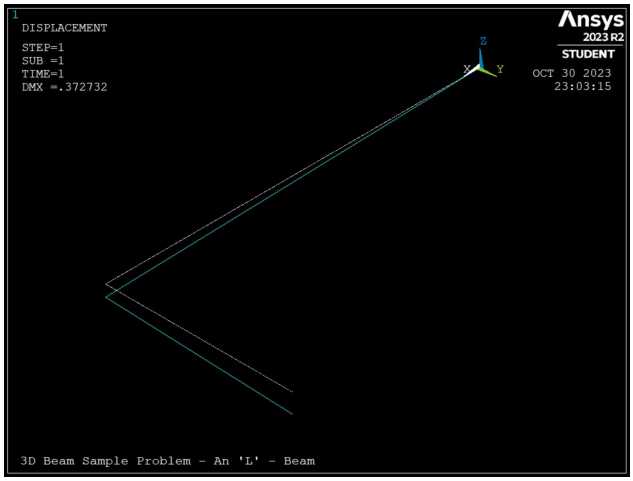
1:

DEFORMED + UNDEFORMED



ELEMENT STRESSES

PRINT S ELEMENT SOLUTION PER ELEMENT

STRESSES AT BEAM SECTION NODAL POINTS

ELEMENT = 1 SECTION ID = 1

ELEMENT NODE = 1

SEC NODE	SXX	SXZ	SXY
1	-5760.0	126.16	-1045.7
3	-5760.0	-5.0000	-1855.9
13	-0.17075E-010	-35.000	-0.44753E-011
11	0.52603E-009	796.06	0.25322E-011
5	-5760.0	-136.16	-1045.7
15	-0.56058E-009	-866.06	-0.39373E-011
23	5760.0	-5.0000	1855.9
21	5760.0	126.16	1045.7
25	5760.0	-136.16	1045.7
Max=	5760.0	796.06	1855.9
Min=	-5760.0	-866.06	-1855.9

ELEMENT NODE = 2

SEC NODE	SXX	SXZ	SXY
1	-0.95965E-004	126.16	-1045.7
3	-0.95965E-004	-5.0000	-1855.9
13	-0.17130E-010	-35.000	-0.44489E-011
11	0.48263E-009	796.06	0.25295E-011
5	-0.95966E-004	-136.16	-1045.7
15	-0.51676E-009	-866.06	-0.39364E-011
23	0.95965E-004	-5.0000	1855.9
21	0.95966E-004	126.16	1045.7
25	0.95965E-004	-136.16	1045.7
Max=	0.95966E-004	796.06	1855.9
Min=	-0.95966E-004	-866.06	-1855.9

STRESSES AT BEAM SECTION NODAL POINTS

ELEMENT = 2 SECTION ID = 1

ELEMENT NODE = 2

SEC NODE	SXX	SXZ	SXY
1	-2880.0	-5.0000	0.49659E-008
3	-2880.0	-5.0000	0.88404E-008
13	0.13049E-012	-35.000	0.35728E-010
11	0.53367E-009	-35.000	0.51341E-011
5	-2880.0	-5.0000	0.49658E-008
15	-0.53357E-009	-35.000	0.50872E-011
23	2880.0	-5.0000	-0.87690E-008
21	2880.0	-5.0000	-0.49557E-008
25	2880.0	-5.0000	-0.49556E-008
Max=	2880.0	-5.0000	0.88404E-008
Min=	-2880.0	-35.000	-0.87690E-008

ELEMENT NODE = 3

SEC NODE	SXX	SXZ	SXY
1	-0.47940E-004	-5.0000	-0.43063E-007
3	-0.47940E-004	-5.0000	-0.76435E-007
13	-0.18576E-013	-35.000	-0.47713E-011
11	-0.93987E-010	-35.000	-0.65156E-012
5	-0.47940E-004	-5.0000	-0.43064E-007
15	0.93983E-010	-35.000	-0.69843E-012
23	0.47940E-004	-5.0000	0.76426E-007
21	0.47940E-004	-5.0000	0.43062E-007
25	0.47940E-004	-5.0000	0.43062E-007
Max=	0.47940E-004	-5.0000	0.76426E-007
Min=	-0.47940E-004	-35.000	-0.76435E-007

