan\frac{y}{x} = \arctan\frac{a\sin kt}{2a\cos^2(\frac{kt}{2})} = \arctan\frac{2a\sin\frac{kt}{2}\cos\frac{kt}{2}}{2a\cos^2\frac{kt}{2}}$

$= \arctan\left(\tan\frac{kt}{2}\right) = \frac{kt}{2}$

$\rho = 2a\left|\cos\frac{kt}{2}\right|$

$\theta = \frac{k}{2}t$.

$\boxed{\text{运动方程 不是 轨道方程}}$