NODAL

```
PRINT U     NODAL SOLUTION PER NODE

 ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

 LOAD STEP=     1  SUBSTEP=     1
  TIME=    1.0000       LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE       UX           UY           UZ           USUM
      1   0.0000       0.0000       0.0000       0.0000
      2 -0.20575E-016 0.76866E-014-0.22126       0.22126
      3 -0.86891E-014 0.76866E-014-0.37273       0.37273

MAXIMUM ABSOLUTE VALUES
NODE          0            0            3            3
VALUE    0.0000       0.0000      -0.37273      0.37273
```

ROTATIONAL NODAL

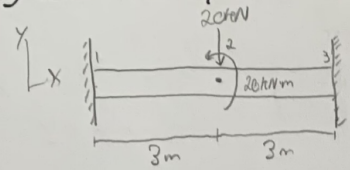
```
PRINT ROT  NODAL SOLUTION PER NODE

 ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

 LOAD STEP=     1  SUBSTEP=     1
  TIME=    1.0000       LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE       ROTX         ROTY         ROTZ         RSUM
      1   0.0000       0.0000       0.0000       0.0000
      2 -0.20631E-001 0.27648E-001 0.12636E-014 0.34497E-001
      3 -0.27543E-001 0.27648E-001 0.15026E-014 0.39026E-001

MAXIMUM ABSOLUTE VALUES
NODE          3            2            0            3
VALUE  -0.27543E-001 0.27648E-001  0.0000       0.39026E-001
```

TEXT INPUT

```
/FILNAM, Tutorial5C
/title, 3D Beam Sample Problem - An 'L' - Beam
/prep7

n, 1, 0.0, 0.0, 0.0
n, 2, 12.0, 0.0, 0.0
n, 3, 12.0, 6.0, 0.0
n, 4, 12.0, 0.0, 4.0

et, 1, beam188

! Material Properties
mp, ex, 1, 1.e7
mp, prxy, 1, 0.3

keyopt, 1, 3, 3

keyopt, 1, 4, 2

sectype, 1, beam, rect
secdata, 1.0, 0.25

! element connection and orientation
en, 1, 1, 2, 4

en, 2, 2, 3, 4

! Displacement boundary conditions
d, 1, ux, 0.
d, 1, uy, 0.
d, 1, uz, 0.
d, 1, rotx, 0.
d, 1, roty, 0.
d, 1, rotz, 0.

! (We could have used d, 1, all, 0.0 to define all root restraints.)

! Applied Force
f, 3, fz, -5.0
finish

/solu
antype, static
solve
save
finish

/post1
```

HW 3

2.] Solve 4.10 by Hand



$E = 210\ GPa$

$I = 4\times10^{-4}\ m^4$

Find displacements and slopes at each node, forces in each element, and the reactions. Also draw the shear force and bending moment diagrams.

$$[K]\begin{Bmatrix} V_2 \\ \phi_2 \end{Bmatrix} = \begin{Bmatrix} F_{2y} \\ M_2 \end{Bmatrix} \qquad [K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & 12 & -6L \\ \text{sym} & & & 4L^2 \end{bmatrix} \begin{matrix} V_1 \\ \phi_1 \\ V_2 \\ \phi_2 \end{matrix}$$

$$\begin{Bmatrix} V_1 \\ \phi_1 \\ V_2 \\ \phi_2 \\ V_3 \\ \phi_3 \end{Bmatrix} = \begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} \qquad [K]$$

$V_1 = 0$

$V_3 = 0$

$\phi_1 = 0$

$\phi_3 = 0$

$$K^{②} = \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{matrix} V_2 \\ \phi_2 \\ V_3 \\ \phi_3 \end{matrix}$$

$$[K]^{①=②} = \frac{EI}{L^3}\begin{bmatrix} 12 & -6L \\ -6L & 4L^2 \end{bmatrix} \begin{matrix} V_2 \\ \phi_2 \end{matrix}$$

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 24 & 0 \\ 0 & 8L^2 \end{bmatrix}$$

$M(x) = EI\,v''(x)$

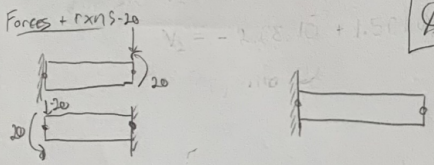
$V(x) = EI\,v'''(x)$

$$\frac{EI}{L^3}(24 V_2) = -20{,}000\ N$$

$$\boxed{V_2 = -2.67\times10^{-4}}$$

$$\frac{EI}{L^3}(8L^2\phi) = 20000\ N\cdot m$$

$$\boxed{\phi_2 = 8.928\times10^{-5}}$$

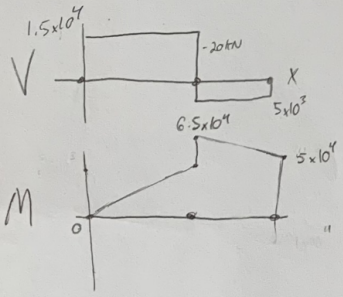


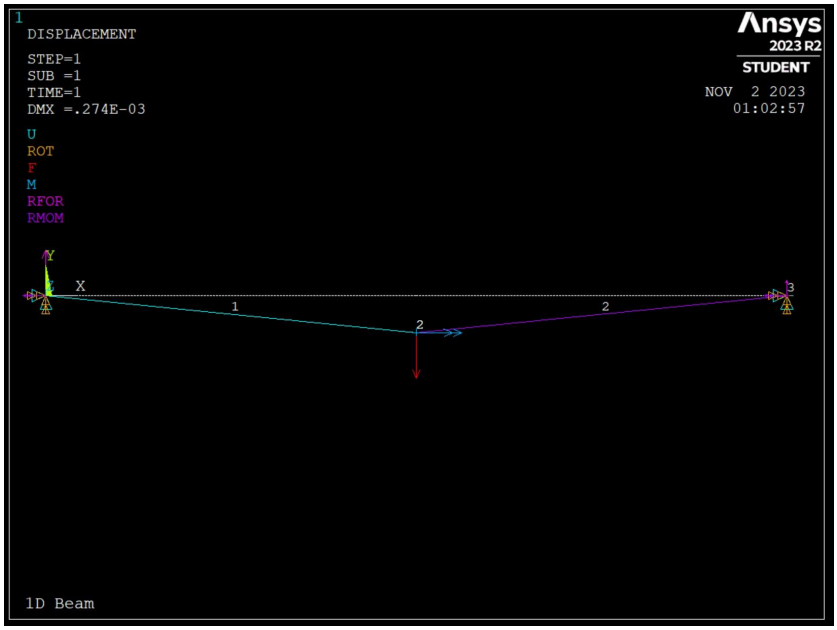
$$R_{1y} = (-12V_2 + 6L\phi_2)\frac{EI}{L^3} \Rightarrow R_{1y} = \boxed{1.5\times10^4\ N}$$

$$M_1 = (-6LV_2 + 2L^2\phi_2)\frac{EI}{L^3} \Rightarrow M_1 = \boxed{2\times10^4\ N\cdot m}$$

$$R_{3y} = (-12V_2 - 6L\phi_2)\frac{EI}{L^3} \Rightarrow R_{3y} = \boxed{5\times10^3\ N}$$

$$M_3 = (6LV_2 + 2L^2\phi_2)\frac{EI}{L^3} \Rightarrow M_3 = \boxed{-1\times10^4\ N\cdot m}$$



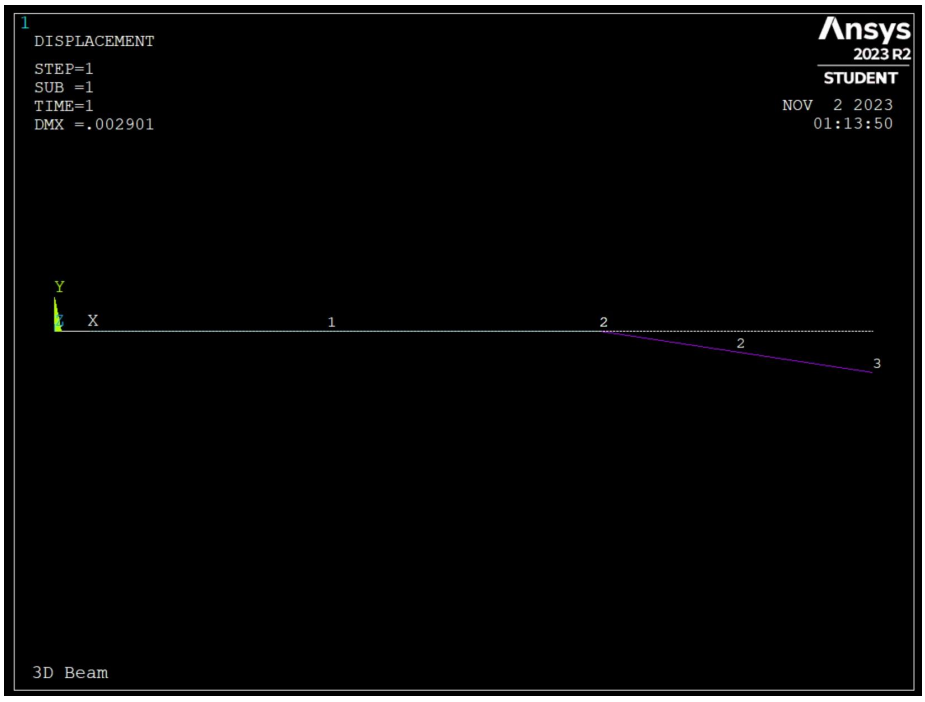



```
 PRINT REACTION SOLUTIONS PER NODE

  ***** POST1 TOTAL REACTION SOLUTION LISTING *****

 LOAD STEP=     1  SUBSTEP=     1
  TIME=    1.0000       LOAD CASE=   0

 THE FOLLOWING X,Y,Z SOLUTIONS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE       FX          FY          FZ          MX          MY          MZ
      1   0.0000      14970.      0.79909E-011 -10000.     -0.36626E-011  19911.
      3   0.0000      5029.6      0.79909E-011 -10000.      0.33787E-011 -10089.

TOTAL VALUES
VALUE    0.0000      20000.      0.15982E-010 -20000.     -0.28394E-012  9822.7
```

```
 PRINT U     NODAL SOLUTION PER NODE

  ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

 LOAD STEP=     1  SUBSTEP=     1
  TIME=    1.0000       LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE       UY
      1   0.0000
      2 -0.27423E-003
      3   0.0000

MAXIMUM ABSOLUTE VALUES
NODE         2
VALUE  -0.27423E-003
```

3:

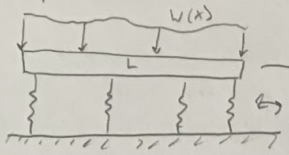
```
/FILNAM, 4.30_Input
/title, 3D Beam
/prep7

n, 1, 0.0, 0.0, 0.0
n, 2, 10.0, 0.0, 0.0
n, 3, 15.0, 0.0, 0.0


et, 1, beam188

! Material Properties
mp, ex, 1, 200e9
mp, prxy, 1, 0.3

keyopt, 1, 3, 3
keyopt, 1, 4, 2

sectype, 1, beam, rect
secdata, 0.6, 0.2410142264

! element connection and orientation
en, 1, 1, 2
en, 2, 2, 3

! Displacement boundary conditions
d, 1, all, 0.0
d, 2, uy, 0.0
d, 2, uz, 0.0


! (We could have used d, 1, all, 0.0 to define all root restraints.)

sfbeam, 2, 2, pres, 10000    !Distributed Pressure Load

/solu
antype, static
solve
save
finish

/post1
```



```
PRINT U     NODAL SOLUTION PER NODE

 ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

 LOAD STEP=      1  SUBSTEP=      1
  TIME=     1.0000       LOAD CASE=    0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

    NODE        UY
       1    0.0000
       2    0.0000
       3 -0.27296E-002

MAXIMUM ABSOLUTE VALUES
NODE            3
VALUE  -0.27296E-002


PRINT S    ELEMENT SOLUTION PER ELEMENT

 STRESSES AT BEAM SECTION NODAL POINTS

ELEMENT =         1  SECTION ID =         1

ELEMENT NODE = 1

    SEC NODE        SXX           SXZ           SXV
       1        0.42860E+007  0.81895E-007   32325.
       3        0.16669E-009 -0.69539E-009  0.22628E+006
      13        0.16669E-010 -0.26599E-009  0.22628E+006
      11        0.42860E+007  0.46392E-006   32325.
       5       -0.42860E+007 -0.79511E-007   32325.
      15       -0.42860E+007 -0.46294E-006   32325.
      23       -0.13335E-009  0.56060E-010  0.22628E+006
      21        0.42860E+007  0.79777E-007   32325.
      25       -0.42860E+007 -0.80233E-007   32325.

    Max=        0.42860E+007  0.46392E-006  0.22628E+006

    Min=       -0.42860E+007 -0.46294E-006   32325.

ELEMENT NODE = 2

    SEC NODE        SXX           SXZ           SXV
       1       -0.86440E+007  0.81997E-007   32325.
       3       -0.21173E-010 -0.69539E-009  0.22628E+006
      13       -0.21173E-011 -0.26599E-009  0.22628E+006
      11       -0.86440E+007  0.46452E-006   32325.
       5        0.86440E+007 -0.79613E-007   32325.
      15        0.86440E+007 -0.46354E-006   32325.
      23        0.16938E-010  0.56060E-010  0.22628E+006
      21       -0.86440E+007  0.79880E-007   32325.
      25        0.86440E+007 -0.80336E-007   32325.

    Max=        0.86440E+007  0.46452E-006  0.22628E+006

    Min=       -0.86440E+007 -0.46354E-006   32325.

 STRESSES AT BEAM SECTION NODAL POINTS

ELEMENT =         2  SECTION ID =         1

ELEMENT NODE = 2

    SEC NODE        SXX           SXZ           SXV
       1       -0.77796E+007  0.51950E-007  -86440.
       3        0.10842E-018  0.18595E-008 -0.60508E+006
      13        0.54210E-019  0.71128E-009 -0.60508E+006
      11       -0.77796E+007  0.34030E-006  -86440.
       5        0.77796E+007 -0.58325E-007  -86440.
      15        0.77796E+007 -0.34290E-006  -86440.
      23        0.86736E-018 -0.14991E-009 -0.60508E+006
      21       -0.77796E+007  0.57614E-007  -86440.
      25        0.77796E+007 -0.56393E-007  -86440.

    Max=        0.77796E+007  0.34030E-006  -86440.

    Min=       -0.77796E+007 -0.34290E-006 -0.60508E+006

ELEMENT NODE = 3

    SEC NODE        SXX           SXZ           SXV
       1        0.86440E+006  0.58668E-007 -0.14394E-002
       3        0.0000        0.30960E-016 -0.10075E-001
      13        0.0000        0.11843E-016 -0.10074E-001
      11        0.86440E+006  0.34230E-006 -0.14392E-002
       5       -0.86440E+006 -0.58668E-007 -0.14394E-002
      15       -0.86440E+006 -0.34230E-006 -0.14392E-002
      23        0.0000       -0.24963E-017 -0.10074E-001
      21        0.86440E+006  0.58668E-007 -0.14390E-002
      25       -0.86440E+006 -0.58668E-007 -0.14390E-002

    Max=        0.86440E+006  0.34230E-006 -0.14390E-002

    Min=       -0.86440E+006 -0.34230E-006 -0.10075E-001
```

4:

4.] Do problem 4.49 by hand



$\{V\} = [N]\{\delta\}$

$\varepsilon_x = -y\left[\frac{d^2N}{dx^2}\right]\{\delta\} = -y[B]\{\delta\}$

$\rightarrow [B] = \left[\frac{d^2N}{dx^2}\right]$

$\sigma_x = -y[E][B]\{\delta\}$

$V(x) = -\frac{dV}{dx}$

$= -EI\frac{dV'''}{dx}$

$= -EIV'''$

$\Pi_p = \int_0^L \frac{1}{2}EI(V'')^2 dx + \underbrace{\int_0^L \frac{k_f V^2}{2} dx}_{\text{springs from}} - \int_0^L wv\,dx$

$\Pi_p = \int_x\int_A \frac{1}{2}[\sigma_x]^T[\varepsilon_x] dA\,dx - \int_0^L b\,T_y[v]^T dx - \{d\}^T\{P\} + [\text{springs}]$

$[N]^2 = [N]^T[N]$

$\Pi_p = \int_0^L \frac{EI}{2}\{d\}^T[B]^T[B]\{d\}dx - \int_0^L v\{d\}^T[N]^T dx - \{d\}^T\{P\} + \int_0^L \frac{k_f}{2}[d][N][N][d]\,dx$

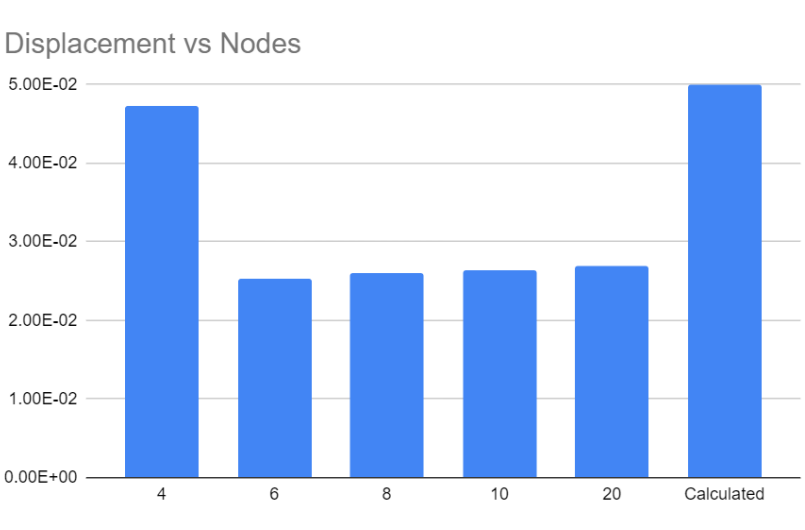
$\frac{\partial \Pi_p}{\partial \{\delta\}}$

$0 = \underbrace{EI\int_0^L [B]^T[B]dx\{\delta\}}_{[k]} - \underbrace{\int_0^L [N]^T v\,dx}_{\{f\}} + \underbrace{k_f\int_0^L [N]^T[N]\,dx}_{\text{springs} \rightarrow [k]}$

$[k] = EI\int_0^L [B]^T[B]\,dx + k_f\int_0^L [N]^T[N]\,dx$

5:

```
/FILNAM, 5.64_Input
/title, 3D Piston Ring
/prep7

n,      1,      0,      0,
n,      2,      0.2105, 0.8932,
n,      3,      0.4211, 1.2276,
n,      4,      0.6316, 1.4586,
n,      5,      0.8421, 1.6307,
n,      6,      1.0526, 1.7614,
n,      7,      1.2632, 1.8593,
n,      8,      1.4737, 1.9295,
n,      9,      1.6842, 1.9749,
n,      10,     1.8947, 1.9972,
n,      11,     2.1053, 1.9972,
n,      12,     2.3158, 1.9749,
n,      13,     2.5263, 1.9295,
n,      14,     2.7368, 1.8593,
n,      15,     2.9474, 1.7614,
n,      16,     3.1579, 1.6307,
n,      17,     3.3684, 1.4586,
n,      18,     3.5789, 1.2276,
n,      19,     3.7895, 0.8932,
n,      20,     4,      0,


et, 1, beam188

! Material Properties
mp, ex, 1, 18e6
mp, prxy, 1, 0.3

keyopt, 1, 3, 3
keyopt, 1, 4, 2

sectype, 1, beam, rect
secdata, 0.3, 0.2

! element connection and orientation
en, 1, 1, 2
en, 2, 2, 3
en, 3, 3, 4
en, 4, 4, 5
en, 5, 5, 6
en, 6, 6, 7
en, 7, 7, 8
en, 8, 8, 9
en, 9, 9, 10
en, 10, 10, 11
en, 11, 11, 12
en, 12, 12, 13
en, 13, 13, 14
en, 14, 14, 15
en, 15, 15, 16
en, 16, 16, 17
en, 17, 17, 18
en, 18, 18, 19
en, 19, 19, 20



! Displacement boundary conditions
d, 1, all, 0.0
d, 20, uz, 0.0

f, 20, fy, -5.3625

/solu
antype, static
solve
save
finish

/post1
```


Displacement vs Nodes
5.00E-02
4.00E-02
3.00E-02
2.00E-02
1.00E-02
0.00E+00
4
6
8
10
20
Calculated

5.] Solve 5.64 using ANSYS

$b = 0.3$

$h = 0.2$

$$\delta = \frac{3\pi FR^3}{EI} + \frac{\pi FR}{EA} + \frac{6\pi FR}{5GA}$$

$R = 2.0\text{in}$

$\delta = 0.1\text{in}$

$$0.1 = \underbrace{\frac{3\pi F(2)^3}{18\times10^6 \cdot 4.5\times10^{-4}}}_{} + \underbrace{\frac{\pi F(2)}{18\times10^6 \cdot 0.06}}_{} + \underbrace{\frac{6\pi F(2)}{5.7\times10^6 \cdot 0.06}}_{}$$

$F = 10.715 \text{ lbs}$

Max stress $= 89.29 \text{ psi}$